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Many-body stabilization of a resonant p-wave Fermi gas in one dimension

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Using the asymptotic Bethe Ansatz, we study the stabilization problem of the one-dimensional spin-polarized Fermi gas confined in a hard-wall potential with tunable p-wave scattering length and finite effective range. We find that the interplay of two factors, i.e., the finite interaction range and the hard-wall potential, will stabilize the system near resonance. The stabilization occurs even in the positive scattering length side, where the system undergoes a many-body collapse if any of the factors is absent. At p-wave resonance, the fermion system is found to feature the "quasi-particle condensation" for any value of effective range, which is stabilized if the range is above twice the mean particle distance. Slightly away from resonance, the correction to the stability condition linearly depends on the inverse scattering length. Finally, a global picture is presented for the energetics and stability properties of fermions from weakly attractive to deep bound state regime. Our results raise the possibility for achieving stable p-wave superfluidity in quasi-1D atomic systems, and meanwhile, shed light on the intriguing s- and p-wave physics in 1D that violate the Bose-Fermi duality.

Introduction. Experimental realization of p-wave ultracold Fermi gases with tunable interactions[1–6] offers great opportunities for investigating many intriguing p-wave phenomena, such as the rich orbital pairing and the topological superfluids[7–11]. However, the detection of p-wave superfluidity of fermions near a Feshbach resonance has encountered great difficulties, due to the severe atom loss which prohibits the many-body equilibration in a reasonably long time scale. In this context, the (quasi-)1D geometry emerges as a promising candidate to overcome the atom loss due to few-body collisions[12, 13]. Nevertheless, there can be another source for the loss in 1D, i.e., the many-body collapse due to negative compressibility. This is inferred from the theorem of Bose-Fermi duality [14], which maps the wave functions and the energies between p-wave fermions and s-wave bosons with inverse coupling strengths [14–16]. Accordingly, the p-wave fermions with a positive scattering length (where a two-body molecule is supported) can be mapped to attractive bosons, which implies the fermions immediately undergo a many-body collapse once across resonance to the molecule side. To ensure the stability, most of previous studies on the 1D p-wave Fermi gas have been carried out in the negative scattering length side[22–24], and there has been rare discussions on many-body physics of 1D fermions with positive scattering length.

In this work, we show that a finite effective range of p-wave interaction can save the 1D fermion system from collapsing in both sides of the resonance. Our work is simply motivated by the fact that the Bose-Fermi duality breaks down in the presence of a finite range, as shown from a simple two-body analysis[17]. In reality, the p-wave range of quasi-1D atomic system is reasonably large[18–21], as it is reduced from the large 3D effective range near p-wave Feshbach resonances[2, 6]. Hence, it is also practically important to consider its effect.

Here, using the asymptotic Bethe Ansatz we study the ground state of 1D fermions across p-wave resonance in both positive and negative sides, with a finite interaction range and in a hard-wall potential. It is found that the interplay of the finite range and the hard-wall potential helps to stabilize the system even in the positive scattering length side, where the system would undergo a many-body collapse if any of the two factors is absent. In particular, at p-wave resonance, the fermion system features the "quasi-particle condensation" for any value of effective range, where all quasi-momenta condense at a single value solely determined by the range and the density. Such a strongly correlated state is found to be stable against collapsing if the range is above twice the mean particle distance. We further extract the stability condition for fermions slightly away from resonance, and finally present a global picture for the energetics/stability property of the system from weakly attractive to deep bound state regime. These results suggest a many-body stable p-wave Fermi gas is within the reach of current cold atoms techniques.

Formalism. To determine the low-energy physics in the quasi-1D regime, i.e., when $E \ll \omega_\perp$ ($\omega_\perp$ is the frequency of transverse harmonic confinement), we utilize the following boundary condition for the many-body wave function when a pair of fermions come close to each other[22, 23] (we set $\hbar = 1$ throughout the paper):

$$\lim_{x\equiv x_j - x_i \to 0^+} \left(\frac{1}{l} + \partial_x - \xi\partial_x^2\right)\Psi(x_1, x_2, ...x_N) = 0. \quad (1)$$

Here $\{x_i\}$ ($i = 1, ...N$) are the coordinates of N spin-polarized fermions; l and ξ are, respectively, the reduced 1D p-wave scattering length and effective range, and near the 1D resonance, l is highly tunable while ξ stays at a positive constant[18–21].

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The many-body wave function takes the form:

$$\Psi(x_1,\cdots,x_N)=\sum_Q \theta(x_{q_N}-x_{q_{N-1}})\cdots\theta(x_{q_2}-x_{q_1}) \times \varphi(x_{q_1},x_{q_2},...,x_{q_N}), \quad (2)$$

where θ is the Heaviside step function, $\varphi(x_{q_1},x_{q_2},...,x_{q_N})$ is the wave function for the region $0\leq x_{q_1}\leq x_{q_2}\leq\cdots\leq x_{q_N}\leq L$ (with L the system length), and $Q=(q_1,...,q_N)$ presents a permutation of the position index of N particles. The antisymmetry of Ψ requires $\varphi(...x_i,...x_j,...)=-\varphi(...x_j,...x_i,...)$. Therefore we will only calculate the wave function in the region $x_1<x_2...<x_N$, i.e., $\varphi(x_1,x_2,...,x_N)$, and the wave function in other regions can be easily deduced according to the antisymmetry requirement. Here we consider the system confined in a hard-wall potential, which satisfies the open boundary condition(OBC)

$$\varphi(0,x_2,...,x_N)=\varphi(x_1,x_2,...,L)=0 \quad (3)$$

According to the Bethe Ansatz, we expand φ by plane-waves:

$$\varphi(x_1,x_2,...,x_N)=\sum_{P,\{r_j\}}\left[A_{P,\{r_j\}}\exp\left(i\sum_j r_jk_{p_j}x_j\right)\right] \quad (4)$$

Here $k_j(>0)$ $(j=1,...N)$ presents the quasi-momentum, and $r_j=+1(-1)$ denotes the plane-wave of the j-th particle (with coordinate x_j) moving from left(right) to right(left) in coordinate space; $P=(p_1,p_2,\cdots,p_N)$ presents a permutation of the momentum index of N particles, and $A_{P,\{r_j\}}\equiv A(k_{p_1},k_{p_2},\cdots,k_{p_N};r_1,r_2,\cdots,r_N)$ is the superposition coefficients. Substituting Eq.(4) into Eq.(2) and applying the boundary conditions Eqs.(1,3), we arrive at the following Bethe Ansatz equations(BAEs):

$$e^{i2k_jL}=\prod_{l\neq j}\frac{i(k_j-k_l)+\left[\frac{\xi}{2}(k_j-k_l)^2+\frac{2}{l}\right]}{i(k_j-k_l)-\left[\frac{\xi}{2}(k_j-k_l)^2+\frac{2}{l}\right]} \times\frac{i(k_j+k_l)+\left[\frac{\xi}{2}(k_j+k_l)^2+\frac{2}{l}\right]}{i(k_j+k_l)-\left[\frac{\xi}{2}(k_j+k_l)^2+\frac{2}{l}\right]} \quad (5)$$

The eigen-energy of the system follows $E=\sum_{j=1}^N k_j^2/(2m)$. Note that the second term in the right side of Eq.5 is due to the reflection of particles at the hard-wall boundary, which is uniquely present for the open boundary but absent for periodic boundary condition (PBC)[22, 23, 25]. Because of such term, the open boundary fermions can have distinct properties compared to the periodic ones, as will show in this paper.

Accordingly, the wavefunction $\varphi\left(x_1,x_2,\cdots,x_N\right)$ is

$$\varphi=\sum_P(-1)^PA_P\exp\left[i\left(\sum_{l<j}^{N-1}\omega_{p_lp_j}\right)+ik_{p_N}L\right]\sin(k_{p_1}x_1)\prod_{1<j<N}\sin\left(k_{p_j}x_j-\sum_{l<j}\omega_{p_lp_j}\right)\sin\left(k_{p_N}(x_N-L)\right) \quad (6)$$

with $\omega_{ab}=\arctan\frac{\frac{\xi}{2}(k_a-k_b)^2+\frac{2}{l}}{k_a-k_b}-\arctan\frac{\frac{\xi}{2}(k_a+k_b)^2+\frac{2}{l}}{k_a+k_b}$, and $A_P\equiv A(k_{p_1},k_{p_2},\cdots,k_{p_N};r_1=r_2\cdots=r_N=1)=\mathcal{N}\prod_{j<l}^N\left(ik_{p_j}-ik_{p_l}+[\frac{\xi}{2}(k_{p_j}-k_{p_l})^2+\frac{2}{l}]\right)$, where $\mathcal{N}$ is the normalization factor. Here $(-1)^P=\pm1$ is the sign factor associated with even/odd permutations of $P=(p_1,p_2,\cdots,p_N)$. Apparently φ satisfies the OBC (3).

For zero-range case ($\xi=0$), one can see that Eq.5 directly reduces to the BAE of identical bosons with coupling $c=-2/l$[26], hence the two systems have the same quasi-k distribution and thus the same E, as is exactly predicted by the Bose-Fermi duality[14]. However, when we turn on a finite range ($\xi>0$), Eq.5 can no longer be reduced to the BAE of the finite-range bosons, due to distinct forms of the energy-dependent coupling strengths for two systems[17], and thus the duality breaks down. As a result, the physics we will address below for the finite-range fermions will have no correspondence in bosons with zero or finite range[27–30]. In the rest of the paper, we use L and $E_0=(2mL^2)^{-1}$ as the unit of length and energy respectively.

At resonance. We first analyze the range effect at resonance ($1/l=0$). In this case, we find the BAEs (5) support the ground state with all equal $k_j\equiv k$, which lead to a single closed equation for k:

$$k=\pi-(N-1)\arctan(\xi k). \quad (7)$$

Accordingly, the wave function simply reduces to

$$\varphi\left(x_1,x_2,\cdots,x_N\right)\sim\prod_{j=1}^N\sin\left[kx_j-(j-1)\frac{k-\pi}{N-1}\right]. \quad (8)$$

In the case of $\xi=0$, we have $k=\pi$ and $\varphi\sim\prod_j\sin(\pi x_j)$, which means all quasi-particles condense at the lowest single-particle state with zero-point energy $E=N\pi^2$. This can be easily understood in the framework of Bose-Fermi duality, namely, the bosons condense at the lowest energy state in the non-interacting limit. As increasing ξ from 0, the Bose-Fermi duality breaks down, while, remarkably, the picture of "quasi-particle condensation" still holds true in that all the quasi-k of fermions change

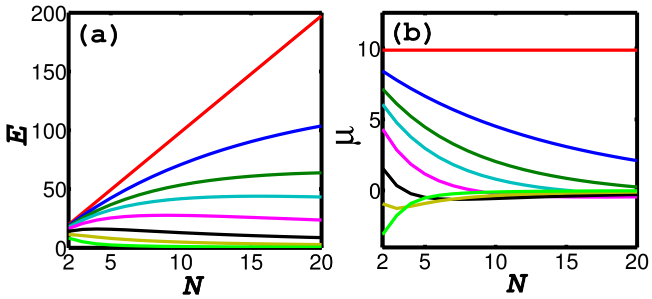

FIG. 1. (color online) Ground state energy E (a) and chemical potential μ (b) as functions of N at p-wave resonance. Several different ranges are chosen as (from top to bottom): $\xi = 0,\ 0.02,\ 0.04,\ 0.06,\ 0.1,\ 0.2,\ 0.4,\ 1.0$. The units of length and energy are L and E_0, respectively.

synchronously from π. Accordingly, in the wave function (8) each quasi-particle catches a different phase shift from left to right in the coordinate space, in order to match the OBC (3). These interesting properties uniquely reflect the finite range effect to 1D resonant fermions, which, as far as we know, has not been discovered in any fermion system before.

For large N, Eq.(7) gives the solution $k = \pi/\big[1+(N-1)\xi\big]$, and the total energy is

$$E = \frac{N\pi^2}{\Big[1+(N-1)\xi\Big]^2}. \tag{9}$$

Therefore E will decrease as increasing ξ. This can be understood from the energy-dependent scattering length $l(k)$, which follows $l^{-1}(k) = l^{-1} + \xi k^2$. Because of the zero-point energy in a hard-wall potential, the lowest collision energy ($\sim k^2$) are generally positive, which gives a stronger effective interaction [i.e., a larger $l^{-1}(k)$] as increasing ξ, and results in a lower energy E. This is to be contrast with the PBC case where the zero-point energy is absent, and we have checked that in this case the fermion energy always stays at zero regardless of ξ[31].

Importantly, the energy expression (9) also suggests a tunable stability by ξ. From (9), one can easily derive the chemical potential $\mu = \partial E/\partial N$ and the inverse compressibility $\chi^{-1} = \partial^2 E/\partial N^2$ as:

$$\mu = \frac{\pi^2\big[1-(N+1)\xi\big]}{\Big[1+(N-1)\xi\Big]^3}; \tag{10}$$

$$\chi^{-1} = \frac{2\pi^2\xi\big[(N+2)\xi-2\big]}{\Big[1+(N-1)\xi\Big]^4}. \tag{11}$$

The system is stable when $\chi > 0$, which, thus, occurs when

$$\xi > \xi_c = \frac{2}{N+2}. \tag{12}$$

In large large N limit, $\xi_c \to 2d$ where $d = 1/N$ is the mean inter-particle distance (recalling that the length unit is the system size L). This means that with an open boundary, the system can be tuned to be stable once the effective range is above twice the inter-particle distance. Again, this is very different from the PBC case[22, 23] or zero-range case[24], where the system is only stable for negative scattering lengths and the stability is not tunable by effective range. Hence, our system is stabilized by the interplay of two essential factors, namely, the presences of a finite range and an external (hard-wall) confinement.

In Fig.1(a,b), we plot $E(N)$ and $\mu(N) = E(N+1) - E(N)$ of the ground state by solving Eq.(7) for different values of ξ. We see that as ξ increases from zero, E gradually decreases from the zero-point energy $N\pi^2$, and even changes the curvature as a function of N. Accordingly, μ changes from a constant π^2 to a varying function of N. When ξ is above a critical value ξ_c, the slope of $\mu(N)$ changes from negative to positive, implying the system become stable with a positive compressibility. In Fig.2(a), we show the numerically extracted ξ_c as a function of N, which fits well with the analytical prediction (12) for large $N(\geq 5)$.

Near resonance. We now turn to the near resonance regime, i.e., $1/l \to 0^{\pm}$, where we will extract corrections to the energy and the stability condition up to the lowest order of $1/l$. Away from resonance, the quasi-

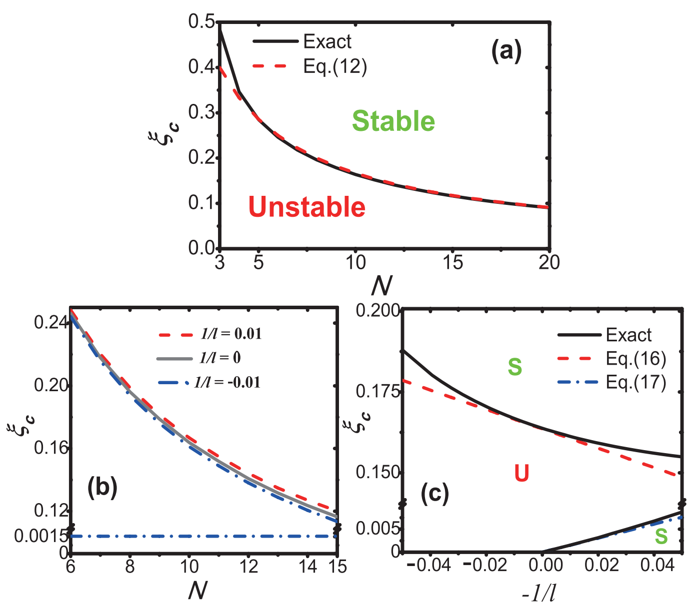

FIG. 2. (color online) (a) Critical effective range ξ_c as functions of N at resonance. The system is stable (unstable) when ξ is above (below) ξ_c. Red dashed line shows the analytical fit according to Eq.(12). (b) ξ_c as functions of N near resonance at $1/l = 0.01$ (red dashed) and at $1/l = -0.01$ (blue dash-dot). For comparison, ξ_c at resonance are shown as gray line. (c) ξ_c as changing $1/l$ for $N = 10$. "S" and "U" denote respectively the stable and unstable regions. The red dashed and blue dash-dot lines show linear fits to Eq.(16) and Eq.(17) respectively.

momenta k_j are no longer identical but depend on j. In the regime $1/l \to 0^-$, all k_j are real and we find that to the leading and next-leading orders it can be expanded as $k_j = k + c_j|l|^{-1/2} + d_j|l|^{-1}$; here k is the solution at $1/l = 0$ following Eq.(7). By expanding the BAEs (5) up to the order of $1/l$, we obtain the following equations for $\{c_j\}$ and $\{d_j\}$:

$$c_j = \sum_{l\neq j}\left(\frac{2}{c_j - c_l} - \frac{\xi}{2}(c_j - c_l) - \frac{\xi}{2}\frac{c_j + c_l}{1+(k\xi)^2}\right); \quad (13)$$

$$d_j = \sum_{l\neq j}\left(F_{jl} + G_{jl} + \frac{1}{1+(k\xi)^2}\left[\frac{1}{k} - \frac{\xi}{2}(d_j + d_l)\right]\right) \quad (14)$$

with $F_{jl} = -(d_j - d_l)(\xi/2 + 2(c_j - c_l)^{-2})$, $G_{jl} = (c_j + c_l)^2 k\xi^3/\left[4(1+k\xi^2)^2\right]$. In the regime $1/l \to 0^+$, k_j has an imaginary part scaling as $l^{-1/2}$ and we find it can be expanded as $k_j = k + ic_j l^{-1/2} - d_j l^{-1}$, where $\{c_j\}$ and $\{d_j\}$ satisfy the same sets of equations as (13,14). Therefore the energy correction up to the order of $1/l$ has a unified form for different sides of resonance, i.e., $\Delta E = (-1/l)\sum_j(c_j^2 + 2kd_j)$.

In the case of $\xi = 0$, we have $c_j = \sum_{l\neq j}\frac{2}{c_j - c_l}$ and $d_j = (N-1)/\pi$, thus the energy correction $\Delta E = (-1/l)3N(N-1)$ is identical to the interaction energy of weakly interacting bosons[28], consistent with the Bose-Fermi duality. When $\xi \neq 0$, the situation can be simplified in large N limit, where $k\xi \ll 1$ and F_{jl}, G_{jl} are negligible compared to the last term in Eq.(14). In this case, we have $\left[1 + (N-1)\xi\right]c_j = \sum_{l\neq j}\frac{2}{c_j - c_l}$ and $d_j = (N-1)/\left(k[1 + (N-1)\xi]\right)$, and the energy correction is

$$\Delta E = -\frac{1}{l}\frac{3N(N-1)}{\left[1+(N-1)\xi\right]}. \quad (15)$$

Given ΔE, we obtain the stability condition near resonance:

$$\xi > \xi_c^{\rm res} + \frac{9N^2}{2\pi^2(N+2)^2}\frac{1}{l}; \quad (16)$$

here $\xi_c^{\rm res}$ is the critical value at resonance. In addition, for $1/l \to 0^-$ the system has an extra stable region:

$$\xi < -\frac{3}{2\pi^2}\frac{1}{l}. \quad (17)$$

These results suggest that even in the positive side of resonance, the system can still be stable against collapse in a sizable region of ξ. In Fig.2(b), we plot the critical ξ_c as function of N for small $1/l$ at different sides of resonance, and we find ξ_c indeed only slightly deviate from the resonance value $\xi_c^{\rm res}$ (the gray line). In Fig.2(c), we show ξ_c for a given $N = 10$ as changing $1/l$, and the numerical results fit well with analytical predictions in Eqs.(16,17) near resonance.

Far from resonance. Further departing from resonance, above results will become invalid when the next-order correction to the energy ($\sim N^3/l^2$) dominates over the lowest one ($\sim N^2/l$) roughly at $|l| < N$. In the weak coupling limit $l \to 0^-$, the BAEs (5) give the linear expansion of k_j around the non-interacting value: $k_j = j\pi(1 + (N-1)l)$, and this leads to

$$E = E^{(0)}\left[1 + 2(N-1)l + o(l^2)\right], \quad (18)$$

where $E^{(0)} = \pi^2 N(N+1)(2N+1)/6$ is the non-interacting Fermi sea energy. Indeed, such an energy expansion (18) can also be obtained through the first-order perturbation theory based on the pseudo-potential $U = \sum_{i,j}(2l/m)\partial_{x_{ij}}\delta(x_{ij})\partial_{x_{ij}}$ and an unperturbed Fermi sea. In this limit, the effective range plays negligible role and the system is always stable.

In the deep bound state limit $l \to 0^+$, we generalize the N-string solution of attractive bosons[24, 26] to the present system as $k_j = \alpha + i(N+1-2j)\sqrt{E_{2b}}$, where $E_{2b} = (1 + 2\xi/l - \sqrt{1 + 4\xi/l})/(2\xi^2)$ is the two-body binding energy. For large N, the total energy is mainly contributed from the imaginary part of the string solution and is given by

$$E = -\frac{N(N^2-1)}{3}E_{2b}, \quad (19)$$

which represents a cluster bound state. In this limit, the system is always unstable for any values of ξ.

In Fig.3, we plot E as function of $-1/l$ for a finite range $\xi = 0.1$ at $N = 10$. We can see that E increases all along with $-1/l$, and the exact solutions can be well fitted by analytical results in the weak-coupling limit (18), near resonance (15), and the cluster bound state limit (19). Given all above analyses, we conclude that a finite range will play the most essential role in the resonance regime, where both the energetics (9,15) and the stability property (12,16,17) of the system can be significantly modified by the range effect.

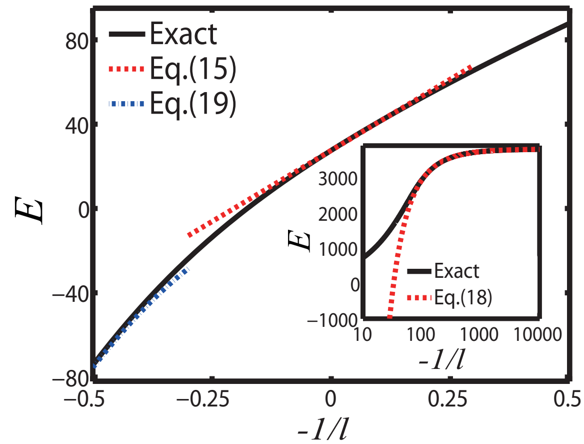

FIG. 3. (Color online) Ground state energy E as a function of interaction strength $-1/l$ for $\xi = 0.1$ at $N = 10$. The red dashed line shows the linear fit near resonance based on Eq.(15), and the blue dash-dot line shows the generalized string solution (19). Inset: E in the extremely weak coupling regime, fitted by the perturbation result (18) (see red dashed).

Discussion and Summary. Our results can be directly tested in the quasi-1D Fermi gas with an additional box-trap potential along the (1D) free direction[32]. Take the ^{40}K Fermi gas near 200G for example, according to the most updated expression for ξ[12], we have $\xi = 250 \sim 950$nm for the transverse confinement length $a_\perp = 60 \sim 120$nm. Therefore ξ can be tuned to stay below or above twice the mean inter-particle distance d, which is typically hundreds of nanometers, and thus the stability can be conveniently tuned by ξ or d (according to Eqs.(12,16)).

In summary, we have shown that a 1D spin-polarized Fermi gas can be stabilized near p-wave resonance (in both sides) in the presence of a finite range and a hard-wall potential, in contrast to the zero-range or periodic boundary condition case where a many-body collapse occurs immediately once across resonance to the positive side. These results can, hopefully, serve as useful guidelines for experiments searching for stable p-wave superfluidity in quasi-1D atomic systems. Moreover, the range-induced physics revealed in this work, including the phenomenon of quasi-particle condensation and the modified energetics at/near resonance, open up a new avenue of research for 1D boson or fermion systems, in particular, in the regime where the Bose-Fermi duality breaks down and new physics comes out.

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[25] Note that the first terms in the right side of our BAEs (Eq.(5) in main text) are slightly different from those in Ref.[22, 23]. The discrepancy may be induced by their improper use of relative momentum of two colliding fermions (with momenta k_i, k_j), which should be $(k_i - k_j)/2$ rather than $k_i - k_j$. Here, we consider that the correct definition of relative momentum of two colliding fermions is $i(k_j - k_l)/2 \sim \partial_x$, which gives rise to Eq.(5) in our paper. However, the BAEs in Ref.[22, 23] are consistent with ours if the transformation $\xi \to \xi/2$, $1/l \to 2/l$ is applied.
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