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Experimental realization of ultracold Yb-7Li mixtures in mixed dimensions

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We report on the experimental realization of ultracold 174 Yb- 7 Li (Boson-Boson) and 173 Yb- 7 Li (Fermion-Boson) mixtures. They are loaded into three dimensional (3D) or one dimensional (1D) optical lattices that are species-selectively deep for the heavy Ytterbium (Yb) and shallow for the light bosonic Lithium (Li) component, realizing novel mixed dimensional systems. In the 1D optical lattice the band structure of 173 Yb is reconstructed in the presence of 7 Li. Spectroscopic measurements of the 174 Yb- 7 Li mixture in the 3D lattice give access to the 174 Yb Mott-insulator structure. Ground state inter-species scattering lengths are determined to be $|a_{\rm bg}(^{174}$ Yb- 7 Li)| = (1.11 ± 0.17) nm and $|a_{\rm bg}(^{173}$ Yb- 7 Li)| = (1.16 ± 0.18) nm. The formation and characterization of an ultracold 173 Yb- 7 Li mixture is a first step towards a possible realization of a topological $p_x + i p_y$ superfluid in this system.

I. INTRODUCTION

In recent years, the strong and interdisciplinary effort towards the realization of topological phases of matter has evolved, particularly bringing topological insulators [1] and superconductors [2] into focus. Here, chiral $p_x + i p_y$ superconductors in two dimensions (2D) are of particular interest, as the appearance of robust Majorana modes might play an important role in realizing faulttolerant quantum computation [3]. Currently, Majorana modes are discussed in 1D nanowires (see e.g. [4, 5]) and other solid-state systems; for example, Sr₂RuO₄ is shown to form a p-wave superconductor [6]. Fine-tuned control of the possible Majorana modes in these systems seems difficult, and it is highly desirable to introduce additional platforms to realize topological superfluids with good control of their properties. Ultracold atom systems could be promising candidates for this application [7]. Additionally, using multi-species systems is a familiar pathway for broadening the range of accessible physical questions and, in particular, realizing mixed-dimensional systems [8].

Following these ideas, it was shown as early as ten years ago that a two-species Fermi gas with one species confined in a 2D plane and immersed into a 3D Fermi sea of the other species could lead to $p_x+i\,p_y$ superfluidity facilitated by inter-species s-wave interaction [9]. Later, using Berezinskii-Kosterlitz-Thouless theory, it was found that Fermi-Bose mixtures in mixed dimensions also support this topological superfluid [10]. More detailed calculations revealed that in considering higher-order contributions, the p-wave gap can be even larger than previously expected [11]. The authors of [11] also provide detailed calculations of the transition behavior of

the two-species fermionic Ytterbium (173 Yb) and bosonic Lithium (7 Li) system. Assuming suitable inter-species scattering lengths, critical temperatures on the order of $0.07\,T_{\rm F}$, with $T_{\rm F}$ the Fermi temperature, are predicted. Most recently, calculations including retardation and self-energy effects [12] predict similar thresholds at about $0.1\,T_{\rm F}$. These limits are certainly challenging. However, they are not too low for state-of-the-art experiments [13]. Indeed, the present work reports on the realization and characterization of a quantum degenerate, mixed dimensional 173 Yb- 7 Li system.

Much effort has been spent in understanding the interactions in the sister system of bosonic ¹⁷⁴Yb and fermionic ⁶Li [14, 15], where even the realization of a double superfluid was reported [16]. In this context, our work strives to expand on these efforts by reversing the roles of the particles, switching out bosonic for fermionic Yb and fermionic for bosonic Li. To complete the picture, we also report on a quantum degenerate ¹⁷⁴Yb-⁷Li mixture.

The present paper is organized as follows. In Sec. II, we introduce the experimental setup and details for reaching double quantum degeneracy. Section III summarizes our first characterization of the mixtures obtained, that is, the measurement of the inter-species scattering lengths (Sec. III A), the creation of a mixed dimensional system (Sec. III B) and the high-resolution spectroscopy of the mixture in a 3D optical lattice (Sec. III C). A final discussion of the results in Sec. IV concludes our work.

II. EXPERIMENT

The experiment proceeds along similar lines as our earlier reported works [17]. In brief, a hot atomic beam is formed starting from a dual species oven heated to about 350 °C, which contains Yb and Li in natural abundances as well as enriched ⁶Li. The isotope shifts of the opti-

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cal transition frequencies are, for Yb, in the few GHz range and, for Li, in the 10 GHz range. Thus, isotopeselective slowing and cooling of both species is possible with only slight adjustments of the cooling laser frequencies. We first slow down Yb in a Zeeman slower operating on the strong ${}^{1}S_{0}$ - ${}^{1}P_{1}$ transition followed by a magneto-optical trap (MOT) on the narrow ${}^{1}S_{0}$ - ${}^{3}P_{1}$ intercombination line. In a second step, Li is slowed and trapped relying on the D2 transition. As a comment, we note that the hyperfine splitting for the ${}^{7}\text{Li}({}^{2}\text{S}_{1/2})$ ground state is, at 803.5 MHz, much larger than that for ⁶Li at 228.2 MHz. Therefore, the inclusion of a ${}^{2}S_{1/2}(F=1)$ - $^{2}P_{3/2}$ repumper laser to the standard $^{2}S_{1/2}(F=2)$ - $^{2}P_{3/2}$ Zeeman slowing light is especially important for slowing sufficient numbers of ⁷Li atoms for loading into the MOT. Compression of the MOT followed by reduction of MOT beam detunings and intensities further cools the atomic clouds. This improves phase matching for loading into our crossed far-off-resonance trap (FORT), where forced evaporative (Yb) and sympathetic (Li) cooling are performed.

For the case of a ¹⁷⁴Yb-⁷Li mixture, we load the Yb and Li MOT for 14 s and 0.5 s, respectively, which results in a typical value of $60 \times 10^5 \text{ Yb}(^1\text{S}_0)$ atoms at 95 μK and $1.5 \times 10^5 \text{ Li}(^2S_{1/2}, \text{ F} = 1)$ atoms at 210 μK in the crossed FORT at the beginning of the evaporation ramp. For ¹⁷³Yb-⁷Li and re-optimizing the settings of the experimental sequence, atom numbers after 20 s and 0.3 s of loading are 46×10^5 for Yb at 80 μ K and 0.7×10^5 for Li at 115 μ K. During forced evaporation and sympathetic cooling, the crossed FORT lasers are reduced from their initial powers to the final values in individually optimized ramps within 7.65 s (9.8 s) for ¹⁷⁴Yb-⁷Li (¹⁷³Yb-⁷Li). We confirmed thermal equilibrium of the two gases during evaporation. All experiments reported in the present work were performed at low magnetic bias fields, typically a few 100 mG, not to involve any Feshbach resonances while still providing a clear quantization axis. We note that in the work described here the ⁷Li atoms are in the F = 1 state where, contrary to the $F=2, m_F=2$ state used in experiments with magnetic traps, the scattering length at low magnetic fields is small but positive [18], about 0.5 nm. In this situation the intra-species interactions are repulsive and formation of a stable ⁷Li BEC for all atom numbers is expected. Development of phase-space density (PSD) and atom number (N) during evaporation normalized by their respective initial values, PSD_0 and N_0 , for both mixtures are shown in Fig. 1. From the double logarithmic representation, we see that the initial stages of the evaporation are well described by a power law and we extract $\gamma = -d \ln(PSD/PSD_0)/d \ln(N/N_0)$, a measure for the evaporation efficiency [19], by linear fit to the data. For the ¹⁷⁴Yb-⁷Li mixture $\gamma_{Yb} = 2.9(1)$ and $\gamma_{Li} = 6.5(2)$ are obtained. Similarly, in the ¹⁷³Yb-⁷Li evaporation sequence $\gamma_{Yb} = 2.7(1)$ and $\gamma_{Li} = 12.8(6)$ are observed. The cooling efficiencies for both Yb isotopes are similar while sympathetic cooling of ⁷Li seems to proceed more effec-

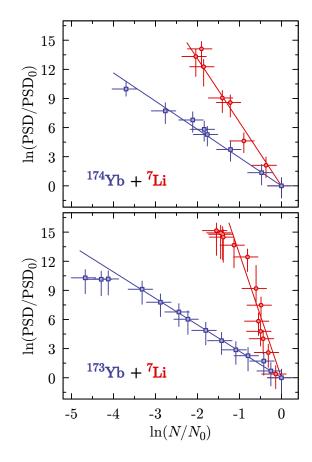


FIG. 1. The path to double quantum degeneracy in mixtures of $^{174}{\rm Yb}$ - $^7{\rm Li}$ (upper panel) and $^{173}{\rm Yb}$ - $^7{\rm Li}$ (lower panel). In each case, the development of the normalized phase-space density (PSD/PSD₀) and the normalized atom numbers (N/N_0) during forced evaporation and sympathetic cooling are shown as points. PSD₀ and N_0 are the respective values at the beginning of the evaporation ramp. Error bars account for uncertainties in the cloud and trap parameters. The trajectories for Yb (blue squares) and Li (red circles) can be roughly described and fitted by power laws (straight lines). See the main text for details on their interpretation.

tively in combination with $^{173}\mathrm{Yb}$. In the latter case, saturation of the $^{173}\mathrm{Yb}$ PSD, in particular, is also visible. The generally large γ_{Li} are a result of the sympathetic cooling where the Li PSD is increased not at the expense of the number of Li atoms, but at the cost of the coolant species, Yb. Accordingly, γ_{Li} should be regarded as a generalized cooling efficiency rather than a single-speciespicture evaporation efficiency. The observed reduction of the Li atom numbers is for the $^{173}\mathrm{Yb}$ - $^{7}\mathrm{Li}$ mixture in line with the background gas collision limited lifetime of the Li atoms in the FORT of 10 to 15 s. In the $^{174}\mathrm{Yb}$ - $^{7}\mathrm{Li}$ case, where evaporation is performed faster, additional losses due to spillage of hot Li atoms from the trap seem to occur.

At the end of the evaporation ramp, due to differences in mass and a factor 2 larger polarizability of Li in the FORT light the trap frequencies for both mixtures are

 $(\omega_x, \omega_y, \omega_z) = 2\pi \times (38, 58, 221)$ Hz for Yb and correspondingly $2\pi \times (250, 395, 1795)$ Hz for Li, where z is in the vertical direction. (The uncertainties of those values are on the order of 10%.) The obtained quantum degenerate ¹⁷³Yb-⁷Li mixture is shown in Fig. 2. We have $N \approx 62\,000^{-173} \text{Yb}$ atoms at T=87 nK and from the fugacity of the Fermi gas distribution, we find by fit $T/T_{\rm F} \approx 0.4$, where $T_{\rm F}$ is the Fermi temperature. No optical pumping techniques have been performed on ¹⁷³Yb during evaporation, and we expect about equal populations of the six spin ground states. A precise determination of the condensate fraction using an unrestricted bimodal fit for the ⁷Li cloud is difficult. We therefore opted to fix the temperature to the value obtained from the fit to the $^{173}{\rm Yb}$ cloud and obtain $N_{\rm BEC} \approx 4\,200$ atoms in the Bose-Einstein condensate (BEC) and $N_{\rm th} \approx 7\,400$ atoms in the thermal component for ⁷Li, that is a condensate fraction of 0.36. In this situation a critical temperature for reaching BEC of $T_{\rm C,Li} \approx 400$ nK is expected. The time-of-flight absorption image of ⁷Li, see right panel of Fig. 2, shows a slight fragmentation of the atomic cloud, the origin of which is not yet understood. By application of a magnetic field gradient after release of the atoms from the trap, we determined that all Li atoms are actually spin polarized in the $m_F = 0$ state and remaining field gradients cannot cause this splitting. We therefore surmise that the probable cause is some technical instability of the experiment leading to additional dynamics as the crossed FORT is being turned off. Turning our attention to the Boson-Boson ¹⁷⁴Yb-⁷Li mixture (not shown), we report $N_{\rm BEC} \approx 76\,000$, $N_{\rm th} \approx 47\,000$, T=110 nK for ¹⁷⁴Yb and $N_{\rm BEC}\approx 12\,000,\,N_{\rm th}\approx 14\,000$ with $T_{\rm C,Li} \approx 500$ nK for ⁷Li again assuming equal temperatures for both species. The condensate fractions are 0.62 and 0.46, respectively.

III. ANALYSIS AND RESULTS

Towards the long-term goal of forming a topological superfluid, three main ingredients are of particular importance [11]: (i) the creation of a Fermi-Boson quantum degenerate mixture at only few nK, (ii) the formation of a mixed-dimensional system and (iii) information on the inter-species s-wave scattering length. In the present initial survey, the experimental setup was not designed to reach the required temperatures to address the first point. Instead, we concentrate on the remaining points by both determining the inter-species background scattering length, Sec. III A, and realizing and probing two different mixed-dimensional systems, Secs. III B and III C.

A. Inter-species scattering length

Similar to earlier determinations of the $^{174}{\rm Yb}$ - $^6{\rm Li}$ interspecies scattering length [20, 21], we perform cross-

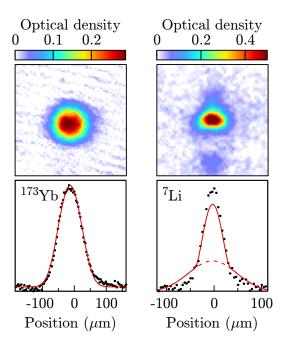


FIG. 2. A quantum degenerate mixture of ¹⁷³Yb (left panels) and ⁷Li (right panels). The clouds have been imaged 15 ms (5 ms) after release from the trap for Yb (Li). The top panels show false-color representations of the obtained absorption images. The lower panels show projections of the data (points) and fit results (lines) on the horizontal axis. Fermionic ¹⁷³Yb was fitted by a Fermi gas distribution and bosonic ⁷Li by a bimodal distribution (dashed line gives thermal component). The apparent discrepancy between fit and data seen in the Li projection is due to some Li atoms being ejected from the main cloud and giving rise to an additional contribution to the projection, see main text for further discussion.

thermalization measurements in cold, but thermal mixture samples of ¹⁷⁴Yb-⁷Li and ¹⁷³Yb-⁷Li mixtures. The experiment starts by loading either mixture into the crossed FORT. The trapping frequencies are about $2\pi \times$ (137, 564, 3200) Hz for Yb and correspondingly $2\pi \times$ (917, 3785, 21285) Hz for Li. Trap depths of about 1 mK and 2 mK, respectively, are expected. After confining the sample for 3 s a steady state of 60–80 μK is reached. Then, by slight power modulation of the horizontal FORT laser beam at 9.5 kHz for 1000 ms, the Li sample is selectively heated to an isotropically equilibrated temperature of 150–200 μ K. Through keeping the temperature-imbalanced mixture in the trap for a variable time before releasing it, we gain access to the temperatures by taking a series of absorption images at different expansion times and the result for the ¹⁷³Yb-⁷Li case is shown in Fig. 3. During the measured thermalization times, the Yb temperature T_{Yb} is found to vary little, while the Li temperature $T_{\rm Li}$ quickly approaches T_{Yb} . This shows that Yb can be treated as a sufficiently large heat bath at constant temperature. We attribute the gap in final temperatures, that is accounted for in

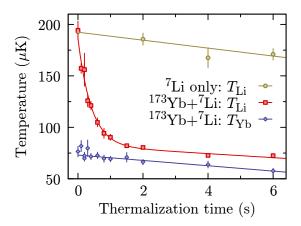


FIG. 3. Overview of experimental data to determine the ¹⁷³Yb-⁷Li inter-species scattering length. After preparation of a temperature-imbalanced thermal mixture, the temperatures of Yb (blue diamonds) and Li (red squares) have been measured for thermalization times of up to 6 s. For reference, the temperature evolution of a Li-only sample has also been taken (yellow circles). While the temperature of Li in the mixture has been fitted by an exponential decay (red line), the other data have been approximated by straight line fits (blue, yellow lines). For each thermalization time, the individual temperatures have been estimated from a series of measurements at different expansion times and the error bars correspond to the uncertainties in the temperature estimation from these data.

the error budget of the data analysis, to a residual small miscalibration of the two independent imaging systems. A control measurement in which Yb is blasted by resonant ${}^{1}S_{0}$ – ${}^{1}P_{1}$ light at zero holding time confirms that all cooling of the Li is due to Yb.

Standard cross-thermalization analysis [20] then gives access to the modulus of the inter-species scattering length. In the error analysis uncertainties of the atom numbers by 20%, the temperatures by 10% and the densities by 30% are considered. For the Fermi-Bose mixture, $|a_{\rm bg}|^{173}{\rm Yb}^{-7}{\rm Li}| = (1.16 \pm 0.18)$ nm is obtained. In the corresponding experiment for the Bose-Bose mixture, we find $|a_{\rm bg}|^{174}{\rm Yb}^{-7}{\rm Li}| = (1.11 \pm 0.17)$ nm. These values should be compared to previous calculations, albeit done for different Li hyperfine states, where scattering lengths of +1.80 nm and +1.74 nm have been reported [22]. This shows that while good order-of-magnitude agreement is achievable, the details of the inter-species interaction potentials are quite challenging to model correctly.

B. Band structure of ¹⁷³Yb in a 1D optical lattice

The aspired realization of a $p_x + ip_y$ superfluid heavily depends on the formation of a novel mixed-dimensional system. Here, by means of a strong 1D optical lattice, we realize an array of 2D 173 Yb fermionic systems in a 3D 7 Li bosonic bath. The 1D optical lattice is formed

by two horizontally counter-propagating laser beams [17] with wavelength $\lambda_{\rm L} = 532$ nm. To reduce the impact of the differential gravitational sag between Yb and Li of about 4 μ m in this and the following experiments, a compensating beam at the same wavelength, focused slightly above the atoms, is utilized [17]. The lattice depth for Yb is set to 15 E_R^{Yb} , with $E_R^{\text{Yb}} = \hbar^2 k_{532}/(2m_{\text{Yb}})$ being the Yb recoil energy, where m_{Yb} is the Yb atomic mass and $k_{532} = 2\pi/\lambda_{\rm L}$. In this situation the Li lattice depth is only 0.7 $E_R^{\rm Li}$ which is too shallow to support a bound state in the optical lattice, permitting the Li atoms to still freely move in 3D space. To confirm formation of the mixed-dimensional system, we then perform lattice modulation spectroscopy [23] to reveal the Yb band structure. After adiabatically ramping up the optical lattice, its depth is modulated by about 5 $E_R^{
m Yb}$ for 0.3 ms and then ramped down to zero in 0.2 ms converting higher band excitations via band mapping to real momenta that can be observed after time-of-flight absorption imaging.

The process is schematically depicted in Fig. 4(a). where the first four bands of the 1D lattice for Yb are shown. The Fermions initially occupy the complete first band (lower dots) and modulation of the lattice at the proper frequency leads to a particle-hole pair (filled and empty circles) between predominantly the first and third band. Experimental data is shown in Fig. 4(b), where the real momentum distribution after time-of-flight is given versus the modulation frequency. The formation of particle-hole pairs is visible as areas of increaseddecreased density. The black lines give the expected structure for a lattice at 14 $E_R^{
m Yb}$. This indicates that the calibration of the lattice depth using Kapitza-Dirac scattering of interacting bosonic ¹⁷⁴Yb before changing the setup to work with ¹⁷³Yb slightly overestimated the actual potential depth. Also the weak signals at $\pm 4 \, \hbar k_{532}$ could indicate imperfections in the band mapping procedure. A repetition of the experiment with ⁷Li blasted away before ramping up the optical lattice (not shown) yields identical results.

C. Spectroscopy in a 3D optical lattice

We now proceed to further reduce the dimensionality of Yb in the mixed dimensional Yb- $^7\mathrm{Li}$ systems. This can be done in almost the same manner for the $^{173}\mathrm{Yb-}^7\mathrm{Li}$ Fermi-Bose and the $^{174}\mathrm{Yb-}^7\mathrm{Li}$ Bose-Bose mixtures. For experimental ease, we choose here the $^{174}\mathrm{Yb-}^7\mathrm{Li}$ system and load in into a 3D cubic optical lattice at $15\,E_R^{\,\mathrm{Yb}}$, where Yb forms a Mott-insulating state while Li remains non-localized. In the case of $^{174}\mathrm{Yb}$, by exploiting the narrow $^{174}\mathrm{Yb}(^1\mathrm{S}_0 \to ^3\mathrm{P}_2)$ transition, we are able to energetically distinguish lattice sites with different Yb occupation numbers as previously demonstrated for both a pure $^{174}\mathrm{Yb}$ system [24] and the $^{174}\mathrm{Yb-}^6\mathrm{Li}$ mixture [15, 17]. The results are summarized in Fig. 5, where a measurement in the presence of $^7\mathrm{Li}$ (red) is compared to a situation where the Li atoms were removed from the trap just

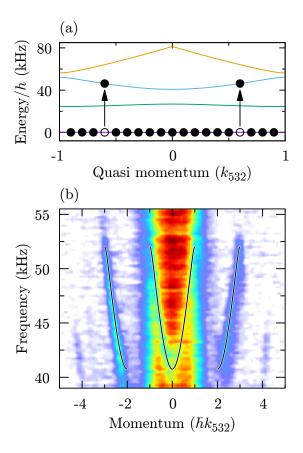


FIG. 4. Lattice modulation spectroscopy of a double quantum degenerate $^{173}\mathrm{Yb}^{-7}\mathrm{Li}$ mixture in a deep 1D optical lattice. (a) Calculated first four bands of the lattice at $14~E_R^{\mathrm{Yb}}$ lattice depth (bottom purple to top orange lines). The population of the first band by Yb Fermions (filled circles) and the excitation of a particle to the third band by lattice modulation (arrows), creating a hole in the first band population (open circle) are schematically indicated. (b) Experimental modulation spectroscopy data. The momentum distribution recorded by a band mapping procedure for different modulation frequencies reveals the creation of particle-hole pairs in the first and third lattice bands. The expected momentum-frequency dependence for this process is indicated (black lines). In the false-color representation light blue (dark red) corresponds to a lower (higher) momentum population probability.

before loading the $^{174}{\rm Yb}$ atoms into the lattice (blue). The well-resolved resonances corresponding to occupation numbers up to n=4 demonstrate the successful formation of a Mott-insulating state in presence of a $^7{\rm Li}$ bosonic background gas. We also note that possible shifts of the resonance frequencies due to different interaction strengths of the Yb $^3{\rm P}_2$ and $^1{\rm S}_0$ states with Li have not been observed.

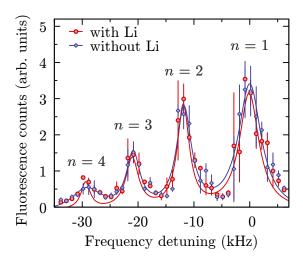


FIG. 5. Measured $^{1}\text{S}_{0} \rightarrow ^{3}\text{P}_{2}(m_{J}=0)$ excitation spectrum (red circles) of ^{174}Yb with ^{7}Li in a 3D optical lattice at 15 E_{R}^{Yb} . At this lattice depth, Li is not localized, while Yb forms a Mott-insulator state and atoms in lattice sites with different occupation numbers (n=1,2,3,4) are separated due to interatomic interaction. For comparison, the experiment was repeated with the Li atoms removed before excitation (blue diamonds). No significantly different excitation behavior is found. The lines are Lorentzian fits to the resonances.

IV. DISCUSSION AND CONCLUSION

With the present set of experimental results, we demonstrate the realization of doubly quantum degenerate mixtures of either ¹⁷⁴Yb or ¹⁷³Yb and ⁷Li. In cross-thermalization measurements the background elastic scattering lengths have been determined. roughly agree with earlier theoretical considerations and are generally, similar to the same mixtures involving ⁶Li, quite small. By then loading the Fermi-Bose mixture into a 1D optical lattice, a mixed dimensional regime has been achieved. As confirmation serve measurements of the ¹⁷³Yb band structure and of the ¹⁷⁴Yb Mottinsulator state. Thus, an important step towards topological $p_x + i p_y$ superfluids has been taken. As expected, inter-species interaction effects are found to be small and enhancement mechanisms such as suitable Feshbach resonances are advantageous for reaching the required Fermi-Bose interactions. The current setup strictly relies on sympathetic cooling to reach the Li quantum degenerate regime. It is desirable to improve this situation by e.g., implementation of Li gray molasses cooling techniques [25] and possibly the use of ⁷Li Feshbach resonances to reduce initial temperature and to enhance evaporation efficiency. Finally, suitable lattice geometries that support the necessary bosonic excitations, while remaining experimentally feasible to realize, need to be explored.

The experiments detailed in the present work serve to establish additional quantum degenerate mixtures in the toolbox of ultracold atomic physics. It is the first large mass-imbalance Bose-Fermi system for the realization of mixed dimensional geometries with a 3D bosonic background and a fermionic component of reduced dimensionality.

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