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**Dynamically Modulated Perfect Absorbers**

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We introduce the concept of multichannel Dynamically Modulated Perfect Absorbers (DMPAs) which are periodically modulated lossy interferometric traps that completely absorb incident monochromatic waves. The proposed DMPA protocols utilize a Floquet engineering approach which inflicts a variety of emerging phenomena and features: reconfigurability of perfect absorption (PA) for a broad range of frequencies of the incident wave; PA for infinitesimal local losses, and PA via critical coupling with high-Q modes by inducing back-reflection dynamical mirrors.

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### I. INTRODUCTION

The quest of new methods and technologies that can lead to a perfect absorption (PA) of an incident wave-form is an interdisciplinary research theme in classical wave physics. It spans a range of frequencies from optics$^1$–$^{13}$ and microwaves$^{14–17}$ to radio-frequencies$^{18,19}$, acoustics$^{20–23}$ and electronics$^{24–26}$. A successful outcome can revolutionize a variety of wave physics applications including energy conversion$^{27,28}$ and photovoltaics$^{29–31}$, imaging techniques$^{32–35}$ and medical therapies$^{36}$, stealth technologies$^{37,38}$ and soundproofing$^{23,39}$. A desirable feature for many of the above applications is an “on-the-fly” reconfigurability of the structure i.e. the possibility to absorb on demand an incoming monochromatic wave at a specific frequency, without altering the fabrication characteristics of the structure itself. Another requirement, either due to cost or design considerations, is to incorporate minimal losses inside the structures while at the same time achieve a perfect absorption. This second requirement has been recently addressed, in the frame of multi-channel systems, by the so-called Coherent Perfect Absorber (CPA) scheme. This is an interferometric protocol employing two counter-propagating waves which destructively interfere outside a weakly lossy cavity (the target) – in analogous manner to time-reversed laser - to achieve coherent perfect absorption$^{3,5}$. The original concept, involving only two channels coupled to a simple cavity has been realized in various frameworks$^{5,13,26,40}$ and further extended to include multi-channel complex cavities$^{41–43}$. CPA is a “generalization” of an older scheme, applicable only for single-channel cavities, which employs a back-reflection mirror and critical coupling to the resonances of the lossy cavity. Unfortunately, two (or multi) -sided coherent wave injection in some applications might be challenging to implement, while back-reflection mirrors are often either lossy (e.g. metallic mirrors) or require additional fabrication effort (e.g. distributed Bragg reflectors). To make things worst, none of the above proposals addresses the “on-the-fly” reconfigurability issue. Along these lines, the recent work$^{44}$ stands out, since it is the first one that proposes to use parametric amplification for the implementation of reconfigurable CPAs. This approach requires periodic modulation of the refractive index at frequencies which are twice the frequency of the incident signal – thus imposing some limitations, specifically in the optical domain. Obviously, a more flexible scheme is extremely desirable.

In this paper we introduce PA in a completely different framework associated with Floquet time-modulated systems. The latter has gain in recent years a lot of momentum– specifically in connection with non-reciprocal transport$^{45–48}$ Here, using a Floquet scattering formalism we show that the proposed Dynamically Modulated Perfect Absorber (DMPA) protocols are “on-the-fly” reconfigurable and can PA any specified frequency of the incoming waves by altering the external driving characteristics – a feature that is absent from any static PA scheme whose properties are fixed during the fabrication process. Importantly, we show that the implementation of Floquet engineering methods provide powerful means.

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**Figure 1:** (Color Online) Schematics of various physical set-ups that can be used for DMPA: (a) A single-mode fiber coupled to a time-periodic phase modulator; (b) A network of four oscillators (resonators) with modulated coupling constants; (c) Two single-mode fibers with a time- modulated coupler; and (d) an acoustic cavity with two cantilevers where the in-between air domain is periodically modulated in time. Weakly lossy elements are indicated with green.
for the implementation of PA protocols with unconventional characteristics. For example, we demonstrate perfect absorption via coherent multi-channel illumination for infinitesimal local losses – a phenomenon which we refer to as Floquet-enhanced perfect absorption – occurring at appropriate values of the frequency/amplitude of the periodic driving scheme. Finally, we show that our scheme can be flexible and interpolate between coherent DMPA and PA via critical coupling associated with single-channel illumination. We demonstrate that the latter scenario can occur even in the absence of physical back-reflecting mirrors. The latter is induced dynamically via Floquet engineering.

The structure of the paper is as follows. In the next section II we present the theoretical model which is based on coupled mode theory. We also derive the associated Floquet scattering matrix and identify conditions under which our formalism is applicable. In section III we derive the necessary conditions for perfect absorption in the case of dynamically modulated targets. Moreover, we theoretically derive expressions for the incident frequency and loss-strength for which perfect absorption occurs. In section IV we discuss the case of DMPA for coherent incident waves. Specific examples are analyzed in detail – both theoretically and numerically. In section V we analyze the scenario where DMPA is induced via dynamical mirrors imposed to the system due to the Floquet driving. Finally our conclusions are presented at the last section VI. At the Appendix we provide some details associated with the numerical models that have been used to demonstrate the numerical models.

II. FLOQUET SCATTERING FORMALISM

We consider a cavity, or a network of \( N_s \) coupled single-mode cavities. Hereafter we refer to the cavities as “sites” with the corresponding resonant mode attached to each site. The mode frequencies, and generally the couplings, are periodically modulated by an electric field which is periodic in time with period \( T = 2\pi/\omega \). For some physical set-ups see Fig. 1. In the context of coupled-mode theory, the time-periodic Floquet resonator network can be described by an effective time-periodic Hamiltonian \( H(t) = H_0(t) - i\Gamma \),

\[
H(t) = H_0(t) - i\Gamma, \quad \Gamma = \sum_{\mu} \gamma_\mu \langle e_\mu | e_\mu \rangle. \tag{1}
\]

Above \( H_0(t) \) is a \( N_s \times N_s \) Hermitian matrix, \( \gamma_\mu \) quantifies the loss in the \( \mu \)-th cavity and \( \{ e_\mu \} \) is the basis of the mode space where \( H_0(t) \) is represented. We turn the system of Eq. (1) to a scattering set-up by attaching to it two static semi-infinite leads \( \alpha = L, R \), each of which is supporting plane waves with a dispersion relation \( \Omega(k) \). We shall assume, for demonstration purposes, that the leads consist of an one-dimensional array of coupled resonators. We further assume that we can identify in each of these resonators one well isolated high-Q resonant mode. Under these assumptions the leads can be described by an one-dimensional tight-binding dispersion \( \Omega = -2C \cos(kx_0) \) where \( x_0 \) represents the distance between the resonators. The constant \( C \) represents the coupling strength between the sites of the tight-binding elements of the leads and controls the band-width of the propagating channels. We emphasize that around the center of the band i.e. \( \pm x_0 \approx \pi/2 \) the tight-binding dispersion will be reduce to the free-space like dispersion under the wide-band approximation. Specifically, we will have that \( \Omega \approx 2Cx_0k \). In this case, and for relatively small driving frequency compared with the band-width, our discussion below will be also applicable to the study of DMPA in the free space. Below we will adopt a natural unit system by setting \( C = x_0 = 1 \). The loss strength of the lossy resonator(s) are assumed to be the same \( \gamma_\mu = \gamma \).

When an incident wave with frequency \( \Omega_0 = \Omega(k_0) \in [-\omega/2, \omega/2] \) is engaged with the periodically-modulated target, it is scattered to an infinite number of outgoing channels (including evanescent ones) supporting frequencies \( \Omega_n = \Omega_0 + \omega = -2\cos k_n \) \( (n \) is an integer). The evanescent channels (that are supported in the semi-infinite leads) with \( \Omega_n \not\in (-2,2) \) do not carry flux. We, therefore, consider the scattering matrix \( S \) which connects only the \( N_p \) incoming with the outgoing propagating channels \( \Omega_n \in (-2,2), k_n \in (0, \pi) \) at each of the \( \alpha = L, R \) leads. Following Ref. we write the flux-normalized scattering matrix \( S \) as

\[
S = -I_{2N_p} + iWG_sW^T, \quad G_s = \frac{1}{\Omega_0 - H_Q + W^T\tilde{W}^T}
\]

where \( I_{2N_p} \) is the \( 2N_p \times 2N_p \) identity matrix. The quasi-energy operator \( H_Q \) is defined in the Floquet-Hilbert space and has elements \( (H_Q)_{ns,n's'} = H_{ss'}^{(n,n')} - \delta_{nn'}\omega \) where the Fourier components are \( H_{ss'}^{(n,n')} = \frac{1}{T} \int_0^T dt H_{ss'}(t) \exp(i\omega nt) \) and \( n, n' \) are integers while \( s \) labels the sites (resonators) of the system. \( W \) is the coupling matrix that describes the coupling between the propagating-channels (at the leads) and the system. Its matrix elements are \( (W)_{n_p,n,n'} = \sqrt{\delta_{n_p,n}} c_\alpha \delta_{n,n'} \delta_{\alpha+n+s} \) where \( n_p = \partial\Omega/\partial k|_{k_n} = 2\sin k_n \) and the sub-index \( n_p \) labels only the propagating channels. The matrix \( (W_c)_{n,n,n'} = c_\alpha \delta_{n,n'} \delta_{\alpha+s} \) and \( c_\alpha \) describes the bare coupling between the lead \( \alpha \) and the Floquet system where we define \( \delta_{\alpha+s} = 1 \) when lead \( \alpha \) is coupled with the site \( s \) directly or \( \delta_{\alpha+s} = 0 \) otherwise. Finally the matrix \( K \) takes the form \( (K)_{n,n,n'} = \delta_{n,n'} \delta_{\alpha+s} \exp(i\alpha k_n) \).

We point out that our analysis involves scenarios for which strong non-Hermitian effects (like EPs etc) associated with the driving\(^44\)\(^50\) are secondary as opposed to non-Hermiticity associated with material losses. Moreover, we assume that the material-related (non-radiative) losses are relatively weak and can be considered a small perturbation to the Hamiltonian of the isolated system. In such cases it is a standard practice to incorporate such
effects to the diagonal part of $H(t)$ which in Eq. (1) is indicated as $\Gamma$. In this case the diagonal elements $\gamma_{\mu}$ can be obtained by estimating the shift of the imaginary part of the eigenfrequencies with respect to the lossless case, see Refs. 51, 52.

III. NECESSARY CONDITIONS FOR DYNAMICALLY MODULATED PERFECT ABSORPTION

The scattering matrix $S(\Omega_0, \gamma, \omega)$, Eq. (2), relates the incoming wave (in the propagating channel representation) $|I\rangle$ to an outgoing wave $|O\rangle$ emerging after the scattering with the periodically time-modulated target. In other words we have that $S(\Omega_0, \gamma, \omega) |I\rangle = |O\rangle$. The condition for perfect absorption follows by requiring that the outgoing wave is the null vector, i.e. $|O\rangle = 0$. The latter is satisfied for a set of real valued scattering parameters $(\Omega_{DMPA}, \gamma_{DMPA}, \omega_{DMPA})$ for which $\det [S(\Omega_{DMPA}, \gamma_{DMPA}, \omega_{DMPA})] = 0$. While the reality of the driving frequency $\omega$ and the loss-strength parameter $\gamma$ are dictated by the formulation of the problem itself, the requirement for real incident frequencies $\Omega_0 = \Omega_{DMPA}$ is based on physical considerations; namely the fact that the incoming wave has to be a propagating wave. Using Eq. (2) we are able to recast the above condition for DMPA to the following form

$$\det (\Omega_{DMPA} - H_Q + W_Q^T KW_c - iW^T W) = 0 \quad (3)$$

which resembles a generalized eigenvalue problem associated with an effective non-Hermitian Hamiltonian $H_{\text{eff}} = H_Q - W_Q^T KW_c + iW^T W$.

In the small coupling limit $c_\alpha \to 0$, a first-order perturbation approach allows us to evaluate theoretically $(\Omega_{DMPA}, \gamma_{DMPA})$ from Eq. (3)53. We have, accidently the eigenvectors $|\psi_n^{(0)}\rangle$ of the quasi-energy operator $H_Q(\gamma = 0)$ are related to the Floquet mode associated with the Hamiltonian $H_0(t)$. In the case of one driven resonator, we can write the eigenvalues $\Omega_n^{(0)}$ and the corresponding eigenvectors $|\psi_n^{(0)}\rangle$ of $H_Q(\gamma = 0)$ explicitly using the driving $h(t)$. Specifically, we have

$$\Omega_n^{(0)} = \Omega_n^{(0)} + n\omega, \quad \Omega_n^{(0)} = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} dt^* (u(t') - u(0))$$

where $u(t) = \exp \left( -i \int_0^t dt' h(t') + i\Omega_n^{(0)} t \right) u(0)$ is the Floquet state of the time-periodic Hamiltonian $h(t)$ with the initial condition being $u(0)$.

In order to make further progress, we now consider a specific example where $h(t) = \beta \cos \omega t$. Using Eq. (5), we obtain $\Omega_n^{(0)} = n\omega$ and $\langle n' | \psi_n^{(0)} \rangle = J_{n'+n} / \omega u(0)$, where $J_{n'+n}$ denotes the Bessel function of first kind of the integer order $n'+n$. Under the assumption that there exists only one propagating channel $\Omega$ in each lead, we get from Eq. (4)

$$\Omega_{DMPA} \approx 0, \quad \gamma_{DMPA} \approx (\alpha^2 + c_R^2) J_2^2 (\beta / \omega) \quad (6)$$

From Eq. (6), we see that due to the factor $J_2^2$, the lossy strength $\gamma_{DMPA}$ required for the realization of DMPA
The driving frequency is \( \Omega = 0 \) if there is perfect absorption when \( \gamma = 0 \). The position of the zeros for \( \gamma = 0 \) are indicated with filled circles. The two set of trajectories correspond to strong \( c \approx -1 \) (red-dashed lines) and weak \( c \approx -0.2 \) (black-solid lines) coupling of the resonators with the leads. The crosses indicate the predictions of perturbation theory Eq. (4). Right inset: The DMPA can be reconfigured to occur with filled circles. The two set of trajectories correspond to changing the driving frequency \( \omega \) for an extremely small value of \( \gamma \) (4). Right inset: The DMPA can be reconfigured to act as PAs for a dense set of incident waveforms with channel amplitudes dictated by the eigenvalues of the absorption operator are 0 (used in the left inset of Fig. 2) together with the relatively small driving amplitudes \( \beta \approx 0.2 \) and driving frequency \( \omega \approx 0.3 \) (more than one order smaller than the band-width), opens up the possibility of achieving PA via the proposed DMPA scheme in the optical domain where one has to work with relatively slow modulation rates and amplitudes.

Let us finally comment on an alternative formulation that allows us to evaluate the coherent DMPA values \( (\Omega_{\text{DMPA}}, \gamma_{\text{DMPA}}) \) together with the corresponding incident waveform \( |I_{\text{DMPA}}(\Omega')\rangle \). This approach involves the notion of the absorption matrix \( A(\Omega, \gamma, \omega) \equiv I_{2N_p} - S \dagger S = A \dagger \). The eigenvalues \( \alpha(\Omega, \gamma, \omega) \) of the absorption operator indicate the amount of absorption that a coherent incident waveform, with channel amplitudes dictated by the components of the associated eigenvector, will experience once it encounters the modulated target. Obviously the eigenvalues of the absorption operator are \( 0 \leq \alpha \leq 1 \); when \( \alpha(\Omega, \gamma) = 0 \) the incident waveform is not absorbed, while \( \alpha(\Omega = \Omega_{\text{DMPA}}, \gamma = \gamma_{\text{DMPA}}) = 1 \) indicates complete absorption. Using Eq. (2), we can re-write the absorption matrix \( A \) in a simpler form

\[
A = 2\gamma \sum_{n} \sum_{\mu} |u_{n\mu}| \langle u_{n\mu}, |u_{n\mu}\rangle \equiv WG_{\text{DMPA}}^\dagger |e_{n\mu}\rangle. \tag{7}
\]

In the weak-coupling limit \( \gamma_{\text{DMPA}} \) and \( \Omega_{\text{DMPA}} = \Omega_{\text{DMPA}}^\prime, \gamma = \gamma_{\text{DMPA}} \), we have \( G_{\text{DMPA}} \approx \langle \psi_1^{(0)} | G_{\text{DMPA}} | \psi_0^{(0)} \rangle |\psi_0^{(0)}\rangle \langle \psi_0^{(0)} | \) and thus \( |I_{\text{DMPA}}(\Omega')\rangle \propto \langle \psi_1^{(0)} | G_{\text{DMPA}} | \psi_0^{(0)} \rangle |\psi_0^{(0)}\rangle \). Therefore, the study of coherent DMPA in the weak-coupling limit boils down to the eigenvalue problem of the operator \( H_Q(\gamma) \).
V. DMPA BASED ON DYNAMICAL MIRRORS AND CRITICAL COUPLING

The implementation of PA protocols that do not need a coherent multi-sided illumination or a back-reflection mirror, is highly attractive for many applications and could open up many engineering possibilities. One way to achieve this goal is by utilizing the presence of accidental degeneracies of critically coupled modes with opposite symmetries\[^{34}\] - a quite demanding scheme in terms of organizing appropriately resonant modes and their quality factors. Below we propose an altogether different approach which utilizes critical coupling to resonances. In this case, the physical back-reflection mirrors are absent and instead, we utilize appropriate Floquet driving schemes that generate dynamical mirrors.

The basic idea can be demonstrated using the simple system of Fig. 1c, described by the effective Hamiltonian Eq. (1) with

$$H_0(t) = \left( \begin{array}{cc} \varepsilon_L & -e^{i\omega t} \\ -e^{-i\omega t} & \varepsilon_R \end{array} \right)$$

and $\Gamma = \left( \begin{array}{cc} 0 & 0 \\ 0 & \gamma \end{array} \right)$ (8)

due to a “right” lossy resonator. In the equation above $\varepsilon_{L/R}$ is the frequency of the left/right resonator respectively. To this end, we consider that an incident wave with frequency $\Omega_0$, impinges the driven target from the left lead. The driving will couple the propagating channel $\Omega_1$ only with the $\Omega_1 = \Omega_0 + \omega$ channel in the right lead\[^{55}\].

To realize a DMPA in this framework, we consider $\Omega$ and $\omega$ values such that $\Omega_1 > 2$ (band edges) corresponding to an evanescent channel carrying zero flux. Essentially in this scenario, the Floquet scheme generates an impenetrable wall (dynamical mirror) at the right lead which enforces total reflection of the impinging wave. Consequently, the reflected wave exits the scattering domain at the same frequency $\Omega_0$, as the incident wave. Using Eq. (2), we obtain the reflection amplitude (in case of perfect coupling $c = -1$)

$$r_0 = \frac{1 - \left( \varepsilon_L + e^{ik_0} \right) \left( \varepsilon_R - i\gamma + e^{-ik_0} \right)}{1 - \left( \varepsilon_L + e^{-ik_0} \right) \left( \varepsilon_R - i\gamma + e^{ik_0} \right)}.$$  \hspace{1cm} (9)

Using Eq. (9) together with the DMPA condition $r_0 = 0$ we can evaluate the DMPA points ($\Omega_{DMPA}, \gamma_{DMPA}, \omega_{DMPA}$).

For example, for an incident wave at frequency $E_0 = E_{DMPA} = 0$ (middle of the band) we find that an DMPA occurs at loss strength $\gamma_{DMPA} \approx 1/\varepsilon_L^2$ and driving frequency $\omega_{DMPA} \approx 2 \left( \varepsilon_R - \sqrt{\gamma_{DMPA}} \right)$\[^{36}\]. These DMPA points have a simple physical interpretation: In the limit of small losses $\gamma_{DMPA}$, the on-site potential $\varepsilon_L \sim (1/\sqrt{\gamma})$ at the left site has to take large values. At the same time the Floquet driving creates a dynamical wall that forbids the incident wave to escape from the right lead. In other words, the scattering system turns to a high-Q cavity. When the incoming wave is at resonant with the modes of the cavity (i.e. perfect impedance matching) then it can be trapped for large times and eventually absorbed completely – even if the absorption strength $\gamma$ is infinitesimally small. The above scenario is nothing else than the so-called impedance matching condition, which, once expressed in terms of losses, indeed states that radiative and material losses must be equal\[^{57}\].

The Floquet-induced critical coupling scenario of PA can be realized for more complicated cavities like a network of four fully connected resonators (see Fig. 1b). The model Hamiltonian that has been used in these simulations is shown in the Appendix B and assumes that two of the resonators have local losses $\gamma$. In Fig. 3 we show the parametric evolution of two of the complex zeroes of the scattering matrix $S$ as the driving amplitude $\beta$ increases. We find that there is an DMPA value at $\beta_{DMPA}$ for a specific driving frequency $\omega_{DMPA}$ and loss strength $\gamma_{DMPA}$ for which the zeroes cross the real axis. We, therefore, conclude that at this real frequency an incident wave exists which is completely absorbed by the network.

VI. CONCLUSIONS

We have introduced a class of perfect absorbers that rely on Floquet engineering schemes. These Dynamically Modulated Perfect Absorbers (DMPA’s) are easily reconfigurable, and can host a variety of new phenomena, including PA in the presence of infinitesimal local losses, and unidirectional reconfigurable PA’s that operate with-
out the use of physical mirrors. The latter are now sub-
stituted by dynamical (reconfigurable) mirrors realized us-
ing appropriate Floquet drivings schemes. Dynamically
modulated PA’s and other Floquet photonic systems is an
emerging field which currently is in it’s infancy. They can
be proven useful in a variety of applications ranging from
radar cloaking, sensing and photexcitation control in
photonic nanostructures, to linear optical switches, mod-
ulators, and coherence filtering of optical signals (for a
review on potential applications see for example, Ref.58).
Along these lines, promising future directions include the
implementation of our DMPA’s proposal in existing Flo-
quet platforms (see for example59) and the implementa-
tion of DMPA protocols to realistic systems beyond the
coupled mode theory approximation and under vectorial
conditions. This will be the theme of future investiga-
tions.

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Appendix A: Network system associated with Fig. 2

A network of coupled resonators has been used for the
data shown in Figure 2. The network consists of four
coupled resonators with time-modulated couplings (for a
mechanical analog of such network see Fig. 1b). The cor-
responding isolated system is described by the following
effective time-dependent Hamiltonian $H(t)$:

$$
H(t) = \begin{pmatrix}
0 & -1 + \beta \sin(\omega t) & -1 + \beta \cos(\omega t) & -1 + \beta \sin(\omega t) \\
-1 + \beta \sin(\omega t) & 0 & -1 + \beta \cos(\omega t) & -1 + \beta \sin(\omega t) \\
-1 + \beta \cos(\omega t) & -1 + \beta \cos(\omega t) & -i\gamma & -1 + \beta \cos(\omega t) \\
-1 + \beta \sin(\omega t) & -1 + \beta \sin(\omega t) & -1 + \beta \cos(\omega t) & 0
\end{pmatrix}
$$

(A1)

where we have assumed that there are two types of (out-
of-phase) driving couplings $-1 + \beta \sin(\omega t)$ and
$-1 + \beta \cos(\omega t)$.

In order to study the DMPA phenomena discussed in
the main text, we have coupled this system with one-
dimensional leads of coupled resonators. The left lead
is coupled directly with the first site (resonator) while
the right lead is directly coupled to the fourth site (res-
onator). Both (bare) couplings $c$ are assumed to be equal.
The loss, with loss-strength $\gamma$, has been included in the
third resonator.

The data shown in the main panel of Fig. 2 correspond
to the following parameters: $\omega = 1$, $\beta = 0.9$ for two
different couplings $c = 1$ (strong coupling) and $c = 0.2$
(weak coupling). In this case the loss-strength $\gamma$ was
increasing in order to obtain the parametric evolution of
the zeros of the $S$ matrix.

The data associated with the right inset of Fig. 2
correspond to driving parameters ($\beta = 0.01; \omega = 1$)
and coupling constant $c = 0.5$. Finally the data asso-
ciated with the left inset of Fig. 2 correspond to $\gamma_{\text{DMPA}} = 0.0322; c = 0.5$ and driving amplitude $\beta$ rang-
ing between [0.1781, 0.472] with $\omega_{\text{DMPA}}$ and $\Omega_{\text{DMPA}}$ vary-
ing as shown.

Appendix B: Network system associated with Fig. 3

The results for Figure 3 are obtained when considering
the time dependent Hamiltonian $H(t)$:

$$
H(t) = \begin{pmatrix}
0 & \beta \cos(2\omega t) & \beta \cos(\omega t) & \beta \cos(3\omega t) \\
\beta \cos(2\omega t) & -i\gamma & \beta \cos(\omega t) & \beta \cos(2\omega t) \\
\beta \cos(\omega t) & -i\gamma & \beta \cos(\omega t) & 0 \\
\beta \cos(3\omega t) & \beta \cos(\omega t) & \beta \cos(2\omega t) & 0
\end{pmatrix}
$$

(B1)

where $\gamma = 0.4749$, $\omega = 5$ while the driving amplitude has
been increased from $\beta = 0.15$ to 0.3. The coupling to the
leads is the same as the one used in the modeling for Fig.
2, with a coupling constant which is $c = 0.25$. 
Hall, Englewood Cliffs, NJ, 1984)
