Thermal-Error Regime in High-Accuracy Gigahertz Single-Electron Pumping

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Single-electron pumps based on semiconductor quantum dots are promising candidates for the emerging quantum standard of electrical current. They can transfer discrete charges with part-per-million (ppm) precision in nanosecond time scales. Here, we employ a metal-oxide-semiconductor silicon quantum dot to experimentally demonstrate high-accuracy gigahertz single-electron pumping in the regime where the number of electrons trapped in the dot is determined by the thermal distribution in the reservoir leads. In a measurement with traceability to primary voltage and resistance standards, the averaged pump current over the quantized plateau, driven by a 1-GHz sinusoidal wave in the absence of magnetic field, is equal to the ideal value of \( ef \) within a measurement uncertainty as low as 0.27 ppm.

I. INTRODUCTION

Single-electron (SE) pumps can generate quantized electrical current by controlling the transport of individual electrons with an external periodic drive [1, 2]. These devices relate the pumped direct current, \( I \), to the elementary charge, \( e \), and the driving frequency, \( f \), through the expression, \( I = n e f \), where \( n \) is an integer. As an on-demand SE source, they can be useful in the context of quantum information processing as well as in the study of fermionic optics [3–5]. Arguably, the most important application of this technology is to realize a quantum standard of electrical current [6].

Single-electron pumps and turnstiles have been realized in various physical systems, including normal-metal tunnel junction devices [7, 8], surface acoustic wave devices [9, 10], superconducting devices [11–13], hybrid superconductor-normal-metal-tunnelstiles [14], quantum dots [15–27] and single dopants or traps [28–32]. The tunable-barrier SE pumps based on semiconductor quantum dots (QDs) stand out from the competing technologies for providing a good balance between low pumping error and high output current [17, 25, 27, 33, 34].

Three different designs of GaAs pumps have achieved relative errors close to or below 1 part per million (ppm) in high-accuracy measurements traceable to primary standards [17, 18, 33–35]. These GaAs pumps transport a fixed number of electrons per cycle following a series of sequential back-tunneling events, known as the decay cascade [36]. Previous studies indicate that strong magnetic fields, tailored waveform drives, and subkelvin temperatures are required for the GaAs pumps to achieve ppm level accuracy at gigahertz pumping frequencies [17, 33, 34]. These requirements render the realization of the quantum current standard demanding and restrict the user base of GaAs pump technology.

In contrast, QD pumps in silicon alleviate some of these burdens. Compared to depletion mode GaAs QDs, the gate-voltage-induced silicon QDs tend to have a larger addition energy due to their smaller physical size. This feature of the compact silicon devices enables accurate high frequency SE pumping in the decay-cascade regime without arbitrary waveform drives or high magnetic fields [25, 27]. The remarkable results recently achieved in silicon devices not only demonstrate the universality of SE pumping in tunable-barrier QDs at sub-ppm uncertainty, but also clearly indicate that a compact silicon SE pump may pave the way towards a more practical quantum standard of electrical current [27]. From a pragmatic point of view, it is advantageous to implement the quantum current standard in silicon, since it is compatible with the metal-oxide-semiconductor (MOS) technology widely employed in industry. Through well-established fabrication techniques, silicon SE pumps can be seamlessly integrated with peripheral control circuits to deliver a cost-effective on-chip current standard.

One challenge for the SE pumps is that the large rf drive amplitude usually required at gigahertz pumping frequency may heat the electron reservoir up to several kelvins and result in excessive thermal errors [24, 37]. When the electron reservoir temperature increases, forward tunneling of thermally excited electrons from the reservoir into the QD becomes significant during the charge capturing process, and the number of electrons trapped in the QD reflects the Fermi distribution of electrons in the leads [38, 39]. To the best of our knowledge, gigahertz high-accuracy SE pumping in the thermal regime has not been achieved among the silicon devices.

In this work, we use a silicon QD, fabricated employing a MOS planar gate stack technology [40, 41], to demonstrate high-accuracy SE pumping in the regime where the number of pumped electrons in each cycle is
determined by the thermal distribution of electrons in the reservoir leads. We investigate whether the accuracy of our SE pump significantly deteriorates due to drive-induced heating in the electron reservoir, as reported in previous studies [24, 37]. Fits of the measurement data to the thermal model of electron capture yield a theoretical lower bound of 4 parts per billion (ppb) for the thermal error on the $e_f$ current plateau at $f = 1$ GHz. In addition, we experimentally measure the pumped current using a high-accuracy measurement set-up, which compares the pumped current to a reference current derived from primary voltage and resistance standards [17]. We find that the averaged current on the plateau, induced by a sine wave drive in the absence of magnetic field, matches the $e_f$ value within the measurement uncertainty of $\sim 0.3$ ppm. This is the most accurate measurement of the current from a silicon electron pump to date.

II. DEVICE ARCHITECTURE AND EXPERIMENTAL METHODS

The sample used in the experiments was fabricated on a high-purity near-intrinsic silicon wafer. We thermally grow 7-nm high-quality SiO$_2$ gate oxide on top of the substrate. Three layers of aluminium gate electrodes are lithographically defined on top of the gate oxide. Between each layer, the sample is heated up to 150°C in air to form an aluminium oxide coating on the electrode surface. This coating provides good electrical insulation between different metal layers [40, 41].

A scanning electron microscope image of the aluminum gate stack of a device similar to the one used in the experiments is shown in Fig. 1(a). These metal gates, connected to programmable dc voltage sources through 200-Hz low-pass filters, can locally induce two dimensional electron gas (2DEG) channels or potential barriers at the Si/SiO$_2$ interface. By tuning the individual gate voltages, a quantum dot containing a few conduction electrons can be defined below the plunger gate (PL) as shown in Fig. 1(a). Electron reservoirs are accumulated below the source lead (SL) and the drain lead (DL), electrically connecting the quantum dot to the ohmic contacts.

We optimized the pump performance using a normal-accuracy measurement set-up shown in Fig. 1(a). The pumped current, $I_p$, is measured by a low-noise transimpedance amplifier (Femto DDPCA300) connected to the drain contact. The reference current source used in high-accuracy measurement is also connected to the drain, but it is switched off ($V = 0$) in the normal-accuracy measurement set-up. We operate the SE pump with a sinusoidal excitation. As shown in Fig. 1(b), each pumping cycle begins with the rf drive lowering the potential barrier between the QD and the source reservoir and loading the QD with electrons. Then the rf drive raises the barrier to trap electrons and eject some or all of them to the drain reservoir. Gate B1 was driven by a microwave source (HP8341B) through a room temperature bias-tee followed by a 9-dB attenuator. The source was synchronized to a 10-MHz reference frequency derived from a primary caesium frequency standard. All RF power levels quoted in this paper refer to the power after the 9-dB attenuator. All measurements presented in this work were carried out on a single device in the absence of a magnetic field with a small ($\sim 250$ μV) stray bias across the pump due to the current preamplifier. The sample was cooled in a helium-3 cryostat with a base temperature of 300 mK.

We take the following approach to search for a stable low-error current plateau: First, the capacitive coupling strength of the quantum dot to each gate is obtained.
from the period of the corresponding Coulomb blockade oscillations. Second, the two gate voltages that have the strongest capacitive coupling to the dot potential, namely, \(V_{B1}\) and \(V_{PL}\), are selected to be the main sweep parameters. Third, a sinusoidal excitation with a relatively low frequency, starting from 500 MHz, is applied to \(B1\). We gradually increase the rf drive power, \(P_{B1}\), until a plateau structure, shown in Fig. 2(a), appears in the \(V_{B1} - V_{PL}\) plane. Finally, we decrease \(V_{C1}\) and \(V_{C2}\) to obtain a flatter current plateau [25]. We verify the robustness of the well optimized current plateau at high pumping frequencies. As shown in Fig. 2(b), the \(ef\) current plateau is well pronounced up to 2 GHz without changing the gate voltages or rf power.

The search time is determined by the scan speed of the normal-accuracy measurement set-up, which is limited by the 200-Hz low-pass filters connected between the dc voltage sources and the metal gates in this study. The tune-up process lasted a few hours and was performed only once during the whole measurement campaign. The fine-tuned current plateau, presented in Fig. 2(b), was stable throughout the high-accuracy measurement over a time period of a few weeks. Using this tune-up procedure, tens of devices with identical design have showed high frequency current plateaux. Although these devices showed extremely low theoretical error rates and excellent stability over time, due to the limited access to the high-accuracy measurement setup, this latest study is the only one where we could experimentally determine the pumping accuracy at the sub-ppm level.

III. RESULTS

A. SE Pumping in the Thermal Regime

The shape of the current staircase between two adjacent plateaux as a function of the QD depth-tuning gate \(V_{PL}\) (Fig. 2(b)) provides information about the process by which the QD is decoupled from the source lead. To date, the reported accurate semiconductor

FIG. 2. (a) Coarsely tuned current plateaux at \(f = 500\) MHz measured using the normal-accuracy set-up. In the notation \((m,n)\), \(m\) (or \(n\)) represents the ideal number of captured (ejected) electrons. Here, \(V_{SL} = V_{B2} = 1.5\) V, \(V_{DL} = 1.75\) V, \(V_{C1} = -1\) V, \(V_{C2} = 0.2\) V, and \(P_{B1} = 2\) dBm. (b) Normal-accuracy measurements (black crosses) of pumped current as a function of the plunger gate voltage for different pumping frequencies and fits to the thermal model (red solid lines). Data have been horizontally shifted for clarity. Parameter settings: \(V_{SL} = V_{B2} = 1.5\) V, \(V_{B1} = 0.45\) V, \(V_{DL} = 1.9\) V, \(V_{C1} = -1.04\) V, \(V_{C2} = 0.187\) V and \(P_{B1} = 3\) dBm.

FIG. 3. (a) Normal-accuracy measurement (black crosses) of pumped current as a function of plunger gate voltage and its fits to the thermal (red solid line) and decay-cascade models (blue solid line). Insets: selected data from the main panel on expanded axes. (b) The reduced \(\chi^2\) fit error as a function of pumping frequency for both thermal and decay-cascade models. (c) The thermal error at the center of the first plateau predicted according to the fit to the thermal model at different frequencies. The plateau center is defined as the point of inflection of the thermal fit. The red error bar represents the typical error of the fit (~10%). For (a), (b), and (c), all gate voltage settings and rf power level are identical to Fig. 2(b).
pumps [17, 18, 27, 33, 35] have operated in the decay-cascade regime [36], where the final number of electrons in the QD is determined by a one-way cascade of back-tunneling events [36]. Consequently, the average number of captured electrons, \( \langle m \rangle \), is characterized experimentally by an asymmetric staircase modelled using a double-exponential function of the QD depth-tuning gate voltage, which in our device is \( V_{PL} \),

\[
\langle m \rangle = \sum_n \exp(-\exp(-aV_{PL} + \Delta_n)),
\]

(1)

where \( a \) and \( \Delta_n \) are fit parameters.

In this work, we consider the possibility that the electron reservoir is heated by the large-amplitude sinusoidal drive, leading to charge capture in the thermal regime. In this regime, electrons are exchanged between the dot and the leads during the initialisation, so that the average number of captured electrons, \( \langle m \rangle \), follows the grand canonical distribution [37] and can be expressed as

\[
\langle m \rangle = \sum_n 1/(1 + \exp[E_{add}^{(n)}/(k_BT)]),
\]

(2)

where \( k_B \) is the Boltzmann constant, \( T \) is the electron temperature of the source resevoir, and \( E_{add}^{(n)} \) is the addition energy of the \( n \)th electron. We assume that the addition energy is approximately a linear function of \( V_{PL} \) within the small voltage range swept for pumping in the single electron regime. Therefore, Eq. (2) can be further expressed as a function of \( V_{PL} \),

\[
\langle m \rangle = \sum_n 1/(1 + \exp(A_n + B_nV_{PL})],
\]

(3)

where \( A_n \) and \( B_n \) are the fit parameters for the \( n \)th current plateau. Assuming the ejection error is negligible during pumping, the normalized current, \(-I_p/ef\), measures the average number of captured electrons. In this work, the normalized pumped current, \(-I_p/ef\), is used in the numerical fit of \( \langle m \rangle \) for both decay-cascade and thermal models.

As shown in Fig. 3(a), the current staircase of our device is more accurately described by the thermal model than the decay-cascade model. The reduced \( \chi^2 \) fit error for the thermal model, displayed in Fig. 3(b), is significantly lower than that for the decay-cascade model for all studied pumping frequencies. This strongly suggests our device is indeed operating in the thermal regime.

We estimate the reservoir electron temperature for the measurement in Fig. 3 using the following method. We extract the ratio of \( E_{add}/(k_BT) \) from the thermal fits presented in Fig. 3. Along with an addition energy of 17 meV, calculated based on a conduction band profile simulated in the commercial semiconductor software package ISE-TCAD[42], we deduce the local electron temperature near the SE pump to be around 9 K at a \( f = 1 \) GHz. We need to estimate the addition energy using a simulation because the tunnel barriers are made completely opaque in the SE pumping regime in order to prevent co-tunneling errors [43], which prevents the direct observation of \( E_{add} \) in conductance measurements. More details on the TCAD simulation and the estimation of the QD addition energy are presented in the supplementary information [44].

In our previous work [25], a device with similar design driven by a much smaller rf signal, roughly \(-6 \) dBm, demonstrated SE pumping in the decay-cascade regime. This suggests that the thermal regime observed in the present experiments is indeed due to heating of the electron reservoirs by the large rf drive signal. A similar heating effect has been observed in a SE shuttle fabricated employing the same silicon technology [24]. An effective electron temperature of 7 K, attributed to rf-induced heating, has also been reported in another SE pumping study employing a silicon nanowire device [37].

Next, we investigate whether the accuracy of our SE pump will, as reported in previous studies using silicon devices [24, 37], significantly deteriorate due to such severe localized heating in the electron reservoir. Since our pump operated in the thermal regime, the main cause of capture error is expected to be thermal fluctuations of the QD electron number during its decoupling from the source reservoir [37]. The thermal error rate at the optimal working point of the \( I = ef \) plateau can be estimated as \( \Delta_error^{thermal} = 1 - 1/[1 + \exp(E_{add}^{(1)}/(k_BT))] \) [37], with the optimal working point given by the point of inflection of the fit line. Figure 3(c) shows the thermal error rate as a function of frequency. Despite the elevated electron temperature, we find the thermal error is as low as 4 ppb for \( f = 1 \) GHz. However, this should only be considered a lower bound for the overall error rate. Other error mechanisms, such as the non-adiabatic excitation of the captured electron [27, 28, 45, 46] may be present and are not considered in the above analysis.

**B. High-Accuracy Measurement**

To experimentally investigate the quantised current accuracy, we employ the high-accuracy measurement scheme described in Ref. [17]. We compare the pumped current, \( I_p \), to a reference current, \( I_{ref} \), with traceability to primary voltage and resistance standards. The transimpedance amplifier is used to measure the difference between these currents, \( I_{null} \). Because it measures a very small signal, the drift in gain of the transimpedance amplifier, for example due to temperature fluctuations, introduces only a small contribution to the overall uncertainty.

In previous studies employing the same measurement set-up [17, 27, 33], the 0.8 ppm systematic uncertainty in the calibration of the 1 GΩ resistor was by far the dominant contribution to the uncertainty budget. In this work, we introduce a revised uncertainty budget following a first-principles re-evaluation of the cryogenic current comparator (CCC) bridge used to calibrate the re-
sistor [47]. In the revised uncertainty budget, the largest systematic term is 0.1 ppm, due to the 10 MΩ reference resistor used in the calibration, and the statistical uncertainty in the resistor calibration is also of order 0.1 ppm. A recent comparison of precision reference current sources [48] has highlighted problems with short-term drift affecting high-value standard resistors. To reduce the impact of this drift on the pump measurements to well below 0.1 ppm, in this work we calibrated the 1 GΩ resistor very frequently, with an interval between 0.01 ppm. A recent comparison of precision reference current sources [48] has highlighted problems with short-term drift affecting high-value standard resistors. To reduce the impact of this drift on the pump measurements to well below 0.1 ppm, in this work we calibrated the 1 GΩ resistor very frequently, with an interval between calibrations as short as 2 days.

We carried out our high-accuracy measurement on an optimized $I = \varepsilon f$ plateau at 1 GHz. The pumped current as a function of $V_{PL}$ is shown in Fig. 4(a), where the fractional deviation of the pumped current from $\varepsilon f$ is defined as $\Delta I_P \equiv (I_P - \varepsilon_9 f) / \varepsilon_9 f$. We use $\varepsilon_9 = 2/(R_{JK-90} K_{1-90})$ to maintain consistency of units, since $I_{ef}$ is derived from primary voltage and resistance standards using the conventional 1990 values $K_{1-90}$ and $R_{JK-90}$ for the Josephson and von Klitzing constants respectively [49]. The normalised difference between $\varepsilon_9$ and the latest SI (CODATA 2014) value of $\varepsilon$ is $(\varepsilon_9 - \varepsilon) / \varepsilon_9 = -8.06 \times 10^{-8}$ [50], so consistency of unit systems is an important consideration as the total measurement uncertainty approaches the 0.1 ppm level. The detailed breakdown of the uncertainty budget for the measurement of $I_P$ is shown in Table I. More information about the precision measurement technique, including a detailed description of each term in Table I, is given in the supplementary information [44].

We define the plateau as the region where the fit to the thermal model deviates from the true $\varepsilon f$ value by less than 0.03 ppm. We show these 8 data points on the plateau in the inset of Fig. 4(a). We performed an additional statistical test, detailed in the supplementary information [44], to verify that the scatter of the selected data points is consistent with the data being drawn from the same distribution - in other words, that there is no structure on the plateau within our experimental resolution [35]. Averaging these $N_P = 8$ points, we obtain $\Delta I_P = -0.013$ ppm, with standard deviation $\sigma(\Delta I_P) = 0.672$ ppm. We take the error of the mean over the data points as the relative statistical (type A) uncertainty for the measurement of the pumped current, $U_A = \sigma(\Delta I_P) / \sqrt{N_P} = 0.237$ ppm. The total relative measurement uncertainty of the pumped current, $U_{total} = 0.31$ ppm, is given by the root-sum-square of the eight terms listed in Table I, of which $U_A$ is the largest. Thus the pumped current averaged over the plateau can be expressed as $\Delta I_0 = (-0.013 \pm 0.31)$ ppm. To verify the robustness of our device, we also carried out another high-accuracy scan by stepping $V_{B1}$. In this scan, we find the pumped current averaged over the plateau, shown in the inset of Fig. 4(b), to be $\Delta I_P = (-0.257 \pm 0.27)$ ppm.

![Fig. 4](image-url)
TABLE I. Breakdown of the uncertainty budget for the measurements of the pumped current $I_P$ in Fig. 4. All reported uncertainties are dimensionless 1σ relative uncertainties.

<table>
<thead>
<tr>
<th></th>
<th>PL plateau</th>
<th>B1 plateau</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Voltmeter calibration (type A)</td>
<td>0.01 ppm</td>
<td>0.01 ppm</td>
</tr>
<tr>
<td>2. Voltmeter linearity (type B)</td>
<td>0.05 ppm</td>
<td>0.05 ppm</td>
</tr>
<tr>
<td>3. Voltmeter drift (type B)</td>
<td>0.068 ppm</td>
<td>0.068 ppm</td>
</tr>
<tr>
<td>4. 1 GΩ calibration (type B)</td>
<td>0.1 ppm</td>
<td>0.1 ppm</td>
</tr>
<tr>
<td>5. 1 GΩ drift (type B)</td>
<td>0.01 ppm</td>
<td>0.01 ppm</td>
</tr>
<tr>
<td>6. 1 GΩ calibration (Type A)</td>
<td>0.15 ppm</td>
<td>0.07 ppm</td>
</tr>
<tr>
<td>7. $I_{c_{\text{null}}}$ (type B)</td>
<td>0.01 ppm</td>
<td>0.01 ppm</td>
</tr>
<tr>
<td>8. $I_{c_{\text{e}}}$ (type A)</td>
<td>0.237 ppm</td>
<td>0.229 ppm</td>
</tr>
<tr>
<td>Total</td>
<td>0.31 ppm</td>
<td>0.27 ppm</td>
</tr>
</tbody>
</table>

The deviation of the pumped current from $e f$ is within the measurement uncertainty, and represents the most accurate measurement to date on a silicon SE pump. This work, along with the previous high-accuracy study of silicon devices in the decay-cascade regime [27], indicates that the silicon-based single-electron pump can lead to a more practical and transferable quantum standard of electrical current.

IV. SUMMARY AND OUTLOOK

Despite severe heating in the electron reservoir, the electron pump presented in this work generated a pumped current equal to $e f$ within the $\sim 0.3$ ppm measurement uncertainty at $f = 1$ GHz. Furthermore, fitting the data to a thermal-capture model indicates a theoretical lower bound for the pumping error of 4 ppb at the center of the first current plateau. This suggests that our pump may satisfy the stringent accuracy requirements for a metrological current source [1, 6, 7]. In addition, the fact that strong magnetic fields or tailored waveform drives are not required for the accurate operation of our pump, could greatly simplify the experimental implementation of the new standard of electrical current.

Note that, by adopting a three-waveform pumping scheme [26], one can potentially reduce the reservoir electron temperature and hence significantly improve the accuracy of our pump in the thermal regime. The three waveform scheme may reduce the rf amplitude required to pump the electrons and mitigate the drive-induced heating in the source reservoir.

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[44] See Supplemental Material at [URL will be inserted by publisher] for more information about the addition energy extraction, TCAD simulation as well as the precision measurement technique.