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Phys. Rev. Applied **21**, 014041 — Published 22 January 2024

DOI: [10.1103/PhysRevApplied.21.014041](https://doi.org/10.1103/PhysRevApplied.21.014041)

# Effective model analysis of intrinsic spin Hall effect with magnetism in stacked-kagome Weyl semimetal $\text{Co}_3\text{Sn}_2\text{S}_2$

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We theoretically study the spin Hall effect in a simple tight-binding model of stacked-kagome Weyl semimetal  $\text{Co}_3\text{Sn}_2\text{S}_2$  with ferromagnetic ordering. We focus on the two types of the spin Hall current: one flowing in the in-plane direction with respect to the kagome lattice (in-plane spin Hall current), and one flowing in the stacking direction (out-of-plane spin Hall current). We show the spin Hall conductivities for those spin currents drastically change depending on the direction of the magnetic moment. Especially, the out-of-plane spin Hall current may induce surface spin accumulations, which are useful for magnetization switching via spin-orbit torque.

## I. INTRODUCTION

The generation and control of spin current, namely the flow of spin angular momentum, are important objectives in spintronics. Spin Hall effect (SHE) [1–3] is one of the most fundamental phenomena for generating spin current; the spin current is driven transversely to an applied electric field. Spin-orbit coupling (SOC) plays a significant role in obtaining the spin-dependent electron motion and thus the SHE. The SHE-based highly-efficient manipulation of magnetization, such as spin-orbit torque [4–9], has recently been studied. Conventionally, non-magnetic materials with a strong SOC, such as Pt [10] and Ta [8], had been examined as a spin Hall current generator. Meanwhile, in such a system, the direction of the accumulated spin at the interface is constrained to be perpendicular to both the direction of the flow of spin current and the applied electric field [1, 3].

In addition to non-magnetic materials, recent studies explore the possibility of magnetic materials as spin Hall systems [11–18]. Particularly, some magnetic systems exhibit a peculiar SHE where the direction of the accumulated spin is parallel to that of flow of spin current, depending on the magnetic configurations, such as antiferromagnetic ordering [18–20]. Finding spin Hall systems without restrictions on the direction of induced spin accumulation may help us design functional spintronic devices.

Recently, from viewpoint of spintronic functionalities, Weyl semimetals with magnetism, magnetic Weyl semimetals have been studied [21–26]. Originating from the Berry curvature generated by the band crossing points (Weyl points) [27, 28], distinctive electromagnetic responses, such as the intrinsic anomalous Hall effect (AHE) [28], occur. ABC-stacked kagome-lattice fer-

romagnet  $\text{Co}_3\text{Sn}_2\text{S}_2$  is a promising candidate for magnetic Weyl semimetal [29–32]. The giant AHE arises because the Weyl points are very close to the Fermi level. Additionally, the giant anomalous Hall angle is realized due to the small longitudinal conductivity, i.e., the small Fermi surfaces [29]. Other characteristic responses, for example, the anomalous Nernst effect [33] and magneto-optical Kerr effect [34] are also studied. According to the above features, we expect  $\text{Co}_3\text{Sn}_2\text{S}_2$  might be useful for the efficient and functional manipulations of the spin current for the following reasons. First, the fully-polarized current is realized because of the half-metallicity [29, 35]. Thus the giant anomalous Hall angle implies highly-efficient SHE. Second, owing to SOC, the band topology and AHE depend on the magnetic configurations [36–38]. Similarly, the characteristic relations between the SHE and the direction of the magnetic moments are expected. As a matter of fact, a recent experiment reported a SOT with high spin-change conversion efficiency in the ferromagnetic phase of  $\text{Co}_3\text{Sn}_2\text{S}_2$  [39]. However, the theoretical understanding of the role of magnetic orderings for spin transport in this system is still limited.

In this paper, we theoretically investigate the SHE and AHE in an effective tight-binding model [40] of magnetic Weyl semimetal  $\text{Co}_3\text{Sn}_2\text{S}_2$  with ferromagnetic ordering. First, we review that this model can describe the Dirac semimetal state and the SHE in a non-magnetic state. Then we show that spin Hall conductivities (SHCs) drastically change depending on the direction of the magnetic moment. Especially, the out-of-plane SHE is enhanced by tilting the magnetic moment from the  $z$  axis, in the presence of a certain SOC. Lastly, the possibility of this system as a spin-current generator for magnetization switching is discussed.

## II. TIGHT-BINDING MODEL

First, we explain the minimal tight-binding model of magnetic Weyl semimetal  $\text{Co}_3\text{Sn}_2\text{S}_2$  introduced in our previous study Ref. 40. This model describes the three pairs of the Weyl points by using the  $d$  orbital of Co and the  $p$  orbital of interlayer Sn, as shown in the fol-

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lowing section. Figure 1(a) shows the crystal structure of  $\text{Co}_3\text{Sn}_2\text{S}_2$ . The kagome layers consist of Co, being responsible for magnetism. Sn atoms are located at the center of these hexagons of the kagome lattice. Triangular lattice layers formed by one Sn and two S are sandwiched by the kagome layers. To distinguish two different Sn, we use the notation Sn1 and Sn2 for inter-layer Sn and intralayer Sn, respectively. In our effective model, one  $d$  orbital from Co forming the kagome layers (red) and one  $p$  orbital from interlayer Sn1 (blue) are extracted. We neglect the rest of the orbitals from another Sn2 (cyan) and S (green) for simplicity. The effect of those neglected sites is substantially incorporated into the model as an electric field yielding a SOC, as discussed in the following. Therefore the unit cell consists of the (3+1) sublattices. The primitive translational vectors are defined as  $\mathbf{a}_1 = (\frac{a}{\sqrt{3}}, 0, \frac{c}{3})$ ,  $\mathbf{a}_2 = (-\frac{a}{2\sqrt{3}}, \frac{a}{2}, \frac{c}{3})$ ,  $\mathbf{a}_3 = (-\frac{a}{2\sqrt{3}}, -\frac{a}{2}, \frac{c}{3})$ . Here,  $a$  and  $c$  are lattice parameters of the conventional unit cell, given as  $a = 5.37\text{\AA}$  and  $c = 13.18\text{\AA}$  [29], respectively.

The total Hamiltonian of this model is given by,

$$H_0 = H_{\text{hop}} + H_{\text{soc}} + H_{\text{exc}}. \quad (1)$$

Here,  $H_{\text{hop}}$  is the spin-independent hopping term,  $H_{\text{soc}}$  is the SOC term, and  $H_{\text{exc}}$  is the exchange coupling term between electron spins and the magnetic moment.  $H_{\text{hop}}$  is given by,

$$H_{\text{hop}} = - \sum_{ijs} t_{ij} d_{is}^\dagger d_{js} + t_{\text{dp}} \sum_{\langle ij \rangle s} (d_{is}^\dagger p_{js} + p_{is}^\dagger d_{js}) + \epsilon_p \sum_{is} p_{is}^\dagger p_{is}, \quad (2)$$

where  $d_{is}$  and  $p_{is}$  are the annihilation operators of the Co- $d$  and Sn- $p$  orbitals, respectively. The index  $s$  is for the spin, and  $i$  for the site.  $t_{ij}$  includes the first and second-nearest neighbor hopping,  $t_1$  and  $t_2$ , in the kagome layer, inter-kagome layer hopping  $t_z$ . The summation for  $\langle ij \rangle$  is taken over the nearest neighbor hopping between the Co and Sn sites.  $t_{\text{dp}}$  represents the  $dp$  hybridization between the Co- $d$  and Sn- $p$  orbitals. The lattice vectors for the intra-layer nearest-neighbor are calculated by  $\mathbf{b}_{AB} = (\mathbf{a}_2 - \mathbf{a}_1)/2$ ,  $\mathbf{b}_{BC} = (\mathbf{a}_3 - \mathbf{a}_2)/2$ , and  $\mathbf{b}_{CA} = (\mathbf{a}_1 - \mathbf{a}_3)/2$ . In the same manner, the lattice vectors for intra-layer second-nearest-neighbor are calculated by  $\mathbf{d}_{AB} = (\mathbf{b}_3 - \mathbf{b}_2)/2$ ,  $\mathbf{d}_{BC} = (\mathbf{b}_3 - \mathbf{b}_2)/2$ , and  $\mathbf{d}_{CA} = (\mathbf{b}_1 - \mathbf{b}_3)/2$ . The lattice vectors for the inter-layer nearest-neighbor are calculated by  $\mathbf{c}_{AB} = (\mathbf{a}_1 - \mathbf{a}_2)/2$ ,  $\mathbf{c}_{BC} = (\mathbf{a}_2 - \mathbf{a}_3)/2$ , and  $\mathbf{c}_{CA} = (\mathbf{a}_3 - \mathbf{a}_1)/2$ . Here, A, B and C are the sublattice indices [40].

The SOC term is given by  $H_{\text{soc}} = H_{\text{KM}} + H_{\text{SR}}$ , where

$$H_{\text{KM}} = -it_{\text{KM}} \sum_{\langle\langle ij \rangle\rangle ss'} \nu_{ij} \cdot d_{is}^\dagger \sigma_{ss'}^z d_{js'}, \quad (3)$$

$$H_{\text{SR}} = -it_{\text{SR}} \sum_{\langle ij \rangle ss'} \boldsymbol{\lambda}_{ij} \cdot d_{is}^\dagger \boldsymbol{\sigma}_{ss'} d_{js'}. \quad (4)$$

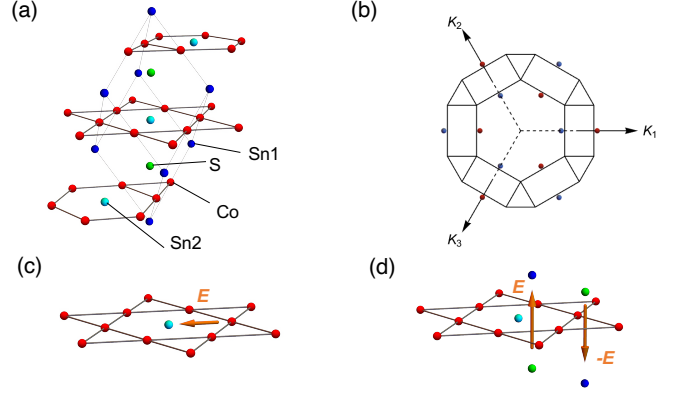


FIG. 1. (Color online) (a) Crystal structure of  $\text{Co}_3\text{Sn}_2\text{S}_2$ . (b) The Weyl points configuration computed by our model. (c) Electric field generated by Co nucleus (red) and Sn nucleus (cyan) at the center of the hexagon in the kagome layer, arising intra-layer-kagome type SOC. (d) Electric field generated by Sn nucleus (blue) and S nucleus (green) in between the kagome layers, arising the staggered-Rashba type SOC.

$H_{\text{KM}}$  describes the intra-kagome-layer Kane-Mele type SOC [41, 42] with strength  $t_{\text{KM}}$ .  $\boldsymbol{\sigma}$  is the vector of Pauli matrices, corresponding to the electron spin. The sign is  $\nu_{ij} = +1(-1)$  when the electron hops counterclockwise (clockwise) to reach the second-nearest-neighbor site on the kagome plane. The summation  $\langle\langle ij \rangle\rangle$  is taken for intralayer second-nearest-neighbor sites. As shown in Fig. 1(b), this SOC originates from the potential of Sn at the center of hexagons of the kagome lattice. Then we explain the staggered-Rashba type SOC  $H_{\text{SR}}$  as introduced in Ref. 43. In  $\text{Co}_3\text{Sn}_2\text{S}_2$ , this SOC originates from the local inversion symmetry breaking of the Sn and S sites, as discussed below. As shown in Fig. 1(c), when we focus on one of the triangular plaquettes of the kagome lattice, Sn is located on the top while S is located on the bottom. Its nearest neighboring plaquettes have the reversed configuration. Therefore, a perpendicular electric field penetrates each triangular plaquette in a staggered pattern.  $\boldsymbol{\lambda}_{ij}$  denotes the effective magnetic field vector from the electric field when the electron hops from the site  $j$  to  $i$ .  $\boldsymbol{\lambda}_{AB} = (\frac{1}{2}, \frac{\sqrt{3}}{2}, 0)$ ,  $\boldsymbol{\lambda}_{BC} = (-1, 0, 0)$ ,  $\boldsymbol{\lambda}_{CA} = (\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0)$ . The summation  $\langle ij \rangle$  is taken for intralayer nearest-neighbor sites. This staggered-Rashba type SOC has a significant role in obtaining the finite out-of-plane AHE and SHE, as discussed later.

The exchange coupling between the itinerant electron's spins and magnetic moments on the kagome lattice is given by,

$$H_{\text{exc}} = -J \sum_{iss'} \mathbf{m}_i \cdot (d_{is}^\dagger \boldsymbol{\sigma}_{ss'} d_{is'} + p_{is}^\dagger \boldsymbol{\sigma}_{ss'} p_{is'}). \quad (5)$$

$J$  is the exchange coupling constant and  $\mathbf{m}_i$  is the vector of magnetic moment. We note that  $\mathbf{m}_i$  should be calculated self-consistently with the Coulomb interaction [44]. We set the same value of exchange coupling on Co and Sn

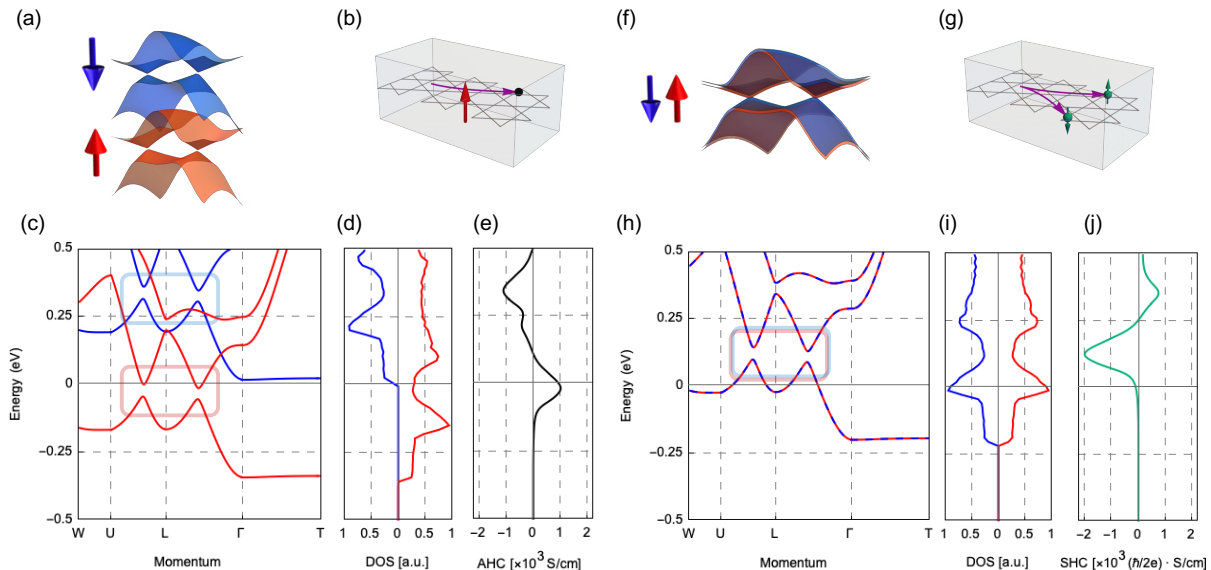


FIG. 2. (Color online) For magnetic Weyl semimetal state in this model, (a) schematic figure of band structure and (b) anomalous Hall effect, (c) band structure **along high-symmetry line**, (d) density of states, and (e) anomalous Hall conductivity. For Dirac semimetal state, (f) schematic figure of band structure and (g) spin Hall effect, (h) band structure **along high-symmetry line**, (i) density of states, and (j) spin Hall conductivity. Fermi level is calculated with a constant electron number  $n_e = 3$  per unit cell.

site for simplicity. In the following, we set  $t_1$  as a unit of energy,  $t_2 = 0.6t_1$ ,  $t_{dp} = 1.8t_1$ ,  $t_z = -1.0t_1$ ,  $\epsilon_p = -7.2t_1$ ,  $t_{KM} = -0.1t_1$ ,  $t_{sR} = 0.1t_1$ , and  $J = 1.2t_1$ . These parameters are also chosen to fit the band structure to the result obtained by the first-principles calculations [29, 45]. In the following, we study the SHC and AHC based on this tight-binding model.

### III. FERROMAGNETIC WEYL SEMIMETAL STATE AND NON-MAGNETIC DIRAC STATE

To keep this paper self-contained, here we review the ferromagnetic Weyl state and the paramagnetic Dirac state in the model as discussed in Ref. 46. In Fig. 2(a), the energy spectrum of the spin-splitting Weyl state is schematically shown. The band structure in the ferromagnetic state is shown in Fig. 2(c). The red and blue lines are the spin-up band and spin-down band, respectively. **We should note that the spin-up band and spin-down band cannot be identified when the staggered-Rashba type SOC is considered. However, for visibility of the spin-polarized electronic structure, here we consider only the Kane-Mele type SOC, which preserves spin  $s_z$ . In the following sections, we discuss the role of staggered-Rashba type SOC for the SHE.** We do not show the lowest and second-lowest bands, because those bands are energetically far from  $E_F$ . The Weyl pairs near  $E_F$  are formed with spin-up (red) bands, indicated by the red box. The Weyl points configuration seen from the  $k_z$  axis, computed by our model, is shown in Fig. 1(b). The three-fold symmetry with respect to the  $k_z$  axis and the

number of the Weyl points in the Brillouin zone are consistent with first-principles calculations [40]. Figure 2(d) is the density of states (DOS) as a function of the energy, computed with  $\mathbf{m} = (0, 0, 1)$ . We use  $t_1 = 0.15$  eV as a unit of energy.  $E = 0$  eV is set as  $E_F$  obtained by the electron number per unit cell  $n_e = 3$ , corresponding to  $E_F$  of non-doped  $\text{Co}_3\text{Sn}_2\text{S}_2$  [40, 44]. The DOS shows a local minimum near  $E_F$ , corresponding to the energy of the Weyl points. In the Weyl semimetal state, the AHE occurs, as schematically shown in Fig. 2(b). Figure 2(e) shows  $\sigma_{yx}^{\text{AHE}}$  as a function of energy. Here, the AHC is calculated by the Kubo formula [47] as

$$\sigma_{yx}^{\text{AHE}} = e^2 \hbar \sum_{n \neq m} \text{Im} \int_{\text{BZ}} \frac{d^3 k}{(2\pi)^3} \frac{f(E_{n\mathbf{k}}) - f(E_{m\mathbf{k}})}{(E_{n\mathbf{k}} - E_{m\mathbf{k}})^2} \times \langle n\mathbf{k} | \hat{v}_y | m\mathbf{k} \rangle \langle m\mathbf{k} | \hat{v}_x | n\mathbf{k} \rangle. \quad (6)$$

$\hat{v}_i = \frac{1}{\hbar} \frac{\partial H(\mathbf{k})}{\partial k_i}$  ( $i = x, y$ ) is the velocity operator.  $f$  is the Fermi-Dirac distribution function with  $k_B T = 0.01t_1$ . The AHC is maximized near the  $E_F$ , originating from the Berry curvature generated by the Weyl points. As we discussed in our previous study [40], the value  $\sim 1000$  [S/cm] is very close to the result obtained by the first-principles calculations and experiment [29]. We note that, in addition to the electronic structure, our model also quantitatively explains the doping effect for magnetic orderings, as discussed in Ref. 44. The perpendicular ferromagnetic ordering in the non-doped regime and suppression of the magnetism in the hole-doped regime obtained by the self-consistent method also agree with the first principles calculations [45]. Therefore, our model is reasonable

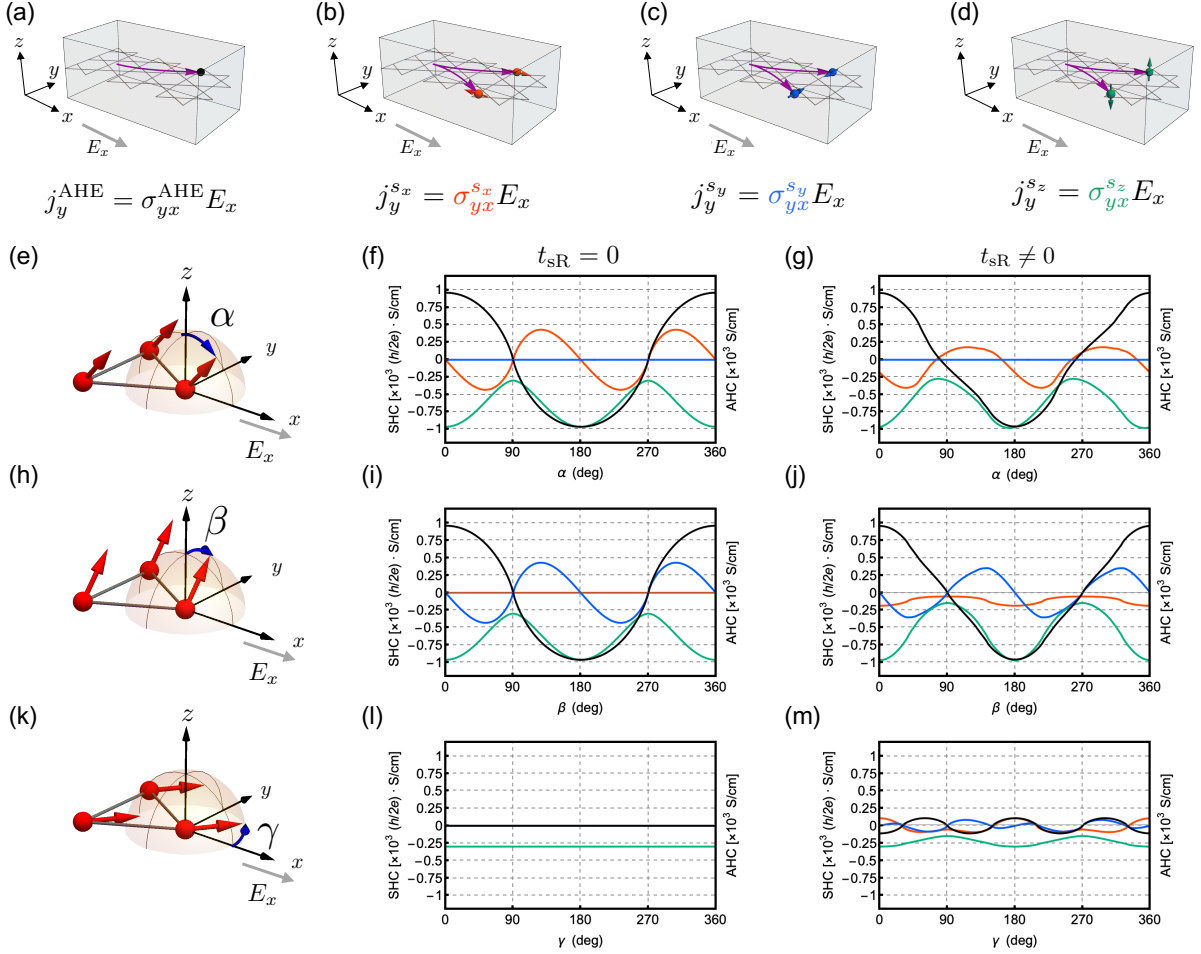


FIG. 3. (Color online) Schematic figures of in-plane (a) anomalous Hall effect and (b-d) spin Hall effect induced by electric field  $E_x$ . (a) Anomalous Hall current flows to  $y$ -direction and is characterized by the Hall conductivity  $\sigma_{yx}^{\text{AHE}}$ . Spin Hall currents with spin angular momentum (b)  $s_x$ , (c)  $s_y$ , and (d)  $s_z$ , flow to  $y$ -direction and are characterized by the Hall conductivities  $\sigma_{yx}^{s_x}$ ,  $\sigma_{yx}^{s_y}$ , and  $\sigma_{yx}^{s_z}$ , respectively.  $\sigma_{yx}^{\text{AHC}}$  (black),  $\sigma_{yx}^{s_x}$  (red),  $\sigma_{yx}^{s_y}$  (blue), and  $\sigma_{yx}^{s_z}$  (green) (f) without and (g) with staggered Rashba SOC  $t_{\text{sR}}$ , as a function of tilting angle (e)  $\alpha$  (in  $xz$ -plane). (i)(j) The same for tilting angle (h)  $\beta$  (in  $yz$ -plane). (l)(m) The same for tilting angle (k)  $\gamma$  (in  $xy$ -plane).

to describe the electronic state including the Weyl points near the Fermi level in  $\text{Co}_3\text{Sn}_2\text{S}_2$ .

$x$ -direction. The SHCs can be calculated as

$$\sigma_{\nu x}^{s_\mu} = \hbar \sum_{n \neq m} \text{Im} \int_{\text{BZ}} \frac{d^3k}{(2\pi)^3} \frac{f(E_{n\mathbf{k}}) - f(E_{m\mathbf{k}})}{(E_{n\mathbf{k}} - E_{m\mathbf{k}})^2} \times \langle n\mathbf{k} | \hat{j}_\nu^\mu | m\mathbf{k} \rangle \langle m\mathbf{k} | (-e\hat{v}_x) | n\mathbf{k} \rangle, \quad (7)$$

Next, we explain the paramagnetic Dirac semimetal state and the SHE [46]. In Fig. 2(f), the energy spectrum of the paramagnetic Dirac state is schematically shown. Figures 2(h) and (i) show the spin degenerate band structure and DOS as a function of the energy, using the parameter  $\mathbf{m} = (0, 0, 0)$ . We emphasize that the energy of the Dirac points is located on  $E \sim 0.1$  eV, deviating from  $E_F$ . The Dirac semimetal state the SHE occurs, as schematically shown in Fig. 2(g). We focus on the spin current driven by an electric field pointing in the

where  $j_\nu^\mu$  is a spin-current operator with a spin polarization  $\mu$  and spatial direction  $\nu$ , given by  $j_\nu^\mu = \frac{1}{2} \{ \frac{\hbar}{2} \sigma_\mu, v_\nu \}$  [3]. We neglect an extrinsic contribution from impurities for simplicity. Figure 2(j) shows the  $\sigma_{yx}^{s_\mu}$  as a function of the energy. Near the energy of the Dirac points, corresponding to the minimum of the DOS,  $\sigma_{yx}^{s_\mu}$  is maximized. To obtain the maximized SHC originating from the Dirac points, an appropriate tuning of  $E_F$  is required [46].

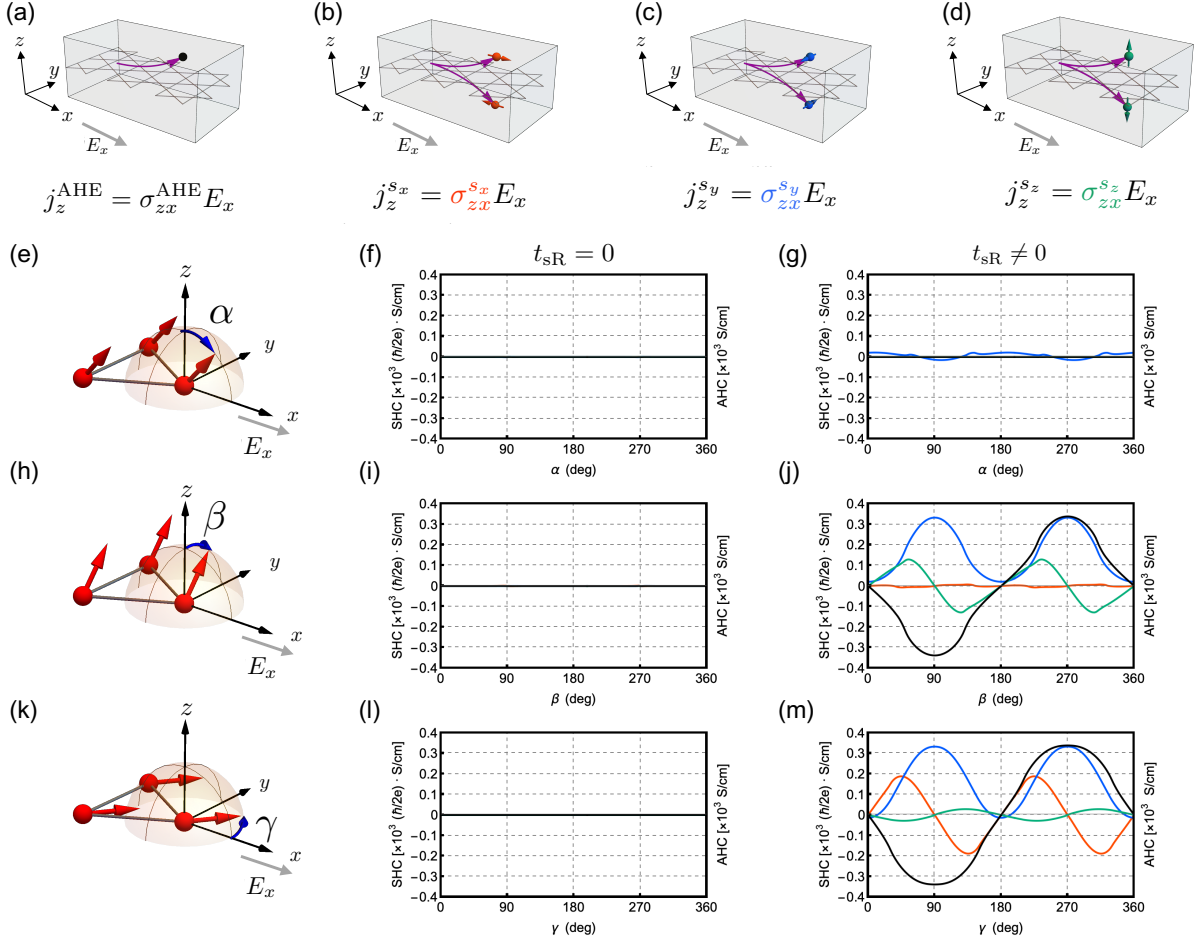


FIG. 4. (Color online) Schematic figures of out-of plane (a) anomalous Hall effect and (b-d) spin Hall effect induced by applied electric field  $E_x$ . (a) Anomalous Hall current flows to  $z$ -direction and is characterized by the Hall conductivity  $\sigma_{zx}^{\text{AHE}}$ . Spin Hall currents with spin angular momentum (b)  $s_x$ , (c)  $s_y$ , and (d)  $s_z$ , flow to  $z$ -direction and are characterized by the spin Hall conductivities  $\sigma_{zx}^{s_x}$ ,  $\sigma_{zx}^{s_y}$ , and  $\sigma_{zx}^{s_z}$ , respectively.  $\sigma_{zx}^{\text{AHC}}$  (black),  $\sigma_{zx}^{s_x}$  (red),  $\sigma_{zx}^{s_y}$  (blue), and  $\sigma_{zx}^{s_z}$  (green) (f) without and (g) with staggered Rashba SOC  $t_{\text{sR}}$ , as a function of tilting angle (e)  $\alpha$  (in  $xz$  plane). (i)(j) The same for different tilting angle (h)  $\beta$  (in  $yz$  plane). (l)(m) The same for different tilting angle (k)  $\gamma$  (in  $xy$  plane).

#### IV. IN-PLANE SPIN/ANOMALOUS HALL EFFECT

In the previous section, we reviewed that our model describes the SHE and AHE, originating from the non-magnetic Dirac state and ferromagnetic Weyl state, respectively. As we mentioned in the introduction, the recent experiment reported the SOT with high spin-charge conversion efficiency in a device based on  $\text{Co}_3\text{Sn}_2\text{S}_2$  and  $\text{CoFeB}$  film [39]. In the device, an in-plane magnetized  $\text{CoFeB}$  is attached to the top surface of  $\text{Co}_3\text{Sn}_2\text{S}_2$ . Additionally, it was predicted that  $\text{Co}_3\text{Sn}_2\text{S}_2$  of this device possesses an in-plane component of the magnetization near the interface [39], although the single system has a strong perpendicular magnetic anisotropy [48]. Motivated by these experimental results, we study the relation between the SHCs and the direction of the magnetization. In this section, we study the in-plane

SHE (AHE), where the spin (anomalous) Hall current  $j_y^{s_\mu}$  ( $j_y^{\text{AHE}}$ ) is induced by an electric field  $E_x$ , as schematically shown in Figs. 3(a)-3(d). The in-plane SHCs and AHC are described as  $\sigma_{yx}^{s_\mu}$  and  $\sigma_{yx}^{\text{AHE}}$ , respectively. Then, we introduce the magnetic moments with different directions, which are characterized by the parameter  $\mathbf{m}_i$  in the exchange coupling term Eq. (5). For simplicity, we focus on the uniform tilting of the magnetization in the bulk model and introduce the tilting angles as tunable parameters. We use uniform magnetization with three tilting angles in (e)  $xz$  plane  $\alpha$ , (h)  $yz$  plane  $\beta$ , and (k)  $xy$  plane  $\gamma$ , as shown in Fig. 3. These magnetic configurations are given by, (e)  $\mathbf{m}_A = \mathbf{m}_B = \mathbf{m}_C = m(\sin \alpha, 0, \cos \alpha)$ , (h)  $\mathbf{m}_A = \mathbf{m}_B = \mathbf{m}_C = m(0, \sin \beta, \cos \beta)$ , (k)  $\mathbf{m}_A = \mathbf{m}_B = \mathbf{m}_C = m(\cos \gamma, \sin \gamma, 0)$ , respectively. In all the following calculations,  $E_F$  is computed at each angle with the constant electron number ( $n_e = 3$ ), corresponding to the



non-doped  $\text{Co}_3\text{Sn}_2\text{S}_2$  [40, 44].

First, we study the SHCs (AHC) for the changes in the magnetization angle  $\alpha$  shown in Fig. 3(e). In Fig. 3(f),  $\sigma_{yx}^{s\mu}$  and  $\sigma_{yx}^{\text{AHE}}$  are computed as a function of  $\alpha$  by considering only the Kane-Mele type SOC  $t_{\text{KM}}$ . When  $\alpha = 90^\circ$ , the magnetization is parallel to the applied electric field. Red, blue, green, and black lines indicate  $\sigma_{yx}^{s_x}$ ,  $\sigma_{yx}^{s_y}$ ,  $\sigma_{yx}^{s_z}$ , and  $\sigma_{yx}^{\text{AHE}}$ , respectively. The sign of the AHC  $\sigma_{yx}^{\text{AHE}}$  (black line) changes when the magnetization flips [40]. One finds that when  $\alpha = 0^\circ$  and  $\alpha = 180^\circ$ ,  $|\sigma_{yx}^{\text{AHE}}| \sim |\sigma_{yx}^{s_z}|$  (green line) holds except for the unit  $\frac{\hbar}{2e}$  because the spin Hall current is equal to the spin-polarized anomalous Hall current.  $\sigma_{yx}^{s_x}$  (red line) can be finite and has a periodicity of  $180^\circ$ . On the other hand,  $\sigma_{yx}^{s_y}$  (blue line) vanishes. In Fig. 3(g), the staggered-Rashba SOC  $t_{\text{SR}}$  is considered in addition to  $t_{\text{KM}}$ . We note  $\sigma_{yx}^{\text{AHE}}$  can be finite even without the out-of-plane component of the magnetization at  $\alpha = 90^\circ$  and  $270^\circ$ . The periodicities of the Hall conductivities remain unchanged even with  $t_{\text{SR}}$ .

Next, we study the SHCs (AHC) for the changes in the magnetization angle  $\beta$ , as shown in Fig. 3(h). In Fig. 3(i),  $\sigma_{yx}^{s\mu}$  and  $\sigma_{yx}^{\text{AHE}}$  are computed as a function of  $\beta$  by considering only  $t_{\text{KM}}$ . When  $\beta = 90^\circ$ , the magnetization is parallel to the spin Hall current. The behaviors of  $\sigma_{yx}^{\text{AHE}}$  (black line) and  $\sigma_{yx}^{s_z}$  (green line) are equivalent to those for the angle  $\alpha$  [Fig. 3(f)]. We find  $\sigma_{yx}^{s_y}$  (blue line) can be finite, whereas  $\sigma_{yx}^{s_x}$  (red line) vanishes. In Fig. 3(j),  $t_{\text{SR}}$  is considered in addition to  $t_{\text{KM}}$ . In contrast to the case with  $\alpha$ , all the components of SHCs become finite.  $\sigma_{yx}^{\text{AHE}}$  (black line) vanishes at  $\beta = 90^\circ$  and  $270^\circ$ , while  $\sigma_{yx}^{\text{AHE}}$  can be finite at  $\alpha = 90^\circ$  and  $270^\circ$ .

Then, we study the SHCs (AHC) for the changes in the magnetization angle  $\gamma$  shown in Fig. 3(k). When  $\gamma = 0^\circ$ , the magnetization is parallel to the applied electric field. As shown in Fig. 3(l), all the conductivities are independent of  $\gamma$  by considering only  $t_{\text{KM}}$ . We note that only  $\sigma_{yx}^{s_z}$  (green line) is finite. In Fig. 3(m),  $t_{\text{SR}}$  is considered in addition to  $t_{\text{KM}}$ . All of the conductivities can be finite and show angular dependences.

## V. OUT-OF-PLANE SPIN/ANOMALOUS HALL EFFECT

In the SOT experiment based on  $\text{Co}_3\text{Sn}_2\text{S}_2$  mentioned previously [39, 46], it is favorable to study the spin current flowing to the stacking ( $z$ -) direction. Motivated by such experiments, we study the out-of-plane SHE (AHE), where the spin (anomalous) Hall current  $j_z^{s\mu}$  ( $j_z^{\text{AHE}}$ ) is induced by an electric field  $E_x$ , as shown in Figs. 4(a)-4(d). We find the characteristic angular dependences and enhancements of the Hall conductivities. The out-of-plane SHCs and AHC are described as  $\sigma_{zx}^{s\mu}$  and  $\sigma_{zx}^{\text{AHE}}$ , respectively. We again consider the changes in the angles of the magnetization  $\alpha$ ,  $\beta$ , and  $\gamma$  [see Figs. 4(e), 4(h), and 4(k)]. Here, red, blue, green, and black lines indicate  $\sigma_{zx}^{s_x}$ ,  $\sigma_{zx}^{s_y}$ ,

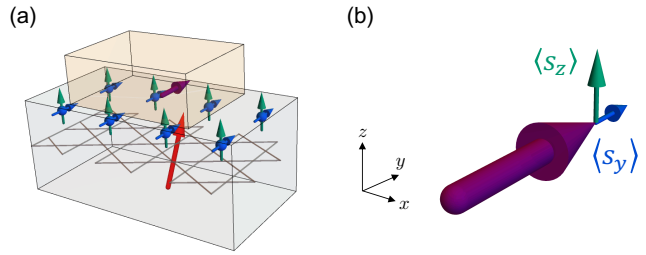


FIG. 5. (Color online) (a) Possible configurations of spin accumulation  $\langle s_y \rangle$  and  $\langle s_z \rangle$  induced by out-of-plane spin Hall effect. (b) Relation between the in-plane magnetization and spin accumulations.

$\sigma_{zx}^{s_z}$ , and  $\sigma_{zx}^{\text{AHE}}$ , respectively. Figure 4 shows the out-of-plane Hall conductivities as a function of the angles as in Fig. 3. We notice that all the Hall conductivities vanish irrespective of the angles when only  $t_{\text{KM}}$  is finite [see Figs. 4(f), 4(i), and 4(l)]. Then, we consider  $t_{\text{SR}}$  in addition to  $t_{\text{KM}}$ . In Fig. 4(g), for the angle  $\alpha$ ,  $\sigma_{zx}^{s_y}$  (blue line) can be finite, and others vanish. In Figs. 4(j) and 4(m), for the angles  $\beta$  and  $\gamma$ , all of the Hall conductivities can be finite.  $\sigma_{zx}^{s_y}$  (blue line) and  $|\sigma_{zx}^{\text{AHE}}|$  (black line) are maximized at  $\beta = \gamma = 90^\circ$  and  $270^\circ$ . At those angles, we recall the magnetization is perpendicular to the applied electric field and induced Hall currents. This enhancement of the out-of-plane AHC  $\sigma_{zx}^{\text{AHE}}$  is consistent with that obtained by first-principles calculations [36]. In Fig. 4(j), for the angle  $\beta$ ,  $\sigma_{zx}^{s_x}$  (red line) is negligible. On the other hand, in Fig. 4(m), for the angle  $\gamma$ ,  $\sigma_{zx}^{s_x}$  (red line) and  $\sigma_{zx}^{s_y}$  (blue line) are comparable. In particular, in both angles  $\beta$  and  $\gamma$ ,  $|\sigma_{zx}^{s_z}|$  (green line) is enhanced around  $60^\circ$ ,  $120^\circ$ ,  $240^\circ$ , and  $300^\circ$  while it vanishes at  $0^\circ$ ,  $90^\circ$ ,  $180^\circ$ , and  $270^\circ$ . We emphasize that the distinct angular dependences of the Hall conductivities are originating from the presence of the staggered-Rashba type SOC.

Before concluding this paper, we discuss the possibility of this system as a functional spin current generator. Here, we show that the SHE studied in this work might be useful for efficient magnetization switching. In the previous paragraph, we found the enhancement of the SHCs by tilting the magnetization. In particular, we showed the enhancement of  $\sigma_{zx}^{s_y}$  (blue line) and  $\sigma_{zx}^{s_z}$  (green line) as shown in Fig. 4(j). This implies that the  $y$ - and  $z$ -components of the spin accumulations ( $\langle s_y \rangle$  and  $\langle s_z \rangle$ , respectively) are induced at the surface, as shown in Fig. 5(a). As mentioned previously, the recent experiment studied SOT in the devices of  $\text{Co}_3\text{Sn}_2\text{S}_2$  attached to the in-plane magnetized CoFeB film [39]. Motivated by this experiment, we consider the same geometry where a ferromagnet is attached to the top of  $\text{Co}_3\text{Sn}_2\text{S}_2$ . In the attached ferromagnet, we assume that the macroscopic magnetization points in the  $y$ -direction, which is represented by a single purple arrow in Fig. 5(a). Additionally, when the ferromagnetic coupling exists near

the interface, the magnetization of  $\text{Co}_3\text{Sn}_2\text{S}_2$  (red arrow) may be tilted slightly even with the perpendicular magnetic anisotropy. This may yield an enhancement of the SHCs as mentioned previously. With this situation, let us discuss the SOT from  $\langle s_z \rangle$  and  $\langle s_y \rangle$  exerting on the magnetization, as shown in Fig. 5(b). First, we discuss two types of SOTs from  $\sigma_{zx}^{sz}$ . The damping-like torque tends to align magnetization parallel to  $z$ -axis. The field-like torque drives the precession motion of the magnetization around  $z$ -axis. When the easy-plane magnetic anisotropy is present, the field-like SOT drives the precession motion of the magnetization in the easy plane, while the damping-like SOT generates the hard-axis component. We note that the flip of the electric field changes the direction of the field-like SOT. Nevertheless, the torque results in the precession motion of the magnetization. Then we discuss the SOTs from  $\sigma_{zx}^{sy}$ . The damping-like torque can stabilize the magnetization pointing in the  $+y$  axis. When the direction of the electric field is flipped, the anti-damping-like torque points in the  $-y$  direction, helping the switching of the magnetization. We note the field-like torque, pointing in  $+z$  or  $-z$  direction, is ineffective when the easy plane magnetic anisotropy is present. In summary, a combination of the field-like torque from  $\sigma_{zx}^{sz}$  and the (anti-) damping-like torque from  $\sigma_{zx}^{sy}$  may switch the magnetization efficiently, only with an electric field

but without an external magnetic field.

## VI. CONCLUSION

In this paper we theoretically studied the SHE in an effective model of magnetic Weyl semimetal  $\text{Co}_3\text{Sn}_2\text{S}_2$ . We showed the drastic changes of the SHCs depending on the direction of the magnetic moment. Especially, for the out-of-plane SHE, the enhancements of the SHCs were found. Our finding may help us design an efficient SOT generator for the in-plane magnetization switching based on the magnetic Weyl semimetal.

## ACKNOWLEDGMENTS

The authors would like to appreciate Y. Araki, K. Fujiwara, Y. Kato, Y.-C. Lau, Y. Motome, K. Nakazawa, T. Seki, A. Tsukazaki, and Y. Yahagi for valuable discussions. This work was supported by JST CREST, Grant No. JPMJCR18T2 and by JSPS KAKENHI, Grant Nos. JP20H01830 and JP22K03446. A. O. was supported by GP-Spin at Tohoku University and by JST SPRING, Grant No. JPMJSP2114.

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