Dynamic effects in electron momentum spectroscopy of sulfur hexafluoride

Xing Wang,1 Shen Yue Xu,1 Chuangang Ning,2 O Al-Hagan,3,4 Pengfei Hu,1 Yongtao Zhao,1 Zhongfeng Xu,1 Jingkang Deng,2 Enliang Wang,5 Xueguang Ren,1,5 Alexander Dorn,5 and Don Madison3

1 School of Science, Xi’an Jiaotong University, 710049 Xi’an, China
2 Department of Physics, State Key Laboratory of Low-Dimensional Quantum Physics, Tsinghua University, Beijing 100084, China
3 Physics Department, Missouri University of Science and Technology, Rolla, Missouri 65409, USA
4 Department of Physics, King Khalid University, Abha, 61421, Saudi Arabia
5 Max-Planck-Institut für Kernphysik, 69117 Heidelberg, Germany

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Electron momentum spectroscopy (EMS) results are presented for the sulfur hexafluoride (SF₆) molecule using a high-resolution binary (e, 2e) spectrometer at incident energies (Eᵢ) of 600 eV, 1200 eV, and 2400 eV plus the binding energy. The valence orbital momentum profiles were measured with a binding energy resolution of 0.68 eV and angular resolutions of Δθ=±0.6°, Δφ=±0.85°. Whereas the two higher incident energies are in the range where normally EMS measurements do not exhibit an impact-energy dependence, the current experimental data display a dynamic dependence on the impact-energies. The measured momentum profiles are compared with predictions from a plane wave impulse approximation (PWIA) calculation using molecular orbitals obtained from a density functional theory quantum chemistry calculation. The PWIA calculations are in fairly good agreement with experiment only for 2400 eV impact-energy particularly for the summed 1τ₂u and 5τ₁u orbitals. We have also compared the experimental results for the 5a₁g state with the molecular three-body distorted-wave (M3DW) approach using the orientation-averaged molecular orbital (OAMO) approximation. Unlike the PWIA, the M3DW results are in very good agreement with the experimental data at all three measured incident energies for small momenta which indicates that dynamical distortion effects are important for this molecule.
I. INTRODUCTION

It is now well documented that electron momentum spectroscopy (EMS) serves as a sensitive probe of the electronic structure of atoms and molecules [1][4]. The EMS process is electron-impact ionization of the target where the projectile electron and ionized electron are detected in coincidence. The standard impact-energy range for these experiments is 1~2 KeV [2]. The residual ion acts as a spectator, the target electron momentum is equal in magnitude but opposite in sign to the recoil ion momentum. Within the plane wave impulse approximation (PWIA), the measured \((e, 2e)\) cross section is directly proportional to the momentum space wavefunction of the ionized electron [1][4]. As a result, the measured cross sections do not depend on the incident-electron impact-energy.

Many experimental and theoretical works have shown that the PWIA is usually valid for impact-energies in the KeV range [1][2]. However, it was recently found that the PWIA is not valid for some cases, e.g. ionization of the atomic nd orbitals [5][6], the \(^2\Pi_g\) orbital in molecular oxygen (\(O_2\)) [7], the \(1E_g\) orbital in ethane (\(C_2H_6\)) [8], and the \(1b_3\)g orbital in ethylene (\(C_2H_4\)) [9]. For these cases, experiment found a higher intensity than was predicted by the PWIA in the low momentum region, as well as an impact-energy dependent effect. For atomic orbitals, the observed higher intensity at low momenta can be well reproduced by distorted-wave calculations [5][6][10]. For molecules, calculations considering molecular vibration indicate that the higher intensity at the low momenta can be partly, but not totally, attributed to vibrational effects, see e.g. [11][13].

Recently, the Hefei EMS group [14] and Sendai EMS group [15] have performed experiments on sulfur hexafluoride (\(SF_6\)) at the impact-energy of ~1200 eV to study the interference effect or bond oscillation for the five outermost molecular orbitals of \(SF_6\), which are each constructed from the 2p atomic orbital of the F atoms. The experimental momentum distributions show higher intensity at the low momentum region than predicted by the PWIA calculations. This increased low momentum intensity has been labeled the ‘turn-up’ effect in EMS [5][7][9]. Even considering the vibrational effects, the ‘turn-up’ effect has not been satisfactorily explained [15]. An open question then is whether this observed effect can be explained by distortions of the continuum wavefunctions which has not yet been investigated.

In the present work, we conducted high resolution EMS experiments for the valence orbitals of \(SF_6\) at various projectile energies of ~600 eV, ~1200 eV, and ~2400 eV in order to examine the validity of the PWIA and the influence of the distorted-wave effects. Momentum profiles for the \(1t_{1g}, 1t_{2g} + 5t_{1u}, 3e_g, 1t_{2g}, 4t_{1u}, \) and \(5a_{1g}\) states were obtained and compared with the PWIA calculations. The experimental data show higher intensity than the PWIA in the low-momentum region and a dynamic dependence on the impact-energies also contrary to the PWIA calculations. Distorted-wave calculations were performed for ionization of the \(5a_{1g}\) state using the molecular three-body distorted-wave (M3DW) approach with the orientation-averaged molecular orbital (OAMO) approximation. Both the low momentum intensity and energy dependence of the data are rather well reproduced by the M3DW. This is the first time that the distorted-wave method is utilized to model the EMS measurements of the bound electron momentum profile for molecular orbitals.

This paper is organized as follows. After a brief description of the experimental apparatus in Sect. II, and we summarize the essential points of the two theoretical models in Sect. III. The results are presented and discussed in Sect. IV before we finish with the conclusions in Sect. V.

II. EXPERIMENTAL METHOD

The experiment was performed using a high resolution and high efficiency electron momentum spectrometer. The detail of this apparatus has been reported in previous works [16][19] and hence will not be repeated here. Briefly, it utilizes a non-coplanar symmetric geometry, i.e. the two outgoing electrons have almost equal energies and equal polar angles \(\theta_a \approx \theta_b = 45^\circ\) with respect to the direction of the incident electron beam. A double toroidal energy analyzer equipped with two large position sensitive detectors was used to detect the two outgoing electrons in coincidence. The new spectrometer can collect the multi-energy and multi-angle electrons simultaneously, thus the detection efficiency of the coincidence \((e, 2e)\) events was greatly increased. A new electron gun equipped with an oxide cathode was designed to produce the electron beam with a low energy spread and low divergence angle. Compared to the generic filament cathodes, the oxide cathode can work at a much lower temperature (~1100K), and thus a lower energy spread of the electron beam can be achieved. The electron beam size was constrained to 0.3 mm in diameter by a molybdenum aperture. The binding energy resolution in the present work is 0.68 eV, and angular resolutions are \(\Delta \theta = \pm 0.6^\circ, \Delta \phi = \pm 0.85^\circ\) respectively, which were obtained with the calibration measurements of helium and argon. A commercial \(SF_6\) gas sample with 99.9% purity was used in the experiment.

Using energy and momentum conservation, the binding energies \(\varepsilon\) and momenta \(p\) of the bound electron prior to be ejected can be determined. This momentum \(p\) is dependent on the out-of-plane azimuthal angle \(\phi\) between the two
outgoing electrons:

\[ p = \sqrt{(p_i - \sqrt{2}p_a)^2 + 2p_a^2 \sin^2(\phi/2)} \]  

where \( p_i \) and \( p_a \) (\( p_a = p_b \)) are the momenta of the incident electron and the outgoing electrons, respectively.

### III. THEORETICAL MODELS

Within the plane wave impulse approximation (PWIA) framework, and the target Hartee-Fock approximation (THFA) or the target Kohn-Sham approximation (TKSA), the triple-differential cross section (TDCS) for randomly oriented molecules are given by:

\[ \frac{d^3\sigma}{d\Omega_a d\Omega_b dE_b} \propto S^f \int d\Omega |\psi_i(p)|^2 \]  

where \( \psi_i(p) \) is the momentum space representation of a canonical Hartree-Fock or Kohn-Sham orbital wavefunction, and \( S^f \) denotes the associated spectroscopic factor, which accounts for the shake-up processes due to configuration interactions in the final state, and \( \int d\Omega \) denotes the spherical average over the random molecular orientations. The molecular orbitals were calculated using Density Functional theory (DFT) program along with the standard hybrid functional B3LYP with TZ2P Slater type basis set in the ADF program. The resulting molecular orbitals were used to generate the theoretical momentum space wavefunction using our recently developed program named NEMS [20], which formally can process any type \((s, p, d, f, g \ldots)\) of atomic orbital wavefunctions.

It should be noted that the distortion interactions for all continuum electron wavefunctions are neglected in the PWIA calculation. To consider the distorted-wave effects, the molecular three-body distorted wave (M3DW) calculation with the orientation averaged molecular orbital approximation is used to describe the present electron-impact ionization process. Although the M3DW has been described previously [21–24] we summarize the essential ideas and the particular ingredients for the current cases of interest in order to make this paper self-contained. More detailed information can be found in the given references. The direct-scattering amplitude is given by:

\[ T_{dir} = \langle \chi^+_a(k_a, r_0)\chi^-_b(k_b, r_1)C_{ab}(r_{01})|W|\phi_{DY}(R, r_1)\chi^+_i(k_i, r_0) \rangle \]  

where \( k_i, k_a \) and \( k_b \) are the wave vectors for the initial, scattered, and ejected electrons, respectively. \( \chi^+_i(k_i, r_0) \) is an initial state continuum distorted wave and the (+) indicates outgoing wave boundary conditions, \( \chi^-_a(k_a, r_0) \), \( \chi^-_b(k_b, r_1) \) are the scattered and ejected electron distorted waves with incoming wave boundary conditions, and the factor \( C_{ab}(r_{01}) \) is the final state Coulomb-distortion factor between the two electrons normally called the postcollision interaction (PCI). Here we use the exact final state electron-electron interaction and not an approximation for it such as the Ward-Macek factor [25]. The perturbation \( W = V_i - U_i \), where \( V_i \) is the initial state interaction potential between the incident electron and the neutral molecule, and \( U_i \) represents the initial state spherically symmetric approximation for \( V_i \) and \( U_i \) is used to calculate the initial state distorted wave \( \chi^+_i(k_i, r_0) \). Here \( \phi_{DY}(R, r_1) \) is the initial bound-state molecular wavefunction, which is commonly called the Dyson molecular orbital, for the active electron and it depends both on the electron coordinate \( r_1 \) and the orientation of the molecule which is designated by \( R \). The triple differential cross section (TDCS) for a given orientation \( R \) with respect to the laboratory frame can be obtained from

\[ \sigma^{TDCS}(R) = \frac{1}{(2\pi)^5} \frac{k_a k_b}{k_i} \left( |T_{dir}(R)|^2 + |T_{exc}(R)|^2 + |T_{dir}(R) - T_{exc}(R)|^2 \right). \]  

where the exchange-scattering \( T_{exc} \) is calculated similar to \( T_{dir} \) except that the particles 1 and 2 are interchanged in the final state wavefunction. The only term in the integral for the T-matrix that depends on the orientation is the Dyson wavefunction. In the OAMO (orientation averaged molecular orbital) approximation [21, 22], we average the wavefunction over all orientations and then we calculate a single TDCS. This approximation makes these calculations tractable with present computing resources.

The important differences between the PWIA and the M3DW are the following. In the PWIA, all continuum state electron wavefunctions are treated as plane waves and the post collision interaction is treated only to first order. In the M3DW, all continuum electron wavefunctions are distorted waves. A distorted wave is a numerical wavefunction that is a solution of the Schrödinger equation for a numerical distorting potential calculated based upon the location of all the atomic nuclei and the electronic charge density of the molecule. For the interaction between the continuum
electron and the bound electrons, the molecular electronic charge density is averaged over all orientations and then it is used to calculate the radially dependent potential between the bound electrons and the continuum electron. For the interactions with the atomic nuclei, each nuclei is averaged over all orientations which means that each nuclear charge is effectively placed on a sphere centered on the center of mass. For SF₆, this means that there is a charge of 16 (sulfur) at the center of mass and a charge of 54 on a sphere of radius 3 a.u. since all 6 fluorine nuclei are almost the same distance from the center of mass. When the electronic and nuclear parts are combined, we have a screened potential which, for a fixed radius, corresponds to a potential equivalent to the net charge inside a sphere of that radius. For the incoming electron, the asymptotic form of this potential is zero and for the two outgoing electrons, the asymptotic form of this potential corresponds to an effective charge of +1. The other important difference between the PWIA and the M3DW is that the M3DW has the post collision interaction to all orders of perturbation theory instead of just to the first order. This will be important for equal energy electrons leaving the collision at small angular separations but probably is not important for the present kinematics.

IV. RESULTS AND DISCUSSIONS

SF₆ has an octahedral geometry, O₆h symmetry, consisting of six fluorine atoms attached to a central sulfur atom. Within Hartree-Fock theory, the ground state configuration can be written as:

\( \text{core}^{2s}(4a_1)^2(3t_1u)^6(2e_g)^2(5a_1g)^2(4t_1u)^5(1t_2g)^5(3e_g)^4(5t_1u)^5(1t_2u)^5(1t_1g)^5 \).

Figure 1 presents the measured binding energy spectra of SF₆. Here, the binding energy \( \varepsilon \) is equal to the incident electron energy minus the two outgoing electron energies (\( \varepsilon = E_e - E_a - E_b \)). The outer valence orbitals of SF₆ are well resolved in the binding energy spectrum except for the overlap of 5t₁u and 1t₂u, which cannot be resolved even with the high resolution photoelectron spectroscopy (PES) [26]. The binding energy spectra at different \( \phi \) angles can be obtained simultaneously using the spectrometer. From the angle-energy density map displayed in the Fig.1(a), the basic features of EMS for each orbitals can be seen directly. Density minima are observed for each orbital at the azimuthal angle \( \phi = 0^\circ \) (\( \phi \approx 0^\circ \)) for the 5a₁g orbital because the S2s state contributes to the 5a₁g orbital while the others contain mainly the F2p state. The binding energy spectrum in the bottom panel of Fig. 1 was obtained by summing all the energy spectra for different \( \phi \) angles. To obtain the experimental momentum distribution for each orbital, the binding energy spectra at the different \( \phi \) angles were fitted with the multiple Gaussian functions. The peak centers were determined through high resolution PES, and the widths were determined by combining the experimental energy resolution and the vibrational broadening on PES. The experimental momentum distributions were obtained by fitting the intensity for each state plotted as a function of the momentum \( p \).

To compare the experimental momentum distributions with theory, a normalization procedure is needed because the experimental intensity are on a relative scale. A global normalization factor was determined by fitting the summed experimental momentum distributions in Fig. 1 to the corresponding PWIA distributions, i.e. 5a₁g + 4t₁u + 1t₂g + 3e₂ + 5t₁u + 1t₂u + 1t₁g, and then this factor was used to normalize the experimental distributions for each orbitals. As Fig.2 shows, the experimental distributions for different impact-energies agree well with each other for the momentum region \( p > 1.0 \text{ a.u.} \), therefore the data in this region were used to determine the normalization factor. The best fit to the experimental data in this region of momenta was obtained by normalizing the data to the PWIA at \( p \) about 1.5 a.u.. From Fig. 2 one can also see a energy-dependent effect for \( p < 1.0 \text{ a.u.} \). Below we will discuss this structure in more detail for the individual orbitals.

The experimental momentum distributions for projectile energies of 600 eV, 1200 eV and 2400 eV are compared in Fig.3(a)-(f) with the PWIA calculations for the states 1t₁g, 5t₁u + 1t₂u, 3e₂, 1t₂g, 4t₁u and 5a₁g, respectively. The experimental data are generally well described by PWIA in the high momentum range (\( p > 1.0 \text{ a.u.} \)). However at low momenta (\( p < 1.0 \text{ a.u.} \)) there is an unexpected higher intensity observed compared to PWIA, which has been called the ‘turn-up’ effect. Such ‘turn-up’ effects can be qualitatively explained by the distortion of the incoming and outgoing electron waves in the target and the ion potentials, since the size of the effect decreases with increasing impact-energy [5]. For most cases, the ‘turn-up’ effect occurs in the low momentum range and is most evident at impact-energy of 600 eV, becomes smaller at 1200 eV, and is much smaller at 2400 eV. Particularly for the summed 1t₂u and 5t₁u orbitals shown in Fig.3(b), both experiment and the PWIA have a maximum intensity at \( p \approx 0.5 \text{ a.u.} \) and a second shoulder structure at \( p \approx 1.5 \text{ a.u.} \). For this case, the increased intensity is seen at the peak (\( p \approx 0.5 \text{ a.u.} \)). Increased intensity at low momenta have been observed in the atomic nd orbitals where the distorted-wave impulse approximation (DWIA) calculations supported the idea that the ‘turn-up’ effect at low momenta is due to distorted-wave effects [5]. Further analysis of the orbital symmetry indicates that the low momenta can contribute to the electron density in the near nuclear region in d orbitals where distortion effects should be the strongest. The DWIA calculations for atomic nd orbitals at 600 eV, 1200 eV and 2400 eV confirmed that such distortion effects should decrease with increasing impact-energy [5]. Unfortunately, theoretical calculations using the DWIA for molecules have not been reported so far.
FIG. 1. Momentum-energy density map of SF$_6$ (top) and binding energy spectrum summed over all azimuthal angles $\phi$ (bottom), obtained at the impact-energy of 1200 eV plus binding energies. The dashed lines represent Gaussian fits to the individual peaks and the solid curve is the summed fit. The labels of each peak are the orbital assignment.

FIG. 2. Measured and PWIA calculated spherically averaged momentum profiles for the sum of $5a_{1g} + 4t_{1u} + t_{2g} + 3e_g + 5t_{1u} + 1t_{2u} + 1t_{1g}$ orbitals of SF$_6$ at the impact-energies of 600, 1200, and 2400 eV. The solid line is the PWIA results with DFT-B3LYP/TZ2P method.
FIG. 3. Measured and PWIA calculated spherically averaged momentum profiles for the outer valence orbitals of SF$_6$: a) 1t$_{1g}$; b) 5t$_{1u}$ + 1t$_{2u}$; c) 3e$_g$; d) 1t$_{2g}$; e) 4t$_{1u}$; f) 5a$_{1g}$ at impact energies of 600, 1200, and 2400 eV. The solid lines are PWIA calculated results with DFT-B3LYP/TZ2P method, which have been convolved with the experimental resolution at 2400 eV.

For the 1t$_{1g}$, 3e$_g$, 1t$_{2g}$ and 4t$_{1u}$ orbitals, the deviations between the experimental distributions and PWIA at low momenta decrease with increasing impact energy. However, the agreements between the experimental data and PWIA for 2400 eV are not as good as is seen for the case of the 1t$_{2u}$+5t$_{1u}$ orbitals. The influence of molecular vibration is a possible source of the observed disagreement for low momenta which has been analyzed by Watanabe et al. [15]. It was found that for the 1t$_{1g}$ orbital the vibrational effects calculation predicts higher intensity than the equilibrium geometry calculation for low momenta which reduced the deviation (∼50%) from the experimental result. While there are no noticeable difference between the two kinds of calculations for the 3e$_g$ and 1t$_{2g}$ orbitals, showing that influence of nuclear motion on their electron momentum distributions probably cannot explain this effect. Other effects, such as dynamic correlation, have been mentioned in the literature [7] as a possible reason for the higher intensity at low momenta in the experiment.

The PWIA theory predicted a $s-p$ type momentum distribution for the 5a$_{1g}$ orbital (Fig. 3(f)), which is generally consistent with the shape of the experimental distributions. We now focused on the results of the 5a$_{1g}$ orbital since the M3DW calculations can be performed for this orbital. Figure 4 shows the molecular orbitals for SF$_6$. It can be seen from Fig. 4 that, except for the 5a$_{1g}$ orbital, all other orbitals will produce a zero wavefunction with the OAMO approximation due to the anti-symmetry. Thus, only the 5a$_{1g}$ orbital can be calculated using the M3DW model. In Fig. 5, the M3DW results are compared with experiment and the PWIA where both theory and experiment are normalized to unity at $p ≈ 0$ a.u.. As can be seen from the figure, the M3DW is in reasonably good agreement with experiment for low momenta for all three impact energies. The PWIA, on the other hand, is the same for all
energies and agrees best with the data for the highest energy. The M3DW results are not in very good agreement with experiment for the peak observed near a momentum of unity. However, this is a well-known problem associated with the OAMO approximation for lower energy TDCS work. In the PWIA, the momentum is equal (in magnitude) to the momentum of the recoil ion. The low energy TDCS results have a peak (called the binary peak) which would correspond to low recoil ion momenta and a second much smaller peak (called the recoil peak) which would correspond to high recoil ion momenta. The OAMO results typically are in reasonably good agreement with the binary peak and significantly underestimate the recoil peak (very similar to the results seen in Fig. 5) [27–32]. In spite of the fact that the M3DW underestimates the observed peak at $p$ around 1.1 a.u., it is interesting to note that M3DW predicted the same trend for the impact-energy dependence of the 1.1 a.u. intensity as the experimental observation with the intensity increasing with increasing energy.

The most likely problem with the OAMO approximation can be seen from an examination of the $5a_{1g}$ wavefunction. As Fig. 4(f) shows, there are six anti-symmetric lobes, which will produce zero intensity at a larger $r$ region with OAMO approximation. The central ball in $5a_{1g}$ orbital is mainly attributed to the $S2s$ electron, while the six outer lobes are from the $F2p$ and $2s$ electron. Molecular orbital population analysis predicted that the $S2s$ contributes about 33% of the gross orbital electron density, while the contribution from the F atoms is 67%. The OAMO approximation probably underestimates the contributions from the six F atoms, which are mainly at $p \approx 1$ a.u.. Recent $(e, 2e)$ studies of CH$_4$ [33] and H$_2$O [34] indicate that it is more accurate to perform a proper average (PA) over orientation-dependent cross sections than to use the OAMO. The computational cost of the PA method, however, is much higher than the OAMO and we do not presently have sufficient computational resources to perform a PA calculation for energies this high. A recent multicenter distorted-wave method (MCDW) [35, 36] developed for high impact energies is expected to become a suitable model for EMS for investigating the distorted-wave effect in the future.

In the M3DW model, the continuum wavefunctions (distorted waves) are elastic scattering waves. The potential

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**FIG. 4.** The molecular orbital patterns of SF$_6$ with density contour value 0.1.
V. CONCLUSIONS

We have reported a combined experimental and theoretical study of electron momentum spectroscopy (EMS) of sulfur hexafluoride (SF$_6$). The electron momentum profiles of SF$_6$ valence orbitals were measured using a high resolution binary (e, 2e) spectrometer at various projectile energies of 600 eV, 1200 eV, and 2400 eV. The binding energy resolution of $\Delta \varepsilon = 0.68$ eV allows us to nicely resolve all the valence orbitals except for the overlap of the 5t$_{1u}$ and 1t$_{2u}$ orbitals. The purpose of this study was to provide a direct test of the distorted-wave effect as a function of impact-energy.

The experimental momentum distributions for the individual orbitals were compared with plane wave impulse approximation (PWIA) results and it was found that the experimental data are generally well described by the PWIA in the high momentum range ($p > 1.0$ a.u.). For low momenta, the experimentally observed intensity was higher than expected and also showed an energy dependence not predicted by the PWIA. This discrepancy can be qualitatively explained by the distortion of the incoming and outgoing electron waves in the target and the ion potentials. This
explanation is supported by the fact that the discrepancy between experiment and theory decreases with increasing impact-energy, particularly for the summed $1t_{2u}$ and $5t_{1u}$ momentum profile. Additionally, the higher intensity at low momenta for the low impact-energy of 600 eV becomes smaller at 1200 eV, and, for most cases, is either small or gone when the projectile energy is further increased to 2400 eV.

Distorted-wave calculations for electron momentum spectroscopy of molecules were reported for the first time using the molecular three-body distorted-wave (M3DW) approach coupled with the orientation-averaged molecular orbital approximation (OAMO) for the $5a_{1g}$ orbital of SF$_6$. Unlike the PWIA, the M3DW properly predicts the low momentum features and energy dependent change in the momentum profiles. This is the first direct demonstration of the influence of distorted-wave effects on the momentum profiles for molecules.

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