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Phys. Rev. A **98**, 042131 — Published 26 October 2018

DOI: 10.1103/PhysRevA.98.042131

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August 27th, 2018.

Quantum states are the ultimate touchstone to produce sequences of random numbers. Spatially spread entangled states allow the generation of correlated random sequences in remote locations. The impossibility of observing a quantum state, without disturbing it, ensures that the messages encoded using these sequences cannot be eavesdropped. This is the basis of Quantum Key Distribution. It is then of crucial importance knowing whether the sequences generated in the practice by spatially spread entangled states are truly random, or not. Yet, that knowledge is not immediate. One of the obstacles is the very definition of randomness. "Statistical" randomness is related with the frequency of occurrence of strings of data. "Algorithmic" randomness is related with compressibility of the sequence, what is given by Kolmogorov complexity. Sequences generated by entangled pairs of photons are analyzed, focusing on estimations of their complexity. Standard tests of statistical randomness are also applied.

PACS: 03.67.Dd Quantum cryptography and communication security. 05.45.Tp Time series analysis. 03.65.Ud Entanglement and quantum non-locality (EPR paradox, Bell's inequalities, etc.).

1. Introduction.

Sequences of random numbers are a basic supply in many applied sciences of information, from statistics to cryptography. Yet, the randomness of a given sequence is difficult to establish in the practice. Even the very definition of "random" is controversial. All definitions agree that "predictable" $\Rightarrow$ "not random", hence "random" $\Rightarrow$ "unpredictable". But, the unpredictability of an event is, in general, an ambiguous property. It depends on the available information. Some consensus has been reached, that appropriate measurements performed on quantum systems guarantee randomness. This is a consequence of von Neumann's axiom: quantum measurements violate the principle of sufficient reason. Or, in other words, a quantum measurement produces one or another outcome *without cause*. It is intuitive to conclude that a sequence of such outcomes is unpredictable, although this conclusion is difficult to formalize [1]. Note that it is still logically possible the existence of sequences that are both "unpredictable" and "not random". Depending on the precise definitions of "random" and "predictable" involved, chaotic sequences may be an example of this case.

There are at least two definitions of randomness that are relevant from a practical point of view [2]:

i) "Statistical" randomness. Imagine a sequence of 1 and 0. The sequence is "statistically random" if the number of strings of 1 and 0 of different length n (say, 110101 for n=6), in the total sequence, coincides with the number one would get if the sequence had been produced by tossing an ideal coin (statistical spread is taken into account, of course). Yet, certifying this property for any value of n and/or different ways of choosing the strings is not easy. Other tests of statistical randomness involve the decay of the self-correlation or the mutual information. They all involve measuring probabilities. The battery of tests provided by the National Institute of Standards and Technology (NIST) is mostly based on this approach.

ii) "Algorithmic" randomness. A sequence can pass the tests mentioned above and still be predictable, hence, not random. A well known example is the sequence of the digits of π (or any other transcendental number). A sequence is "algorithmically random" if there is no algorithm or program code able to generate the sequence using a number of bits shorter than the said sequence. Note that this definition does not employ probabilities. It applies even to sequences that are not statistically stationary. By the way, practical tests of randomness often include subroutines aimed to recognize the digits of the best known transcendental numbers.

Algorithmic randomness is directly related with the definition of *complexity* developed by Kolmogorov [3], Chaitin [4] and Solomonoff [5]. In few words, the complexity K of a binary sequence of length N is the binary length of the shortest program (running on a classical Turing machine) whose output is the said binary sequence. A sequence is "algorithmically random" if $K \approx N$. As there is no way of expressing the sequence using less bits that the sequence itself, the sequence is said to be *incompressible*. This definition is intuitive and appealing, but it has two main drawbacks. One: it is possible to demonstrate that all sequences are (partially) compressible; hence, the precise condition $K \approx N$ cannot be reached. This is solved by appropriately rescaling the definition. Two: K cannot be actually *computed*, for one can never be sure that there is no shorter program able to generate the sequence. Nevertheless, K can be *estimated* from the compressibility of the sequence using, f.ex., the algorithm devised by Lempel and Ziv [6].

Randomness of quantum origin has been proved to be non-computable [7], which is a condition weaker than algorithmic randomness. Algorithmic randomness of sequences produced by quantum devices based on detecting single photons after a beam splitter has been compared with sequences produced by others means [8,9]. It was found that the quantum-device generated sequences were not always the most random ones. This result is probably caused by detectors' deficiencies (see Conclusions).

Measurements performed on quantum entangled states can generate correlated outcomes in two remote stations. The sequences of outcomes, assumed random, allow the encryption of messages in a secure way using one-time-pad Vernam's cipher. This technique is known as Quantum Cryptography [10], or Quantum Key Distribution (QKD). The purity of the achieved entanglement puts a minimum bound on the entropy of the generated sequences [11] and hence, to the degree of statistical randomness. The loophole-free verification of the violation of Bell's inequalities was required as a necessary step to certify the randomness of the sequences and the invulnerability of QKD [1]. This loophole-free verification has been recently achieved by several groups using different techniques [12-15] (for a sort of critical review, see [16]). Loophole-free generated sequences have been recently used to produce series of numbers of "quantum certified" statistical randomness [17,18]. Yet, the algorithmic randomness of quantum-produced finite sequences is controversial. An experimental approach has been proposed to explore this problem [1,9].

In this paper, we carry out that proposal. We study the algorithmic randomness of time series generated in Bell's experiments by using the realization of Lempel and Ziv algorithm developed by Kaspar and Schuster [19] and implemented by D.Mihailovic *et al.* [20]. We also use part of the battery of tests of NIST to evaluate statistical randomness of the same files. It is evident that the results of these tests, performed on *actual* Bell's sequences, are crucial to ensure the in-vulnerability of QKD in the practice. In Section 2, we briefly describe the idea in Lempel and Ziv algorithm. We also review some previous attempts to detect deviations from randomness in Bell's experiments. In Section 3 we report results for the main set of data of the experiment performed in Innsbruck in 1998 [21], generously provided by Prof. G.Weihs. We also include some data recorded in our own setup. Although our experiment is far more modest, it puts light on the probable cause of the regularities found in some runs of the Innsbruck experiment.

2. Background.

2.1 Lempel and Ziv algorithm.

Complexity has advantages over other methods of detecting regular behavior. Regarding the statistical methods, complexity does not need to assume stationary probabilities. Regarding non-statistical methods, as the ones extracted from the theory of nonlinear dynamical systems (Takens' theorem), complexity does not need to assume the existence of a low dimensional object in phase space. On the other hand, complexity cannot be properly calculated; it can only be estimated.

Assuming a time series of elements $\{s_1,s_2\ldots s_N\}$ the Lempel and Ziv algorithm adds a new "word" to its memory every time it finds a substring of consecutive elements not previously registered. The size of the compiled vocabulary, and the rate at which new words are found, are the basic ingredients to evaluate complexity. In the realization of the algorithm [19,20], the time series is encoded so that a binary string is produced. Then the complexity counter *c(N)*, which is defined as the minimum number of distinct words in a given sequence of length *N*, is calculated. As $N\to\infty$, $c(N) \to N/log_2(N)$ in a random sequence. The normalized complexity measure *K* is then defined as:

$$K(N) \equiv c(N) \times log_2(N)/N \qquad (1)$$

The value of *K*(*N*) is designed to be near to 0 for a periodic or regular sequence, and near to 1 for a random one, if the available value of *N* is large enough. For a chaotic sequence it is typically halfway between 0 and 1. For a "strongly" random sequence of relatively short length, *K(N)* can be considerably larger than 1. As references, the sequence of the digits of π has $K(27{,}000)$ = 0.95. A typical chaotic time series (dimension of embedding d_E = 4, one positive Lyapunov exponent) recorded from a solid-state laser with modulated losses [22] has $K(10^5)$ = 0.4. A numerically generated quasi-periodical (2-torus) time series has $K(10^6) < 0.02$.

2.2 Some previous studies on deviations from randomness in Bell's experiments.

Some years ago, we looked for regularities in the time series generated in the Innsbruck experiment. This experiment is not only crucial to the foundations of Quantum Mechanics, but also is a superb realization of the quantum channel of a QKD setup.

In that experiment, each *run* includes 4 *files*, that is: for each of the two stations, one has the time of photon detection, and also a code for the angle setting and detector that fired (see Fig.1). We firstly looked for periodicities in one of the runs (named *longdist35*) using standard linear transforms [23], finding none. Later, we sought for low dimensional objects in phase space, using Takens' reconstruction theorem, on the whole set of available data [24]. We found a chaotic attractor with d_E=10, and four positive Lyapunov exponents, in the longest run in real time. It is named here *longtime*, and is made of the runs originally named *longtime1* and *newlongtime2*. It was possible to reconstruct the attractor and to predict the outcomes in the sequence roughly up to the inverse of the largest positive Lyapunov exponent, as expected. Remarkably, the same was possible for the 16 subsets corresponding to the different settings in spite of their shorter length. If the files in *longtime* were used for QKD, it would be possible to predict until 20 bits of the key, what is a vulnerability of a new kind [24].

The run *longtime* was the only one where d_E was reliably measured. In order to check if the cause was its time length, we perform a simpler Bell's experiment, but with an unusually long continuous time of observation. It amounts to more than half an hour, about five times longer than *longtime*. In this run, named here *SL1722*, no value of d_E is reliably measured. The cause of the regularity in *longtime* is believed to be a drift between Alice and Bob's clocks. File *SL1722* is recorded with a single clock instead, so that the obtained result is consistent with this belief. Unfortunately, when the attractor in *longtime* was found the Innsbruck experiment had been dismantled, so it is impossible to know its cause by sure.

Inequalities involving algorithmic complexity of series of outcomes produced by local realistic theories (bounds that are violated by the quantum mechanical predictions) have been derived [25-27]. Some of these inequalities are valid even if the series are not independent and identically distributed, which is a condition often stated as necessary for the validity of the usual Bell's inequalities. The violation of a measurable version of an algorithmic inequality has been experimentally verified [28]. Be aware that these inequalities involve series of *outcomes* (regardless the time each outcome is measured), while our study here deals mainly with the *time* elapsed between measurements (regardless the outcome, to get longer series). Nevertheless, a relevant result regarding the complexity of series of outcomes is briefly commented at the end of Section 3.2.

3. Results.

3.1 Structure of the Innsbruck experiment's runs.

The Innsbruck experiment includes fast switching of the analyzers' settings, driven by independent and quantum-based random number generators, and spatially distant stations, what is named the "remote, switched" condition. Most of the results obtained in this condition are the set of runs named *longdist**. We also study some preparatory runs with the stations close to each other and slowly varying settings (condition "local, static"). Also, with close stations and fast and random switched settings (condition "local, switched"). There are no "remote, static" runs. We discard most of the runs that do not violate the involved Bell's inequality ($S_{CHSH} \leq 2$).

The structure of the runs is shown in Figure 1. For each of the two stations, there are two files: the one with extension *_V.dat (left column in the Figure) is the (always increasing) series of photon detection times, in seconds. The one with the same name but extension *_C.dat (right column in the Figure) indicates the setting of the analyzer and the detector that fired at that time, using a two-bit code. Both files have the same length. There is a pair of similar files for the other station. The files' length is the number of single photons detected. It is, in general, different in each station.

A coincidence occurs when the difference of the values in the *_V.dat for each station is smaller than a certain value, what is called "time coincidence window" T_w. Once a coincidence is found, we pick up the time value in *_V.dat of station *Alice* (this choosing is arbitrary, it may well to be *Bob*, or the average between them, in any case the difference is small) to write down a time series of coincidences. The corresponding codes in the two *_C.dat files allow calculating the value of the S_{CHSH} parameter.

Regarding the algorithmic inequality mentioned in the previous Section, the subsets corresponding to the different settings are, in general, too short to allow a reliable estimation of their complexity. Besides, the basic angle setting here is $\theta = 22.5^{o}$, a value for which quantum mechanical predictions do not violate the inequality. It is therefore impossible testing the violation of the algorithmic inequality with the available data.

2.1634050886170270e-006	1
8.0075823256314910e-006	1
1.2668053500635000e-005	2
4.9706368996378400e-005	2
5.2854269101605610e-005	0
9.7272343207776580e-005	2
1.2815431751139350e-004	3
1.3008972522198680e-004	0
1.4427547709393630e-004	0
1.5615472722963800e-004	3
2.0560825198241920e-004	0
2.1648761420145820e-004	3
2.1938141279290700e-004	2
2.8082761420145820e-004	3
2.9832825198241920e-004	0
3.1709738886421220e-004	0
3.2956761420145820e-004	3
3.3093472722963800e-004	2
3.4241627025778890e-004	0
3.5539167101916270e-004	1
3.6150337933573870e-004	2
3.6763245357770900e-004	2
3.8984212241744120e-004	1
4.3617738345535160e-004	3
4.4365097917360800e-004	2
4.5388049708441920e-004	1
4.9135137159675660e-004	1
4.9907703051256600e-004	2
5.2821928900505510e-004	1

Figure 1: Structure of the files of the Innsbruck experiment [21]. The left column (*_V.dat file) indicates the time photons were detected, in seconds. The right column (*_C.dat file) indicates the detector that fired and the analyzer's orientation, according to a code. The displayed files belong to *Alice* station of run *longdist35*.

In time stamped setups like these, the value of T_w can be chosen at will after the experiment has ended. Due to different response times of detectors and electronic channels, cable lengths, etc. a time delay between the files in each station must be added. The value of the delay is found by maximizing the number of coincidences for a given value of T_w. This leads to some ambiguity in the definition of the coincidences' file. Here we use the values of T_w and delay reported by the Authors of the experiment.

3.2 Algorithmic and statistic randomness.

The time stamped files are translated into binary sequences assigning the value 1 (0) if the time difference between two successive inputs is above (below) the average for the whole file [20]. We calculate *K* of these sequences and also submit them to statistical tests developed by NIST. The complete battery includes 15 tests. Here we use the simplest 6, namely: Frequency (Monobit) Test, Frequency Test within a Block, Runs Test, Tests for the Longest-Run-of-Ones in a Block, Binary Matrix Rank Test and Discrete Fourier Transform (Spectral) Test. We say a run to have positive ("yes") NIST randomness only if passes the 6 tests. The calculation of *K* and these 6 tests form a set of relatively simple and fast running programs that are feasible to be included as a control of randomness in a QKD setup in the practice. As will be seen, all runs that are discarded by the complexity criterion (arbitrarily, $K<0.9$) are also discarded by NIST tests. Yet, it must be kept in mind that this result is specific for the set of runs included in this study. Given the different nature of the two types of randomness, the safe criterion is that a sequence can be considered random only if it passes *both* types of tests.

The main results are summarized in the Table 1. The last column is the length of the sequence. It corresponds to the number of coincidences, excepting for the "singles" files, in which case they correspond to the *Alice* station. There are three groups of sequences with different complexities (Fig.2): the ones with $K<0.9$, which are not considered random, the ones with $K\approx 1$, which are "normally" random, and the ones with $K>1$, which are "strongly" random, what means that the normalization factor in eq.(1) is insufficient. Most of the sequences belong to the latter two groups, meaning that Bell's experiments often generate sequences of algorithmically random numbers (as expected).

The run *longtime* has not only low K and is discarded by NIST statistical tests (3 over 6), but is even partially predictable, as it was discussed before. The subset corresponding to analyzers' settings and firing detectors *Alice*=0, *Bob*=3 shows a slightly higher K than the complete sequence (probably because it is shorter), and is also unable to pass NIST. Runs *longdist22* and *longdist35* have low K and, correspondingly, they do not pass NIST. Runs *longdist10* and *12* have high K but do not pass NIST. None of these five runs can be considered random despite they violate the involved Bell's inequality. They are indicated by open circles in Figure 2.

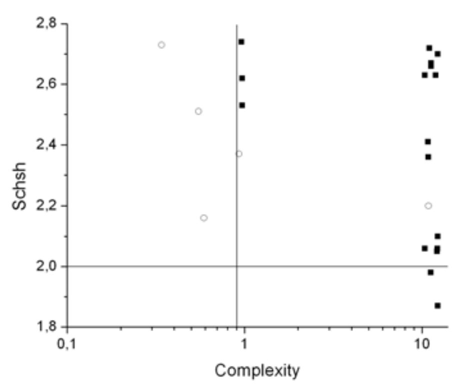

Figure 2: Graphical representation of main data in Table 1. Open circles (full squares) indicate the runs that do not pass (do pass) NIST tests. Horizontal line indicates Bell's limit, vertical line K=0.9.

All runs with $K>1$ pass NIST excepting *longdist10*. As a reference, run *Conlt3* is obtained from the coincidences between detectors observing uncorrelated fields, it has K=7.29 and passes NIST.

It seems that higher K may correspond to lower S_{CHSH}. F.ex., the three runs with higher K (*longdist30, 32* and *37,* all with $K>12$) have an average S_{CHSH} = 2.28, while the three ones with lower K (but still $K\approx 1$, *longdist0, 36* and *31*) have an average S_{CHSH} = 2.62. Runs *longdist10* and *12* are not included in this set because they do not pass NIST. Run *longdist34*, which has the highest complexity of all (12.26), is also discarded because it has $S_{CHSH} = 1.87 < 2$.

The complexity of single files and coincidence files is, in general, nearly the same. There are exceptions: in runs *longdist22* and *35* the value of K in coincidence files is smaller than in singles ones, down to the point that they cannot be considered random. On the other hand, in *longdist36* the value of K is larger for coincidences than for singles, although both can be considered random.

All runs obtained in the "local" condition (regardless if "switched" or "static") have high K and pass NIST. Finally, the complete sequences of outcomes (the columns on the right in Fig.1) have high K and also pass NIST. This confirms the reliability of the random number generators used to drive the settings in the Innsbruck experiment. This also experimentally confirms the main result theoretically implied in [25]: if the experimenter is able to generate an incompressible string (the one that drives the settings) then the measured photons come up with a noncomputable behavior as well.

Summary.

An estimation of Kolmogorov complexity of sequences recorded in Bell's experiments has been performed. Almost all sequences have complexity $K\approx 1$ or $K>1$, what means they are algorithmically random, as expected. The few with low K belong to the "remote, switched" condition. They do not pass NIST tests either. This deviation from randomness is presumably caused by a drift between the clocks in each station, an effect that had been independently detected. It is worth mentioning that in random number generators based on detecting photons after a beam-splitter, the deviation from randomness is caused mostly by detectors' different efficiencies, blind time and spurious after pulses. In the Innsbruck experiment these deficiencies have negligible impact, because of relatively low detection rate, and filtering provided by the time-coincidence selection.

Even though low K does not allow, by itself, to predict outcomes, it implies that the involved sequences are compressible, and hence potentially vulnerable. In our opinion, the main conclusion of this study is that, although random sequences are generated in many cases, it is not safe taking randomness *for granted* in experimentally generated sequences, even if they violate the involved Bell's inequality by a wide margin with a maximally entangled state. Deviations from randomness are observed even in the controlled conditions of the Innsbruck experiment, which are very difficult (perhaps impossible) to achieve in a QKD setup operating in a real world situation. Therefore, applying additional statistical and algorithmic tests and, if necessary, using distillation and extraction techniques are advisable before coding a message in the practice.

Acknowledgments.

Many thanks to Prof. Andrei Khrennikov and Prof. Jorge G. Hirsch for the encouragement, observations and advices. Also, to Prof. Dragutin Mihailovic for his help to use the algorithms and to interpret their outputs. Many thanks again to Prof. Gregor Weihs for providing the files of the Innsbruck experiment. This work received support from the grants: "Search for vulnerabilities of fundamental origin in QKD" N62909-18-1-2021 Office of Naval Research Global (USA), and "Desarrollo de láseres de estado totalmente sólido y de algunas aplicaciones" PIP 2017 0100027C CONICET (Argentina).

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Filename (description)	Complexity	NIST (RND=?)	S_{CHSH}	*N*
Longtime (remote, switched)	0.55	NO	2.51	95801
Longtime, subset {0,3}	0.65	NO	Not applicable	2122
Longdist0 (remote, switched)	0.97	yes	2.53	15501
Longdist0, singles	0.96	NO	Not applicable	471017
Longdist1	11.94	yes	2.63	16168
Longdist2	11.21	yes	1.98	26675
Longdist3	11.25	yes	2.67	24335
Longdist4	11.24	yes	2.66	25402
Longdist10	10.88	NO	2.20	26529
Longdist11	10.82	yes	2.41	25573
Longdist12	0.93	NO	2.37	27158
Longdist12, singles	0.97	yes	Not applicable	934979
Longdist13	10.84	yes	2.36	27160
Longdist20	10.37	yes	2.06	41549
Longdist22	0.59	NO	2.16	39915
Longdist22, singles	0.96	yes	Not applicable	1237058
Longdist23	10.37	yes	2.63	41058
Longdist30	12.24	yes	2.10	14145
Longdist31	0.97	yes	2.62	13022
Longdist32	12.24	yes	2.70	10992
Longdist33	12.18	yes	2.06	13004
Longdist34	12.26	yes	1.87	14289
Longdist35	0.34	NO	2.73	14562
Longdist35, singles	0.96	yes	Not applicable	388455
Longdist36	11.0	yes	2.72	14573
Longdist36, singles	0.96	yes	Not applicable	388573
Longdist37	12.16	yes	2.05	14661
Loccorr1 (local, switched)	0.96	yes	2.74	72533
Loccorr3	0.96	yes	2.74	73269
Loccorr3, singles	0.96	yes	Not applicable	853985
Bluesin1 (local, static), $\alpha= 0^o$, $\beta= 7.5^o$	0.98	yes	Not applicable	6797
Bluesin2, $\alpha= 0^o$, $\beta=15^o$	0.97	yes	Not applicable	6815
Bluesin3, $\alpha= 0^o$, $\beta =22.5^o$	0.97	yes	Not applicable	6822
Bluesin4, $\alpha= 0^o$, $\beta=30^o$	0.96	yes	Not applicable	6824
Bluesin5, $\alpha= 0^o$, $\beta=37.5^o$	0.97	yes	Not applicable	6784
SL1722 (local, static) $\alpha=0^o$, $\beta=22.5^o$	0.96	yes	Not applicable	56913
Conlt3 (local, static, uncorrelated)	7.29	yes	Not applicable	4950

TABLE 1: Summary of results. They correspond to total coincidences between stations, unless indicated otherwise. The condition of the experiment is indicated for the first run with the same name, f.ex.: the condition of being "remote, switched" applies to all runs whose names start with *Longdist*. The "static" runs have fixed settings, which are indicated. All runs belong to the Innsbruck experiment [21], excepting *SL1722* and *Conlt3*, which are ours. The second column is "yes" only if the run passes all the 6 test of NIST named in the main text.