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Relation between absorption and emission directivities for dipoles coupled with optical antennas

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Optical antennas have been widely used to control the absorption and emission properties of dipole emitters. Understanding the relationship between the absorption and emission processes of such dipole emitters in the presence of optical antennas is therefore of fundamental importance. Here we provide an in-depth analysis of the relationship between the absorption and emission directivities of dipoles coupling to optical antennas. We show that for reciprocal systems, the absorption and emission directivities are identical for a single dipole emitter, and also in the case of multiple dipole emitters when one consider incoherent emission from the emitters. On the other hand, in general, the coherent emission directivities of the dipoles are always different from the absorption directivities in the multiple dipole case. But there are special situations in the multiple dipole case where the coherent emission has directivities that are approximately the same as the absorption directivity. This study clarifies the relation between absorption and emission directivities for emitters coupled with optical antennas, and may provide useful insights in the design and characterization of optical antennas for various applications.

I. INTRODUCTION

In recent years, the studies of optical antennas have opened up a wide range of applications. Progress in optical antenna research has enabled light localization, field enhancement, emission and absorption manipulation [1–5]. Both metallic and dielectric optical antennas are intensively studied to control the spontaneous decay rate and the emission directivity of the quantum emitters [3, 4, 6–10]. Moreover, optical antenna thermal emitters are investigated for nano-scale thermal radiation engineering [11–15]. The concepts in optical antennas are also applied to the design of photovoltaics for energy harvesting purposes [16].

As an important application, optical antenna has been extensively applied to enhance the interaction of light with single or a few dipole emitters such as color centers and molecules [3, 4, 6–10, 17, 18]. A typical setup is shown in Fig. 1, where an emitter is placed in the immediate vicinity of a dielectric or metallic structure acting as an antenna. This system can operate in two modes. In the 'emission' mode, one excites the dipole emitter to generate the emission. The antenna can be used to enhance such emission or to control the direction of such emission. In this case, one can define an emission directivity as the ratio of the emission power per unit solid angle at a certain direction to the angular averaged emission power [19]. Alternatively, in the 'absorption' mode, a plane wave is incident from a specific direction, and one considers the absorption of the dipole emitter, assumed to be in its ground state in the absence of the externally incident plane wave. The antenna can be used to funnel light to the dipole emitter to enhance the absorption of the emitter. In this case, one can define an absorption cross section as the power absorbed by the dipole divided by the intensity of the incident plane wave. The corresponding absorption directivity is defined as the ratio of the absorption cross section for a particular angle of incidence to the angle-averaged absorption cross section.

To understand the absorption and emission properties of an antenna, then, it is of fundamental importance to understand the relationship between the directivity in the absorption and emission processes. In this paper, we will assume that both the antenna structure and the emitter satisfy the Lorentz reciprocity [20], which is the typical situation in optical antennas. Intuitively, one typically claims that the absorption and emission processes are reciprocal to each other. And hence one might expect that the absorption and emission directivities should be identical to each other. This statement is indeed true for typical application scenarios involving radio-frequency antennas [19]. As we will show in this paper, however, for typical application scenarios in optical antenna, the absorption and emission processes may not be strictly reciprocal to each other, and consequently the absorption and emission directivities in general may be quite different. Such a difference is particularly pronounced in systems with multiple emitters, since in general there are substantial degrees of freedom in choosing the relative phases between the dipole emitters in the emission process while these degrees of freedom are absent in the absorption process.

In this paper, we provide an in-depth discussion of the relation between the absorption and emission directivities in various scenarios. In Sections II and III, we show that the two directivities are identical only in the scenarios when the absorption and emission processes are strictly reciprocal. Such scenarios include: (1) The case of a single dipole, as discussed in Section II, where the emission involves a single point dipole with a well-defined polarization, and the absorption is provided by the same dipole with the same polarization; (2) The case of multiple thermal emitters, as discussed in Section III, where one considers the absorption of multiple emitters and the thermal

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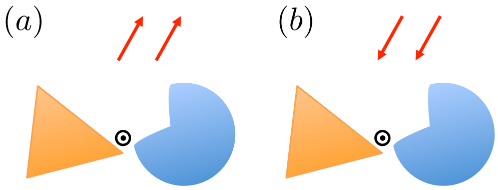

FIG. 1. Setup of the optical antenna system. The orange and blue structures represent an optical antenna with an arbitrary shape. The dipole located in the vicinity of the optical antenna functions as an emitter in the 'emission' mode (a), and an absorber in the 'absorption' mode (b).

emission from the same emitters. In both scenarios, the absorption directivity and emission directivity are identical independent of the structure of the optical antenna. In Section IV, we consider the general case where multiple emitters are coupled with an optical antenna. We show that in the typical cases where the optical antenna supports more than one optical mode, if one considers the coherent emission where the phase relations between the multiple emitters are fixed, the emission directivity in general does not match with the absorption directivity, for *any chosen phase configuration*. For the multiple emitter case, we also show that the emission directivity and absorption directivity can be approximately equal either with a single-mode optical antenna, or by using a specially configured multi-mode optical antenna. We conclude in Section V.

II. SINGLE DIPOLE

In this section, we study the emission and absorption directivity in the situation where the optical antenna contains only a single dipole source with a well-defined polarization, as shown in Fig. 2. This situation is similar to the operation condition for radio-frequency (RF) antennas. For emission, we consider only the radiation from this dipole, and only the electromagnetic power absorbed by this dipole is considered in the absorption process. We prove that the emission and absorption have the same angular and polarization dependence for arbitrary dielectric distribution.

To start, the Maxwell's equations describing the optical antenna excited by a current source oscillating at frequency ω is [21, 22]

$$(\hat{H}_0 - \hat{V} - \frac{\omega^2}{c^2}\hat{I})\vec{E} = -i\omega\mu_0\vec{J}, \tag{1}$$

where $\hat{H}_0 = \nabla\times\nabla\times$, and $\hat{V} = \frac{\omega^2}{c^2}(\hat{\epsilon}-\hat{I})+\nabla\times(\hat{I}-\frac{1}{\hat{\mu}})\nabla\times$ is an operator representing the 'potential' introduced by the structure of the optical antenna. Throughout the paper we use $\exp(i\omega t)$ convention, where ω is the operating frequency. We assume that the system consists of only reciprocal materials such that the relative permittivity and permeability tensors $\hat{\epsilon}$ and $\hat{\mu}$ are symmetric. The current

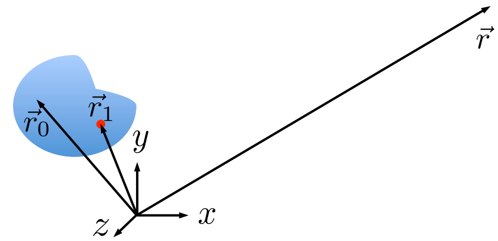

FIG. 2. Coordinate system used in the theoretical study of an optical antenna coupled to a single dipole emitter. The reference position $\vec{r}_0$ is an arbitrary position inside the optical antenna. The position of the dipole source ($\vec{r}_1$) can be either inside or close to the optical antenna. The observation position $\vec{r}$ is in the far field of the optical antenna.

density $\hat{J}$ describes the current generated by a dipole emitter that is located at a position r_1 in the emission process. For a dipole moment $\vec{p}_1$ oscillating at the frequency ω, the generated current is $J(\vec{r}) = i\omega\vec{p}_1\delta(\vec{r}-\vec{r}_1)$. The electric field generated by this dipole is:

$$\vec{E}(\vec{r}) = \frac{\omega^2}{\epsilon_0 c^2}\hat{G}(\vec{r},\vec{r}_1)\vec{p}_1, \tag{2}$$

where $\hat{G}$ is the Green's function of the antenna. It can be formally expressed as $\hat{G} = (\hat{H}_0 - \hat{V} - \frac{\omega^2}{c^2}\hat{I} + i\eta)^{-1}$, where $\eta > 0$, and is related to the Green's function $\hat{G}^0$ of free space as [23]:

$$\hat{G} = \hat{G}^0 + \hat{G}^0\hat{V}\hat{G} = \hat{G}^0 + \hat{G}\hat{V}\hat{G}^0. \tag{3}$$

For an emitting dipole located at r_1 that is near a chosen reference position r_0, the free space Green's function has an approximated form [20, 24] if the observation position $\vec{r}$ is in the far field zone, as illustrated in Fig. 2:

$$\hat{G}^0(\vec{r},\vec{r}_1) \approx \frac{e^{-ikR+i\vec{k}\cdot(\vec{r}_1-\vec{r}_0)}}{4\pi R}(\hat{I}-\hat{k}\hat{k}), \tag{4}$$

where $R = |\vec{r}-\vec{r}_0|$, $\vec{k} = k\hat{k}$ is the wave vector in free space, and $\hat{k}$ is the unit vector along the direction of $\vec{r}-\vec{r}_0$. Using Eqs. 2-4, we can write the electric field in the far field of the optical antenna:

$$\begin{aligned}\vec{E}(\vec{r}) &= \frac{\omega^2}{\epsilon_0 c^2}\Big[\hat{G}^0(\vec{r},\vec{r}_1) \\ &\quad + \int d\vec{r}_a d\vec{r}_b \hat{G}^0(\vec{r},\vec{r}_a)\hat{V}(\vec{r}_a,\vec{r}_b)\hat{G}(\vec{r}_b,\vec{r}_1)\Big]\vec{p}_1 \\ &= \frac{\omega^2}{\epsilon_0 c^2}\frac{e^{-ik|\vec{r}-\vec{r}_0|}e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_0)}}{4\pi|\vec{r}-\vec{r}_0|}(\hat{I}-\hat{k}\hat{k})\Big[\hat{I} \\ &\quad + \int d\vec{r}_a d\vec{r}_b e^{i\vec{k}\cdot(\vec{r}_a-\vec{r}_1)}\hat{V}(\vec{r}_a,\vec{r}_b)\hat{G}(\vec{r}_b,\vec{r}_1)\Big]\vec{p}_1.\end{aligned} \tag{5}$$

Since $\hat{V}$ represents the potential introduced by the optical antenna, its value in the position space representation

$\hat{V}(\vec{r}_a, \vec{r}_b)$ is zero unless both r_a and r_b are inside the antenna. Thus, the integrations over $\vec{r}_a$ and $\vec{r}_b$ in Eq. 5 are inside the optical antenna.

Let $\hat{e}_{\hat{k},\sigma}$ denote a unit vector representing the polarization of a plane wave traveling in direction $\hat{k}$ with polarization σ. The power radiated in $\hat{k}$ direction and in polarization σ per unit solid angle is:

$$\begin{aligned} P_e(\hat{k}, \sigma) = & \lim_{|\vec{r}-\vec{r}_0|\to\infty} \frac{|\hat{e}^{\dagger}_{\hat{k},\sigma}\vec{E}(\vec{r})|^2|\vec{r}-\vec{r}_0|^2}{2\eta_0} \\ = & (\frac{\omega^2}{4\pi\epsilon_0 c^2})^2 \frac{1}{2\eta_0}\Big|\hat{e}^{\dagger}_{\hat{k},\sigma}\Big[\hat{I} \\ & + \int d\vec{r}_a d\vec{r}_b e^{i\vec{k}\cdot(\vec{r}_a-\vec{r}_1)}\hat{V}(\vec{r}_a,\vec{r}_b)\hat{G}(\vec{r}_b,\vec{r}_1)\Big]\vec{p}_1\Big|^2, \end{aligned} \tag{6}$$

where we substitute in the electric field expression in the far field (Eq. 5) and use the property that the electric field of a plane wave is transverse to the propagation vector such that $(\hat{I} - \hat{k}\hat{k})\hat{e}_{\hat{k},\sigma} = \hat{e}_{\hat{k},\sigma}$, η_0 is the impedance of vacuum. Equation 6 provides the polarization and angular distribution of the emission.

We proceed to investigate the absorption directivity of the optical antenna. We consider the absorption by a dipole located at the same position r_1 as the emitting dipole considered above, coupled with the same optical antenna structure. Consider that the incident plane wave propagates in $\hat{k}$ direction with polarization σ, as described by an incident field $\vec{E}_0(\vec{r}) = \vec{E}_i \exp(-i\vec{k}\cdot\vec{r})$ with $\vec{E}_i = E_i\hat{e}_{\hat{k},\sigma}$. The total field $\vec{E}$, due to the interaction of the incident wave with the optical antenna, is obtained using the Lippmann-Schwinger equation [23, 25]:

$$\vec{E} = \vec{E}_0 + \hat{G}_0\hat{V}\vec{E} = \vec{E}_0 + \hat{G}\hat{V}\vec{E}_0. \tag{7}$$

To obtain the absorption by the dipole, we calculate the electric field at the position of the dipole:

$$\begin{aligned} \vec{E}(\vec{r}_1) = & e^{-i\vec{k}\cdot\vec{r}_1}\Big[\hat{I} \\ & + \int d\vec{r}_a d\vec{r}_b \hat{G}(\vec{r}_1,\vec{r}_a)\hat{V}(\vec{r}_a,\vec{r}_b)e^{-i\vec{k}\cdot(\vec{r}_b-\vec{r}_1)}\Big]\vec{E}_i. \end{aligned} \tag{8}$$

In describing the emission process, we have assumed that the dipole can only oscillate in the direction $\hat{e}_{p_1}$. As we will see throughout the paper, in order for the directivity of the absorption and the emission processes to be identical, the two processes must be strictly reciprocal to each other. Therefore, here, to describe an absorption that is exactly reciprocal to the emission process as described above, we consider the dipole with an anisotropic polarizability $\alpha\hat{e}_{p_1}\hat{e}^T_{p_1}$. The power absorbed by such a dipole is [2]:

$$\begin{aligned} P_a(\hat{k}, \sigma) = & -\frac{1}{2}\omega Im(\alpha)|\hat{e}^T_{p_1}\vec{E}(\vec{r}_1)|^2 \\ = & -\frac{1}{2}\omega Im(\alpha)|\vec{E}^T(\vec{r}_1)\hat{e}_{p_1}|^2. \end{aligned} \tag{9}$$

Combining Eqs. 8 and 9 results in:

$$\begin{aligned} P_a(\hat{k}, \sigma) = & -\frac{1}{2}\omega Im(\alpha)|E_i|^2\Big|\hat{e}^T_{\hat{k},\sigma}\Big[\hat{I} \\ & + \int d\vec{r}_a d\vec{r}_b e^{-i\vec{k}\cdot(\vec{r}_b-\vec{r}_1)}\hat{V}^T(\vec{r}_a,\vec{r}_b)\hat{G}^T(\vec{r}_1,\vec{r}_a)\Big]\hat{e}_{p_1}\Big|^2. \end{aligned} \tag{10}$$

In a reciprocal system, $\hat{V}^T(\vec{r}_a, \vec{r}_b) = \hat{V}(\vec{r}_b, \vec{r}_a)$, $\hat{G}^T(\vec{r}_a, \vec{r}_b) = \hat{G}(\vec{r}_b, \vec{r}_a)$. Substituting these reciprocity relations into Eq. 10, we get the expression of the absorption at the dipole:

$$\begin{aligned} P_a(\hat{k}, \sigma) = & -\frac{1}{2}\omega Im(\alpha)|E_i|^2\Big|\hat{e}^{\dagger}_{-\hat{k},\sigma}\Big[\hat{I} \\ & + \int d\vec{r}_a d\vec{r}_b e^{-i\vec{k}\cdot(\vec{r}_a-\vec{r}_1)}\hat{V}(\vec{r}_a,\vec{r}_b)\hat{G}(\vec{r}_b,\vec{r}_1)\Big]\hat{e}_{p_1}\Big|^2, \end{aligned} \tag{11}$$

where we note that for two plane waves propagating in the opposite directions, they would have the same polarizations if their polarization vectors are complex conjugate of each other, i.e., $\hat{e}_{\hat{k},\sigma} = \hat{e}^*_{-\hat{k},\sigma}$.

Comparing Eq. 6 and 11, we obtain the relation between emission and absorption directivities, which is the main conclusion of this section:

$$P_a(\hat{k}, \sigma) = -\omega Im(\alpha)|E_i|^2 \frac{P_e(-\hat{k}, \sigma)}{\frac{1}{\eta_0}(\frac{\omega^2}{4\pi\epsilon_0 c^2})^2 p_1^2}. \tag{12}$$

We note that the term $Im(\alpha)$ is a scalar quantity which has no angular dependence. The right hand side of Eq. 12 is thus proportional to the emission directivity at direction $-\hat{k}$ with polarization σ. Equation 12 here provides a connection between $P_a(\hat{k}, \sigma)$ and $P_e(-\hat{k}, \sigma)$. To summarize, in the case that the reciprocal optical antenna couples to a single dipole, Eq. 12 indicates that the emission and absorption directivities are the same provided that the polarizability of the dipole in the absorption process has a form that is reciprocal to the polarization of the dipole used in the emission process.

We proceed to provide a numerical demonstration of Eq. 12. We use the Finite-Difference Frequency Domain (FDFD) method and eigenmode solver in frequency domain [26] to study a two-dimensional resonant optical antenna coupling to a point dipole. The optical antenna has a race-track shape and is made of silicon with a relative permittivity $\epsilon_r = 12$. The radius of the two half disks is 0.4 μm and the width of the rectangle between the two half disks is 0.2 μm, as shown in Fig. 3(a). Since this optical antenna will also be used in the discussions in Section III and IV, we comment first about its properties in the absence of dipoles coupling with it. For the TM polarization, which has its electric field polarized along the z-direction as shown in Fig. 3(a), the optical antenna supports two resonant modes at the wavelength of 1.351 and 1.354 μm. The electric field distribution of these two modes are shown in Fig. 3(c) and (d), respectively. Figure 3(b) shows the scattering cross-section spectrum of

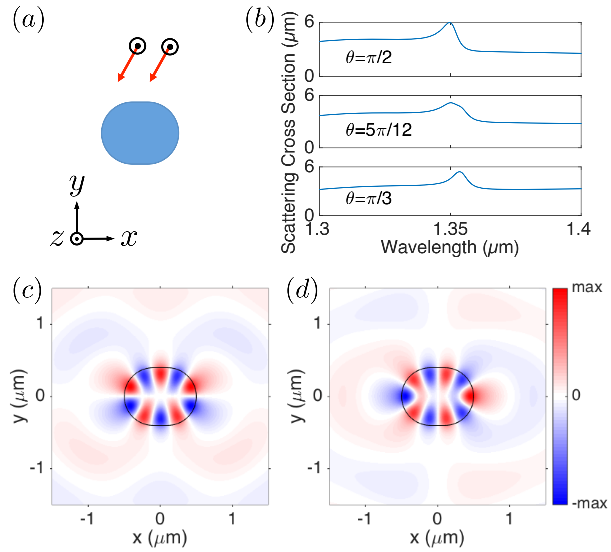

FIG. 3. (a) Schematic of the optical antenna under illumination of a plane wave with TM-polarization. The red arrows indicate the direction of the incident wave vector $\hat{k}$. The optical antenna has a race-track shape where the radius of the half disks is 0.4 μm and the width of the middle rectangle is 0.2 μm. Its relative permittivity is 12. (b) The scattering cross section as a function of the wavelength of the incident wave under several incident angles. The incident angle θ is defined as the angle between $-\hat{k}$ and x-axis. (c) (d) Electric field distributions of the resonant modes supported by the optical antenna at wavelength 1.351 and 1.354 μm respectively.

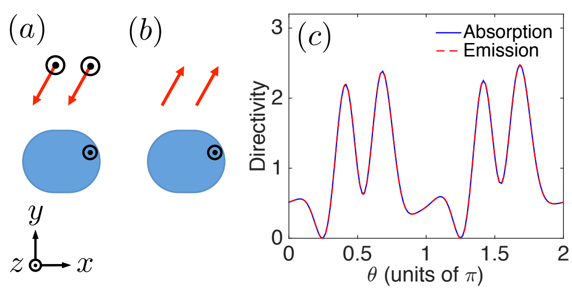

FIG. 4. Directivity of the optical antenna with a single dipole. The optical antenna is the same as in Fig. 3. The dipole locates at (0.45, 0.05) μm relative to the center of the optical antenna and oscillates along z-axis. (a) and (b) illustrate the absorption and emission respectively. The emission and absorption directivity are respectively shown in red dashed and solid blue curve in (c). The angle θ is defined as the angle between $-\hat{k}$ and x-axis for the incident wave, and the angle between $\hat{k}$ and x-axis for the emission wave.

the optical antenna in the wavelength range near the two resonances under different incident angles. We define the angle θ of the incident wave as the angle between $-\hat{k}$ and x-axis. When the wave is incident in the direction along the $-y$-axis ($\theta = \pi/2$), the mode in Fig. 3(c) can be excited but the mode in Fig. 3(d) is not. Thus, the peak of the scattering cross-section spectrum for $\theta = \pi/2$ is close

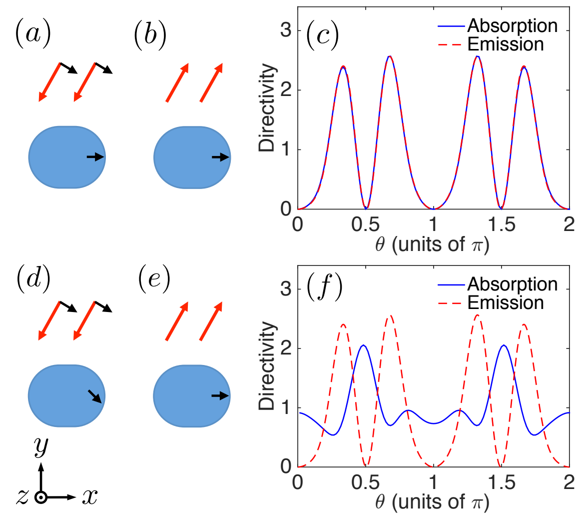

FIG. 5. Directivity of the optical antenna for TE-polarization. The optical antenna is the same as in Fig. 3. The dipole locates at (0.45, 0) μm relative to the center of the optical antenna. The chosen wavelength is 1.37 μm. (a) and (b) respectively illustrate the absorption and emission when the dipole oscillates along x-axis in both cases. (c) shows the corresponding absorption and emission directivity. (d) and (e) respectively illustrate the absorption and emission when the emission dipole oscillates along x-axis but the induced dipole in the absorption process oscillates in a direction parallel to the local field. (f) shows the corresponding absorption and emission directivity.

to the resonant wavelength of the mode in Fig. 3(c). At the incident angle $\theta = \pi/3$, the mode in Fig. 3(d) is dominant and the peak of the scattering spectrum is close to its resonant wavelength 1.354 μm. The incident wave at $\theta = 5\pi/12$ can resonantly excite both modes and therefore the scattering cross-section spectrum shows a broader peak.

We now couple a dipole emitter with the dielectric antenna structure as characterized above. We choose the operating wavelength to be 1.352 μm and place the dipole at (0.45, 0.05) μm relative to the center of the optical antenna, such that both resonant modes can be excited. Here we have intentionally chosen the general case where both resonant modes are excited, since there are subtle aspects about antennas supporting only a single resonant mode as we comment later in Section IV. The dipole is chosen to oscillate only along z-axis in both emission and absorption processes, as illustrated in Fig. 4(a-b). Figure 4(c) shows the numerically determined emission and absorption directivity. We see that the two directivies are identical to each other, validating the theoretical derivation above.

In our proof above equating the absorption and emission directivity, the use of an anisotropic polarizability is essential to ensure the strict reciprocity between the absorption and emission processes. As an illustration, we show the emission and absorption directivity of the same optical antenna for the TE-polarization in Fig. 5. The dipole is placed at (0.45, 0) μm relative to the center of the optical antenna and assumed to oscillate along the x-axis in the emission process. The operating wavelength is 1.37 μm. When the polarizability is anisotropic such that the induced dipole in the absorption process is restricted along the x-axis (Fig. 5(a-b)), we find that the absorption and emission directivity are identical (Fig. 5(c)). However, if the polarizability is scalar such that, in the absorption process, the induced dipole always oscillates along a direction parallel to the local electric field (Fig. 5(d-e)), the absorption and emission directivity become different (Fig. 5(f)), since in this case, the absorption process and the emission process are not strictly reciprocal to each other.

III. INCOHERENT EMISSION FROM MULTIPLE DIPOLES

For the rest of the paper we consider the more general cases where the optical antenna is coupled to multiple dipoles in its vicinity. In these cases, the study of the absorption process is straightforwardly carried out: We describe each of the dipole in terms of its polarizability. And we compute the total absorption of these dipoles as a function of the direction of the incident plane waves. The emission process however is more subtle. To define the emission process we will need to specify the amplitudes and the phases for the oscillation of each dipole moment. In the case of multiple dipole emitters, the angular distribution of the total emission depends on such amplitudes and phases. This is in contrast with the single dipole emitter case where the angular distribution of the emission does not depend on the amplitude or the phase of the dipole oscillation as seen in Eq. 6. In the case of multiple dipoles, therefore, the relevant question in order to elucidate the connection between absorption and emission directivities is then: For a given collection of such dipoles, in the emission process, is there a choice of the amplitudes and phases of the oscillation of the dipole moments, such that the emission directivity matches the absorption directivity?

In this section, we prove rigorously that, in the incoherent emission case, where there is no phase coherence between the dipole emitters, there always exists a particular choice of amplitudes for the dipole moments, such that the emission from these dipoles has a directivity that matches the absorption directivity. This result is related to the well-known Kirchhoff's law of thermal radiation, which states that for thermal emitters, the absorptivity must match the emissivity at every angle [11, 27]. Our result here is a slight generalization of the Kirchhoff's law since we do not assume thermal equilibrium for the emitters.

Similar to the derivation in Section II, we consider an optical antenna that is coupled with n dipoles located at $\vec{r}_j$, $j = 1, 2, \ldots, n$. The absorption by all the dipoles is equal to the sum of absorption by each dipole. Suppose the incident light is a plane wave in direction $\hat{k}$ and has polarization σ, we can obtain the absorption power by extending Eq. 11 to the multiple dipole case:

$$\begin{aligned} P_a(\hat{k},\sigma) = &-\frac{\omega}{2}|E_i|^2 \sum_j Im(\alpha_j)\Big|\hat{e}^\dagger_{-\hat{k},\sigma}\Big[\hat{I} \\ &+\int d\vec{r}_a d\vec{r}_b e^{-i\vec{k}\cdot(\vec{r}_a-\vec{r}_j)}\hat{V}(\vec{r}_a,\vec{r}_b)\hat{G}(\vec{r}_b,\vec{r}_j)\Big]\hat{e}_{p_j}\Big|^2. \end{aligned} \tag{13}$$

In the incoherent emission process, the interference between the waves emitted by different dipoles is zero under ensemble averaging, i.e. $\langle \vec{p}_i \vec{p}_j^\dagger \rangle = 0$ for $i \neq j$. Thus, the ensemble-averaged power emitted in direction $\hat{k}$ with polarization σ is

$$\begin{aligned} \langle P_e(\hat{k},\sigma)\rangle = &\lim_{|\vec{r}-\vec{r}_0|\to\infty} \frac{1}{2\eta_0}\Big|\frac{\omega^2}{c^2\epsilon_0}\Big|^2|\vec{r}-\vec{r}_0|^2 \\ &\sum_j \hat{e}^\dagger_{\hat{k},\sigma}\hat{G}(\vec{r},\vec{r}_j)\langle\vec{p}_j\vec{p}_j^\dagger\rangle\hat{G}^\dagger(\vec{r},\vec{r}_j)\hat{e}_{\hat{k},\sigma}. \end{aligned} \tag{14}$$

In order to have the absorption and emission directivity to be equal to each other, we can choose the amplitude of each dipole such that

$$\langle \vec{p}_j \vec{p}_j^\dagger \rangle = A \cdot Im(\alpha_j)\hat{e}_{p_j}\hat{e}^\dagger_{p_j}, \tag{15}$$

where the coefficient A is the same for every dipole. This condition is consistent with the fluctuation-dissipation theorem which governs the relation between thermal emission and absorption [28, 29]. With this choice of the dipole amplitudes, the ensemble averaged emission is:

$$\begin{aligned} \langle P_e(\hat{k},\sigma)\rangle = &\Big(\frac{\omega^2}{4\pi\epsilon_0 c^2}\Big)^2\frac{A}{2\eta_0}\sum_j Im(\alpha_j)\Big|\hat{e}^\dagger_{\hat{k},\sigma}\Big[\hat{I} \\ &+\int d\vec{r}_a d\vec{r}_b e^{i\vec{k}\cdot(\vec{r}_a-\vec{r}_j)}\hat{V}(\vec{r}_a,\vec{r}_b)\hat{G}(\vec{r}_b,\vec{r}_j)\Big]\hat{e}_{p_j}\Big|^2. \end{aligned} \tag{16}$$

Comparing Eqs. 13 and 16, we find that the absorption and incoherent emission have the same directivity. The derivation above is for the general case where the polarizability is an anisotropic tensor. It is, indeed, also applicable to the case of the scalar polarizability, since both the absorption and the incoherent emission processes can be described by simply summing over the independent contributions of three dipole moments along the principal axes.

To illustrate the analytic results above, we provide a numerical simulation of the incoherent emission directivity for the multi-mode optical antenna, same as that in Section II, coupling with two dipoles. The two dipoles are located at (0.45, 0) and (0, 0.32) μm with respect to the center of the optical antenna and oscillates along z-direction. Only TM-polarization is considered and the operating free-space wavelength is chosen as 1.352 μm.

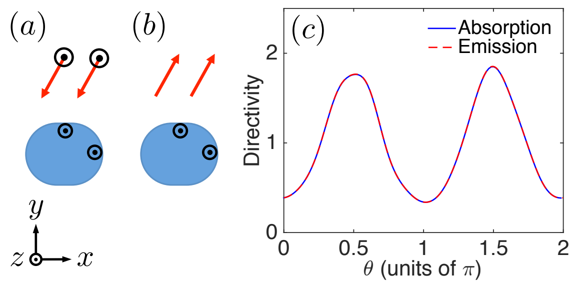

FIG. 6. The absorption and incoherent emission directivity. (a) and (b) illustrate the absorption and emission processes respectively. The optical antenna is the same as that in Fig. 3. The two dipoles are at (0.45, 0) and (0, 0.32) μm respect to the center of the optical antenna. The wavelength is 1.352 μm and only TM-polarization is considered. (c) shows the absorption directivity (solid blue curve) and the incoherent emission directivity (dashed red curve).

In the absorption calculation, we assume that the polarizability of the two dipoles are the same. Correspondingly, according to Eq. 15, we choose the amplitudes of the two dipoles to be the same in the emission process. Figure 6(c) shows the absorption and incoherent emission directivities, which agree with each other perfectly as expected.

IV. COHERENT EMISSION FROM MULTIPLE DIPOLES

In this section, we discuss relation between emission and absorption directivities when the emission from multiple dipoles is coherent. In contrast to the incoherent emission case, here we shall prove that in typical situations, the directivity from such coherent emission cannot match the absorption directivity, for *any possible choice* of the amplitudes and phases of the emitting dipoles.

To give a quantitative discussion of this phenomenon, we follow a similar approach presented in Section II and III. We consider an optical antenna that couples with n dipoles located at $\vec{r}_j$, $j = 1, 2, \ldots, n$. The absorption process is described in Section III and the absorption power is shown in Eq. 13. In the coherent emission process, the electric field is

$$\vec{E}(\vec{r}) = \frac{\omega^2}{\epsilon_0 c^2} \sum_j \hat{G}(\vec{r}, \vec{r}_j)\vec{p}_j, \tag{17}$$

which is a vectorial sum of the electric field generated by each dipole. Using Eq. 17, the power emitted in direction $\hat{k}$ with polarization σ is

$$\begin{aligned} P_e(\hat{k}, \sigma) = & \lim_{|\vec{r}-\vec{r}_0|\to\infty} \frac{1}{2\eta_0} (\frac{\omega^2}{c^2\epsilon_0})^2 |\vec{r}-\vec{r}_0|^2 \\ & \Big\{ \sum_j \hat{e}^\dagger_{\hat{k},\sigma} \hat{G}(\vec{r}, \vec{r}_j)\vec{p}_j\vec{p}_j^\dagger \hat{G}^\dagger(\vec{r}, \vec{r}_j)\hat{e}_{\hat{k},\sigma} \\ & + \sum_{i>j} Re\big[\hat{e}^\dagger_{\hat{k},\sigma} \hat{G}(\vec{r}, \vec{r}_i)\vec{p}_i\vec{p}_j^\dagger \hat{G}^\dagger(\vec{r}, \vec{r}_j)\hat{e}_{\hat{k},\sigma}\big] \Big\}. \end{aligned} \tag{18}$$

The first term in Eq. 18 is the same as in the incoherent emission (Eq. 14) and has no dependence on the relative phases of the oscillation dipoles. As shown in Section III, the first term has the same angular dependence as the absorption power if the amplitudes of the dipoles are chosen according to Eq. 15. The second term in Eq. 18, referred to as the 'interference term' below, represents the interference between the fields generated by multiple dipoles. In typical situations, the interference term does not vanish for all angles and has an angular dependence different from the incoherent term. Thus, the emission directivity is in general different from the absorption directivity.

Equation 18 also indicates that there are special antenna structures for which the absorption and emission directivities can be approximately equal. This can happen, for example, in two cases. Case (1): The interference term has the same angular distribution as the incoherent term. Case (2): The interference term vanishes. Below, we discuss each of these cases separately.

Case (1) occurs when the optical antenna supports a single resonant mode around the frequency of the emitters. We assume that each dipole excites the resonance [7], and the field then escape from the resonance into the far field. The emission pattern of the dipole is then determined by the far-field radiation pattern of the resonator. Thus the emission pattern of all dipoles are the same independent of their positions or relative phases or amplitudes. And hence the interference term in Eq. 18 will have the same angular distribution as the incoherent term since they are both dominated by the same far field radiation pattern of the resonant mode. A quantitative discussion of the single mode optical antenna based on the coupled-mode theory is presented in the Appendix.

We note however that the argument here is approximate. Certainly the argument relies upon the dominance of a single resonant mode which is an approximation in itself. Moreover, if some of the dipoles do not couple to the resonant mode, their emission patterns will not be controlled by the far field radiation of the resonant mode.

For case (2), in general, as mentioned above, when the optical antenna supports multiple modes and more than one mode is excited by the dipoles, the interference term usually has a different angular dependence compared with the incoherent term in Eq. 18. Nevertheless, as we will show in the numerical example below, in a specially configured optical antenna, the interference term

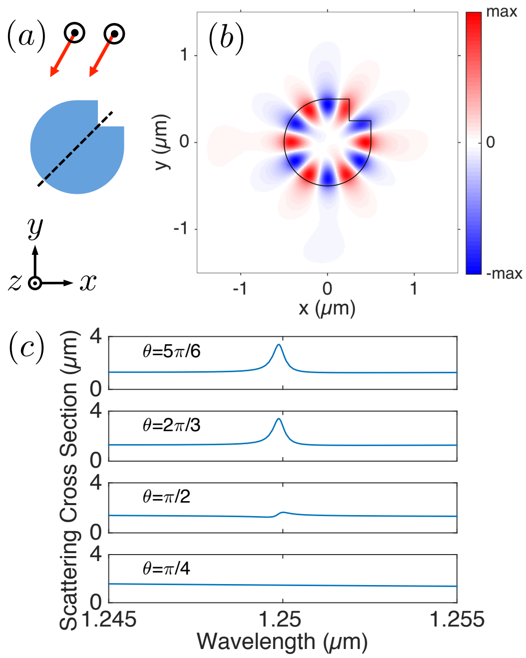

FIG. 7. (a) Schematic of the optical antenna under the illumination of a plane wave with TM-polarization. The optical antenna consists of 3/4 of a disk with radius 0.5 μm and a square with size 0.5 μm subtracting a square with size 0.25 μm. Its relative permittivity is 12. The dashed line represents the axis of mirror symmetry. The electric field of the resonant mode at a wavelength 1.2499 μm is shown in (b). (c) Spectra of scattering cross-sections for different incident angles.

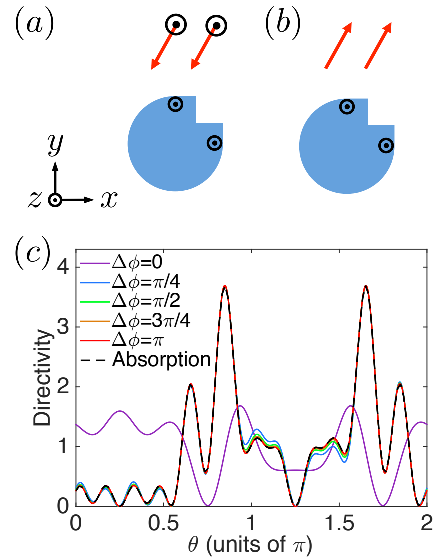

FIG. 8. Directivity of a single mode optical antenna. The optical antenna is the same as in Fig. 7. The dipoles locate at (0.45, 0) and (0, 0.45) μm relative to the center of the optical antenna. They oscillate along z-axis and have the same permittivity in absorption and same oscillation amplitude in emission. The simulation wavelength is 1.2499 μm. (a) and (b) illustrate the absorption and emission respectively. (c) shows the absorption directivity in the dashed black curve. The emission directivity when the relative phases are 0, $\pi/4$, $\pi/2$, $3\pi/4$, and π are respectively shown in purple, blue, green, orange, and red.

can vanish by carefully choosing the complex excitation amplitudes of the dipole emitters.

We now proceed to provide numerical demonstrations, for both the general case, as well as the two special cases as discussed above. We first study the special case (1) where the optical antenna supports a single resonant mode. We consider a 2D optical antenna as shown in Fig. 7(a). The optical antenna consists of 3/4 of a disk with radius 0.5 μm and a square with side length 0.5 μm subtracting a square with side length 0.25 μm. The relative permittivity is 12. This structure has a mirror plane that is tilted at 45 degree and passes through the center of the disk. This optical antenna supports a resonant mode with TM-polarization at wavelength near 1.25 μm as shown in Fig. 7(b). The mode has an odd symmetry with respect to the mirror plane mentioned above. Figure 7(c) shows the scattering cross-section spectra for incident waves at different incident angle θ, defined as the angle between the inverse of the incident wave direction and the x-axis. The scattering spectra at $\theta = \pi/2$, $2\pi/3$ and $5\pi/6$ all show a peak at the resonant wavelength. On the other hand, at incident angle $\theta = \pi/4$, the peak is absent. The resonant mode has an odd symmetry with respect to the mirror plane, and thus cannot be excited at this incident angle. These scattering spectra confirm that the optical antenna supports a single resonant mode at wavelength around 1.25 μm.

Our theory above indicates that for a single mode antenna the absorptivity and emissivity has approximately the same angular dependence in the presence of multiple dipoles. To illustrate this, we embed two dipoles in the single mode optical antenna at (0.45, 0) and (0, 0.45) μm with respect to the center of the optical antenna (Fig. 8(a-b)). We assume that the polarizability of the two dipoles are identical. The absorption directivity at the resonant wavelength is shown in the dashed black curve in Fig. 8(c). In the coherent emission process, we assume that the amplitudes of the two dipoles are the same, in consistency with the two dipoles having the same polarizability in the absorption process, while the relative phase between the two dipoles can vary. The solid curves with different colors in Fig. 8(c) represent the emission directivity for different relative phases.

We find that the emission directivity almost overlap with the absorption directivity for most relative phases. The only exception is the special case where $\Delta\phi = 0$. In this case, the current sources have an even symmetry with respect to the mirror plane mentioned above, and hence cannot excite the resonant mode with an odd symmetry. As a result, the emission directivity is completely different from the absorption directivity. For all other cases except for this special case, we observe a small discrepancy between the absorption and emission directivi-

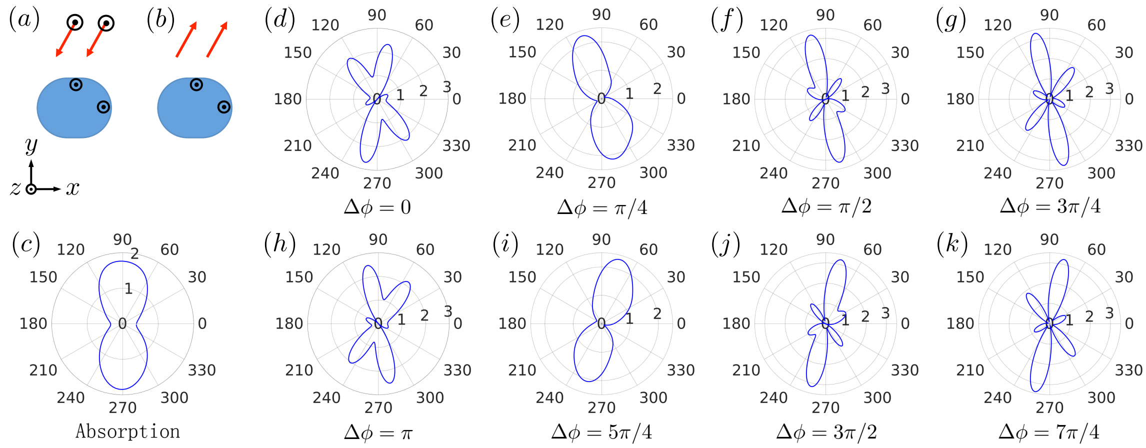

FIG. 9. Directivity of a multi-mode optical antenna coupling with two dipoles. The optical antenna and the dipoles are the same as described in Fig. 6. (a) and (b) respectively illustrate the absorption and emission. (c) shows the absorption directivity when the two dipoles have the same polarizability. (d)-(k) show the emission directivity when the amplitudes of the two dipoles are the same but the relative phase is tuned from 0 to $7\pi/4$ as indicated by $\Delta\phi$ in each figure.

ties. This small discrepancy arises from the dipole emission that does not couple to the resonant mode. In the presence of the resonant mode, such a non-resonant contribution is relatively small and hence as long as the resonant mode is excited, the discrepancy between the absorption directivity and the emission directivity is small. These numerical results validate that the emission and absorption directivities are approximately the same for a single mode optical antenna and also demonstrate the situations when the single-mode approximation breaks down.

In the general case, when multiple dipoles are coupled to an optical antenna, the absorption directivity, and the coherent emission directivity assuming that the dipoles have fixed relative phases, are different. As a numerical illustration, we use the same optical antenna studied in Section II, which supports two resonant modes with TM-polarization at the wavelength of 1.351 and 1.354 μm respectively. We consider the same absorption process as described in Fig. 6, with the incident wave at a wavelength of 1.352 μm, and with the same dipole configurations as shown in Fig. 6 where the two dipoles have the same polarizability. The absorption directivity, which was shown in Fig. 6, is replotted as a polar plot in Fig. 9(c) to facilitate the comparison with the emission directivity. In the emission process, we set the oscillation amplitudes of the two dipoles to be identical, which is consistent with the dipoles having the same polarizability in the absorption process, and allow the relative phase to vary. With different relative phases, the coherent emission directivities are shown in Fig. 9(d-k). None of the emission directivities match the absorption directivity.

In this numerical example, the dipole at (0.45, 0) μm can only excite the mode shown in Fig. 3(d), and the dipole at (0, 0.32) μm can only excite the mode shown in Fig. 3(c). These two modes have different far-field

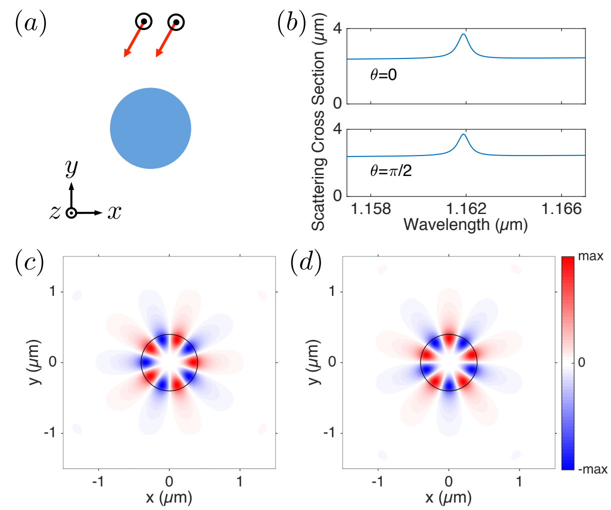

FIG. 10. (a) is a schematic of a circular optical antenna, with radius 0.4 μm, under the illumination of a plane wave with TM-polarization. Its relative permittivity is 12. The optical antenna supports a pair of degenerate resonant modes at a wavelength 1.1619μm. (b) shows scattering spectra for different incident angles. The electric fields of the resonant modes with angular dependence $\cos(m\theta)$ and $\sin(m\theta)$, where $m = 5$, are shown in (c) and (d) respectively.

radiation pattern and the interference term in Eq. 18 is non-zero regardless of the relative phase between the two dipoles. Numerical calculation also shows that the angular dependence of the interference term is different from the incoherent term in Eq. 18. Thus, for any relative phase between the two dipole oscillations, the emission directivity always differs from the absorption directivity.

In the last numerical example, we demonstrate the special case (2) as mentioned above, when the interference term in Eq. 18 can vanish approximately. We numer-

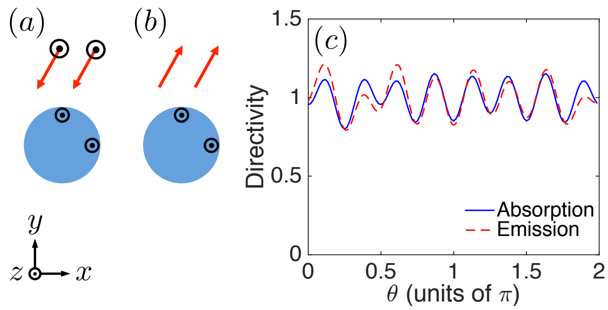

FIG. 11. Directivity of an optical antenna supporting a pair of degenerate resonant modes. The optical antenna is the same as in Fig. 10. The dipoles locate at (0.35, 0) and (0, 0.35) μm relative to the center of the optical antenna. They oscillate along z-axis and have the same permittivity in absorption and same oscillation amplitude in emission. The simulation wavelength is 1.1619 μm. (a) and (b) illustrate the absorption and emission respectively. The absorption directivity and the emission directivity when the relative phase is $\pi/2$ are respectively shown in the solid blue curve and the dashed red curve in (c).

ically study a 2D optical antenna in a circular shape with radius 0.4 μm. The relative permittivity is 12. The optical antenna supports two degenerate resonant modes with TM-polarization at wavelength near 1.1619 μm. These modes have an angular dependence $\cos(m\theta)$ (Fig. 10(c)) and $\sin(m\theta)$ (Fig. 10(d)) respectively, where $m = 5$. Figure 10(b) shows the scattering spectrum for incident waves at different incident angles. At incident angle $\theta = 0$, only the mode shown in Fig. 10(c) is excited, while only the mode shown in Fig. 10(d) is excited at incident angle $\theta = \pi/2$. The identical scattering spectrum at different incident angles results from the fact that the optical antenna supports a pair of degenerate modes.

We place two dipoles in this circular optical antenna at $(0.35, 0)$ and $(0, 0.35)$ μm with respect to the center (Fig. 11(a-b)). We assume that the polarizability of the two dipoles are identical. The absorption directivity at the resonant wavelength is shown in the solid blue curve in Fig. 11(c). In the coherent emission process, we assume that the amplitudes of the two dipoles are the same, in consistency with the dipoles having the same polarizability in the absorption process. The dipole at $(0.35, 0)$ μm excites only the resonant mode shown in Fig. 10(c) while the dipole at $(0, 0.35)$ μm excites only the resonant mode shown in Fig. 10(d). When the relative phase between the two dipoles is $\pi/2$, the amplitudes of the two resonant modes differ by $\pi/2$. Hence, the interference term in Eq. 18 vanishes, if the emission entirely results form the far-field radiation of the resonances. The emission directivity for such an excitation configuration is shown in the dashed red curve in Fig. 11(c). We find that the emission directivity and absorption directivity indeed agree reasonably well. The deviation arises from the non-resonant contributions in the dipole emission and absorption.

In summary of this section, we show that when an optical antenna couples to multiple dipoles, the coherent emission directivity and the absorption directivity are in general different. We also show two special situations where the emission directivity and absorption directivity approximately match with each other.

V. CONCLUSION

In conclusion, we study the relation between emission and absorption directivities in an optical antenna satisfying Lorentz reciprocity. We prove that the emission directivity and the absorption directivity are identical either in the case when the optical antenna couples to a single dipole, or in the case of multiple thermal emitters. When the optical antenna couples to multiple emitters, we show that the coherent emission directivity and absorption directivity are in general different. For the multiple emitter case, we also show that the coherent emission directivity approximately matches the absorption directivity either with a single-mode optical antenna, or by using a specially configured multi-mode optical antenna. This work clarifies a fundamental issue in the understanding of the properties of optical antenna, and may prove useful in the design and characterization of optical antenna for various applications.

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APPENDIX: A SINGLE-MODE OPTICAL ANTENNA COUPLING TO MULTIPLE DIPOLES

In this appendix, we discuss the case in which an optical antenna supports a single resonant mode near the frequency of interest. We apply the coupled mode theory to quantitatively show that the emission and absorption have the same directivity regardless of the number of the dipoles coupled to the antenna, as long as the resonant mode can be excited in the absorption and emission processes. We also show that the emission directivity remains approximately the same in both coherent and incoherent emission.

The dynamics of the resonant mode under incident light excitation and dipole excitation can be described by the coupled mode theory formalism [30, 31] for a single resonant mode.

$$\frac{d}{dt}c = (i\omega_0 - \gamma_0 - \gamma)c + \boldsymbol{d}^T\boldsymbol{a}_+ + \boldsymbol{d}_p^T\boldsymbol{p}, \tag{19a}$$

$$\boldsymbol{a}_- = B\boldsymbol{a}_+ + B_p\boldsymbol{p} + c\boldsymbol{d}, \tag{19b}$$

where c is the amplitude of the resonant mode, ω_0 is the resonant frequency, γ_0 and γ are the non-radiative and radiative loss rates, respectively. $\boldsymbol{a}_+$ and $\boldsymbol{a}_-$ are column vectors representing the amplitudes of the incoming and outgoing waves respectively, with each element of the vector representing the wave amplitude in a particular propagating wave channel. $\boldsymbol{p} = [p_1, p_2, \ldots, p_n]^T$ represents the amplitudes of the dipole excitations. The normalization is chosen such that $|c|^2$ represents the energy stored in the resonant mode, and $|\boldsymbol{a}_+|^2$ ($|\boldsymbol{a}_-|^2$) is the power of the incoming (outgoing) wave. $\boldsymbol{d}$ and $\boldsymbol{d}_p$ respectively represent the coupling strength from the incoming wave and the dipole to the resonant mode. Since the system has Lorentz reciprocity, the coupling strength from the resonant mode to the outgoing wave is also represented by $\boldsymbol{d}$. B and B_p respectively represent the background scattering matrix for the incoming waves and the background radiation patterns of the dipoles in the absence of the resonant mode. In the following derivation, we consider a single frequency ω near the resonant frequency ω_0 of the mode. When the resonant mode is excited, the total field can be approximately described by the resonant mode [7]. Thus, we can assume that the radiation from the resonant mode dominates the background radiation from the dipoles, i.e. $B_p\boldsymbol{p}$ is negligible compared with $c\boldsymbol{d}$ in Eq. 19, as long as the resonant mode is excited.

In the emission process, the incident light is absent $\boldsymbol{a}_+ = 0$, while the dipole is excited $\boldsymbol{p} \neq 0$. The outgoing wave is determined by the radiative decay of the resonant mode in the limit of negligible $B_p\boldsymbol{p}$, i.e. $\boldsymbol{a}_- = c\boldsymbol{d}$. Let $\boldsymbol{f}_{\hat{k},\sigma}$ be a normalized vector representing the plane wave propagating in $\hat{k}$ direction with polarization σ. The power emitted into direction $\hat{k}$ and polarization σ is:

$$\begin{aligned} P_e(\hat{k},\sigma) &= |\boldsymbol{f}^\dagger_{\hat{k},\sigma}\boldsymbol{a}_-|^2 \\ &= \frac{|\boldsymbol{d}_p^T\boldsymbol{p}|^2}{(\omega-\omega_0)^2 + (\gamma_0+\gamma)^2}|\boldsymbol{f}^\dagger_{\hat{k},\sigma}\boldsymbol{d}|^2. \end{aligned} \tag{20}$$

One can observe that the angular and polarization dependence are entirely contained in the term $|\boldsymbol{f}^\dagger_{\hat{k},\sigma}\boldsymbol{d}|^2$, as long as the radiation from the resonant mode dominates the background radiation from the dipoles. Equation 20 describes the situation when the contribution of each dipole add up coherently, as indicated by the term $|\boldsymbol{d}_p^T\boldsymbol{p}|^2$. If the dipole excitations are incoherent such that the phase of each dipole oscillation is purely random, the ensemble-averaged emitted power in direction $\hat{k}$ and polarization σ becomes

$$\langle P_e(\hat{k},\sigma)\rangle = \frac{\sum_j |d_{p,j}p_j|^2}{(\omega-\omega_0)^2 + (\gamma_0+\gamma)^2}|\boldsymbol{f}^\dagger_{\hat{k},\sigma}\boldsymbol{d}|^2. \tag{21}$$

Comparing with Eq. 20, the angular and polarization dependent term $|\boldsymbol{f}^\dagger_{\hat{k},\sigma}\boldsymbol{d}|^2$ is unchanged, although the value of the emitted power could be different. Thus, the emission directivities are the same for both the coherent and incoherent emission processes. Moreover, the interference of the field generated by different dipoles, obtained by subtracting Eq. 21 from Eq. 20, also has an angular dependence described by $|\boldsymbol{f}^\dagger_{\hat{k},\sigma}\boldsymbol{d}|^2$. Hence, the single mode situation belongs to the special case (1) as discussed in Section IV.

In the absorption process, the incident wave is a plane wave with normalized amplitude $\boldsymbol{a}_+ = \boldsymbol{f}_{\hat{k},\sigma}$. We set $\boldsymbol{p} = 0$, since the dipole emitters are described by the local polarizabilities in the absorption process and no longer function as sources of the resonant field. We assume that the total field close to the optical antenna is approximately described by the resonant mode, i.e. $\vec{E}(\vec{r}) \approx c\tilde{E}(\vec{r})$, where c is the amplitude of the resonant mode and $\tilde{E}(\vec{r})$ is the normalized field of the resonant mode [7]. Under this assumption the power absorbed by the dipoles is

$$\begin{aligned} P_a(\hat{k},\sigma) &= -\frac{1}{2}\omega\sum_j Im(\alpha_j)|\hat{e}^T_{p_j}\vec{E}(\vec{r}_j)|^2 \\ &= -\frac{1}{2}\omega\sum_j Im(\alpha_j)|\hat{e}^T_{p_j}\tilde{E}(\vec{r}_j)|^2|c|^2 \\ &= \frac{-\frac{1}{2}\omega\sum_j Im(\alpha_j)|\hat{e}^T_{p_j}\tilde{E}(\vec{r}_j)|^2}{(\omega-\omega_0)^2 + (\gamma_0+\gamma)^2}|\boldsymbol{f}^\dagger_{-\hat{k},\sigma}\boldsymbol{d}|^2, \end{aligned} \tag{22}$$

where we apply the relation that the plane wave with same polarization propagating in $\hat{k}$ and $-\hat{k}$ directions are time reversal pairs, i.e. $\boldsymbol{f}_{\hat{k},\sigma} = \boldsymbol{f}^*_{-\hat{k},\sigma}$. Equation 22 shows that the angular dependence of the absorption power is entirely captured in the term $|\boldsymbol{f}^\dagger_{-\hat{k},\sigma}\boldsymbol{d}|^2$. Comparing Eq. 20-22, we find that $P_e(\hat{k},\sigma)$ (or $\langle P_e(\hat{k},\sigma)\rangle$) is proportional to $P_a(-\hat{k},\sigma)$. Therefore, the emission directivity and the absorption directivity are identical under the single-mode approximation.

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