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# ***General Impedance Matching via Doped Epsilon-Near-Zero Media***

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## **Abstract**

The emerging technique of photonic doping endows epsilon-near-zero (ENZ) media with a broadly tunable effective magnetic permeability. In this work, we theoretically and experimentally demonstrate that a finite-size doped ENZ region counterintuitively behaves as a lumped circuit element, modelled as a controllable series reactance. Based on this concept, a general matching network is constructed to match a load with arbitrary complex impedance, while interestingly its operating bandwidth can also be modified by fine-tuning the dopants' properties. To demonstrate the universality of the concept, different kinds of loads are matched, including microwave circuits, antennas, and absorbing particles. Since this technique is not limited to a specific type of load, nor a specific geometry, and can be readily transplanted from microwave to optical regimes, the proposed methodology facilitates impedance matching for maximum usage of power in quite general scenarios, **and thus exhibits** promising potential for broad applications.

## I. Introduction

The past decades have witnessed the exciting development of the metamaterial paradigm [1-4], synthetic structures which exhibit unconventional properties and advanced functionalities, such as cloaking [5,6], superresolution imaging [7,8], anomalous transmission and reflection [9], analog computing [10], and digital coding [11], to name only a few. Recently, media with the refractive index close to zero, near-zero-index (NZI) media [12,13], have drawn growing interest due to their exotic wave-matter interaction phenomenon. As the permittivity and/or permeability approach zero, the wavelength, as well as the phase velocity of light within the medium, tend to infinity, leading to spatially static while temporally dynamic field configurations. **These materials with small constitutive relative parameters can give access to a number of intriguing phenomena and applications [13,14], such as supercoupling of wave through arbitrarily shaped channels [15-20], manipulating wave-matter interaction [21, 22], and implementing on-chip wave-guiding devices [23]. In addition, NZI media were also introduced to the design of geometry-invariant resonators [24], controlling the emission of quantum emitters [25-27], enhancement of nonlinear effects [28,29], and trapping of light in open resonators [30,31].**

The emerging technique of photonic doping [32], where macroscopic dielectric impurities are introduced to an epsilon-near-zero (ENZ) host, provides a new gateway for realizing ENZ media with a designable effective permeability. In fact, to an external observer, an ENZ medium “doped” with one or very few arbitrarily located dielectric particles excites exactly the same external fields (both near and far) as a homogeneous ENZ medium with the designed effective permeability. This includes extreme cases, such as perfect magnetic conductors (PMC), with an infinitely large effective permeability, and epsilon-and-mu-near-zero (EMNZ) media, with near-

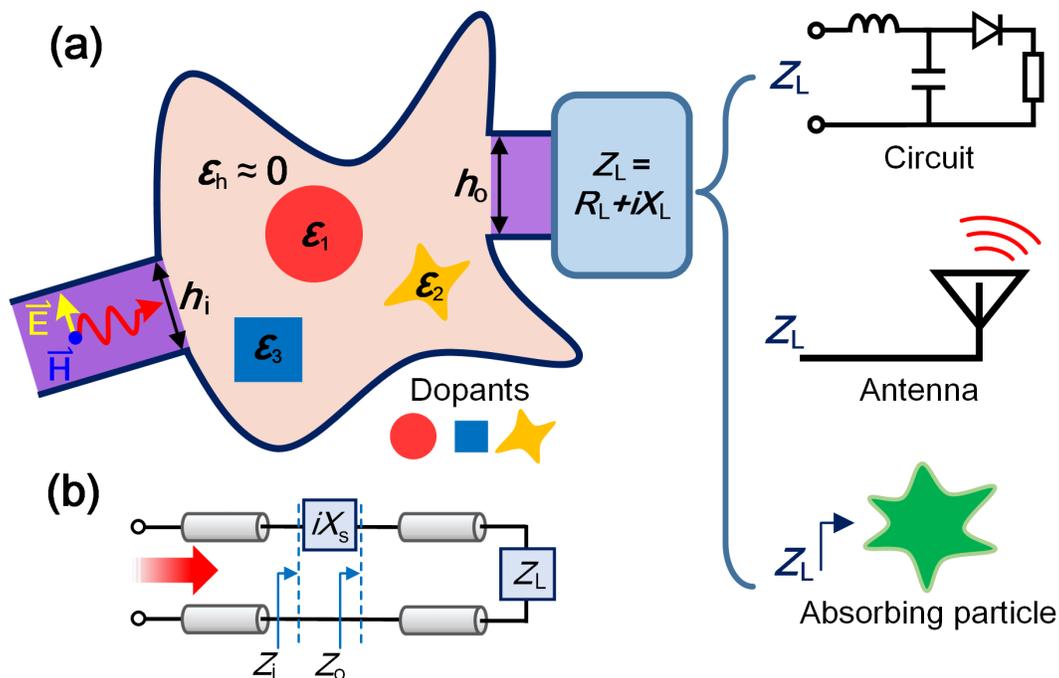
zero effective permeability. Therefore, this intriguing property may lead to some exceptional functionalities unattainable from other material systems.

In this work, we utilize doped ENZ building blocks to enable optimal transmission of power, thus proposing the concept coined “general impedance matching”. It is theoretically and experimentally demonstrated that, a doped ENZ medium, with an arbitrary cross-sectional shape, can be modeled exactly equivalent to a series lumped reactive element, despite being a wavelength size structure containing one or more resonant particles. The specific value of its reactance is directly associated with its effective permeability and can range from negative infinity to positive infinity via tuning of one or more arbitrarily located dopants. Subsequently, general impedance matching networks are established based on the doped ENZ building blocks. As an important subject in engineering, impedance matching [33] — where a load impedance is transformed to the characteristic impedance of the feeding transmission line (or to the complex conjugate of the source impedance) — serves as the purposes of minimizing the reflected power from the load and maximizing the transfer of power between the source and the load. Available matching schemes are usually defined for a specific frequency region and customized designs are necessary for different kinds of loads. For example, the networks constructed by lumped inductors [33] and capacitors can offer appreciable degrees of freedom to match electrical devices. However, this method is restricted to low frequency ranges, since traditional lumped elements need to be much smaller than their operating wavelength. Another popular technique is the quarter-wavelength film [34], which is a standard approach in optics to reduce the reflection of light. However, its applicability is restricted to layered structures with different refractive indices. These structural restrictions do not apply in ENZ media, which can be realized anywhere from the microwave regime to the optical regime via naturally occurring plasma materials [35]

and artificially synthetic near-zero-index materials [13, 15, 19, 23]. The “infinite” wavelength makes an electrically large area electromagnetically shrink to a “point”, where all bodies within it can be described by an effective impedance. Therefore, this technique can be applied to match systems with a complex geometry (which may include bends) which are not necessarily electrically small (when compared to the free-space wavelength).

To demonstrate this concept and its design scheme, exemplary two-component and three-component matching networks based on doped ENZ regions are constructed and analyzed, where we use a microwave component with complex input impedance as the load. As a property of the proposed matching network, a tunable bandwidth is readily obtained by engineering the dispersion of the effective permeability of the doped ENZ material. Three proof-of-concept implementations are presented, including (i) the matching of a stepped waveguide, (ii) feeding an aperture antenna, and (iii) efficiently transferring power to an absorbing particle. The first two are experimentally verified by the use of the platform of substrate-integrated photonic doping [36], and the latter associated with the absorbing particle is numerically demonstrated. The obtained results validate the theory of general impedance matching. Since this technique can be adapted to different frequency regimes and applications, the proposed general impedance matching scheme sets a new direction in the paradigm of optical circuit/metatronics [37], and it might have important implications in different areas, such as electronics, material science, and energy management.

## II. Concept and theory



**Fig. 1. Conceptual sketch of general impedance matching using doped ENZ regions.** (a) A two-dimensional (2D) ENZ medium comprising dielectric dopants is introduced within a transmission line to match a load of arbitrary complex impedance  $Z_L$ , which can take on various forms in practice, such as circuits/waveguides, antennas and absorbing particles. The heights of the input and output ports are denoted as  $h_i$  and  $h_o$ , respectively. The incident wave is polarized with the magnetic field along the out-of-plane axis. (b) The equivalent transmission line model, where the doped ENZ channel is modelled as a series reactance  $iX_s$ .

As conceptually depicted in Fig. 1(a), we consider a two-dimensional (2D) ENZ channel surrounded by perfect electric conducting (PEC) walls, with arbitrary cross-sectional geometry and area  $A$ , containing one or more arbitrarily located dielectric dopants with cross-sectional area  $A_d$  ( $d = 1, 2, 3, \dots$ ). This doped ENZ channel is inserted into a transmission line terminated by a load with an arbitrary complex impedance  $Z_L$ , and excited by an incident wave with the magnetic

field polarized along the out-of-plane axis. To obtain the impact of the doped ENZ channel on this configuration, we apply Faraday's law along the boundary of the ENZ channel:

$$\int_{\partial A} \mathbf{E} \cdot d\mathbf{l} = E_o h_o - E_i h_i = i\omega\mu_0 H [(A - \sum_d A_d) + \sum_d \iint_{A_d} \psi^d(\mathbf{r}) ds] \quad (1)$$

, where  $E_o$  and  $E_i$  denote the electric field at the output and input ports with heights  $h_o$  and  $h_i$ , respectively,  $\omega$  is the operating angular frequency,  $H$  is the uniform magnetic field in the ENZ host, and  $\psi^d$  is the magnetic field distribution within a dopant normalized to unity on its boundary. The  $e^{-i\omega t}$  time convention has been assumed, and we have used the fact that the tangential component of the electric field vanishes along the PEC boundary of the channel and the magnetic field is spatially invariant throughout the ENZ host [15-17]. The impedance seen at the input and output ports of the doped ENZ channel toward the load are denoted as  $Z_i$  and  $Z_o$ , being equal to  $E_i/H$  and  $E_o/H$ , respectively. Therefore, with the help of Eq. (1) the relationship between  $Z_i$  and  $Z_o$  yields:

$$Z_i = Z_o h_o / h_i - i\omega\mu_0\mu_{\text{eff}} A / h_i \quad (2)$$

, where  $\mu_{\text{eff}}$  is the effective permeability of the doped ENZ channel, defined as:

$$\mu_{\text{eff}} = (A - \sum_d A_d + \sum_d \iint_{A_d} \psi^d(\mathbf{r}) ds) / A \quad (3)$$

As demonstrated in Eq. (2), if the heights of the ports  $h_i$  and  $h_o$  are set equal, the doped ENZ channel contributes a purely imaginary additive impedance, proportional to the effective permeability of the doped ENZ region. Equation (2) also implies the interesting physical effect that although the doped ENZ can be electrically large, it simply behaves as a series lumped impedance component, with the following reactance value:

$$X_s = -\omega\mu_0\mu_{\text{eff}} A / h_i \quad (4)$$

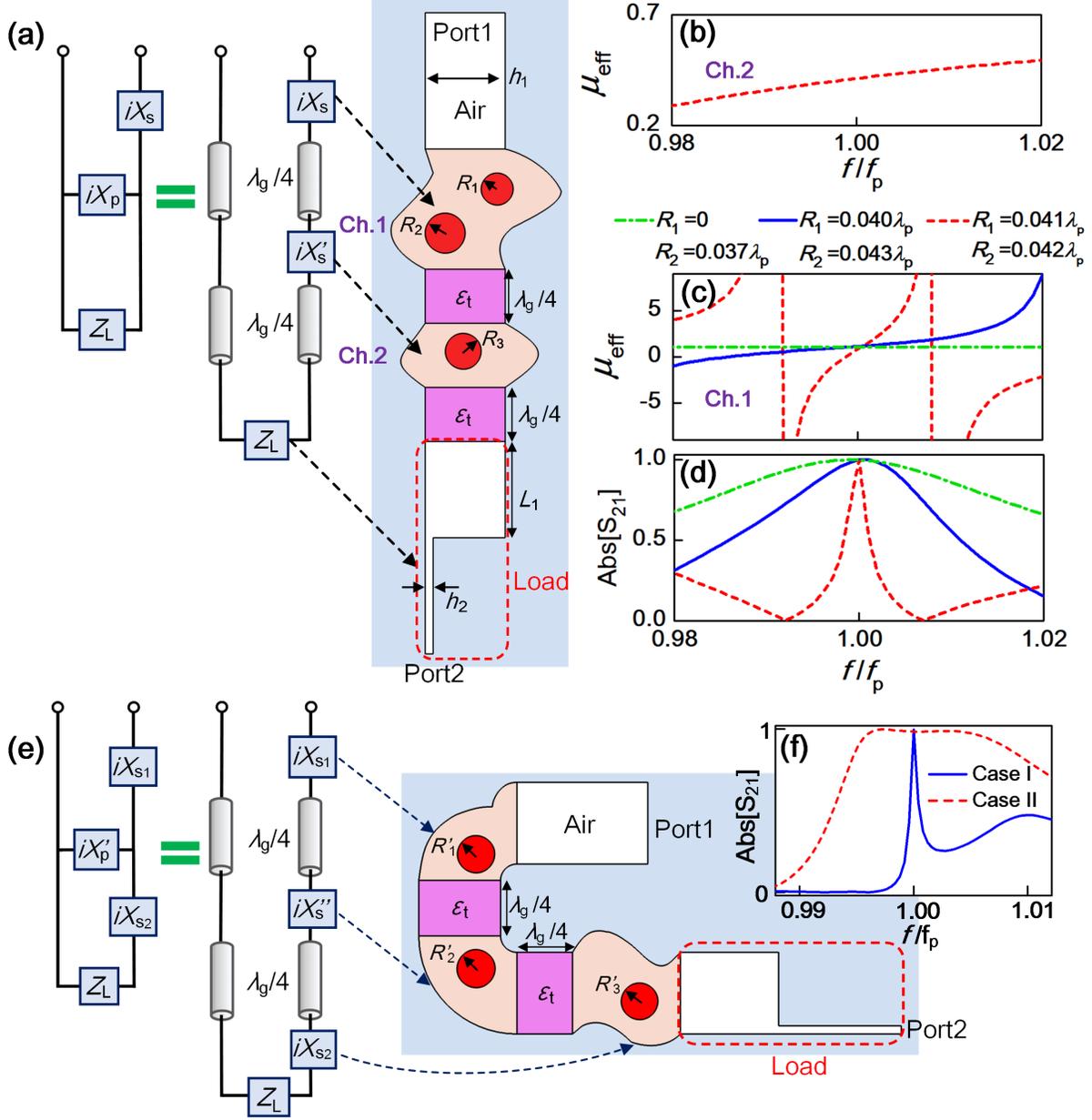
The equivalent circuit diagram of Fig. 1(a) is displayed in Fig. 1(b). An ENZ region containing no dielectric particles is characterized by  $\mu_{\text{eff}} = 1$ , and behaves as a series inductance. In contrast, the value of the lumped reactance emulated by the doped ENZ channel can be adjusted to exhibit nontrivial values and dispersion via engineering the effective permeability characteristics by tuning the dopants' properties.

**Starting with the doped ENZ material as cornerstone**, we demonstrate the design scheme of general impedance matching, we consider an air-filled parallel-plate waveguide with intrinsic impedance of  $\eta_0 = 377\Omega$  as the transmission line, while a stepped waveguide with an abrupt change in height from  $h_1$  to  $h_2$  serves as the load with complex input impedance. The port with the height of  $h_2$  is assumed to be terminated by a well-matched wave port. The detailed geometry and impedance characteristics of the stepped waveguide are presented in the Supplementary Section S1 of the Supplementary Material [38], which shows that the input impedance  $Z_L$  of the stepped waveguide is  $(2.1-4.0i)\eta_0$  at  $f_p = 5.5$  GHz. In order to transform the impedance  $Z_L$  into the intrinsic impedance  $\eta_0$  of the transmission line at the design frequency  $f_p$ , we first consider a two-ENZ-channel matching network, which is presented in the left column of Fig. 2(a). To match the load impedance to the transmission line, the required shunt reactance  $iX_p$  and series reactance  $iX_s$  in Fig. 2(a) are calculated to be  $1.97\eta_0i$  and  $-0.27\eta_0i$ , respectively. The shunt reactance then is replaced by the equivalent series reactance ( $iX'_s = \eta_0^2/(iX_p) = -0.51\eta_0i$ ) cascaded with quarter wave transformers (detailed information about the transformation of series impedance to a shunt one is provided in Supplementary Section S2 in the Supplementary Material [38]), as shown in the middle column of Fig. 2(a). In doing so, the circuit topology may be readily realized by the general impedance matching scheme, with the series reactance being implemented by a doped ENZ channel, as shown in the right column of Fig. 2(a). Here, the ENZ

host for both channels 1 and 2 (Ch.1 and Ch. 2) is assumed to be a plasma with a dispersive permittivity described by the Drude model,  $\epsilon_h(f) = 1 - (f_p/f)^2$ , and for simplicity, a circular dielectric dopant is adopted. The normalized magnetic field distribution,  $\psi^d(r)$ , within the circular dopant is given by  $J_0(k_d r)/J_0(k_d R_d)$  [32], where  $J_n(-)$  denotes the  $n^{\text{th}}$  order Bessel function of first kind,  $k_d = \sqrt{\epsilon_d} \omega / c$  is the wave number in the dopant ( $c$  is the speed of light in vacuum,  $\epsilon_d = 88$  is the relative permittivity of the dopant), and  $R_d$  represents the radius of the circular dopant. Inserting the expression of the magnetic field distribution at hand into Eq. (3), we can obtain the effective permeability of the ENZ host comprising circular dopants<sup>39</sup>:

$$\mu_{\text{eff}} = 1 + (1/A) \sum_d \{ (2\pi R_d / k_d) [J_1(k_d R_d) / J_0(k_d R_d)] - \pi R_d^2 \} \quad (5)$$

Accordingly, the reactance of each doped ENZ channel can be obtained by substituting Eq. (5) into Eq. (4). After obtaining the required reactance from standard circuit analysis, the radii of the dopants can then be found by solving the resulting equation. For the case where each ENZ channel in the two-ENZ-channel network (right column of Fig. 2(b)) contains a single dopant (i.e., dopants with radii  $R_2$  and  $R_3$  are arranged in channels 1 and 2, respectively, while  $R_1$  is assumed zero), the calculated effective permeability of doped ENZ channel 2 for  $R_3 = 0.045\lambda_p$  is shown in Fig. 2(b), and the calculated effective permeability of ENZ channel 1 and the simulated transmission coefficient are plotted with green dash-dot lines in Figs. 2(c) and (d), respectively. The numerical simulation is carried on by RF module of finite-element-method commercial software COMSOL Multiphysics V5.0 [39]. Evidently, total transmission of power is achieved at the predesigned frequency  $f_p$ , indicating that the stepped waveguide is perfectly impedance matched.



**Fig. 2. Bandwidth tailorability with general impedance matching.** (a) The equivalent circuit and the geometry of a two-ENZ-channel matching network based on the photonic doping structures. The cross-sectional areas of the regions filled with ENZ hosts (denoted by Ch.1 and Ch.2) are  $0.073\lambda_p^2$  and  $0.039\lambda_p^2$ , where  $\lambda_p$  is the free-space wavelength at  $f_p = 5.5$  GHz. The relative permittivity of all dopants is chosen as 88, while the quarter-wavelength transmission lines are filled by a material with relative permittivity  $\epsilon_t = 1$ , e.g., air. The heights of port 1 and port 2 (denoted by  $h_1$  and  $h_2$ ), and the parameter  $L_1$  take values of 10 mm, 1 mm, and 12 mm, respectively. Here, the guided wavelength  $\lambda_g$  in the transmission line is equal to  $\lambda_p$ . (b) Effective relative permeability of doped ENZ channel 2. (c) Effective relative permeability of channel 1 for different values of the radii of dopants. (d) Simulation results of the transmission coefficients of the two-ENZ-channel matching network. (e) The equivalent circuit and the

geometry of a three-ENZ-channel matching network based on the photonic doping structures. Three ENZ channels comprising dopants (whose radii are denoted by  $R'_1$ ,  $R'_2$ , and  $R'_3$ ) are designed with cross-sectional areas of  $0.032\lambda_p^2$ ,  $0.037\lambda_p^2$ , and  $0.048\lambda_p^2$ , respectively. (f) Simulation results of the transmission amplitudes in case I:  $(R'_1, R'_2, R'_3) = (4.10, 4.28, 4.09) \times 0.01\lambda_p$ , and case II:  $(R'_1, R'_2, R'_3) = (4.18, 4.22, 4.11) \times 0.01\lambda_p$ .

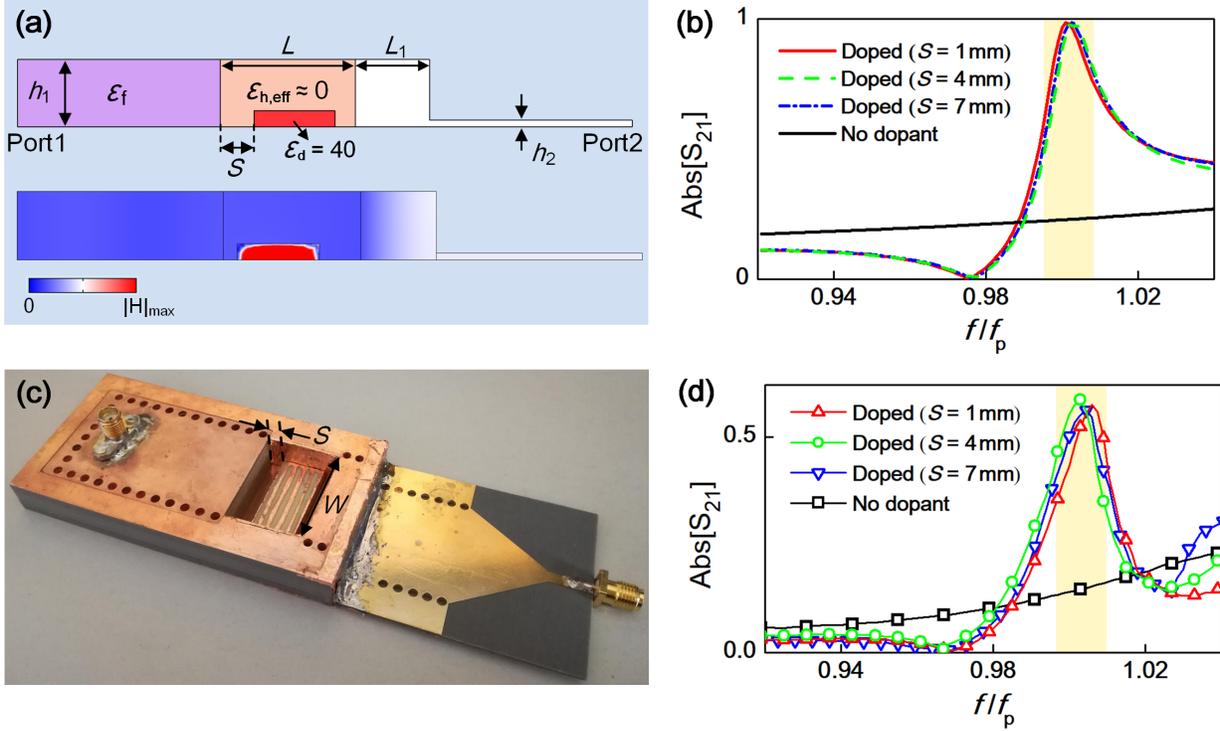
Next, we demonstrate another powerful property of general impedance matching, the convenient tunability of the bandwidth, which can be realized by engineering the slope of the permeability curve of the doped ENZ channel around the operating frequency. To this end, we fix the parameters of the dopant ( $R_3$ ) in channel 2 while introducing two dopants whose radii  $R_1$  and  $R_2$  can be changed individually in ENZ channel 1. The effective permeability of channel 1 for dopants with different radii are shown in Fig. 2(c) (red dash line and blue solid line). Specifically, two closely spaced magnetic resonances are clearly observed in the permeability diagram shown in Fig. 2(c), with the lower frequency resonance being contributed by the rod with larger radius while the higher one provided by the smaller rod. As the difference between the radii of the dopants becomes smaller, the two poles of the permeability function adjacent to  $f_p$  get closer, which leads to a stronger dispersion of the lumped reactance  $iX_s$ . In this manner, the matching bandwidth can be engineered into a transmission peak with a higher quality factor (see Fig. 2(d)).

We can also consider a more complicated matching network. A three-ENZ-channel circuit topology with a T-shape architecture is shown in the left column of Fig. 2(e), where the shunt element is transformed to an equivalent series one, following a procedure similar to that used in the two-element configuration. The architecture of the matching network comprising three doped ENZ channels is shown on the right column of Fig. 2(e). Based on this structure, two cases designed with different radii of the dielectric rods are investigated. In Case I, the reactance

values  $X_{s1}$ ,  $X_s''$ , and  $X_{s2}$  equal  $-1.39\eta_0$ ,  $-0.27\eta_0$ ,  $5.63\eta_0$ , respectively, while in Case II,  $X_{s1}$ ,  $X_s''$ , and  $X_{s2}$  equal  $1.63\eta_0$ ,  $0.22\eta_0$ ,  $2.17\eta_0$ , respectively. It is very interesting to find in Fig. 2(f) that although the simulated transmission amplitudes of both two cases are near 100% at the predesigned frequency  $f_p$ , their bandwidths are dramatically different. The underlying physics is described as follows: to realize the relatively larger (compared with the counterpart in case II) capacitance value of  $iX_{s2}$  for case I, the doped ENZ channel has to offer a large negative effective permeability (according to Eq. (4)) which features a much steeper slope in the dispersive permeability curve. As a result, through simply tuning the size of the dopants while leaving other constituents of the network unchanged, we can conveniently control the bandwidth of the impedance matching. We emphasize that since the electrically large space within index-near-zero media is electromagnetically equivalent to a single point, the behavior of the doped ENZ material as well as the performance of the general impedance matching are independent of the locations of the dopants, as well as the geometry of the ENZ channel.

### III. Experimental verification

As a proof of concept, we experimentally test the performance of doped ENZ medium to compensate the reactance of the stepped waveguide, while the intrinsic impedance of the substrate-filled feeding waveguide is designed equal to the real part of the load impedance, as shown schematically in the first panel of the Fig. 3(a). A rectangular dielectric dopant is chosen for ease of fabrication, and the governing theory of photonic doping for this rectangular dopant has been derived in our earlier work [36]. The input impedance of the load is  $(0.92-2.74i)\eta_0$  at  $f_p = 5.5$  GHz. Detailed information about calculating the load impedance can be found in Supplementary Section S1 in the Supplementary Material [38]. To compensate for the input



**Fig. 3. Experimental verification.** (a) The single-ENZ-channel matching network and the distribution of magnetic field magnitude at  $f_p$ . The dopant is designed with relative permittivity of 40, loss tangent of 0.001, and a cross sectional area of  $12 \text{ mm} \times 2.4 \text{ mm}$  ( $0.22\lambda_p \times 0.044\lambda_p$ ). The length  $L$  of the doped ENZ channel is  $20 \text{ mm}$  ( $0.37\lambda_p$ ), the length  $L_1$  of the load is set as  $11 \text{ mm}$  for the single-ENZ-channel matching, and the relative permittivity of the feeding waveguide  $\epsilon_f$  is 1.5. The heights  $h_1$  and  $h_2$  of port 1 and port 2 are chosen as  $10 \text{ mm}$  and  $1 \text{ mm}$ , respectively. (b) Simulated transmission coefficients for the dopant placed in different locations and in the absence of the dopant. (c) Photograph of the prototype of the single-ENZ-channel matching network, where a channel with width  $W = 27.2 \text{ mm}$  is designed to emulate the plasmonic material following Drude dispersion profile of  $\epsilon_{h,\text{eff}}$  with a plasma frequency at  $f_p = 5.5 \text{ GHz}$ . (d) Measured transmission coefficients for the dopant placed in different positions and in the absence of the dopant.

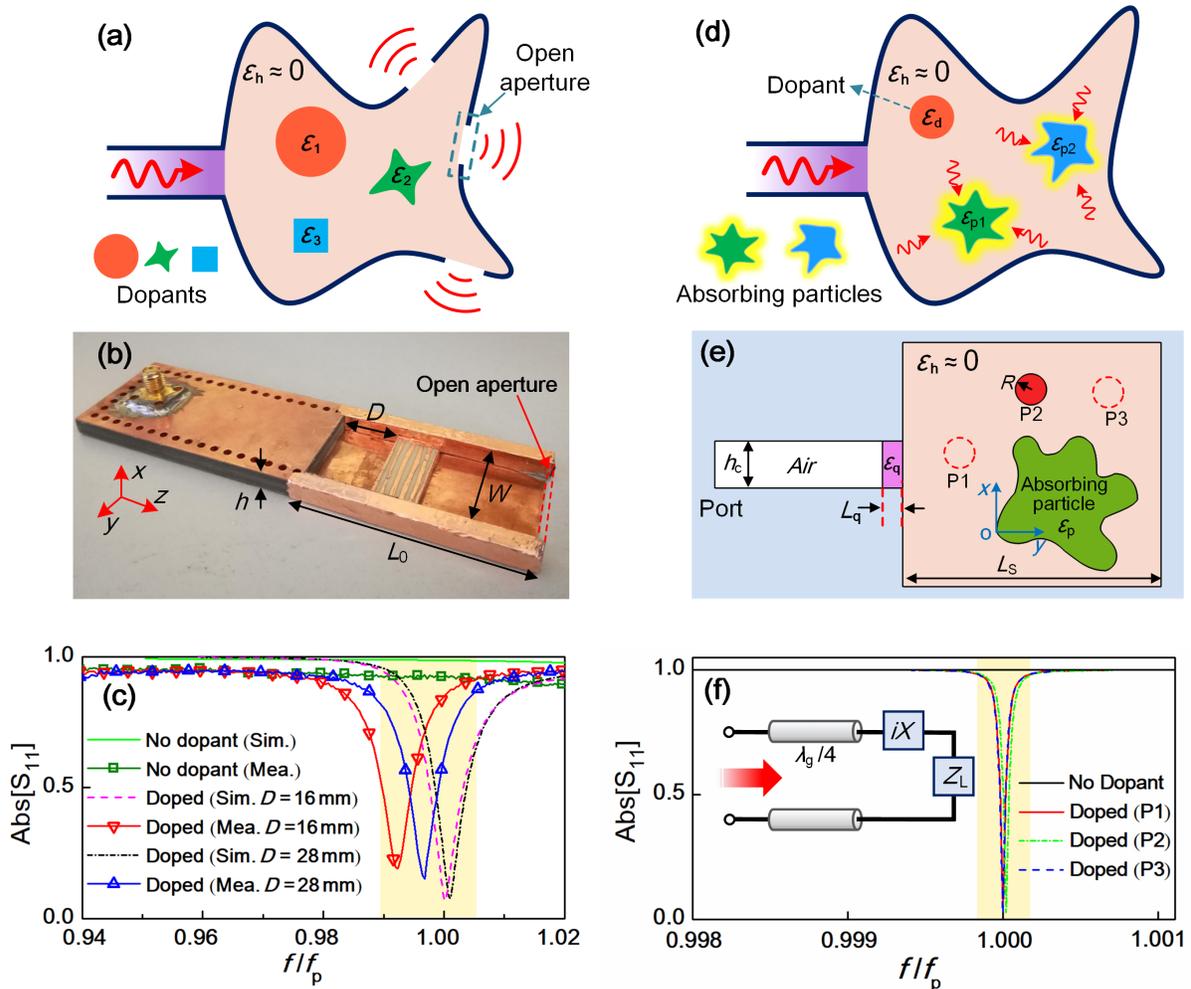
inductance, a series capacitance is needed, corresponding to a doped ENZ channel with a negative effective relative permeability of  $-0.82$ . Hence, a dielectric dopant with a relative permittivity of 40 and a cross-sectional area of  $12 \text{ mm} \times 2.4 \text{ mm}$  ( $0.22\lambda_p \times 0.044\lambda_p$ ,  $\lambda_p$  is the free space wavelength at  $f_p$ ) is chosen so as to satisfy this requirement (Analysis of the ENZ channel

comprising the rectangular dopant is presented in Supplementary Section S3 of Supplementary Material [38]). The simulation result of the distribution of magnetic field magnitude at  $f_p$  is shown in the bottom panel of Fig. 3(a), which confirms the uniform distribution of the magnetic field within the ENZ host and the complete absence of a standing wave in the input and output waveguides, signifying zero reflection. The simulated transmission amplitude spectrum for the dopant placed in different positions is shown in Fig. 3(b), verifying that the matching performance is independent of the location of the dopant. Our prototype of the single-ENZ-channel matching network, constructed using the planar circuit board technique, is shown in Fig. 3(c). **An air-filled waveguide, operating around its cut-off frequency for its TE<sub>10</sub> mode, with a width  $W$  of 27.2 mm is employed to emulate a material with a permittivity dispersion described by Drude model, having a plasma frequency at its cut-off frequency [40]. That is,  $\epsilon_{h,\text{eff}}=1-(f_p/f)^2$  and  $f_p = c/(2W)$ .** The thin metal strip fenced around the dopant is introduced to suppress the undesired coupling to TM modes in the ENZ channel [32], while the metallized vias arranged at the two sides of the feeding waveguide, as well as the stepped waveguide, provide the perfect electric conductor boundary. Additional details on the assembly procedure are presented in the **Appendix A**. The measurement results are shown in Fig. 3(d), where the transmission peak is consistently observed around  $f_p$  for the dopant located with different displacements  $S$ . The lower transmission amplitudes than those in simulations are caused by the dielectric loss of the dopant and substrate. This conclusion is corroborated by the parametric study on the influence of the loss presented in Fig. S4 of the Supplementary Material [38].

#### IV. Applications of general impedance matching

In above sections, we have shown, theoretically and experimentally, an example of the utility of the general impedance matching method for the case of a microwave waveguide. Actually, far beyond that, we can enable maximum transmission of power in many other domains, via exploiting the doped ENZ material as the building block of the matching network. Here, we extrapolate the application of general impedance matching to the design of a near-zero-index antenna. As conceptually depicted in Fig. 4(a), we introduce open apertures at the boundary of the ENZ cavity to enable radiation into free space. However, the input reactance incurred by the evanescent fields at the edge of apertures may lead to a poor matching performance. To address this problem, dielectric dopants can be designed to compensate for the imaginary part of the input impedance of the aperture, thus allowing a better radiation efficiency. The constructed prototype of the near-zero-index antenna is shown in Fig. 4(b), where the air-filled rectangular waveguide at the cut-off frequency  $f_p$  is exploited to emulate the ENZ host, and the end of the ENZ cavity is open for radiation. A detailed description of the assembly procedure is presented in **Appendix A**. The input impedance of the rectangular aperture is extracted and shown in Fig. S5(a), indicating a feeding capacitive reactance of  $2.2\eta_0$ . To cancel that feeding reactance at frequency  $f_p$ , the effective relative permeability of the ENZ cavity (with a cross-sectional area of  $h \times L_0$ ) in Fig. 4(b) has to be 0.35, according to Eq. (4). The size of the dopant is therefore determined by reference to Eq. (S5) of the Supplementary Material [38]. The 2D configuration of the proposed near-zero-index antenna and the simulated magnetic field distribution are illustrated in Figs. S5(b) and S5(c) in the Supplementary Material [38]. As shown in Fig. 4(c), the simulation and measurement results for the reflection amplitude in presence of

the dopant are significantly reduced to below 0.1 at the design frequency. Despite a slight frequency detuning in the measurement, due to the fabrication imperfection (minor unevenness of the width of the waveguide), the insensitivity of matching performance to the dopant's position is also evident from the experimental results, as demonstrated in Fig. 4(c). In addition, the radiation pattern was measured in a standard microwave anechoic chamber, and the results are gathered in Fig. S6 in the Supplementary Material [38], showing a quasi-omnidirectional pattern at the  $x$ - $z$  cut plane and a “doughnut- $\infty$ ” shaped pattern at the  $y$ - $z$  cut plane with a peak realized gain of 2.3 dBi at the boresight ( $+z$  direction).



**Fig. 4. Applications of general impedance matching.** (a) Conceptual sketch of photonic doping to match an antenna. Several apertures are introduced in the walls of the doped cavity, through which an incident guided wave is transformed into a radiating wave. (b) Photograph of the fabricated aperture antenna matched by a doped ENZ medium. The width  $W$ , height  $h$ , and length  $L_0$  of the air-filled cavity are set as 27.2 mm, 5 mm, and 55 mm, respectively. The dopant is designed with a size of 27.2 mm  $\times$  12 mm  $\times$  2.45 mm and a relative permittivity of 40 and loss tangent of 0.001. The upper metallic plate of the cavity is not shown here. (c) The measured and simulated reflection coefficients of the antenna with the dopant placed at different positions. (d) Conceptual sketch of delivering power to arbitrarily shaped absorbing particles via doped ENZ media. (e) Proposed configuration for matching an absorbing particle with a complex permittivity  $\epsilon_p = 10+0.1i$ , detailed geometry information of this absorbing particle is presented in Supplementary Note 4. The dopant is chosen with a permittivity of 40 and a radius  $R$  of 3.3 mm ( $0.061\lambda_p$ ). Other parameters are:  $L_s = 53.5$  mm ( $0.98\lambda_p$ ),  $h_c = 10$  mm,  $L_q = 4.3$  mm, and  $\epsilon_q = 11$ . (f) Simulated reflection coefficient for the absorbing particle located at different positions. The inset depicts the equivalent circuit of the matching scheme.

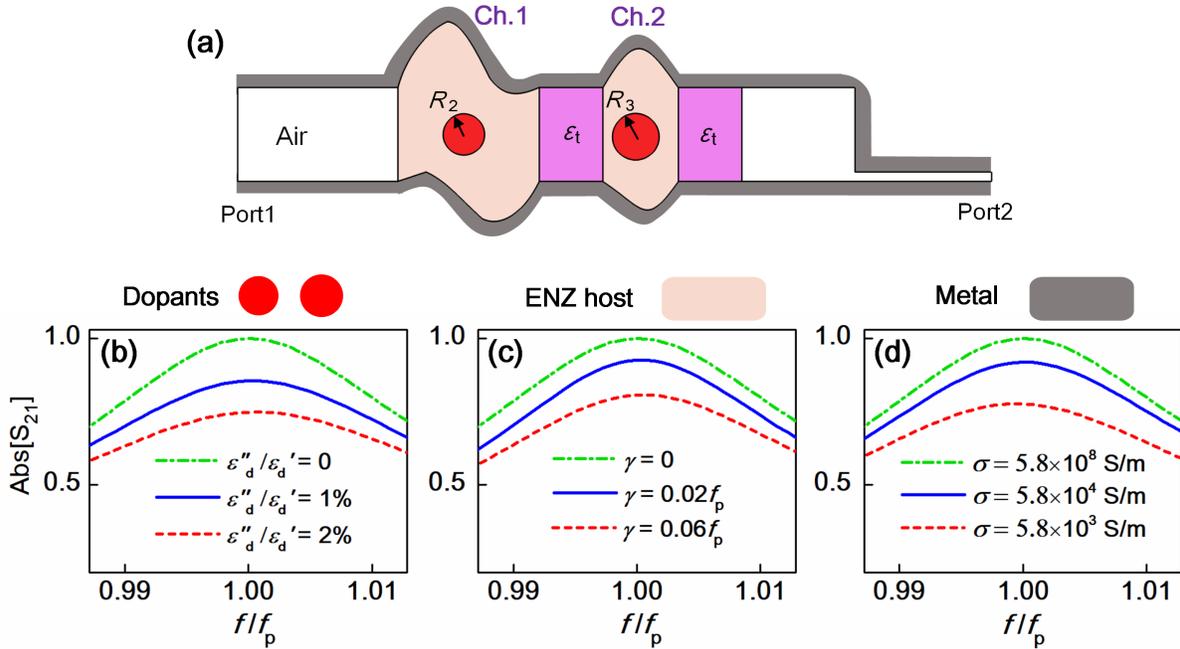
Next, we introduce another interesting application of general impedance matching of delivering power to an arbitrarily shaped lossy dielectric particle. As illustrated in Fig. 4(d), the absorbing particles with arbitrary shape are positioned in the ENZ host, while a dopant is designed to tune the impedance of the whole doped ENZ cavity so as to match the feeding transmission line. In this manner, the input power can be fully sent into the doped cavity and delivered to absorbing particles. A detailed design is presented in Fig. 4(e), where an ENZ cavity with a cross-sectional area of  $0.96\lambda_p^2$  includes an absorbing (i.e., lossy) particle with a relative permittivity of  $10+0.1i$ . The boundary of the absorbing particle is designed with a random-number generator, and for its detailed geometry information, please refer to Table S1 in Supplementary Material [38]. The extracted input impedance at the interface of the ENZ host and feeding waveguide reads  $(0.10-30.85i)\eta_0$ , where the real part of input impedance is due to the loss of the absorbing particle. The imaginary part can be cancelled as before by simply introducing an arbitrarily located dielectric dopant (the detailed matching procedure is presented

in Supplementary Section S6 of Supplementary Material [38]). Once the input impedance is purely resistive, it can easily be transformed to  $\eta_0$  by a simple quarter-wavelength impedance transformer [33], placed between the doped ENZ cavity and the air-filled feeding waveguide. The equivalent circuit is shown in the inset of the Fig. 4(f). As seen from the simulation result, a perfect transmission of power to the absorbing particle is achieved, and changing the position of the dopant as expected does not influence the dip of the reflection coefficient. This method of illuminating or heating an absorbing particle, which can be considered as an efficient “microwave/optical oven”, can be employed in a number of applications, including exciting the transition of a quantum material [41], speeding up a biochemical reaction of, for example, a macromolecular cluster [42], and it may be of special interest to life sciences and material engineering.

## V. Effects of the Material Loss

After providing proof-of-concept demonstrations of the general impedance matching method, next we carry out a systematic study on the role of material loss in the general impedance matching, which will determine the extrapolation of the proposed method to other physical systems. Without loss of generality, the configuration of the two-ENZ-channel matching network (shown in Fig. 5(a)) is used here as a case study to investigate the impact of the loss from the photonic dopants, the ENZ media, and the surrounding metallic walls. The dielectric loss of the dopant directly contributes to the imaginary part of the effective permeability, which can essentially be treated as a resistance in series with the reactive matching component according to Eq. (4). As illustrated in Fig. 5(b), we use the simulation results to

evaluate the transmission amplitudes for the cases of dielectric loss tangent of the dopant set as 0 (thus the imaginary parts of  $\mu_{\text{eff}}$  of doped ENZ channel Ch. 1 and Ch. 2 are 0), 1% (thus the imaginary parts of  $\mu_{\text{eff}}$  of doped ENZ channel Ch. 1 and Ch. 2 calculated by Eq. (5) are  $0.0009\mu_0$  and  $0.0025\mu_0$ , respectively), and 2% (thus the imaginary parts of  $\mu_{\text{eff}}$  of doped channel Ch. 1 and Ch. 2 are  $0.0017\mu_0$  and  $0.0050\mu_0$ , respectively). The reduced transmission amplitude is caused by a slight mismatch and the dissipation within the dopant. Next, we investigate the impact of the loss of the ENZ media. To this end, we use a lossy Drude model  $\epsilon_{\text{h}}(f) = 1 - f_{\text{p}}^2 / (f^2 + if\gamma)$  (where  $\gamma$  denotes the collision frequency) to describe a realistic ENZ host around its plasmonic resonance. As seen in Fig. 5(c), the transmission peak at  $f_{\text{p}}$  is reduced to 0.75 when subjected to a collision frequency of around  $0.06f_{\text{p}}$ . Finally, the influence of the loss in the surrounding metallic walls has to be discussed. To this end, instead of applying a perfect electric conductor boundary, we use a material with finite conductivity to surround the whole structure in Fig. 5(a), and the simulated transmission amplitude responses for several different conductivities of metallic walls are gathered in Fig. 5(d). As seen, although the conductivity of the surrounding metallic material is decreased by five orders of magnitude, the operating band of transmission amplitude higher than 0.6 still remains over a range from  $0.99f_{\text{p}}$  to  $1.01f_{\text{p}}$ . The simulated reflection coefficients considering different losses are reported in Fig. S8 of the Supplementary Material [38]. These parametric studies demonstrate the robustness of the general impedance matching method to the inevitable losses in real applications.



**Fig. 5. Numerical investigation of the influence of material loss.** (a) geometry of a two-element matching network, similar to those in Fig. 2(a) (the radii of the dopants in Ch. 1 and Ch. 2 are  $0.037\lambda_p$  and  $0.045\lambda_p$ , respectively), but considering the influence of loss of dopants, ENZ media, or surrounding metallic walls, which are shown in (b), (c), and (d), respectively.

## VI. Conclusion

In conclusion, we have demonstrated the concept of general impedance matching via doped ENZ media. The theoretical analysis indicates that a doped ENZ region, despite having a finite size, acts as a point-like lumped reactance with a highly controllable value. This property allows us to construct the high-frequency distributed structures via elegant circuit-based design paradigms. As a unique merit of our design, via simply tuning the photonic dopants, we can tailor the dispersion of the effective permeability of doped ENZ medium, thus enabling a flexibly tailorable bandwidth of the matching networks built on doped ENZ regions. We have presented proof-of-concept experimental demonstrations at microwave frequencies, based on substrate

integrated waveguides, to validate the theory and design guidelines. Furthermore, the proposed impedance matching scheme based on doped ENZ medium enables the opportunity to optimally guide power without limiting the implementation to a specific type of load. This capability has been demonstrated to feed a microwave component (the stepped waveguide), to enable efficient coupling of guided wave to free-space radiation, and to fully direct the power into an absorbing particle. With these advantages, our proposed scheme of general impedance matching might find applications in the fields of microwave engineering, optics, biomedicine and material science.

#### **Appendix A : Experimental setup and measurements**

The feeding waveguide in Fig. 3(c) was constructed on a 10 mm substrate with a dielectric constant of 2.5 and loss tangent of 0.025, while the stepped waveguide was fabricated using 10 mm and 1 mm substrates with a dielectric constant of 2.1 and loss tangent of 0.02. The metallized vias were drilled using standard planar circuit board techniques, and then we glued the 10 mm and 1 mm substrates together. For exciting the feeding waveguide, a 50- $\Omega$  SMA connector with a 9.5 mm length inner probe was armed at a distance of  $\lambda_g/4$  ( $\lambda_g$  is the wavelength in the substrate) apart from the shorting vias that end the waveguide. The substrate integrated waveguide on the 1 mm substrate was tapered into a 50  $\Omega$  microstrip line with a width of 2.4 mm and terminated with the receiving SMA connector. The near-zero-index antenna shown in Fig. 4(b) was fabricated on a 5 mm substrate with a dielectric constant of 2.2 and loss tangent of 0.003 via standard planar circuit board techniques, and the 50- $\Omega$  SMA connector with a 4.8 mm length inner probe was armed at a distance of  $\lambda_g/4$  apart from the ending metallic vias.

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