Tunable Planar Josephson Junctions Driven by Time-Dependent Spin-Orbit Coupling

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Tunable planar Josephson junctions driven by time-dependent spin-orbit coupling

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Integrating conventional superconductors with common III-V semiconductors provides a versatile platform to implement tunable Josephson junctions (JJs) and their applications. We propose that with gate-controlled time-dependent spin-orbit coupling, it is possible to strongly modify the current-phase relations and Josephson energy and provide a mechanism to drive the JJ dynamics, even in the absence of any bias current. We show that the transition between stable phases is realized with a simple linear change in the strength of the spin-orbit coupling, while the transition rate can exceed the gate-induced electric field GHz changes by an order of magnitude. The resulting interplay between the constant effective magnetic field and changing spin-orbit coupling has direct implications for superconducting spintronics, controlling Majorana bound states, and emerging qubits. We argue that topological superconductivity, sought for fault-tolerant quantum computing, offers simpler applications in superconducting electronics and spintronics.

In the push to implement beyond-CMOS applications, Josephson junctions (JJs) have found their broad use due to their high-speed switching, low-power dissipation, and intrinsic nonlinearities[1, 2]. In addition to the well-established role of JJs as the key elements for superconducting electronics and superconducting qubits[1–6], there is a growing interest to tailor their spin-dependent properties to enable dissipationless spin currents, cryogenic memory[7–14], and fault-tolerant quantum computing[15–22]. The role of spin-orbit coupling (SOC) has been extensively studied in the normal-state properties and recognized for its importance in spintronics[23–25]. However, the superconducting analogs of the SOC-related effects remain to be understood. They might even be important when their normal-state counterparts are negligibly small[26–32]. Motivated by the recent progress in gate-controlled SOC in planar JJs based on a two-dimensional electron gas (2DEG)[33–35], we reveal how time-dependent SOC tunes many of their key properties and offers an unexplored mechanism to drive JJs.

A common description of a JJ circuit, is given by a Josephson element, resistor, and capacitor connected in parallel, using the resistively and capacitively shunted junction model (RSCJ)[1]. The bias current through the junction, i, is the sum of the supercurrent and the quasiparticle current flowing in the resistor and capacitor. The supercurrent is usually assumed as $I(\varphi) = I_c \sin(\varphi + \varphi_0)$, where I_c is the maximum supercurrent, φ the phase difference between the superconducting regions, and the anomalous phase, $\varphi_0 \neq 0, \pi$, arises from the broken time-reversal and inversion symmetries[36–40].

For a JJ depicted in Fig. 1(a), the interplay between SOC and the effective Zeeman field $\boldsymbol{h}$, yields a more complex current-phase relation (CPR), than $I(\varphi)$ given above, such that for a generalized RSCJ model

$$d^2\varphi/d\tau^2 + (d\varphi/d\tau)/\sqrt{\beta_c} + I(\varphi, \mu, \boldsymbol{h}, \alpha)/I_c = i/I_c, \quad (1)$$

where $\tau = \omega_p t$ is a dimensionless time, expressed using the JJ plasma frequency, $\omega_p = \sqrt{2\pi I_c/\Phi_0 C}$, $\Phi_0 = h/2e$ is the magnetic flux quantum, and C the capacitance. The damping of this nonlinear oscillator is characterized by the Stewart-McCumber parameter, $\beta_c = 2\pi I_c C R^2/\Phi_0$, where R is the resistance[41, 42] and $Q = \sqrt{\beta_c}$ is the quality factor. The generalized CPR can be modified by the chemical potential μ, and $\boldsymbol{h}$, arising from the applied magnetic field or magnetic proximity effect[43]. Since h_z does not induce φ_0[44, 45] and only produces CPR reversals, we focus on $h_z = 0$ [Fig. 1(a)]. The CPR can also be tuned by the Rashba SOC, illustrated in Fig. 1(a), which is parametrized by its strength α, in the Hamiltonian, $H_{\rm so} = \alpha(\boldsymbol{\sigma} \times \boldsymbol{p}) \cdot \hat{\boldsymbol{z}}$. Here $\boldsymbol{\sigma}$ is the Pauli matrix vector, $\boldsymbol{p}$ the in-plane momentum, for 2DEG with the inversion symmetry broken along the z-direction[46].

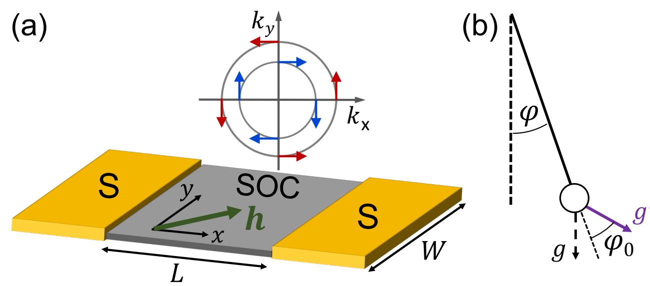

FIG. 1. (a) Schematic of the Josephson Junction (JJ). Two s-wave superconductors (S), are separated by the middle region which hosts the Rashba spin-orbit coupling (SOC), with depicted k-space spin-orbit fields, and an effective Zeeman field, $\boldsymbol{h}$. (b) A mechanical pendulum model of the JJ. The displacement angle φ is analogous to the superconducting phase difference, $\boldsymbol{g}$ is the gravitational acceleration for vanishing SOC and $\boldsymbol{h}$. The pendulum is driven by changing effective $\boldsymbol{g'}$, an interplay between $\boldsymbol{h}$ and time-dependent SOC. This yields a tunable current-phase relation and an anomalous phase, φ_0, equivalent to the displaced pendulum's equilibrium.

While quasistatic, gate-tunable, changes in SOC and φ_0 have been demonstrated in 2DEG-based JJs[33, 34], the implications of dynamically-tuned SOC on the CPR remain unexplored. For a simple CPR $\propto \sin\varphi$, Eq. (1) has a mechanical analog with a driven and damped pendulum, in which φ becomes the displacement angle[41, 42]. A JJ driven by i is equivalent to the pendulum displaced by an external torque from its stable equilibrium, determined by the gravitational acceleration $\boldsymbol{g}$, while ω_p determines the oscillation frequency around a stable equilibrium point[1].

Instead of using i, Fig. 1(b) suggests an entirely different way to drive the pendulum: By changing the orientation of the effective $\boldsymbol{g}'$ and the new equilibrium, resulting from the interplay of the static $\boldsymbol{h}$ and time-dependent α. With JJ advances and gate changes exceeding the GHz range[3], there is a tantalizing prospect for dynamically-controlled CPR by time-dependent SOC. Unlike assuming a specific relation, $I(\varphi) = I_c \sin(\varphi+\varphi_0)$, the CPR can have a more general and anharmonic form which should be obtained microscopically. To this end, a single-particle Hamiltonian, $H(\boldsymbol{p}) = \boldsymbol{p}^2/2m^* + \boldsymbol{\sigma}\cdot\boldsymbol{h} + H_{\mathrm{so}}(\boldsymbol{p})$, where m^* is the effective mass, can be used to solve a BCS model of superconductivity, given by the effective Hamiltonian

$$\mathcal{H}(\boldsymbol{p}) = \begin{pmatrix} H(\boldsymbol{p}) - \mu\hat{1} & \hat{\Delta} \\ \hat{\Delta}^\dagger & -H^\dagger(-\boldsymbol{p}) + \mu\hat{1} \end{pmatrix}, \tag{2}$$

where $\hat{\Delta}$ is a 2×2 superconducting gap in spin space[44].

After diagonalizing the resulting Bogoliubov-de Gennes equations, $\mathcal{H}\hat{\psi} = E\hat{\psi}$, where $\hat{\psi}$ is the four-component wavefunction for quasiparticle states with energy E, we match the wavefunctions and generalized velocities at interfaces ($x = 0, d$), shown in Fig. 1(a). This allows us to obtain the ground-state JJ energy E_{GS}, together with the corresponding CPR using the charge conservation and the quantum definition of current[44]. The CPR is related to the JJ energy, $I(\varphi) \propto \partial E_{\mathrm{GS}}/\partial\varphi$[47].

Our numerical findings are illustrated for the JJ depicted in Fig. 1(a). The normal region (N) has a length $L = 0.3\xi_S$ and a width $W = 10L$, such that lengths are normalized by $\xi_{\mathrm{S}} = \hbar/\sqrt{2m^*\Delta}$, where Δ is the superconducting gap in S. The energies are normalized by Δ and the supercurrent $I_0 = 2|e\Delta|/\hbar$, where e is the electron charge, and $|e\Delta|/\hbar$ is the maximum supercurrent in a single-channel short S/N/S JJ[47].

To explore the tunability of CPRs and JJ energies with SOC, we focus on the parameters for high-quality epitaxial InAs/Al-based JJs, $\Delta_{\mathrm{Al}} = 0.2\,\mathrm{meV}$, g-factor 10 for InAs, while its m^* is 0.03 the electron mass[33, 34]. In these JJs the gate-control of Rashba SOC and thus its magnitude in the range $\alpha \in (0, 180\,\mathrm{meV\AA})$ has been demonstrated[33, 34]. In Fig. 2, at $h_x = (2/3)\Delta \approx 450\,\mathrm{mT}$, we assume gate-control that primarily changes α, not μ. Experimentally, this could be realized with dual-gate schemes[48] to independently tune the carrier density and the electric field, $\boldsymbol{E}$. However, for a continuous change of α, we are unaware that even in a static case the calculated CPR and E_{GS} were given.

In Fig. 2(a), for $\mu = \Delta$, the anharmonic CPR changes significantly with α. There is a competition between $\sin\varphi$, and the next harmonic, $\sin 2\varphi$, resulting in

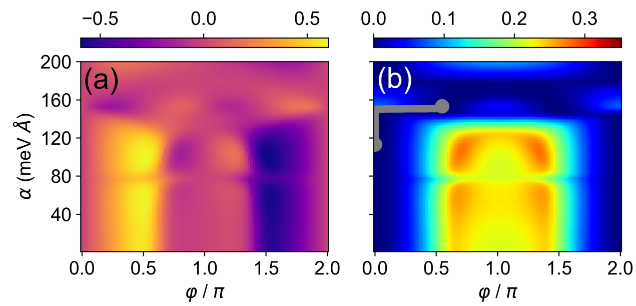

FIG. 2. (a) The evolution of (a) JJ CPR, normalized by $2|e\Delta|/\hbar$, and (b) the JJ energy, normalized by Δ, as a function of the phase φ and the Rashba SOC, α, for chemical potential $\mu = \Delta$ and effective in-plane magnetic field, $h_x = (2/3)\Delta$. The gray curve in (b) denotes the JJ transition from $\varphi = 0$ to $\approx \pi/2$ for a linear increase in α from 112 to 152 meVÅ.

$I(-\varphi) = -I(\varphi)$. However, there is no spontaneous current, $I(\varphi = 0) \equiv 0$, only I_c reversal with α. Such a continuous and symmetric 0-π transition is well studied without SOC in S/ferromagnet/S JJs due to the changes in the effective magnetization or a thickness of the magnetic region[49–57]. The corresponding JJ energy landscape in Fig. 2(b), shifted such that its overall minimum value is 0, corroborates this SOC evolution. By increasing α from 0 to 200 meVÅ, the minimum in E_{GS} changes from $\varphi = 0$ to π, and then goes back to 0. A gray trace indicates that by increasing α in a smaller range, the JJ minimum can transition from $\varphi = 0$ to $\approx \pi/2$.

While we use an exact (complete) CPR with its anharmonicities, their prior descriptions have often relied on an approximate simple harmonic expansion ($\sin n\varphi$, $\cos n\varphi$)[58, 59]. However, this approach is not very efficient with SOC. Instead, it is better to use a compact form where only a small number of terms gives a more accurate description[44]

$$I(\varphi, \mu, \boldsymbol{h}, \alpha) \approx \sum_{n=1}^{N} \sum_{\sigma=\pm} \frac{I_n^\sigma \sin(n\varphi + \varphi_{0n}^\sigma)}{\sqrt{1 - \tau_n^\sigma \sin^2(n\varphi/2 + \varphi_{0n}^\sigma/2)}}, \tag{3}$$

where τ_n^σ is the JJ transparency for spin channel σ and the phase shifts φ_{0n} are additional fitting parameters. This description includes the anomalous Josephson effect $I(\varphi = 0) \neq 0$, revisited in JJ diode effects[60–64]. For a simple picture of a single anomalous phase[44, 45]

$$\varphi_0 \propto h_y \alpha^3, \tag{4}$$

therefore vanishing in Fig. 2, where $\boldsymbol{h} = h_x\hat{x}$.

A quasistatic gate-controlled SOC suggests that more important opportunities are available using fast gate changes, compatible with the advances in JJ circuits[3]. However, the implications of GHz changes in SOC and a different mechanism to drive JJ, as sketched in Fig. 1(b), remain unexplored. To obtain the resulting JJ dynamics

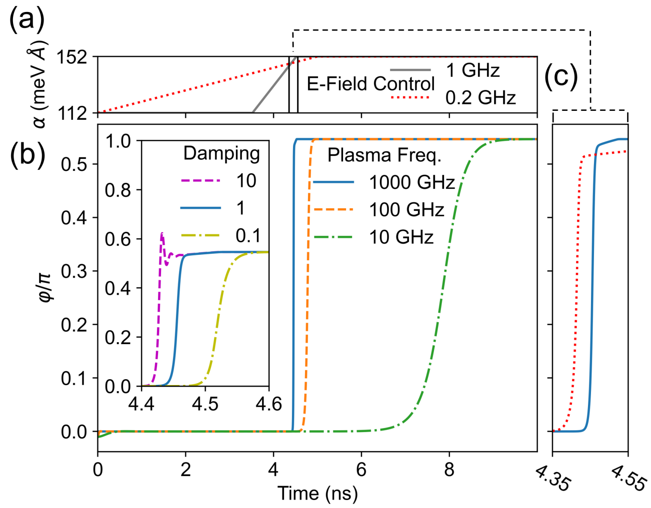

FIG. 3. (a) Time-dependent Rashba SOC, α, controlled by the $\boldsymbol{E}$-field, changing at 0.2 GHz and 1 GHz, used also in (b). (b) Time-dependent phase for different plasma frequencies, ω_p, at damping, $\beta_c = 1$, and for different β_c at $\omega_p = 1000$ GHz (inset). (c) A magnified region for $\varphi = 0$ to $\approx \pi/2$ transition at 0.2 GHz (1 GHz) dotted (solid) changes in α from (a).

we use Eq. (1) with $i \equiv 0$, where the driving arises from $\alpha = \alpha(t)$, viewed as a time-dependent effective $\boldsymbol{g'}$.

Some guidance what to expect for JJ dynamics can be given from the InAs/Al samples, where, in addition to the previous range of α, $I_c \sim 4\,\mu$A, $R \sim 100\,\Omega$, and $C \sim 15$ fF, leading to $\omega_p \sim 900$ GHz and the damping $\beta_c \sim 1$, which is also suitable for the rapid single flux quantum (RSFQ) applications[1, 4]. We keep $h_x = (2/3)\Delta$.

The JJ dynamics is driven by a simple linear variation of $\alpha(t)$ from the gate-controlled $\boldsymbol{E}$, as shown in Fig. 3(a). We first consider in Fig. 3(b) the reduction of ω_p, from 1000 GHz (similar to InAs/Al JJs[33]) to 10 GHz (much faster than the $\alpha(t)$-variation), at $\beta_c = 1$. The results reveal a strong delay in the onset in the $\varphi = 0$ to $\approx \pi/2$ transition, which was indicated from the static picture in Fig. 2(b). Simultaneously, the time for the $\varphi = 0$ to $\approx \pi/2$ transition is increased by an order of magnitude.

We next examine in the Fig. 3(b) inset the influence of reducing β_c from the underdamped and critical ($\beta_c = 10$ and 1) to the overdamped ($\beta_c = 0.1$) regime, at $\omega_p = 1000$ GHz. In addition to the phase oscillation damping, consistent with the pendulum model in Fig. 1(b), we also see a delay in the $\varphi = 0$ to $\approx \pi/2$ transition and its growing, the trends noted from reducing ω_p.

Finally, in Fig. 3(c), the $\varphi = 0$ to $\approx \pi/2$ transition occurs first for the slower $\alpha(t)$-variation, but takes approximately the same time as the faster GHz $\alpha(t)$-variation. This is encouraging for various applications, since (i) $\boldsymbol{E}$-control of SOC allows tailoring the onset of the transition between different states, (ii) a high-frequency switching between different equilibrium states and driving JJs is not limited by the characteristic times for the $\boldsymbol{E}$-variation. $\alpha(t)$-changes at 0.2 GHz give an order-of-magnitude faster transitions between stable phases.

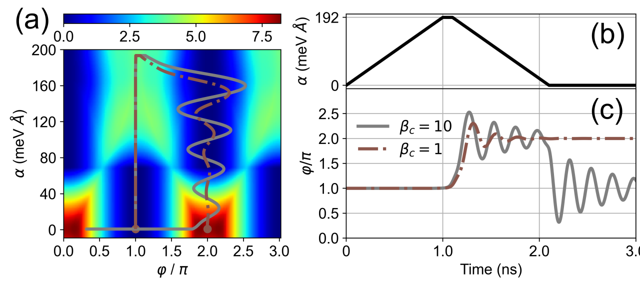

FIG. 4. (a) The JJ energy evolution with φ and α at $\mu = 10\Delta$, $h_x = (2/3)\Delta$. The gray (brown) curve shows the energy variation for $\beta_c = 10$ ($\beta_c = 1$) starting at $\varphi = \pi$ and $\alpha = 0$, for changing α, as given in (b). (c) The corresponding time-dependent φ confirms the decay to different final phase states.

While the $\boldsymbol{E}$-control of α and the evolution of $E_{\rm GS}$ minima in Fig. 2 have largely determined the JJ dynamics in Fig. 3, it helps to identify other opportunities for SOC-driven JJs. In Fig. 4 we consider $\omega_p = 10$ GHz and a triangular-like $\alpha(t)$ at $\mu = 10\Delta$. For an underdamped regime, $\beta_c = 10$, the pendulum analogy from Fig. 1(b) explains the phase evolution of the gray trajectory from Fig. 4(a), reproduced also in Fig. 4(c). By increasing α to the maximum at 192 meVÅ, the pendulum is at an unstable position and will swing towards the $\varphi = 0$ minimum (equivalently shown as $\varphi = 2\pi$), implying that $\boldsymbol{g'}$ points vertically down. With small damping (gray trajectory), the pendulum passes the equilibrium point, even when, with $\alpha < 80$ meVÅ, the equilibrium and the overall minimum shift to $\varphi = \pi$, with $\boldsymbol{g'}$ vertically up. Eventually, with damping it reaches the $\varphi = \pi$ minimum.

For critical damping, with the same starting point [see also Fig. 4(c)], the brown trajectory reveals a very different evolution with α. Instead at the overall $E_{\rm GS}$ minimum $\varphi = \pi$, for $\alpha = 0$, the phase is locked at the local minimum $\varphi = 0$. With a stronger damping, the φ oscillations are insufficient to overcome the SOC-dependent barrier which, for $\alpha = 0$, separates the local minimum at $\varphi = \pi$ from the global one at $\varphi = 2\pi$. The tunability of the SOC-controlled energy landscape alone does not fully determine generalized CPRs. The influence of the JJ circuit parameters can enable different φ-transitions.

In our previous discussion, the tunability of CPRs and $E_{\rm GS}$ did not exploit the anomalous Josephson effect[36–39, 65], which can be understood in analogy with $\boldsymbol{g'}$ pointing sideways and therefore, breaking the symmetry from Figs. 2-4 and $I(-\varphi) \neq -I(\varphi)$. This situation can be simply realized by rotating $\boldsymbol{h}$ along the y-axis, while we retain all the other parameters from Fig. 2(a). The resulting CPR in Fig. 5(a) confirms that the JJ supercurrent is driven not only by φ, but also by φ_0, which is responsible for the stated symmetry breaking and, equivalently, the tilted $\boldsymbol{g'}$. As for SOC cubic in $\boldsymbol{k}$[44], there is a strong anharmonic behavior and the expected diode effect where the sign and magnitude of the supercurrent

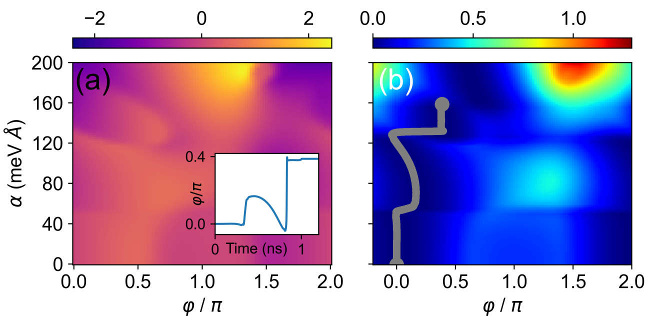

FIG. 5. The evolution of (a) JJ CPR and (b) JJ energy with φ and α, for $\mu = \Delta$ and $h_y = (2/3)\Delta$, rotated by $\pi/2$ from Fig. 2. An anharmonic CPR breaks the $I(-\varphi) = -I(\varphi)$ symmetry in (a) and the corresponding anomalous phase, φ_0, increases with α in (b). Inset (a): $\varphi(t)$ for $\omega_p = 1000\,\mathrm{GHz}$, $\beta_c = 1$ with a linearly increasing α from 0 to 160 meVÅ over 1 ns, then held at maximum,with its JJ energy path in (b).

depends on the polarity of the applied bias [34].

The implications of the combined broken time-reversal and inversion symmetries, responsible for the anomalous Josephson effect, are further illustrated in Fig. 5(b). It shows the SOC-tunable E_{GS}, single-valued for the gray path, and leading to the time-dependent diode effect. This behavior is qualitatively different from the doubly-degenerate φ_0 state in Fig. 2(b), which results from the second harmonic generation in the CPR.

Even with a moderate $h_y \approx 450\,\mathrm{mT}$ for InAs-based JJs, with increasing $\alpha(t)$ we see an evolution of the single global minimum and thus the changes in the corresponding values of φ_0 from $\varphi = 0$ to $\approx 3\pi/4$, in good agreement with the measured values[33]. This suggests that at a larger $\boldsymbol{h}$, for example, in In(As,Sb) with a much larger g-factor[35], it may be possible to fully control the tilt angle of $\boldsymbol{g'}$ and simply swap between 0- and π-states in JJs, further controlling how the JJ dynamics is driven.

The same geometry in Al/InAs JJs at a larger h_y has been experimentally shown to also support topological superconductivity[34]. This is important for several reasons, beyond hosting Majorana bound states[16]. Resulting topological superconductivity is associated with equal-spin p-wave superconductivity which could offer gate-controlled dissipationless spin currents, a key element for superconducting spintronics[7, 8]. Such spin-triplet supercurrents could be extended over a long range[66] and overcome the usual competition between superconductivity and ferromagnetism. A transition to topological superconductivity is accompanied by an extra phase jump, $\approx \pi$[67, 68]. Such a π-jump in Al/InAs JJs was observed at $h_y \approx 600\,\mathrm{mT}$[34], an effective field about 25 times smaller, than expected for the $0 - \pi$ transition

$$B_{0-\pi} = (\pi/2)\hbar v_{\mathrm{F}}/(g\mu_{\mathrm{B}}L), \tag{5}$$

for a spin-polarized system in the absence of SOC[69], where v_{F} is the Fermi velocity, μ_{B} the Bohr magneton, and L the JJ length. Therefore, SOC plays a crucial role in understanding various transitions and that, at larger h_y, the range of an effective φ_0 could exceed 2π[34] and support 2π pendulum rotation from Fig. 1(b), used in RSFQ logic and memory[1, 4]. Therefore, in addition to the prospect of fault-tolerant quantum computing, the search for topological superconductivity also offers a promising platform for superconducting electronics and spintronics.

Without previous studies on SOC-driven JJ dynamics we have focused on a simple model and not considered time-dependent magnetic fields[70] or noise[71]. A more general description could simultaneously include the role of changing μ and other SOC forms, linear and cubic in $\boldsymbol{k}$, shown to give different routes to topological superconductivity and control of Majorana states[44, 72–74]. However, we expect that our focus only on linearized Rashba SOC, easily tunable by $\boldsymbol{E}$-field[33, 34], already clarifies its important role in JJ dynamics. With changing SOC, there are further opportunities for gate-controlled Majorana states and probing their non-Abelian statistics[75, 76] or an added tunability in the implementation of superconducting qubits[3, 77, 78]. This would extend the previously studied qubit tunability by voltage or flux[3, 79] as well as the use of π-phase states for an improved qubit operation[80, 81].

ACKNOWLEDGMENTS

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