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High-Order Qubit Dephasing at Sweet Spots by Non-Gaussian Fluctuators: Symmetry Breaking and Floquet Protection

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Although the Gaussian-noise assumption is widely adopted in studying qubit decoherence, non-Gaussian noise sources, especially the strong discrete fluctuators, have been detected in many qubits. It remains an important task to further understand and mitigate the distinctive decoherence effect of the non-Gaussian noise. Here, we study the qubit dephasing caused by the non-Gaussian fluctuators, and predict a symmetry-breaking effect that is unique to the non-Gaussian noise. This broken symmetry results in an experimentally measurable mismatch between the extremum points of the dephasing rate and qubit frequency, which demands extra carefulness in characterizing the noise and locating the optimal working point. To further enhance the coherence time at the sweet spot, we propose to suppress the second-order derivative of the qubit frequency by the Floquet engineering. Our simulation on a tunable and gapped two-level quantum system, with the parameters from a heavy fluxonium, shows an order of magnitude improvement of the dephasing time, even after including the noise introduced by the drive.

Introduction.– Efficient quantum computing relies on high-coherence qubits and high-fidelity quantum operations [1, 2]. However, further enhancement of the coherence times and gate fidelities toward full quantum error correction, one prerequisite for quantum computation [1–4], has been set back by the decoherence [4–7]. Many solid-state qubits, for example, the superconducting qubits, are especially subjected to the uncontrolled low-frequency environmental fluctuations [5–33], such as the notorious $1/f$ noise [18–33].

This challenge has motivated research efforts on identifying, understanding, and mitigating the contributing noise channels [5–67]. Theoretically, the noise is usually assumed Gaussian, while the realistic noise is more complicated. For example, if strong discrete fluctuators are present [13–18], the decoherence process of the qubits deviates significantly from the prediction by assuming only Gaussian noise [5, 8, 9, 13, 27, 38, 39, 44, 45].

In this Letter, we focus on the special dephasing feature in qubits operated at the sweet spots, where many flagship solid-state qubits reach their maximal dephasing time [23, 26, 31, 52, 55, 62, 63, 67, 68]. We report that the non-Gaussian noise introduces a previously unrevealed dephasing feature, the $\mathbb{Z}_2$ symmetry breaking at the qubit sweet spot. The breaking is predicted to cause an otherwise unexpected mismatch between the extremum points of the qubit dephasing rate and its oscillating frequency. Using a realistic non-Gaussian noise assumption, the strong two-level fluctuators (TLFs), we demonstrate such mismatch in a concrete qubit model by two independent theoretical approaches. This finding provides a simple yet decisive tool to identify the non-Gaussian component in the noise background [53]. To further enhance the sweet-spot coherence time limited by such fluctuators (also Gaussian noise), we propose a triple-protection scheme via the Floquet engineering [48, 49, 69–71], where the qubit is not only protected from the dc fluctuation to the second order, but also from ac fluctuation to the first order.

Model.– The system we study is a tunable qubit subjected to low-frequency fluctuation. The full Hamiltonian in consideration is given by $\hat{H} = \hat{H}_q(\lambda) + \delta\xi(t)\hat{x}$, where $\hat{H}_q(\lambda)$ denotes the bare qubit Hamiltonian, $\delta\xi(t)$ describes the environmental fluctuation and $\hat{x}$ is the operator that the fluctuators are coupled to. The qubit is tuned by an external control parameter λ according to $\hat{H}_q(\lambda) = \hat{H}_q(0) + \lambda\hat{x}$, which for example corresponds to the flux (charge) control of the fluxonium qubit (Cooper-pair box) [35, 58]. We assume that the first-order derivative of this qubit vanishes at $\lambda = 0$, which is a result of a $\mathbb{Z}_2$ symmetry. Specifically, the Hamiltonian satisfies $\hat{R}\hat{H}_q(0)\hat{R}^\dagger = \hat{H}_q(0)$, where $\hat{R}$ is a reflection operation defined by the relation $\hat{R}\hat{x}\hat{R}^\dagger = -\hat{x}$. This symmetry ensures $(\partial\omega_j^{\text{bare}}/\partial\lambda)|_{\lambda=0} = 0$, where ω_j^{bare} is the eigenenergy of the jth eigenstate of the bare Hamiltonian $\hat{H}_q(\lambda)$. Note that the only specification to such modeling is the existence of a symmetry-induced sweet spot, which extends our following discussion to a wide range of qubits exhibiting such working points [22, 31, 35, 55, 63, 67]. For the noise $\delta\xi(t)$, we assume that the external control over the small range of λ under inspection only negligibly affecting its properties.

$\mathbb{Z}_2$ symmetry and its breaking.– The symmetry we address is the invariance of the qubit density matrix elements, $\rho_{ij}(t)|_\lambda$, evaluated in their respective eigenbasis, under the reflection $\lambda \to -\lambda$. Such symmetry is strictly preserved if the fluctuation is Gaussian, and is generally broken for non-Gaussian $\delta\xi(t)$.

To rigorously derive the conclusion above, we first inspect

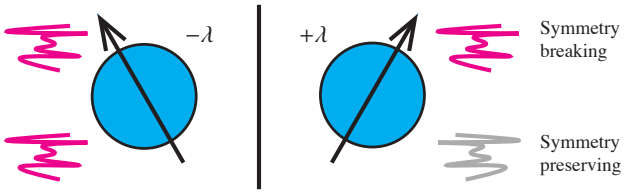


FIG. 1. A cartoon illustrating how noise breaks the reflection symmetry of the qubit. The noise term $\hat{H}_I(t)$ will not undergo a similar reflection (bottom gray curve) as the bare Hamiltonian by the external operation $-\lambda \to \lambda$, which in general breaks the reflection symmetry, unless the statistical effect of $\delta\xi(t)$ is identical to that of $-\delta\xi(t)$.

the relation between the qubit Hamiltonians and eigenstates for $\pm\lambda$. The Hamiltonians are related by $\hat{H}_q(-\lambda) = \hat{R}\hat{H}_q(\lambda)\hat{R}^\dagger$, and the eigenstates can be conveniently chosen to satisfy $|j(-\lambda)\rangle = \hat{R}|j(\lambda)\rangle$. However, the interaction Hamiltonian $\hat{H}_I(t) \equiv \delta\xi(t)\hat{x}$ does not transform similarly due to the independence of $\hat{H}_I(t)$ on λ. This leads to a sign flipping of $\hat{H}_I(t)$ under reflection, i.e., $\hat{R}\hat{H}_I(t)\hat{R}^\dagger = -\hat{H}_I(t)$. As a result, the propagators transform as:

$$\hat{R}\tilde{U}_I^\nu(t)|_\lambda\hat{R}^\dagger = (-1)^\nu\tilde{U}_I^\nu(t)|_{-\lambda}. \tag{1}$$

Above, $\tilde{U}_I^\nu(t)$ is the νth-order term in the Dyson expansion of the interaction propagator $\tilde{U}_I(t) = \mathcal{T}\exp[-i\int_0^t dt'\tilde{H}_I(t')]$, where $\tilde{H}_I(t) = \hat{U}_0^\dagger(t)\hat{H}_I(t)\hat{U}_0(t)$ and $\hat{U}_0(t) \equiv \exp[-i\hat{H}_q(\lambda)t]$. The propagator $\tilde{U}_I(t)$ is expanded as $\tilde{U}_I(t) = \sum_\nu \tilde{U}_I^\nu(t)$, where $\tilde{U}_I^\nu(t)$ contains a product of ν times of $\tilde{H}_I(t)$.

Crucially, the sign flipping is further carried to the density matrix elements, which are expanded as

$$\begin{aligned}\rho_{I,jk}(t)|_\lambda &\equiv \overline{\langle j|\tilde{\rho}_I(t)|k\rangle}|_\lambda = \overline{\langle j|\tilde{U}_I(t)\tilde{\rho}_I(0)\tilde{U}_I^\dagger(t)|k\rangle}\Big|_\lambda \\ &= \sum_{j'k'\nu}\rho_{I,j'k'}(0)\,\Pi^\nu_{jk\leftarrow j'k'}(t)|_\lambda.\end{aligned} \tag{2}$$

Here, $\tilde{\rho}_I(t) = \hat{U}_0^\dagger(t)\hat{\rho}(t)\hat{U}_0(t)$ is the density matrix in the interaction picture, and the Keldysh projector is $\Pi^\nu_{jk\leftarrow j'k'}(t)|_\lambda \equiv \sum_{\nu'+\nu''=\nu}\overline{\langle j|\tilde{U}_I^{\nu'}(t)|j'\rangle\langle k'|\,\tilde{U}_I^{\nu''\dagger}(t)|k\rangle}|_\lambda$. Using Eq. (1), we find $\Pi^\nu_{jk\leftarrow j'k'}(t)|_\lambda = (-1)^\nu\Pi^\nu_{jk\leftarrow j'k'}(t)|_{-\lambda}$, which implies that $\rho_{I,jk}(t)|_\lambda$ and $\rho_{I,jk}(t)|_{-\lambda}$ are unequal in general, unless all odd projectors are irrelevant. The Gaussian noise ensures this condition due to its vanishing odd correlation functions, such as $\overline{\delta\xi_G(t)\delta\xi_G(t_1)\delta\xi_G(0)} = 0$, which leads to $\rho_{I,jk}(t)|_{-\lambda} = \rho_{I,jk}(t)|_\lambda$ given identical initial matrix elements (similarly for lab-frame matrices). On the other hand, non-Gaussian noise does not in general preserve this equality. We summarize the discussion of the symmetry breaking and preserving in FIG. 1.

To theoretically confirm the predicted symmetry breaking, we adopt a simple yet realistic model, i.e., the strong TLFs [27, 38, 39]. This model is motivated by both the microscopic understanding of the materials used in solid-state qubits [5] and abundant experimental evidence [13–18]. To make our results more broadly applicable, the full fluctuation $\delta\xi(t)$ consists of both the Gaussian noise and N_T strong TLFs, which specifies $\delta\xi(t) = \sum_{\mu=1}^{N_T}\delta\xi_{T\mu}(t) + \delta\xi_G(t)$. The TLFs can only take two values, i.e., $\pm|\xi_{T\mu}| - \bar{\xi}_{T\mu}$, where $|\xi_{T\mu}|$ describes the noise magnitude, and $\bar{\xi}_{T\mu}$ is used to cancel the time average of the fluctuator.

The high-order dephasing rates contributed by these fluctuators are derived using the Keldysh diagrammatic technique [40, 59, 60, 72, 73]. It provides an expression of the free-induced (Ramsey) evolution of the off-diagonal matrix element $\rho_{eg}(t) \approx \rho_{eg}(0)\exp[-i\omega_q' t - \Phi(t)]$. Here, the Lamb-shifted qubit oscillation frequency is approximated by $\omega_q' \approx \Delta + D_{2,\lambda=0}\left[\lambda^2 + \int d\omega S(\omega)/2\pi\right]/2$, where the coefficient $D_{2,\lambda=0} \equiv (\partial^2\omega_{ge}^{\rm bare}/\partial\lambda^2)|_{\lambda=0}$ denotes the second-order derivative of the bare qubit frequency and Δ is the extremum qubit bare frequency $\Delta \equiv (\omega_e^{\rm bare} - \omega_g^{\rm bare})|_{\lambda=0}$. (The integration limits in $\int_{-\infty}^{\infty}$ are omitted to save space.) The dephasing profile $\Phi(t)$ are divided into multiple contributions $\Phi(t) \approx \Phi_G(t) + \sum_{\mu=1}^{N_T}\Phi_{T\mu}(t) + \sum_{\upsilon\neq\upsilon'}\Phi_{\upsilon\upsilon'}(t)$, where $\Phi_G(t)$, $\Phi_{T\mu}(t)$ and $\Phi_{\upsilon\upsilon'}(t)$ are contributed by the Gaussian noise, the μth TLF and mutual effect by two different components. Specifically, we find

$$\begin{aligned}\Phi_G(t) &= D_{2,\lambda=0}^2\lambda^2\int\frac{d\omega}{2\pi}S_G(\omega)K^R(\omega,t) \\ &\quad + \frac{D_{2,\lambda=0}^2}{2}\iint\frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}S_G(\omega)S_G(\omega')K^R(\omega+\omega',t), \\ \Phi_{T\mu}(t) &= D_{2,\lambda=0}^2\left(\lambda - \bar{\xi}_{T\mu}\right)^2\int\frac{d\omega}{2\pi}S_{T\mu}(\omega)K^R(\omega,t), \\ \Phi_{\upsilon\upsilon'}(t) &= \frac{D_{2,\lambda=0}^2}{2}\iint\frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}S_\upsilon(\omega)S_{\upsilon'}(\omega')K^R(\omega+\omega',t).\end{aligned} \tag{3}$$

Above, $S_\upsilon(\omega)$ is the Fourier transformation of the two-point correlation functions, $S_\upsilon(\omega) = \int dt\,\overline{\xi_\upsilon(t)\xi_\upsilon(0)}e^{i\omega t}$ ($\upsilon = G, T\mu$ and $\mu = 1, 2, \cdots, N_T$), and the sum spectrum is denoted by $S(\omega) = \sum_\upsilon S_\upsilon(\omega)$. These spectra are sampled time-dependently according to the filter function $K^R(\omega,t) \equiv t^2\mathrm{sinc}^2(\omega t/2)/2$. In Eq. (3), we find that $\Phi_{T\mu}(t)$ does not preserve the $\mathbb{Z}_2$ symmetry, since nonzero $\bar{\xi}_{T\mu}$ leads to $\Phi_{T\mu}(t)|_\lambda \neq \Phi_{T\mu}(t)|_{-\lambda}$. Meanwhile, up to the leading order, the qubit frequency is still symmetric after including the Lamb shift, implying an unexpected mismatch between the extremum points of the dephasing rate and frequency.

In addition to the analytical calculation, we further confirm our prediction independently using the stochastic Schrödinger equation (SSE) [38, 39]. This method averages the ensemble of traces of the qubit evolution according to many realizations of the random fluctuation. We choose to simulate the heavy fluxonium qubit as a concrete example, which can be approximated by a tunnable and gapped two-level quantum system, as $\hat{H}_q(\lambda) \approx \Delta\hat{\sigma}_x/2 + \lambda C_\phi\hat{\sigma}_z/2$ [26, 48] (C_ϕ is an unimportant coefficient). This simple model renders the SSE results also qualitatively applicable to other widely used tunable qubits [22, 31, 52, 63, 67]. The fluctuation used in this simulation is generated according to $\delta\xi(t) = \sum_{\mu=1}^{N_T}\delta\xi_{T\mu}(t) + \delta\xi_G(t)$.

In FIG. 2 (a) and (b), we plot the calculated qubit frequencies and dephasing rates as functions of the control parameter λ for $N_T = 0$ (purely Gaussian) and 1 (non-Gaussian), respectively. Both the dephasing rates γ_2 and the qubit frequency ω_q' are extracted by fitting the simulated Ramsey measurement using a simple exponential function over the same time range. (Note that the dephasing profile is not exactly exponential given the structured noise we use, and this simplest protocol is only chosen for consistency in comparing the simulation results.) For both cases, we find impressive agreement between our analytical prediction (dashed lines) and numerical simulation (squares). In (a), the Gaussian fluctuator $\delta\xi_G(t)$ ensures the $\mathbb{Z}_2$ symmetry under the reflection $-\lambda \to \lambda$. In (b), we observe a measurable mismatch between the minimum points of the

dephasing rate and the qubit frequency. For a more intuitive demonstration of such mismatch, we showcase two simulated Ramsey signals in FIG. 2 (c), taken at the minimum points of

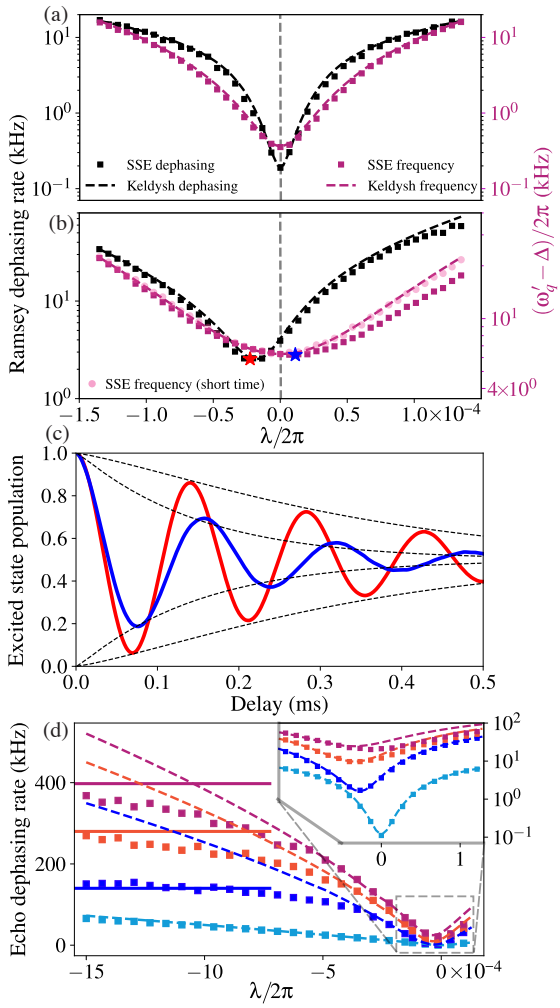


FIG. 2. Noise-mediated evolutions of a tunable qubit around its sweet spot. (a) and (b) show the calculated qubit frequency (purple) and dephasing rates (black) by the Keldysh perturbation theory (dashed lines) and SSE (squares and dots) by simulating the Ramsey measurements. In (c), the red and blue curves are the simulated Ramsey signals at the points marked by red and blue stars (b), respectively. The black dashed curves describe the dephasing profile predicted using Eq. (3). We choose $N_T = 0$ for (a) and $N_T = 1$ for (b) and (c). (d) plots the fitted Echo dephasing rates over a wider range of control parameters, where the dashed curves are from analytical calculation and the squares from SSE. The number of TLFs N_T used in this simulation is varied from 0 to 3 (cyan, blue, red, purple). The solid lines mark the projected saturation rates by the strong TLFs. The qubit used in this simulation is a heavy fluxonium qubit, whose circuit parameters are chosen according to Ref. [26]. The sweet spot we study is located at $\phi_{\rm ext} = \pi$, which gives the choices $\lambda = \phi_{\rm ext} - \pi$ and $\hat{x} \equiv E_L\hat{\varphi}$. The strong TLFs have a magnitude $|\xi_T|/2\pi = 9\times10^{-5}$ and uneven probability distribution $P_{-(+)} = 0.7(0.3)$. The Gaussian noise bath is approximated by 2001 much weaker and independent TLFs, which have a $1/\kappa$ distribution [47, 64] for $\kappa \in [1\,\text{kHz}, 1\,\text{MHz}]$ to mimic the classical $1/f$ noise, and yield a noise magnitude $|\delta\xi_G(t)|/2\pi = 2\times10^{-5}$.

the dephasing rate (red star) and qubit frequency (blue star) in (b). Although the red signal decays slower, it oscillates faster compared to the blue one. This mismatch demands more carefulness in locating the optimal working point experimentally: the extremum point of the qubit frequency is not necessarily the qubit dephasing sweet spot. Our simulation shows that the Ramsey dephasing rates at the two mismatched minimum points differ by a factor of 2. Note that in disagreement with the Keldysh prediction, the SSE results also shows a shift of the minimum of the qubit frequency from $\lambda = 0$ for the non-Gaussian case. We attribute this to the less accuracy of the perturbation theory at long times ($t \sim 0.5$ ms). [Much better agreement is found for the fitting over $0 < t < 20\,\mu$s, as plotted by light pink circles in (b).]

Farther away from the sweet spot, the dephasing rates are also constantly measured for the characterization of the low-frequency noise [23, 26, 30, 55]. In this operating region, the non-Gaussian noise, for example, the strong two-level fluctuators, also dephases the qubit distinctively from its Gaussian counterpart [38, 39, 42, 44, 45]. To reveal the difference in detail, we use the analytical (dashed lines) and numerical methods (squares) to simulate the dephasing rates in this parameter region. We plot the results in FIG. 2 (d). [We switch to the Echo protocol to avoid the complicated beatings.] We find that in the presence of strong TLFs, the dephasing rate contributed by the TLFs saturates toward larger $|\lambda|$ [38, 39, 42, 45]. The saturated dephasing rates are derived as $\bar{\kappa} = \sum_{\vec{\eta}} \bar{\kappa}_{\vec{\eta}} P_{\vec{\eta}}$ [73], where $\vec{\eta}$ is one configuration of the group of strong TLFs, and $\kappa_{\vec{\eta}}$ and $P_{\vec{\eta}}$ are the flipping rate corresponding to that configuration and the probability of finding such configuration, respectively. These predicted saturation rates (solid lines) well capture the behaviors of the dephasing rates at the far left part of the FIG. 2 (d). Such saturation behavior, which is beyond the description of the Gaussian-noise model, caution the use of the Gaussian-noise assumption in noise characterization.

Triple protection by Floquet engineering.– Our calculations point out the crucial role of the second-order derivative $D_{2,\lambda=0}$ in limiting the optimal coherence times, for the presence of both the Gaussian noise and non-Gaussian TLFs. To further enhance the coherence time (while maintaining the qubit controllability), it is ideal to design a qubit with flatter spectrum [35, 36, 50, 55, 66]. However, the experimental implementation of some of these qubits for their full protection encounters challenges from the experimental limits on the circuit parameters [7, 55]. In the following, we present a drive-based protocol to suppress the second-order sensitivity [74], which can circumvent the hardware limitations.

This protocol is based on the Floquet-engineering techniques developed in Refs. [48, 49], while here we require not only the first-order insensitivity $\partial\varepsilon_{01}/\partial\lambda = 0$, but also the second order $\partial^2\varepsilon_{01}/\partial^2\lambda = 0$ (ε_{01} denotes the *bare* quasi-energy difference). In addition to these requirements, it is also crucial to ensure the insensitivity of the qubit to the drive noise, since Refs. [49, 51, 52, 61, 65, 75] point out that the fluctuation of the drive amplitude also causes qubit dephasing. Therefore, the optimal protection scheme should also include $\partial\varepsilon_{01}/\partial A = 0$,

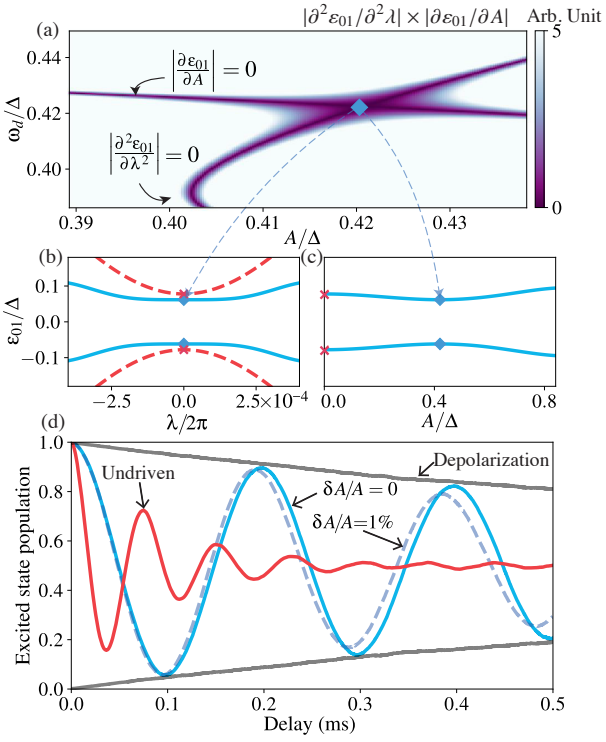


FIG. 3. Floquet engineering of the triple sweet spots. (a) plots the product of the derivatives, i.e., $|\partial\varepsilon_{01}^2/\partial\lambda^2| \times |\partial\varepsilon_{01}/\partial A|$, as a function of the drive amplitude A and frequency ω_d. The two visible dark curves correspond to the vanishing of the two derivatives, respectively, and the crossing hosts the triple sweet spot (cyan diamond). (b) and (c) show the quasi-energy spectra as functions of the dc control parameter λ and the drive amplitude A, respectively. The red dashed lines describe the quasi-energies of the undriven bare fluxonium and the crosses mark the location of static working points ($A = 0$). (d) compares the simulated Ramsey measurements between the Floquet engineered triple sweet spot (cyan) and the static one (red). The simulation shown by the dashed cyan curve includes 1% random fluctuation in A. The gray solid curves are the simulated Floquet excited state population for 1→0 decay (top) and 0→1 excitation (bottom).

where A denotes the drive amplitude. The operating points where the three aforementioned derivatives vanish are referred to as the *triple sweet spots*. Using the two-level approximated qubit model again, we find that these points can be obtained by driving the qubit with $\hat{H}_d(t) = A[\cos\omega_d t + \alpha\cos(2n+1)\omega_d t]\hat{x}$ ($n \in \mathbb{N}^+$). (We choose an odd integer $2n+1$ to ensure the vanishing of $\partial\epsilon_{01}/\partial\lambda$ [73].) This driving protocol is also used in Refs. [51, 52], although an extra second-order protection is targeted. The position of one triple sweet spot is shown in FIG. 3 (a), where we choose $\alpha = 1$ and $n = 1$. In FIG. 3 (b), we plot the quasi-energies of the qubit as a function of the dc control parameter λ, with the ac drive amplitude A fixed at the sweet spot, and the other way around in (c). Both plots show suppressed variation of the qubit frequency around the sweet spot. In (b), the variation of the frequency of the undriven qubit (subtracted by integer multiples of the drive frequency for comparison) is plotted (red-dashed curve) to contrast the second-order suppression. According to Eq. (3), this protection protocol should mitigate dephasing caused by both the Gaussian noise and the non-Gaussian TLFs.

The discussion above only assumes a tunable two-level quantum system. To quantitatively confirm the improved coherence times by SSE, we again take the heavy fluxonium as an example to simulate the Floquet Ramsey evolution [49]. The dephasing time is compared with that in the undriven case in FIG. 3 (d). Using the same noise model $\delta\xi(t)$ ($N_T = 2$ for this simulation), the dephasing time at this triple protection point (solid cyan curve) is improved by 10 times compared to that at the static sweet spot (solid red curve). This is directly related to the suppression of the second-order derivative [FIG. 3 (b)]. The limiting factor for the coherence time at this protection point is the depolarization (solid gray curves) by the noise at the smaller qubit frequency, which can potentially be further mitigated by optimizing the qubit parameters and drive shapes. To emulate the low-frequency drive noise, we further include random fluctuation of the drive amplitude A before each SSE trace. Remarkably, we still find 8 times improvement (cyan dashed curve) for up to 1% of such fluctuation. This experimentally appreciable robustness is owing to the first-order insensitivity of the quasi-energy difference to the drive amplitude [FIG. 3 (c)].

Conclusion.– We study the high-order dephasing effect in a qubit by non-Gaussian fluctuators. Our calculation predicts a symmetry breaking that is unique to the non-Gaussian noise. Concretely, we show that the strong TLFs, a popular non-Gaussian noise model, dramatically change the behavior of the dephasing rates and cause an unexpected mismatch between the minimum points of the dephasing rate and the qubit frequency. These findings challenge the usually assumed equivalence between the two minimum points, call for extra carefulness in locating the optimal working point, and caution the use of the Gaussian model for noise characterization in a variety of widely used qubits. Finally, we propose a triple-protection scheme to suppress both the first-order and the second-order sensitivity of the qubit energy to these fluctuators, where the qubit is also first-order protected from the low-frequency drive noise. We numerically confirm the proposed enhancement with a realistic heavy fluxonium qubit, where we find an order of magnitude improvement in the dephasing time. This approach not only inherits the full compatibility with qubit control and scalability from the previous Floquet-engineering research [48, 70, 71], but also raises the coherence enhancement by an order of magnitude. These advantages make the triple-protection technique a powerful tool toward large-scale and noise-resilient quantum computation.

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