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Phys. Rev. A **97**, 043818 — Published 11 April 2018

DOI: 10.1103/PhysRevA.97.043818

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(Dated: March 5, 2018)

We study an optomechanical transistor, where an input field can be transferred and amplified unidirectionally in a cyclic three-mode optomechanical system. In this system, the mechanical resonator is coupled simultaneously to two cavity modes. We show that it only requires a finite mechanical gain to achieve the nonreciprocal amplification. Here the nonreciprocity is caused by the phase difference between the linearized optomechanical couplings that breaks the time-reversal symmetry of this system. The amplification arises from the mechanical gain, which provides an effective phonon bath that pumps the mechanical mode coherently. This effect is analogous to the stimulated emission of atoms, where the probe field can be amplified when its frequency is in resonance with that of the anti-Stokes transition. We show that by choosing optimal parameters, this optomechanical transistor can reach perfect unidirectionality accompanied with strong amplification. In addition, the presence of the mechanical gain can result in ultra-long delay in the phase of the probe field, which provides an alternative to controlling light transport in optomechanical systems.

I. INTRODUCTION

The interaction between light and mechanical objects in the low-energy scale has been intensively studied both in theory and in experiment during the past two decades. Given the rapid advance in microfabrication [1–3], cavity optomechanical systems have been exploited for both fundamental questions and various applications. Such systems provide an appealing platform to study the quantum behavior of macroscopic system [4]. Meanwhile, applications of optomechanical systems, such as ultra-sensitive measurement in the molecular scale [5–10], weak-force detection [11], quantum wavelength conversion between microwave and optical frequencies [12, 13], and quantum illumination [14], have been investigated. Furthermore, optomachanical systems have also been used to demonstrate quantum optical effects, such as optomechanically induced transparency and absorption [15–24] and optomechanically induced amplification [25, 26].

Among these applications, nonreciprocal transmission and amplification of light fields are of great interest, similar to their analogues in electronic devices. The nonreciprocal devices, which exhibit asymmetric response if the input and output channels are interchanged, can protect unwanted signals from entering into the network, where are essential to signal processing and communications. At the heart of the nonreciprocal devices is an element that breaks the Lorentz reciprocity of the system [44]. Effects that have been used to realize the nonreciprocity include the magneto-optical Faraday effect in ferrite materials [45–48], parametric modulation of system parameters [49–52], optical nonlinearity [53, 54], chiral light-matter interaction [55], and the rotation of device in the real space [56]. It has been shown that the nonreciprocal propagation of light can be realized with optical devices [28–31]. Meanwhile, unconventional propagation of light has been demonstrated by engineering effective non-Hermitian Hamiltonians in optical systems [57–64], which can be used to realize on-chip isolators and circulators [65]. Recently, $\mathcal{PT}$ symmetry breaking in optomechanical systems with coupled cavities, often accompanied by the coalescence of eigenstates at an exceptional point in the discrete spectrum, has been studied [66, 67], and low-power phonon emissions [66], chaos [68], nonreciprocal energy transfer [69], and asymmetric mode switching [70] have been observed. More recently, optomechanical isolators, circulators, and directional amplifiers have been studied in multi-mode systems by modulating the gauge-invariant phases [32–43].

Here we present a scheme for realizing an optomechanical transistor in a cyclic three-mode optomechanical system with finite mechanical gain. In this system, two optical modes are linearly coupled with each other and are also coupled simultaneously to a common mechanical mode. The phase difference between the optomechanical couplings breaks the time-reversal symmetry of this system and ensures nonreciprocity in the state transmission. Meanwhile, amplification arises from the mechanical gain, which induces a phonon-photon parametric process. By combining nonreciprocity and amplification, the transport of signal fields through this system thus resembles that of electrical currents through a transistor. Compared with our previous work [74], this approach does not require the frequency matching between the pump fields on the cavity and the mechanical modes. Furthermore, we show that within the operational parameter window of this optomechanical transistor, an ultra-long delay in the phase of the probe field occurs due to the finite mechanical gain. These findings provide an alternative way to achieving controlled light transport in optomechanical systems and can stimulate future works in light amplifi-

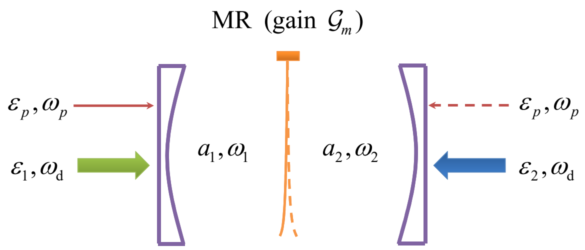

FIG. 1: The schematic of a cyclic three-mode optomechanical system driven by two pump fields of amplitudes ε_1 and ε_2 with frequency ω_d. A probe field with amplitude ε_p and frequency ω_p is applied to one of the cavities (to cavity 1 from the left hand side or cavity 2 from the right hand side). A mechanical gain $\mathcal{G}_m$ is engineered on the mechanical mode of frequency ω_m. The cavities and the mechanical resonator are coupled via radiation-pressure forces and the cavities are directly coupled to each other.

cation with optomechanical devices.

This paper is organized as follows. In Sec. II, we introduce the three-mode optomechanical system with finite mechanical gain. The stability of this system is also discussed in this section. We then derive the transmission coefficients of this system in a generic setting in Sec. III. The behavior of the optomechanical transistor and the ultra-long delay in the phase of the probe field are studied in detail in Sec. IV, Finally, conclusions are given in Sec. V.

II. THE MODEL

Consider an optomechanical system that contains a mechanical mode with frequency ω_m and two cavity modes with frequencies ω_1 and ω_2, respectively, as illustrated in Fig. 1. The Hamiltonian of this system has the form ($\hbar = 1$)

$$\begin{aligned} H &= H_0 + H_I + H_d, && (1)\\ H_0 &= \omega_1 a_1^\dagger a_1 + \omega_2 a_2^\dagger a_2 + \omega_m b^\dagger b, && (2)\\ H_I &= J(a_1^\dagger a_2 + a_1 a_2^\dagger) + \sum_i g_i a_i^\dagger a_i (b + b^\dagger), && (3)\\ H_d &= \sum_i i\varepsilon_i (a_i^\dagger e^{-i\omega_d t} e^{i\theta_i} - \text{H.c.}). && (4) \end{aligned}$$

Here H_0 is the Hamiltonian of the uncoupled cavity and mechanical modes, where $a_i^\dagger$ (a_i) for $i = 1, 2$ and $b^\dagger$ (b) are the corresponding creation (annihilation) operators of these modes. The Hamiltonian H_I describes the linear interaction between the cavities with coupling strength J and the radiation-pressure interactions between the cavity and the mechanical modes with coupling strength g_1 and g_2. The Hamiltonian H_d represents the pump fields applied to the cavities with frequency ω_d, amplitudes $\varepsilon_{1,2}$ and phases $\theta_{1,2}$. Without loss of generality, we assume that the parameters J, $g_{1,2}$, and $\varepsilon_{1,2}$ are real numbers. This system can be realized in various configurations, such as the membrane-in-the-middle setup studied in Refs. [19, 75–78], where one mechanical membrane is inserted between two cavity mirrors and is coupled to two cavity modes simultaneously, and the optomechanical setup where a mechanical resonator is coupled to multiple optical modes in a Fabry Perot cavity [79–81]. In the rotating frame of ω_d, the Hamiltonian becomes

$$\begin{aligned} H_{\text{rot}} &= \sum_i \Delta_i a_i^\dagger a_i + \omega_m b^\dagger b + J(a_1^\dagger a_2 + a_1 a_2^\dagger)\\ &\quad + \sum_i g_i a_i^\dagger a_i \left(b + b^\dagger\right) + i\varepsilon_i (a_i^\dagger e^{i\theta_i} - \text{H.c.}), \qquad (5) \end{aligned}$$

where $\Delta_i = \omega_i - \omega_d$ ($i = 1, 2$) is the detuning of the pump field from the cavity resonance.

We assume the cavity and the mechanical modes are subject to input noise denoted by f_i^{in} ($i = 1, 2$) for the cavity input operators and f_b^{in} for the mechanical input with $\langle f_i^{in} \rangle = \langle f_b^{in} \rangle = 0$. With Hamiltonian (5), the Quantum Langevin equations (QLEs) for the cavity modes are

$$\begin{aligned} \dot{a}_1 &= \left\{-\gamma_1 - i\left[\Delta_1 + g_1\left(b + b^\dagger\right)\right]\right\} a_1 - iJa_2\\ &\quad + \varepsilon_1 e^{i\theta_1} + \sqrt{2\gamma_1} f_1^{in}, && (6)\\ \dot{a}_2 &= \left\{-\gamma_2 - i\left[\Delta_2 + g_2\left(b + b^\dagger\right)\right]\right\} a_2 - iJa_1\\ &\quad + \varepsilon_2 e^{i\theta_2} + \sqrt{2\gamma_2} f_2^{in}, && (7) \end{aligned}$$

where γ_i ($i = 1, 2$) is the decay rate of the corresponding cavity mode. The QLE for the mechanical mode is

$$\begin{aligned} \dot{b} &= \left(\mathcal{G}_m - \gamma_m - i\omega_m\right) b - i\left(g_1 a_1^\dagger a_1 + g_2 a_2^\dagger a_2\right)\\ &\quad + \sqrt{2\mathcal{G}_m} f_b^{in} + \sqrt{2\gamma_m} f_{b,c}^{in}, && (8) \end{aligned}$$

where γ_m represents the intrinsic damping rate and $\mathcal{G}_m$ denotes the controllable gain of the mechanical mode. The noise operators $f_{b,c}^{in}$ and f_b^{in} are associated with γ_m and $\mathcal{G}_m$, respectively. Under the condition $\mathcal{G}_m \gg \gamma_m$, the effect of the mechanical gain dominates over that of the intrinsic damping. Thus we neglect the intrinsic damping of the mode b and the corresponding noise operator. The QLE for the mechanical mode then becomes

$$\begin{aligned} \dot{b} &= \left(\mathcal{G}_m - i\omega_m\right) b - i\left(g_1 a_1^\dagger a_1 + g_2 a_2^\dagger a_2\right)\\ &\quad + \sqrt{2\mathcal{G}_m} f_b^{in}. && (9) \end{aligned}$$

In practice, the mechanical gain can be obtained with various methods, e.g., by phonon lasing or by coupling the mechanical mode to another cavity mode driven with a blue-detuned optical pump field [67].

With strong pumping, the steady-state solutions of the cavity modes $\langle a_i \rangle$ and of the mechanical mode $\langle b \rangle$ can be

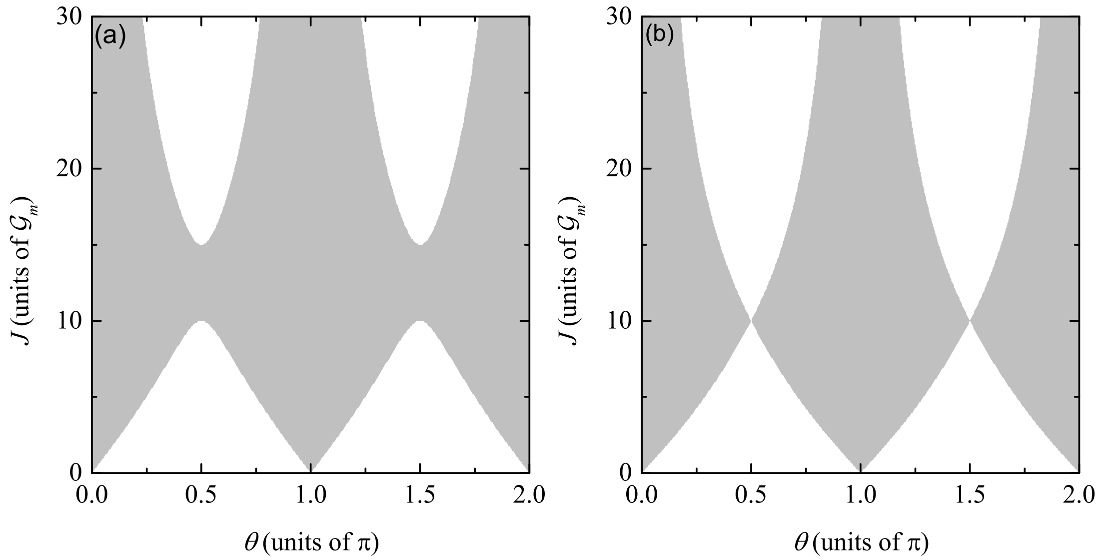

FIG. 2: (Color online) Numerical calculation of the stability of this system with the parameters (a) $\gamma_1 = 10\mathcal{G}_m$, $\gamma_2 = 15\mathcal{G}_m$ and (b) $\gamma_1 = 10\mathcal{G}_m$, $\gamma_2 = 10\mathcal{G}_m$. Other parameters are $G_1 = |G_2| \equiv G = \sqrt{J\mathcal{G}_m/\sin\theta}$, and $\omega_m/\mathcal{G}_m = 10^3$. Each panel contains two regions. The gray (white) regions represent the stable (unstable) regions of this system. In particular, in (b), when $J = 10\,\mathcal{G}_m$, the system is stable with all values θ except for $\theta = \pi/2$.

obtained as

$$\langle a_1\rangle = \frac{(\gamma_2 + i\Delta_2')\,\varepsilon_1 e^{i\theta_1} - iJ\varepsilon_2 e^{i\theta_2}}{(\gamma_1 + i\Delta_1')\,(\gamma_2 + i\Delta_2') + J^2}, \tag{10}$$

$$\langle a_2\rangle = \frac{(\gamma_1 + i\Delta_1')\,\varepsilon_2 e^{i\theta_2} - iJ\varepsilon_1 e^{i\theta_1}}{(\gamma_1 + i\Delta_1')\,(\gamma_2 + i\Delta_2') + J^2}, \tag{11}$$

$$\langle b\rangle = \frac{-i(g_1\,|\langle a_1\rangle|^2 + g_2\,|\langle a_2\rangle|^2)}{-\mathcal{G}_m + i\omega_m} \tag{12}$$

with $\Delta_i' = \Delta_i + g_i[\langle b\rangle + \langle b\rangle^*]$. These coupled equations can be solved self-consistently. By assuming each operator is a sum of the steady-state solution and its quantum fluctuation, i.e., $a_i = \langle a_i\rangle + \delta a_i$ and $b = \langle b\rangle + \delta b$, and neglecting the nonlinear terms, we obtain a set of linearized QLEs:

$$\begin{aligned}
\delta\dot{a}_1 &= (-\gamma_1 - i\Delta_1')\,\delta a_1 - iG_1\left(\delta b + \delta b^\dagger\right) \\
&\quad -iJ\delta a_2 + \sqrt{2\gamma_1}f_1^{in}, &(13)\\
\delta\dot{a}_2 &= (-\gamma_2 - i\Delta_2')\,\delta a_2 - iG_2\left(\delta b + \delta b^\dagger\right) \\
&\quad -iJ\delta a_1 + \sqrt{2\gamma_2}f_2^{in}, &(14)\\
\delta\dot{b} &= (\mathcal{G}_m - i\omega_m)\,\delta b - i(G_1\delta a_1^\dagger + G_1^*\delta a_1) \\
&\quad -i(G_2\delta a_2^\dagger + G_2^*\delta a_2) + \sqrt{2\mathcal{G}_m}f_b^{in}, &(15)
\end{aligned}$$

where $G_i = g_i\langle a_i\rangle$ $(i = 1, 2)$ is the effective linear coupling between the ith cavity and the mechanical mode. We assume that the system is operated in the resolved sideband regime with $\gamma_i, \mathcal{G}_m, G_i \ll \omega_m$ and $\Delta_i' \sim \omega_m$. With these assumptions, we can apply the rotating-wave approximation to the above QLEs and neglect the fast-oscillating counter-rotating terms. The QLEs become

$$\begin{aligned}
\delta\dot{a}_1 &= -\Gamma_{10}\delta a_1 - iG_1\delta b - iJ\delta a_2 + \sqrt{2\gamma_1}f_1^{in}, &(16)\\
\delta\dot{a}_2 &= -\Gamma_{20}\delta a_2 - iG_2\delta b - iJ\delta a_1 + \sqrt{2\gamma_2}f_2^{in}, &(17)\\
\delta\dot{b} &= -\Gamma_{m0}\delta b - iG_1^*\delta a_1 - iG_2^*\delta a_2 + \sqrt{2\mathcal{G}_m}f_b^{in}, &(18)
\end{aligned}$$

where $\Gamma_{i0} = \gamma_i + i\Delta_i'$, and $\Gamma_{m0} = -\mathcal{G}_m + i\omega_m$. For simplicity, we rewrite these QLEs in matrix form with

$$\frac{d}{dt}\lambda = -M\lambda + \Upsilon\lambda^{in}, \tag{19}$$

where the fluctuation vector $\lambda = (\delta a_1, \delta a_2, \delta b)^T$, the input field $\lambda^{in} = (f_1^{in},\ f_2^{in},\ f_b^{in})^T$, the coupling matrix for the input operators $\Upsilon = \mathrm{diag}\big(\sqrt{2\gamma_1}, \sqrt{2\gamma_2}, \sqrt{2\mathcal{G}_m}\big)$, and dynamic matrix

$$M = \begin{pmatrix} \Gamma_{10} & iJ & iG_1 \\ iJ & \Gamma_{20} & iG_2 \\ iG_1^* & iG_2^* & \Gamma_{m0} \end{pmatrix}. \tag{20}$$

The stability of this optomechanical system can be influenced by the mechanical gain. We derive the stability condition for this system using the Routh-Hurwitz criterion, which is equivalent to the requirement that the eigenvalues of matrix M have no positive real part. In Fig. 2, we plot two typical cases that are employed to investigate the optical response of this system in the following sections, where the gray regions are stable and the white regions are unstable. When the system parameters are $J = 11\mathcal{G}_m$, $\gamma_1 = 10\mathcal{G}_m$, and $\gamma_1 = 15\mathcal{G}_m$, the stable region covers all values of θ, which can be seen in Fig. 2(a). However, when $J = \gamma_1 = \gamma_2 = 10\mathcal{G}_m$, as shown in Fig. 2(b), the system is stable with all the possible values of θ except for $\theta = \pi/2$.

Practical parameters of our system can be as follows. For optical modes at the frequency $\omega_i/2\pi = 10^8$ MHz $(i = 1, 2)$, the cavity damping rates can be $\gamma_i/2\pi \sim 1-10$ MHz with current technology. For convenience of discussion, we choose the mechanical frequency $\omega_m/2\pi = 100$ MHz. The strengths of the linearized optomechanical couplings $G_i/2\pi$ $(i = 1, 2)$ can reach a few tens of MHz [79–81]. Moreover, the coupling strength J between the cavity modes can be designed to be the same order of magnitude as the damping rates γ_i [33]. The magnitude of the mechanical gain $\mathcal{G}_m$ depends on specific experimental setups. Small value of $\mathcal{G}_m$ can be utilized to observe OMIT-like spectra and group delay and can be obtained by driving the mechanical mode directly. Meanwhile, tunable and large mechanical gain can be achieved by the phonon lasing approach or by applying a blue-detuning optical pump. The corresponding mechanical gain is proportional to the square of the effective optomechanical coupling, as shown in Ref. [67].

III. TRANSMISSION COEFFICIENTS

Apply a probe field to cavity 1 in the form of $i(\varepsilon_p a_1^\dagger e^{-i\omega_p t} - \text{H.c.})$, as illustrated by the thin solid arrow in Fig. 1. The response to a probe field applied to cavity 2 (the thin dashed arrow in Fig. 1) can be obtained by exchanging the subscripts 1 and 2 in the following results. We assume that the amplitude of the probe field ε_p is much smaller than that of the control field $\varepsilon_{1,2}$, and the steady-state solutions of the operators a_1, a_2, b will not be affected by the probe field. Hence the only change in the QLEs is that one extra term $\varepsilon_p e^{-i(\omega_p-\omega_d)t}$ is added to (16). To solve this set of linear QLEs, we use another interaction picture by transforming $\delta a_i \to \delta a_i e^{-i(\omega_p-\omega_d)t}$, $f_i^{in} \to f_i^{in} e^{-i(\omega_p-\omega_d)t}$ $(i = 1, 2)$, and $f_b^{in} \to f_b^{in} e^{-i(\omega_p-\omega_d)t}$. The corresponding QLEs become

$$
\begin{aligned}
\delta\dot{a}_1 &= -\Gamma_1\delta a_1 - iG_1\delta b - iJ\delta a_2 + \varepsilon_p + \sqrt{2\gamma_1}f_1^{in}, &(21)\\
\delta\dot{a}_2 &= -\Gamma_2\delta a_2 - iG_2\delta b - iJ\delta a_1 + \sqrt{2\gamma_2}f_2^{in}, &(22)\\
\delta\dot{b} &= -\Gamma_m\delta b - iG_1^*\delta a_1 - iG_2^*\delta a_2 + \sqrt{2\mathcal{G}_m}f_b^{in}, &(23)
\end{aligned}
$$

where $\Gamma_i = \gamma_i + i\Delta_i''$ and $\Gamma_m = -\mathcal{G}_m + i\Delta_m$ with $\Delta_i'' = \Delta_i' - (\omega_p - \omega_d)$ and $\Delta_m = \omega_m - (\omega_p - \omega_d)$ being the detunings in the new frame.

The optical response of this system to the probe field can be obtained by solving the steady state of Eqs. (21–23). By setting $\langle\delta\dot{a}_i\rangle = \left\langle\delta\dot{b}\right\rangle = 0$ and neglecting the noise terms, we obtain

$$
\begin{aligned}
\langle\delta a_1\rangle &= \varepsilon_p(\Gamma_2\Gamma_m + |G_2|^2)/D, &(24)\\
\langle\delta a_2\rangle &= -\varepsilon_p\left(G_1^*G_2 + iJ\Gamma_m\right)/D, &(25)\\
\langle\delta b\rangle &= -\varepsilon_p(iG_1^*\Gamma_2 + JG_2^*)/D &(26)
\end{aligned}
$$

with the denominator

$$
\begin{aligned}
D &= J^2\Gamma_m + \Gamma_m\Gamma_1\Gamma_2 + \left(\Gamma_1|G_2|^2 + \Gamma_2|G_1|^2\right)\\
&\quad -iJ\left(G_1^*G_2 + G_1G_2^*\right). &(27)
\end{aligned}
$$

The amplitudes $\langle\delta a_i^{out}\rangle$ of the experimentally accessible cavity output fields are related to the cavity field $\langle\delta a_i\rangle$ by the input-output relation

$$
\left\langle\delta a_i^{out}\right\rangle + \left\langle\delta a_i^{in}\right\rangle = \sqrt{2\gamma_i^e}\left\langle\delta a_i\right\rangle, (i = 1, 2) \tag{28}
$$

where $\left\langle\delta a_1^{in}\right\rangle = \varepsilon_p/\sqrt{2\gamma_1^e}$, $\left\langle\delta a_2^{in}\right\rangle = 0$, and γ_i^e is the external damping rate that describes the coupling between the cavity mode and the input field. We can write $\gamma_i^e = \eta\gamma_i$ with η being the ratio between the external damping rate and the total damping rate. For the coupling parameter $\eta \ll 1$, the cavity is under-coupled; and when $\eta \simeq 1$, the cavity is over-coupled [71, 72]. In this work, we consider the cases of over-coupled cavities with $\eta = 1$ and neglect the cavity intrinsic dissipation.

Using Eqs. (24, 25, 28), the transmission coefficient $t_{21} \equiv \partial\left\langle\delta a_2^{out}\right\rangle/\partial\left\langle\delta a_1^{in}\right\rangle$ can be derived as

$$
t_{21} = -\frac{2\sqrt{\gamma_1\gamma_2}\left(G_1^*G_2 + iJ\Gamma_m\right)}{D}. \tag{29}
$$

By interchanging indices 1 and 2 in Eq. (29), we find that

$$
t_{12} = -\frac{2\sqrt{\gamma_1\gamma_2}\left(G_2^*G_1 + iJ\Gamma_m\right)}{D}. \tag{30}
$$

From (29, 30), we find that by manipulating the phase difference between the optomechanical couplings G_1 and G_2, nonreciprocal propagation of the probe field can be achieved, i.e., $|t_{12}/t_{21}|$ can be adjusted by varying the phase difference. This effect can be understood through the effective Hamiltonian associated with (21–23),

$$
\begin{aligned}
H_{\text{eff}} &= \sum_i \Delta_i''\delta a_i^\dagger\delta a_i + \Delta_m\delta b^\dagger\delta b\\
&\quad + \sum_i G_i\delta a_i^\dagger\delta b + J\delta a_1^\dagger\delta a_2 + \text{H.c.}, &(31)
\end{aligned}
$$

which describes a typical three-mode cyclic system. The propagation of light fields in such a system depends strongly on the interference between different paths in the loop. A non-zero phase difference between the couplings G_1 and G_2 can break the time reversal symmetry of this system and gives rise to nonreciprocal optical response [73]. Compared with our previous work [74], the advantage of this scheme is that it does not require the matching of the pump frequencies between the optical and mechanical fields to achieve nonreciprocal propagation of the probe field. The mechanical gain can be viewed as an effective bath that converts the beam-splitter operation between the mechanical mode and the cavities into phonon-photon parametric processes. We will discuss these points in detail in the following section.

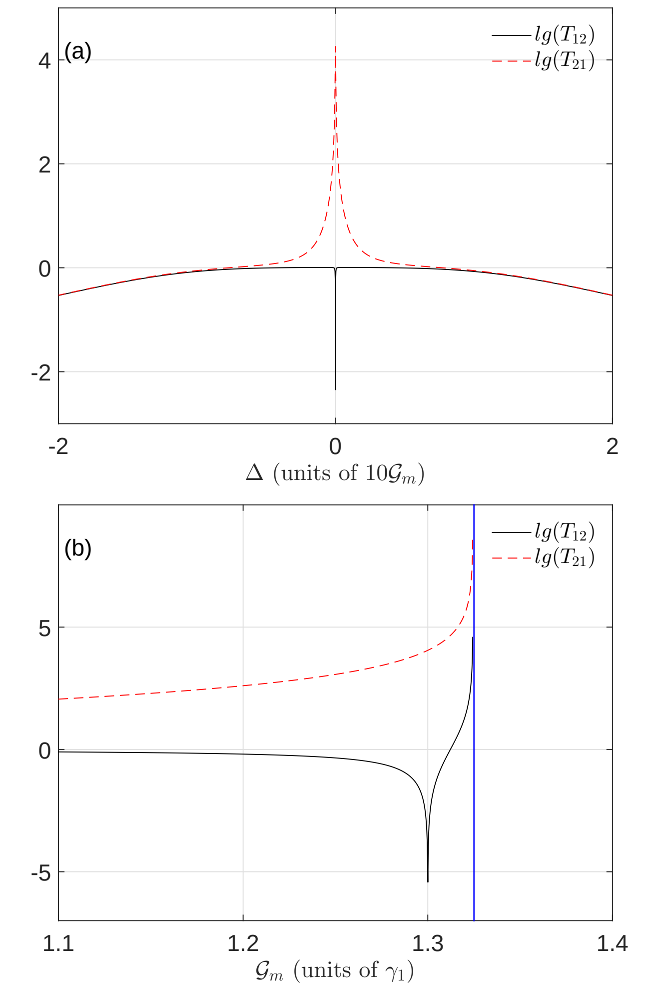

FIG. 3: (Color online) (a) The logarithms of the transmission coefficients T_{12} and T_{21} vs the detuning Δ. Other parameters are $\gamma_1 = 10\mathcal{G}_m$, $\gamma_2 = 15\mathcal{G}_m$, $J = 11\mathcal{G}_m$, $\theta = \pi/2$, and $G_1 = |G_2| \equiv G = \sqrt{J\mathcal{G}_m}$. In the vicinity of $\Delta = 0$, the transmission exhibits unidirectional amplification. (b) The logarithms of the transmission coefficients T_{12} and T_{21} vs the mechanical gain $\mathcal{G}_m$. Other parameters are $\gamma_2 = 1.5\gamma_1$, $J = 1.3\gamma_1$, $\theta = \pi/2$, $G_1 = |G_2| \equiv G = J$, and $\Delta = 0$. Here when $\mathcal{G}_m/\gamma_1 > 1.325$, the system becomes unstable.

IV. NONRECIPROCAL AMPLIFICATION AND OPTICAL DELAY

In this section, we will investigate the properties of the transmission coefficients under a special setup, i.e., when the system acts as an optomechanical transistor. We will show the feasibility of achieving signal amplification and nonreciprocity and study the delayed output response in this three-mode optomechanical system.

A. Optomechanical transistor

We first analyze the behavior of the transmission matrix elements t_{12} and t_{21}, as given by (29) and (30). For simplicity, we assume $G_1 \equiv G$ with $G > 0$, $G_2 \equiv Ge^{-i\theta}$ with a phase difference θ from G_1, and $\Delta_i'' = \Delta_m \equiv \Delta$ with $\Delta_i' = \omega_m$ for the pump fields. The transmission coefficients under these conditions can be written as

$$t_{12} = \frac{2\sqrt{\gamma_1\gamma_2}\left[-iJ\left(-\mathcal{G}_m + i\Delta\right) - G^2 e^{i\theta}\right]}{D_r}, \quad (32)$$

$$t_{21} = \frac{2\sqrt{\gamma_1\gamma_2}\left[-iJ\left(-\mathcal{G}_m + i\Delta\right) - G^2 e^{-i\theta}\right]}{D_r} \quad (33)$$

with the denominator

$$\begin{aligned} D_r = &\left(\gamma_1 + i\Delta\right)\left(\gamma_2 + i\Delta\right)\left(-\mathcal{G}_m + i\Delta\right) \\ &+ G^2\left(\gamma_1 + \gamma_2 + 2i\Delta\right) + J^2\left(-\mathcal{G}_m + i\Delta\right) \\ &- 2iJG^2\cos\theta. \end{aligned} \quad (34)$$

Using (32) for the coefficient t_{12}, we choose the phase difference θ to satisfy the condition $G = \sqrt{J\mathcal{G}_m/\sin\theta}$ and choose the detuning $\Delta = \mathcal{G}_m\cos\theta/\sin\theta$, which yields that $t_{21} \neq 0$ and $t_{12} = 0$, i.e., unidirectional propagation of the probe field can be achieved.

We select a set of parameters that satisfy the stability condition using the result shown in Fig. 2. Using these parameters, we plot the logarithms of the transmission coefficients T_{12} and T_{21} in Fig. 3(a) with $T_{ij} = |t_{ij}|^2$. The result gives a clear feature of unidirectional amplification of the probe field in the vicinity of $\Delta = 0$, which agrees with our theoretical prediction. The physics origin of the amplification arises from the mechanical gain, which can be viewed as a coherent phonon bath that converts the beam-splitter operation between the mechanical and the cavity modes to effective parametric processes between these modes. The parametric processes greatly enhance the photoelastic scattering [67]. This effect is in analogues to the stimulated emission process in atomic systems when the frequency of the probe field is resonant with that of the anti-Stokes field, where amplification of the incident photon field can be achieved. This system can work as an optomechanical transistor at strong mechanical gain with $\mathcal{G}_m \sim \gamma_1$ by choosing appropriate parameters. As shown in Fig. 3(b), strong unidirectional amplification can be achieved at $\mathcal{G}_m = 1.3\gamma_1$. Meanwhile, the increase of the mechanical gain can induce instability to this system. With the parameters in Fig. 3(b), the system becomes unstable for $\mathcal{G}_m/\gamma_1 > 1.325$.

In Fig. 4, we plot the logarithm of the transmission coefficient T_{21} at the optimal conditions for unidirectional propagation, i.e., with $G^2 = J\mathcal{G}_m/\sin\theta$ and $\Delta = \mathcal{G}_m\cos\theta/\sin\theta$. It is shown that in the neighborhood of $\theta = \pi/2$, the transmission coefficient reaches its maximum with $\max\left(T_{21}\right) \approx 10^5$. This strong amplification, together with the nonreciprocity, clearly shows that our system can be used as an optomechanical transistor facilitated by the mechanical gain.

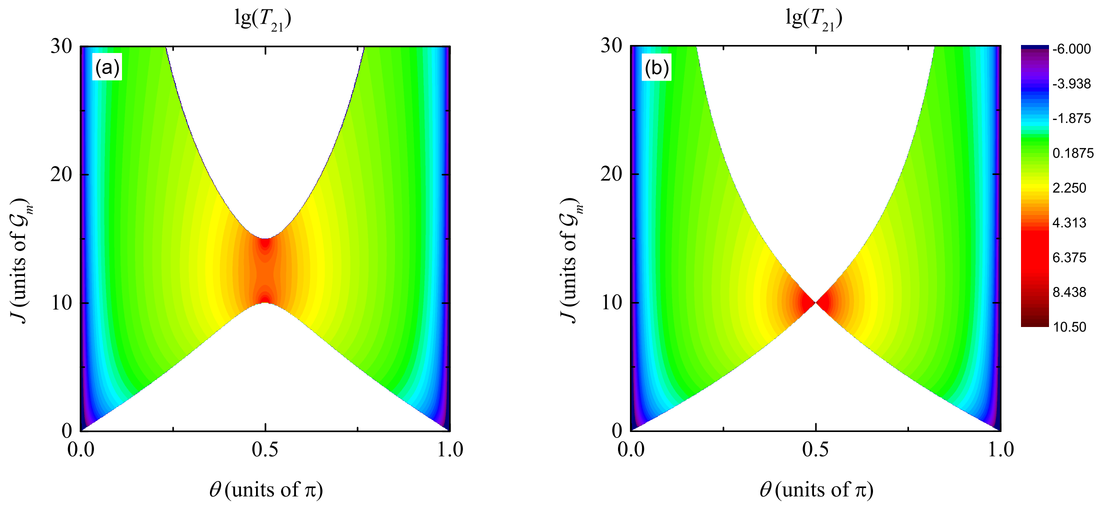

FIG. 4: (Color online) Contour plot of the logarithm of the transmission probability $\lg(T_{21})$ for (a) $\gamma_1 = 10\mathcal{G}_m$, $\gamma_2 = 15\mathcal{G}_m$ and (b) $\gamma_1 = 10\mathcal{G}_m$, $\gamma_2 = 10\mathcal{G}_m$. The optimal unidirectional conditions are used with $G_1 = |G_2| \equiv G = \sqrt{J\mathcal{G}_m/\sin\theta}$ and $\Delta = \mathcal{G}_m \cot\theta$. It is shown that when $\theta \to \pi/2$, the transmission probability reaches its maximum.

B. Ultra-long optical delay

The optical group delay is another important parameter to characterize the optical transmission and responses. It is well known that the optical transmission within an electromagnetically-induced transparency window experiences a dramatic reduction in its group velocity. Similar effects can be expected in the optical transmission in optomechanical systems. Here we investigate the optical delay in our system. We first introduce the optical group delay time defined in terms of the phase of the transmitted probe field as

$$\tau_{ij} = \frac{d\delta_{ij}}{d\omega_p}, \tag{35}$$

where $\delta_{ij} = \arg[t_{ij}(\omega_p)]$ is the phase of the output field at the frequency ω_p [16, 18]. We consider the system operated in the regime of an optomechanical transistor with $|t_{21}| \gg 1$ and $t_{12} = 0$. To ensure unidirectional amplification and similar to the previous subsection, we let the parameters satisfy the relations: $J = 10\mathcal{G}_m$, $G_1 \equiv G$ $(G > 0)$, $G_2 \equiv Ge^{-i\theta}$, $\Delta_i'' \equiv \Delta$, $\gamma_1 = 10\mathcal{G}_m$, $\gamma_2 = 15\mathcal{G}_m$, and $G^2 = J\mathcal{G}_m/\sin\theta$. In Fig. 5, we plot the phase δ_{21} and the group delay τ_{21} as functions of the detuning Δ. It can be shown that strong group delay occurs in the working window of the optomechanical transistor. As θ approaches to the value of $\pi/2$, the group delay exhibits sharp increase. This indicates that the strengthening of the amplification process gives rise to dramatic increase in the group delay. Note that near $\theta = \pi/2$, as shown in Fig. 2(a), the system is close to the boundary between the stable and the unstable regions, and is more fragile to environmental disturbance. Therefore, there is a tradeoff between the amplification and group delay and the stability of this system. By selecting appropriate parameters, one can realize an optomechanical transistor with significant time delay.

C. Noise

In the above discussions, we focus on the nonreciprocal transmission and amplification of the signal field to the designated output channel. However, in our system, the mechanical mode carries thermal fluctuations with thermal occupation number $n_{\text{th}} \gg 1$ and could have a strong impact on the performance of this optomechanical transistor. Below we discuss the transmission of the mechanical noise to the cavity output. Specifically, we study the transmission coefficient t_{23} that describes the forward noise transmission and t_{13} that describes the backward noise transmission [82, 83]. Using Eqs. (24)-(26) and the input-output relations, we derive

$$t_{13} = \frac{2\sqrt{\gamma_1\mathcal{G}_m}\left(-JG_2 - iG_1\Gamma_2\right)}{D}, \tag{36}$$

$$t_{23} = \frac{2\sqrt{\gamma_2\mathcal{G}_m}\left(-iG_2\Gamma_1 - JG_1\right)}{D}. \tag{37}$$

Moreover, with the condition for unidirectionality ($t_{12} = 0$ with $G_1 = |G_1|$, $G_2 = |G_2| e^{-i\theta}$, $J = |G_1||G_2|/\mathcal{G}_m$,

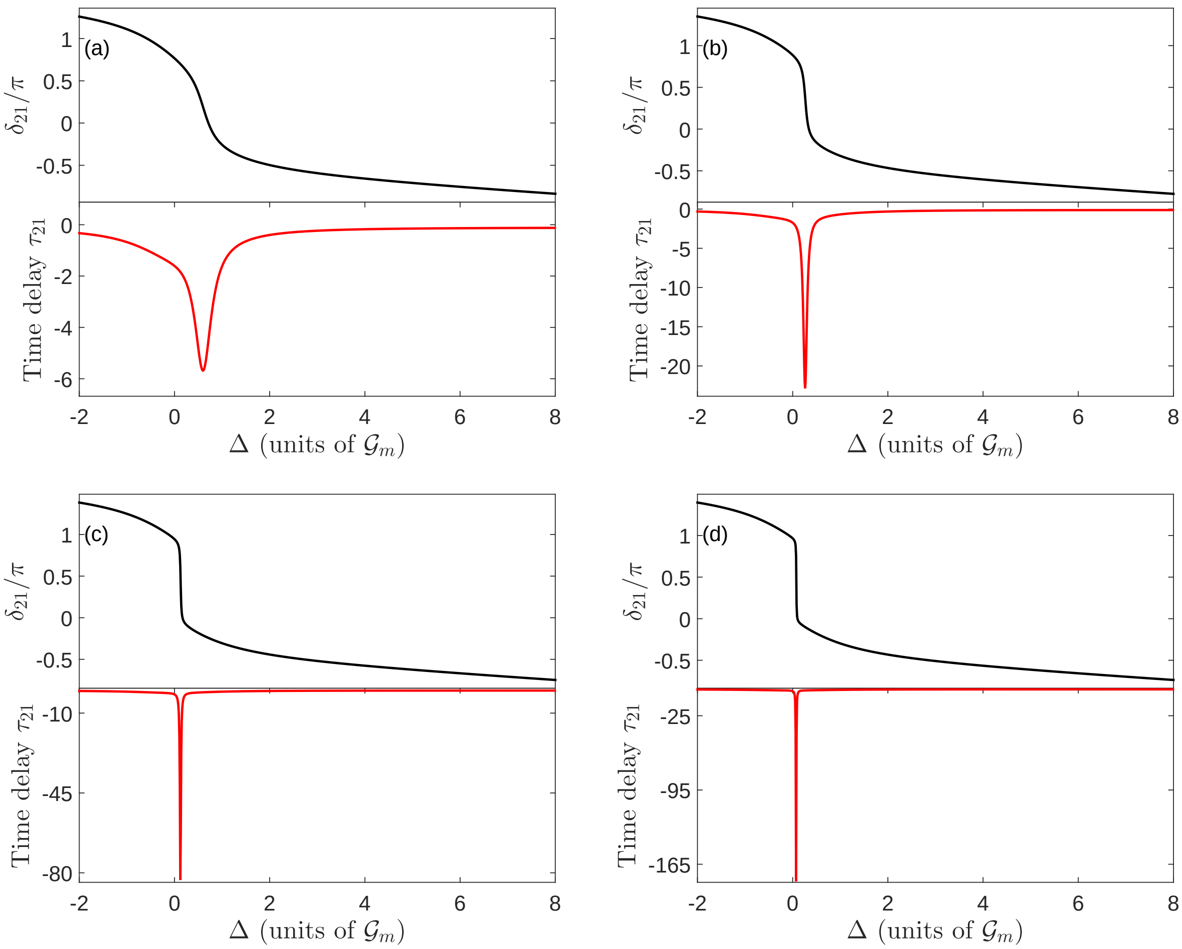

FIG. 5: (Color online) The group delay τ_{21} vs the detuning Δ for (a) $\theta = 0.3\pi$, (b) $\theta = 0.4\pi$, (c) $\theta = 0.45\pi$, and (d) $\theta = 0.47\pi$. The group delay τ_{21} is given in units of $1/\mathcal{G}_m$. Other parameters are $J = 10\mathcal{G}_m$, $\gamma_1 = 10\mathcal{G}_m$, $\gamma_2 = 15\mathcal{G}_m$, and $G = \sqrt{J\mathcal{G}_m/\sin\theta}$.

and $\theta = \pi/2$), these transmission coefficients become

$$t_{13} = -i\frac{2\sqrt{C_1}}{C_1 - 1}, \tag{38}$$

$$t_{23} = \frac{2\sqrt{C_2}\,(1 + C_1)}{(C_1 - 1)\,(C_2 - 1)} \tag{39}$$

with the cooperativity $C_i = |G_i|^2/\mathcal{G}_m\gamma_i$ for $i = 1, 2$. Note that under the unidirectionality condition, the transmission coefficient for the signal field becomes

$$t_{21} = -i\frac{4\sqrt{C_1 C_2}}{(C_1 - 1)\,(C_2 - 1)}, \tag{40}$$

Here the backward propagating noise is determined by the transmission coefficient t_{13}, which can become very large when $C_1 \approx 1$. Similarly, the forward propagating noise is determined by the transmission coefficient t_{23}, which can become very large when $C_2 \approx 1$. To avoid the propagation of the mechanical noise to the input channel, C_1 and C_2 need to be chosen to be well separated from unity. However, as can be seen from Eq. (40), the reduction of the cooperativities results in the reduction of the amplification. Therefore, there is a tradeoff between the suppression of the mechanical noise and the amplification of the signal field in our scheme. Consider the ratio between the mechanical noise and the signal field at the output port. We find that $t_{23}/t_{21} = i\,(1 + C_1)/2\sqrt{C_1} \geq 1$, which indicates that the amplification of the mechanical noise is comparable to or stronger than the amplification of the signal. To suppress the contribution of the mechanical noise, time average on the output signal is required to increase the signal-to-noise ratio.

V. CONCLUSIONS

To conclude, we have shown that an optomechanical transistor can be realized in a cyclic optomechanical system with finite mechanical gain. Similar to our previous work [74], the nonreciprocal behavior of this system arises from the phase difference between the optomechanical couplings G_1 and G_2, which breaks the time-reversal symmetry of this system. The uniqueness of this scheme is that we use an engineered mechanical gain to achieve the amplification for the signal field. The presence of the mechanical gain generates strong parametric processes between the mechanical and the cavity modes, and significantly enhances the photoelastic scattering at the optimal frequency. Combining the phase difference with the mechanical gain thus enables the unidirectional amplification of the signal field. Furthermore, the amplification of the probe field is accompanied by an ultra-long group delay in the output field. Our work hence provides an effective approach to control the light propagation in an optomechanical system and could stimulate future studies of nonreciprocal optomechanical interfaces in nonlinear photonic devices.

Acknowledgments

This work is supported by the National Key R&D Program of China grant 2016YFA0301200, the National Basic Research Program (973 Program) of China (under Grant No. 2014CB921403), the National Natural Science Foundation of China (under Grants No. 11505126, No. 11774024, No. 11534002, No. U1530401), and the Postdoctoral Science Foundation of China (under Grant No. 2016M591055). X.Z.Z. is supported by PhD research startup foundation of Tianjin Normal University under Grant No. 52XB1415. L.T. is supported by the National Science Foundation (USA) under Award Numbers DMR-0956064 and PHY-1720501.

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