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Damping-free collective oscillations of a driven two-component Bose gas in optical lattices

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Phys. Rev. A **97**, 043601 — Published 2 April 2018

DOI: 10.1103/PhysRevA.97.043601

Damping-free collective oscillations of a driven two–component Bose gas in optical lattices

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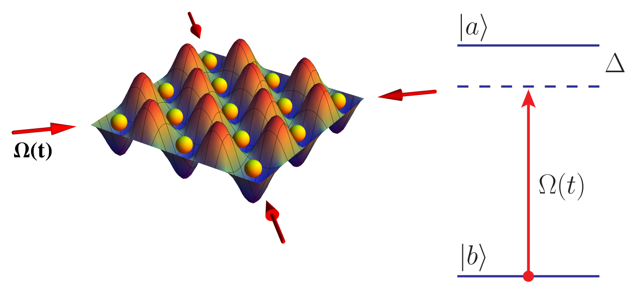
We explore quantum many-body physics of a driven Bose-Einstein condensate in optical lattices. The laser field induces a gap in the generalized Bogoliubov spectrum proportional to the effective Rabi frequency. The lowest lying modes in a driven condensate are characterized by zero group velocity and non-zero current. Thus, the laser field induces roton modes, which carry interaction in a driven condensate. We show that collective excitations below the energy of the laser-induced gap remain undamped, while above the gap they are characterized by a significantly suppressed Landau damping rate.

PACS numbers: 03.75.Kk, 03.75.Mn, 42.50.Gy, 67.85.-d, 63.20.kg

I. INTRODUCTION

Multicomponent Bose-Einstein condensates (BECs) of ultracold gases [1–4] are a superb system for exploring quantum many-body physics and emergent phenomena in a well-controlled macroscopic quantum system. The spinor BEC led to a major advances that include observation of Dirac monopoles [5], exotic magnetism [1], spin Hall effect [6], spontaneous symmetry breaking [7], coherent spinor dynamics [8], dynamic stabilization [9], vortex formation [10], quantum spin mixing [11], spin domain wall formation [12], and realization of topological states [13, 14]. The observation of the low-lying collective modes of a condensate revealed its dynamics and unique spectral signatures [15, 16]. Moreover, collective modes in a two-component condensate were instrumental in the examination of the crossover from a BEC to Bardeen-Cooper-Schrieffer (BCS) superfluid regime of ultracold Fermi gases [17–19]. At low temperatures, the decay of collective modes, caused by the coupling between them, is described by the Landau damping process [20]. In this Letter, we show that the laser field induces a gap in an otherwise gapless Bogoliubov spectrum, which leads to the existence of roton modes in a driven condensate. We show that the laser-induced gap in the spectrum of elementary excitation protects the low-lying collective modes from Landau damping. Above the energy of the gap, the damping is dominated by the laser-induced roton modes and is considerably suppressed compared to the phonon-mediated damping found in a field-free condensate.

We describe a weakly interacting two-component BEC confined in an optical lattice (see Fig.1.(a)) by a driven Bose-Hubbard Hamiltonian [3, 4]. For a driven two-component BEC we obtain the exact results for the elementary amplitudes and find a set of exact symmetries that are inherent among them. Based on the obtained energy spectrum and amplitudes, we explore the near-equilibrium dynamics of a condensate and calculate Landau decay rate of the collective modes. The microwave field that drives the condensate from the ground state to the first excited state operates in a single mode regime



(a) (b)

FIG. 1. *Bose-Einstein condensate in an optical lattice driven by a laser field* (a) Bose-Einstein condensate (yellow spheres) confined in an optical lattice, formed by counter-propagating waves (red arrows) and driven from the ground state by the laser field $\Omega(t)$. (b) Two-component Bose-Einstein condensate is modeled by a set of two-level systems, driven from the ground state $|b\rangle$ to the excited state $|a\rangle$ by the laser field $\Omega(t)$. The laser field is characterized by the Rabi frequency Ω_R and detuning Δ. Initially, the condensate is prepared in the ground state $|b\rangle$.

[21–25], and is characterized by the Rabi frequency and detuning (see Fig. 1.(b)). We find that the applied laser field creates a gap in the energy spectrum that dramatically modifies the interaction in a driven condensate. In a scalar BEC, coupling between the collective modes, carried by the phonons, leads to the absorption (emission) of the collective modes described by Landau damping [26] (Beliaev damping [27]). In contrast, the lowest lying elementary excitations in a driven BEC have zero group velocity and non-zero current. Thus, the interaction in a driven condensate is carried by the laser-induced roton modes. The laser-induced gap in the spectrum of elementary excitation ensures zero Landau damping of the collective modes lying below energy of the gap. Above the gap, it is proportional to the density of the laser-induced roton mode and is considerably slowed down compared to a field-free scalar condensate.

The suppression of the collective modes was previously reported for a number of physical systems. This includes the observation of suppressed Landau damping in the Bose-Fermi superfluid mixture [28, 29], reduced decay rate of the collective excitation in fermionic polar molecules confined in optical lattices due to the quantum Zeno mechanism [30], and the prediction of the absence of damping for quasi 2D dipolar Bose gas at zero temperature [31–33]. However, in all of these systems the energy spectrum is gapless for small momenta, and therefore the interaction in a BEC gas is carried predominantly by the phonons. Thus, the phonon modes contribute the most to the Landau damping of the low-lying collective modes. For larger momenta, the roton modes are damped out, and therefore, Landau damping in scalar or dipolar BECs is carried primarily by the phonon modes [32]. In contrast to these studies, we show that the energy spectrum of elementary excitations of a two-component BEC acquires a gap in the presence of a laser field, which was reported previously [34]. We demonstrate that Landau damping of the collective modes in a driven two-component BEC is governed by the rotons and is considerably suppressed, compared to the phonon-mediated damping in scalar BECs. Moreover, we find that collective oscillations lying below the energy of the gap are damping free. The undamped collective modes of a driven multicomponent condensate, combined with the extremely long coherence time of BECs, will allow one to observe long-lived internal dynamics of nonlinear macroscopic phenomena, such as quantum vortices, quantum turbulence, and solitons [3].

II. MANY-BODY HAMILTONIAN OF A DRIVE TWO-COMPONENT BEC IN OPTICAL LATTICE

We start with the Bose-Hubbard Hamiltonian of the two-component BEC in an optical lattice, [21–23, 35–38]

$$H=\int d\mathbf{r}\sum_{j=a,b}\widehat{\psi}_j^{\dagger}(\mathbf{r})\left(-\frac{\hbar^2}{2m}\nabla^2+V(\mathbf{r})-\mu_j\right)\widehat{\psi}_j(\mathbf{r})$$
$$+\frac{1}{2}\int d\mathbf{r}\sum_{j=a,b}\widehat{\psi}_j^{\dagger}(\mathbf{r})\left(\sum_{j'=a,b}g_{jj'}\widehat{\psi}_{j'}^{\dagger}(\mathbf{r})\widehat{\psi}_{j'}(\mathbf{r})\right)\widehat{\psi}_j(\mathbf{r})$$
$$+\frac{\Omega_R}{2}\int d\mathbf{r}\left(e^{i\Delta t}\widehat{\psi}_a^{\dagger}(\mathbf{r})\widehat{\psi}_b(\mathbf{r})+e^{-i\Delta t}\widehat{\psi}_b^{\dagger}(\mathbf{r})\widehat{\psi}_a(\mathbf{r})\right). \quad (1)$$

Here $\widehat{\psi}_j(\mathbf{r})$ is the field operator, which obeys Bose-Einstein statistics and annihilates a particle characterized by the mass m, location $\mathbf{r}$, and the internal state $j=a(b)$ for a particle in the excited (ground) state. The chemical potential for a particle occupying the internal state j is given by μ_j. The lattice potential is assumed to have a cubic form, $V(\mathbf{r})=V_0\sum_{i=1}^{3}\sin^2(k_L r_i)$, and is given in terms of the lattice vector $k_L=\pi/a_L$, where a_L is the lattice constant. The interaction between particles occupying the internal states j and j' are given by the coupling constants $g_{jj'}$. The laser field that drives the condensate from the ground state $|b\rangle$ to the first excited state $|a\rangle$ is characterized by the Rabi frequency Ω_R and detuning Δ (see Fig.1.(b)) from the excited state.

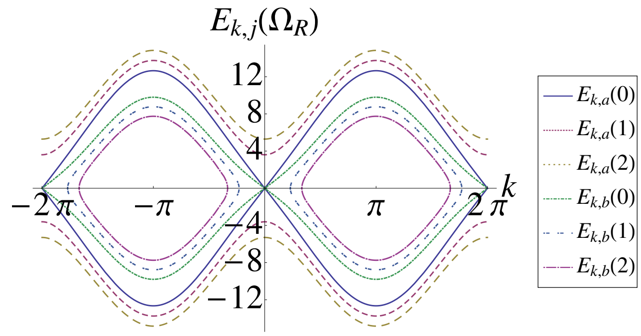


FIG. 2. (Color online) *Rabi-Bogoliubov spectrum for a driven two-component Bose-Einstein condensate in an optical lattice.* The new dispersion law consists of two branches, $E_{ka}(\Omega_R)$ (solid curves) and $E_{kb}(\Omega_R)$ (dashed curves), which are separated by the standard Bogoliubov spectrum. The first branch of the new Rabi-Bogoliubov spectrum that lies above the Bogoliubov spectrum is characterized by the gap and has characteristic quadratic dispersion relation for small momenta k in the units of the lattice constant a_L. The second branch of the spectrum has a real solution only for certain values of momenta, while for the rest of the momenta, has only purely imaginary solutions.

In the tight binding model, and lowest band approximation, which is valid in the long-wavelength limit, one can expand the bosonic field operators $\widehat{\psi}_j(\mathbf{r})$ in the Wannier basis $\widehat{\psi}_j(\mathbf{r})=\sum_n b_{nj}w_j(\mathbf{r}-\mathbf{r}_n)$. Throughout the paper, we will use the index convention, according to which the first argument of the index describes the site in an optical lattice, while the second argument corresponds to the internal state within the site. The expansion of the field operators in the Wannier basis in the driven Bose-Hubbard Hamiltonian 1 directly leads to

$$H=-\sum_{j=a,b}\sum_{\langle m,n\rangle}J_{mn}^{jj}\left(\widehat{b}_{mj}^{\dagger}\widehat{b}_{nj}+\widehat{b}_{nj}^{\dagger}\widehat{b}_{mj}\right) \quad (2)$$
$$-\sum_{j=a,b}\mu_j\sum_n\widehat{b}_{nj}^{\dagger}\widehat{b}_{nj}+\sum_{j,j'=a,b}\frac{U_{jj'}}{2}\sum_n\widehat{b}_{nj}^{\dagger}\widehat{b}_{nj'}^{\dagger}\widehat{b}_{nj'}\widehat{b}_{nj}$$
$$+\frac{\Omega_R}{2}\sum_n\left(e^{i\Delta t}\widehat{b}_{na}^{\dagger}\widehat{b}_{nb}+e^{-i\Delta t}\widehat{b}_{nb}^{\dagger}\widehat{b}_{na}\right),$$

where we considered hopping between nearest neighbors, indicated by $\langle m,n\rangle$. Here the hopping integral is

$$J_{mn}^{ij}=-\int d\mathbf{r}w_i^*(\mathbf{r}-\mathbf{r}_m)\left[-\frac{\hbar^2}{2m}\nabla^2+V(\mathbf{r})\right]w_j(\mathbf{r}-\mathbf{r}_n), \quad (3)$$

and the on-site interaction is

$$U_{jj'}=g_{jj'}\int d\mathbf{r}w_j^*(\mathbf{r})w_{j'}^*(\mathbf{r})w_{j'}(\mathbf{r})w_j(\mathbf{r}). \quad (4)$$

In order to transform the Bose-Hubbard Hamiltonian into the k-space we introduce the Fourier transform of the creation and annihilation operators,

$$\widehat{b}_{nj} = \frac{1}{\sqrt{N_L}} \sum_k \exp\left[-i\mathbf{k}\mathbf{r}_n\right] \widehat{a}_{kj}, \quad (5)$$

where N_L is number of lattice cites. Then we linearize the Fourier-transformed Bose-Hubbard Hamiltonian Eq.(2) by expanding the creation and annihilation operators near their average values, $\widehat{a}_{kj} = \langle \widehat{a}_{0j} \rangle + (\widehat{a}_{kj} - \langle \widehat{a}_{0j} \rangle)$. Here $\langle \widehat{a}_{0j} \rangle$ is the average value of the annihilation operator, which is given in terms of number of the particles occupying zero momentum state N_{0j}, $\langle \widehat{a}_{0j} \rangle = \sqrt{N_{0j}}$. The coupling between particles occupying the internal states $j = \{a, b\}$ is described by the matrix

$$\begin{pmatrix} n_a U_{aa} & \sqrt{n_a n_b} U_{ab} \\ \sqrt{n_a n_b} U_{ba} & n_b U_{bb} \end{pmatrix} \equiv \begin{pmatrix} u_a & s_a \\ s_b & u_b \end{pmatrix}. \quad (6)$$

Here $n_j = N_{0j}/N_L$ are the average filling factors of the particles occupying the internal state j and momentum $k = 0$. In this work we will consider the simplified case, $u = n_a U_{aa} = n_b U_{bb}$ and $s = \sqrt{n_a n_b} U_{ab} = \sqrt{n_a n_b} U_{ba}$. However, the main physical results concerning the Rabi-Bogoliubov spectrum of a driven two-component BEC and the characteristic rate of Landau damping of the collective modes in the symmetric case match the results obtained in the general case Eq.(6). In the special case $u = s$ the matrix Eq.(6) simplifies to the fully symmetric Manakov model, which very closely describes a two-component BEC of ^{87}Rb.

III. RABI-BOGOLIUBOV ENERGY SPECTRUM AND AMPLITUDES OF A DRIVEN TWO-COMPONENT BEC IN OPTICAL LATTICE

The linearized Bose-Hubbard Hamiltonian can be diagonalized via the generalized Bogoliubov transformation. This transformation introduces the quasiparticle creation and annihilation operators, according to $\widehat{a}_{kj} = \mathscr{U}_k \widehat{\alpha}_{ka} + \mathscr{V}_k^* \widehat{\alpha}^\dagger_{-ka} + \mathscr{W}_k \widehat{\beta}_{kb} + \mathscr{Y}_k^* \widehat{\beta}^\dagger_{-kb}$. Here $\widehat{\alpha}_{k,a}$ and $\widehat{\beta}_{k,b}$ are the quasiparticle annihilation operators in the excited $j = a$ (ground $j = b$). We impose the Bose-Einstein commutation relation for these quasiparticle operators, $[\widehat{\alpha}_{kj}, \widehat{\alpha}^\dagger_{-k,j'}] = \delta_{jj'}$, and $[\widehat{\beta}_{kj}, \widehat{\beta}^\dagger_{-k,j'}] = \delta_{jj'}$. This leads to a constraint on the Rabi-Bogoliubov amplitudes, $\mathscr{U}_k^2 - \mathscr{V}_k^2 + \mathscr{W}_k^2 - \mathscr{Y}_k^2 = 1$.

In the quasiparticle basis the Bose-Hubbard Hamiltonian is diagonal and is given by,

$$H_{\text{eff}} = \frac{1}{2} \sum_k E_a(k) \widehat{\alpha}^\dagger_{a,k} \widehat{\alpha}_{a,k} + \frac{1}{2} \sum_k E_b(k) \widehat{\beta}^\dagger_{b,k} \widehat{\beta}_{b,k}. \quad (7)$$

Rabi-Bogoliubov spectrum of elementary excitations is obtained from the condition $\det\left[M - \mathbf{1}(E/2)\right] = 0$,

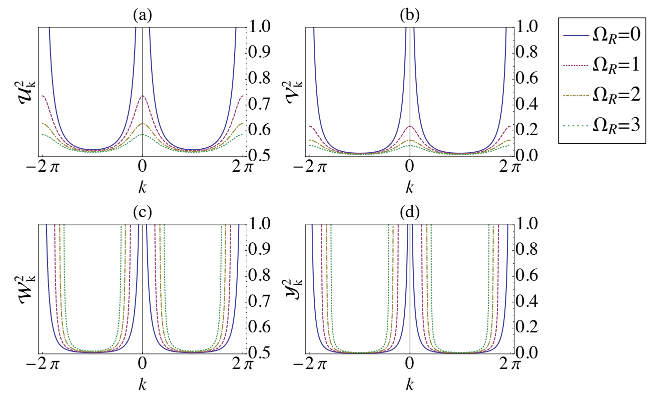


FIG. 3. (Color online) *Rabi-Bogoliubov amplitudes for a two-component Bose-Einstein condensate in an optical lattice driven by a microwave field.* The standard Bogoliubov amplitudes have poles at the location of the roots of Bogoliubov spectrum. The laser field, which drives the condensate, creates a gap in the spectrum of elementary excitations. As a result, Rabi-Bogoliubov amplitudes (a) $\mathscr{U}_k^2$ and (b) $\mathscr{V}_k^2$ are finite for all values of momenta k, given in the units of the lattice constant a_L. For a two-component condensate, one obtains additional Rabi-Bogoliubov amplitudes, (c) $\mathscr{W}_k^2$ and (d) $\mathscr{Y}_k^2$, which are absent for a scalar BEC. These amplitudes ensure Bose-Einstein commutation relation for the quasiparticle creation and annihilation operators.

where the matrix M is given by

$$M = \begin{pmatrix} t_k + u + \frac{\Delta}{2} & s + \frac{\Omega_R}{2} & u & s \\ s + \frac{\Omega_R}{2} & t_k + u - \frac{\Delta}{2} & s & u \\ -u & -s & -t_k - u - \frac{\Delta}{2} & -s - \frac{\Omega_R}{2} \\ -s & -u & -s - \frac{\Omega_R}{2} & -t_k - u + \frac{\Delta}{2} \end{pmatrix}. \quad (8)$$

Here, the tunneling parameter $t_k = 4J \sin^2(k a_L/2)$ is given in terms of tunneling amplitude $J \equiv J^{jj}_{mn}$, momentum k, and the lattice constant a_L. The exact Rabi-Bogoliubov spectrum of a driven two-component condensate is

$$E_a(k) = \sqrt{4t_k\left(t_k + 2u\right) + \Delta^2 + \Omega_R^2 + 4s\Omega_R + 4\sigma}, \quad (9)$$

$$E_b(k) = \sqrt{4t_k\left(t_k + 2u\right) + \Delta^2 + \Omega_R^2 + 4s\Omega_R - 4\sigma} \quad (10)$$

where the parameter σ is defined as the positive branch of the square root,

$$\sigma^2 = 4st_k\left(t_k + u\right)\Omega_R + s^2\left(4t_k^2 - \Delta^2\right) + \left(t_k + u\right)^2\left(\Delta^2 + \Omega_R^2\right). \quad (11)$$

For the Rabi-Bogoliubov amplitudes we find a set of symmetries that hold among them,

$$\begin{aligned} \mathscr{V}_k^2(E_a, \sigma) &= -\mathscr{U}_k^2(-E_a, \sigma), \quad (12) \\ \mathscr{W}_k^2(E_b, \sigma) &= \mathscr{U}_k^2(E_b, -\sigma), \\ \mathscr{Y}_k^2(E_b, \sigma) &= -\mathscr{U}_k^2(-E_b, -\sigma). \end{aligned}$$

The new symmetries are the direct generalization of the intrinsic symmetries of the standard Bogoliubov amplitudes. Indeed, if one reverses the sign of the energy in the square of the first Bogoliubov amplitudes, $v_k^2(E)$, and swaps the sign of the whole expression, one arrives at the second Bogoliubov amplitude, i.e. $u_k^2(E) = -v_k^2(-E)$. As a result of the symmetries Eq.(12), one can obtain all the Rabi-Bogoliubov amplitudes from any one of them, for instance, $\mathscr{U}_k^2(E_a,\sigma)$, which is explicitly given by,

$$\begin{aligned}\mathscr{U}_k^2(E_a,\sigma) = \frac{1}{4E_a\sigma} \times \big(s^2\left(4t_k - 2\Delta\right) \\ + \left(2t_k + 2u + E_a + \Delta\right)\left(\left(t_k + u\right)\Delta + \sigma\right) \\ + 2s\left(2t_k + u\right)\Omega_R + \left(t_k + u\right)\Omega_R^2\big)\end{aligned} \tag{13}$$

We note that in the long-wavelength limit the Rabi-Bogoliubov amplitudes $\mathscr{W}_k(E_a)$ and $\mathscr{Y}_k(E_a)$ are purely imaginary. Therefore, we are left with the real-valued Rabi-Bogoliubov amplitudes $\mathscr{U}_k(E_a)$ and $\mathscr{V}_k(E_a)$, which can be considerably simplified in the special case of a resonant drive, i.e. $\Delta = 0$,

$$\begin{aligned}\mathscr{U}_k(E_a) &= \sqrt{\frac{E_a + 2(t_k + u + s + \Omega_R/2)}{4E_a}}, \\ \mathscr{V}_k(E_a) &= -\sqrt{\frac{-E_a + 2(t_k + u + s + \Omega_R/2)}{4E_a}},\end{aligned} \tag{14}$$

where $E_\alpha = \sqrt{(2t_k + \Omega_R)\left(2t_k + 4u + 4s + \Omega_R\right)}$.

IV. LANDAU DAMPING OF COLLECTIVE EXCITATION IN A DRIVEN TWO-COMPONENT BEC IN OPTICAL LATTICE

In the previous section we derived the exact Rabi-Bogoliubov spectrum and amplitudes of the elementary excitations for a driven two-component BEC in an optical lattice. The collective excitations of a driven BEC represent its vibrational (normal) modes with the dispersion law given by the derived Rabi-Bogoliubov energy spectrum. In this section, we will consider the interaction between the collective modes and the thermally excited modes in a driven two-component BEC. In particular, we will examine the scattering process of an incoming collective mode and a thermal mode that creates a single outgoing thermal mode. In this collision process, the outgoing thermal mode may gain or lose energy, which ultimately depends on the slope of its energy distribution. The negative slope of the Bose-Einstein distribution, which governs the energy distribution of the thermal modes, results in a net loss of energy of the thermal modes. As a result, it leads to the Landau damping process [4, 26, 39] of the collective excitation in a driven two-component BEC. The interaction between the collective modes in given by,

$$V_{\rm int} = \frac{g_{jj}}{2}\int d\mathbf{r}\widehat{\psi}_j^\dagger(\mathbf{r})\widehat{\psi}_j^\dagger(\mathbf{r})\widehat{\psi}_j(\mathbf{r})\widehat{\psi}_j(\mathbf{r}). \tag{15}$$

First, we expand the field operator $\widehat{\psi}_j(\mathbf{r})$ around its equilibrium value, $\langle\widehat{\psi}_j(\mathbf{r})\rangle$, with a perturbation due to the fluctuations in the system, $\delta\widehat{\psi}(\mathbf{r})$,

$$\widehat{\psi}_j(\mathbf{r}) = \langle\widehat{\psi}_j(\mathbf{r})\rangle + \delta\widehat{\psi}_j(\mathbf{r}). \tag{16}$$

Next, we expand the perturbation, $\delta\widehat{\psi}_j(\mathbf{r})$, in a linear superposition of the quasiparticle creation and annihilation operators of thermal and collective modes,

$$\begin{aligned}\delta\widehat{\psi}_j(\mathbf{r}) = \left(\mathscr{U}_{\rm col}\widehat{\alpha}_{\rm col} + \mathscr{V}_{\rm col}^*\widehat{\alpha}_{\rm col}^\dagger + \mathscr{W}_{\rm col}\widehat{\beta}_{\rm col} + \mathscr{Y}_{\rm col}^*\widehat{\beta}_{\rm col}^\dagger\right) + \\ + \sum_m\left[\mathscr{U}_m\widehat{\alpha}_m + \mathscr{V}_m^*\widehat{\alpha}_m^\dagger + \mathscr{W}_m\widehat{\beta}_m + \mathscr{Y}_m^*\widehat{\beta}_m^\dagger\right].\end{aligned} \tag{17}$$

Here the sum goes over all the thermal modes in a driven two-component BEC. The probability of absorption (emission) of a collective mode is given by Fermi Golden Rule [4],

$$W = \pi\sum_{mn}\left|\langle n|V_{\rm int}|m\rangle\right|^2. \tag{18}$$

The next step consists of collecting all the terms that form a product of three quasi-particle operators - the annihilation operator of an incoming collective mode, the annihilation operator of an incoming thermal mode, and the creation operator of an outgoing thermal mode. Thus, we obtain the Landau damping rate of the collective excitation in a driven two-component BEC,

$$\begin{aligned}\Gamma_L = -\pi\sum_{m,n}\left[g^{(0)}(E_n) - g^{(0)}(E_m)\right]\times \\ \Big(|A_{mn}|^2\delta(E_n^{(a)} - E_m^{(a)} - \hbar\omega_q) + \\ + |B_{mn}|^2\delta(E_n^{(b)} - E_m^{(a)} - \hbar\omega_q) + \\ + |C_{mn}|^2\delta(E_n^{(a)} - E_m^{(b)} - \hbar\omega_q) + \\ + |D_{mn}|^2\delta(E_n^{(b)} - E_m^{(b)} - \hbar\omega_q)\Big).\end{aligned} \tag{19}$$

Here the energies $E^{(a)}(k)$ and $E^{(b)}(k)$ are the branches of the Rabi-Bogoliubov spectrum given by Eq.(9) and Eq.(10), respectively; $\hbar\omega_q$ is the energy of the incoming collective mode; $g^{(0)}(E)$ is the thermal distribution of the thermal modes, given by the Bose-Einstein distribution, $g^{(0)}(E) = 1/(\exp[\beta E] - 1)$. Here the amplitudes are given by (with $\widetilde{g}_{jj} = 2\sqrt{N}g_{jj}$)

$$\begin{aligned}A_{mn} &= \widetilde{g}_{jj}\left[\left(\mathscr{U}_{\rm col} + \mathscr{W}_{\rm col}\right)\left(\mathscr{U}_m\mathscr{U}_n^* + \mathscr{V}_m(\mathscr{U}_n^* + \mathscr{V}_n^*)\right)\right. \\ &\quad \left. + \left(\mathscr{V}_{\rm col} + \mathscr{Y}_{\rm col}\right)\left(\mathscr{U}_m(\mathscr{U}_n^* + \mathscr{V}_n^*) + \mathscr{V}_m\mathscr{V}_n^*\right)\right], \qquad (20) \\ B_{mn} &= \widetilde{g}_{jj}\left[\left(\mathscr{U}_{\rm col} + \mathscr{W}_{\rm col}\right)\left(\mathscr{U}_m\mathscr{W}_n^* + \mathscr{V}_m(\mathscr{W}_n^* + \mathscr{Y}_n^*)\right)\right. \\ &\quad \left. + \left(\mathscr{V}_{\rm col} + \mathscr{Y}_{\rm col}\right)\left(\mathscr{U}_m(\mathscr{W}_n^* + \mathscr{Y}_n^*) + \mathscr{V}_m\mathscr{Y}_n^*\right)\right], \qquad (21) \\ C_{mn} &= \widetilde{g}_{jj}\left[\left(\mathscr{U}_{\rm col} + \mathscr{W}_{\rm col}\right)\left(\mathscr{U}_n^*(\mathscr{W}_m + \mathscr{Y}_m) + \mathscr{Y}_m\mathscr{V}_n^*\right)\right. \\ &\quad \left. + \left(\mathscr{V}_{\rm col} + \mathscr{Y}_{\rm col}\right)\left(\mathscr{W}_m\mathscr{U}_n^* + \mathscr{V}_n^*(\mathscr{W}_m + \mathscr{Y}_m)\right)\right], \qquad (22) \\ D_{mn} &= \widetilde{g}_{jj}\left[\left(\mathscr{U}_{\rm col} + \mathscr{W}_{\rm col}\right)\left(\mathscr{W}_m\mathscr{W}_n^* + \mathscr{Y}_m(\mathscr{W}_n^* + \mathscr{Y}_n^*)\right)\right) \\ &\quad \left. + \left(\mathscr{V}_{\rm col} + \mathscr{Y}_{\rm col}\right)\left(\mathscr{W}_m(\mathscr{W}_n^* + \mathscr{Y}_n^*) + \mathscr{Y}_m\mathscr{Y}_n^*\right)\right)\right]. \qquad (23)\end{aligned}$$

In the continuous limit we can replace the sum in Eq.(19) by the integral, which directly leads us to

$$\Gamma_L = \tag{24}$$
$$-\pi\hbar\omega_q \int \frac{d^3p}{(2\pi)^3}|A_{mn}|^2\frac{\partial g^{(0)}(E^{(a)})}{\partial E^{(a)}}\delta(E^{(a)}_{p+q}-E^{(a)}_p-\hbar\omega_q)$$
$$-\pi\hbar\omega_q \int \frac{d^3p}{(2\pi)^3}|B_{mn}|^2\frac{\partial g^{(0)}(E^{(b)})}{\partial E^{(b)}}\delta(E^{(b)}_{p+q}-E^{(a)}_p-\hbar\omega_q)$$
$$-\pi\hbar\omega_q \int \frac{d^3p}{(2\pi)^3}|C_{mn}|^2\frac{\partial g^{(0)}(E^{(a)})}{\partial E^{(a)}}\delta(E^{(a)}_{p+q}-E^{(b)}_p-\hbar\omega_q)$$
$$-\pi\hbar\omega_q \int \frac{d^3p}{(2\pi)^3}|D_{mn}|^2\frac{\partial g^{(0)}(E^{(b)})}{\partial E^{(b)}}\delta(E^{(b)}_{p+q}-E^{(b)}_p-\hbar\omega_q),$$

where we expanded the Bose-Einstein factor, $g^{(0)}(E)$, in a Taylor series and kept the leading non-vanishing term. This expansion is valid only for the low-energy collective modes with energy $\hbar\omega_q \ll k_BT$, where k_B is the Boltzmann constant. In the long-wavelength limit, the leading contribution to Landau damping comes form the real-valued Rabi-Bogoliubov amplitudes $\mathscr{U}_k(E_a)$ and $\mathscr{V}_k(E_a)$. Thus, the Landau damping rate can be simplified into,

$$\Gamma_L \simeq \tag{25}$$
$$-\pi\hbar\omega_q \int \frac{d^3p}{(2\pi)^3}|A_{mn}|^2\frac{\partial g^{(0)}(E^{(a)})}{\partial E^{(a)}}\delta(E^{(a)}_{p+q}-E^{(a)}_p-\hbar\omega_q).$$

In the long-wavelength limit, the scattering amplitude A_{mn} can be reduced to,

$$A_{mn} \simeq 4\sqrt{N}\frac{g_{jj}}{2}\times \tag{26}$$
$$\Big[(\mathscr{U}_{\rm col}+\mathscr{V}_{\rm col})\Big(\mathscr{U}_m\mathscr{U}_n^*+\mathscr{V}_m\mathscr{V}_n^*+\frac{1}{2}(\mathscr{V}_m\mathscr{U}_n^*+\mathscr{U}_m\mathscr{V}_n^*)\Big)$$
$$+(\mathscr{U}_{\rm col}-\mathscr{V}_{\rm col})\,\frac{1}{2}(\mathscr{V}_m\mathscr{U}_n^*-\mathscr{U}_m\mathscr{V}_n^*)\Big].$$

For the low energy excitation, the Rabi-Bogoliubov amplitudes can be simplified to

$$\mathscr{U}_{\rm col}+\mathscr{V}_{\rm col} \simeq \frac{\sqrt{\omega_q}}{\sqrt{2(u+s)}}, \qquad \mathscr{U}_{\rm col}-\mathscr{V}_{\rm col} \simeq \frac{\sqrt{2(u+s)}}{\sqrt{\omega_q}}. \tag{27}$$

As a result, the scattering amplitude acquires a particularly simple form

$$A(E^{(a)}) = $$
$$= \frac{\sqrt{\omega_q}}{\sqrt{2(u+s)}}\left[\left(\mathscr{U}_m^2+\mathscr{V}_m^2+\mathscr{U}_m\mathscr{V}_m\right)+E^{(a)}\frac{\partial\mathscr{U}_m^2}{\partial E^{(a)}}\right]$$
$$\simeq \frac{\sqrt{\omega_q}}{\sqrt{2(u+s)}}\left[\frac{3}{4}\frac{E^{(a)}}{(u+s)}\right]. \tag{28}$$

If we introduce $E^{(a)} \equiv E$, we obtain the Landau decay rate of the collective modes in a driven two-component BEC,

$$\Gamma_L = -\pi\hbar\omega_q\frac{2\pi}{(2\pi\hbar)^3}\left(4\sqrt{N}\frac{g_{jj}}{2}\frac{\sqrt{\omega_q}}{\sqrt{2(u+s)}}\right)^2\frac{1}{q}$$
$$\times\,\beta\frac{\partial}{\partial\beta}\int dp\,\frac{1}{v_g}\frac{p^2}{E}\frac{1}{(e^{\beta E}-1)}\left(\frac{3}{4}\frac{E}{(u+s)}\right)^2, \tag{29}$$

where ω_q and q are the frequency and momentum of the collective mode. Now we define the density of the roton gas,

$$\rho_r = \frac{4\pi}{3(2\pi\hbar)^3}\int dp\,p^2\frac{E^2}{v_g}\left(-\frac{\partial}{\partial E}\frac{1}{(e^{\beta E}-1)}\right)$$
$$= \frac{4\pi}{3(2\pi\hbar)^3}\left(-\beta\frac{\partial}{\partial\beta}\right)\int_{E_0}^{\infty}dE\,\frac{p^2}{v_g^2}\frac{E}{(e^{\beta E}-1)}, \tag{30}$$

where the group velocity is defined as $v_g = \partial E(p)/\partial p$ and $\beta = 1/k_BT$. For small momenta we find laser-induced roton-like spectra, $E \simeq E_0+E_2p^2/2$, where we have introduced the gap $E_0 = \sqrt{\Omega_R\,(4u+\Omega_R)}/2$ and the curvature $E_2 = (2u+\Omega_R)\,/[m^*\sqrt{\Omega_R\,(4u+\Omega_R)}]$, given in terms of the effective mass $m^* = 1/(Ja_L^2)$. Then we immediately obtain Landau damping rate of the two-component BEC in an optical lattice expressed in terms of the density of the laser-induced roton mode,

$$\Gamma_L = \theta(\hbar\omega_q - E_0)\frac{27\pi}{16}\hbar\omega_q\frac{\rho_r}{\rho(\omega_q)}, \tag{31}$$

where the collective mode spectral density $\rho(\omega_q) = q(u+s)^3/(g_{jj}^2N\omega_q)$ and the momentum $q = \sqrt{2(\hbar\omega_q-E_0)/E_2}$. For a collective mode with energy below the gap, i.e. $\hbar\omega_q < E_0$, the Landau damping is zero, $\Gamma_L = 0$. Thus, laser-induced gap in the spectrum protects the collective modes from Landau damping. Above energy of the gap, the Landau damping rate is proportional to the density of the laser-induced roton gas, which scales at low temperatures as $\rho_r \simeq \frac{1}{\beta^2}$. In the special field-free case, i.e. $\Omega_R = \Delta = 0$, Rabi-Bogoliubov spectrum reduces to the conventional Bogoliubov spectrum. Thus, we obtain well-known result [39] of the phonon-mediated Landau damping of the collective modes,

$$\Gamma_L(\Omega_R = 0) = \frac{27\pi}{16}\hbar\omega_q\frac{\rho_n}{\rho} \simeq \frac{1}{\beta^4}. \tag{32}$$

Here ρ_n is the normal density of a phonon gas $\rho_n = 2\pi^2T^4/(45\hbar^3c^5)$, where c is the speed of sound [40]. Therefore, in the presence of a laser field damping of the collective modes in the condensate significantly slows down compared to the laser-free phonon-mediated Landau damping of a scalar BEC.

The damping rate of collective excitations in a driven Bose gas can be verified via two-photon Bragg spectroscopy as it was done in [41]. In the experiment, the quasiparticles in a BEC of ^{87}Rb atoms were excited by tuning the frequency difference and angle between the Bragg beams applied to the condensate. Immediately after the applied Bragg pulses, the magnetic trap, which confined the BEC, was rapidly turned off. Following the free expansion the BEC cloud was imaged via on-resonance absorption, which allowed the number of scattered atoms as a function of energy and momentum to be extracted. The measurement revealed a significant suppression of collisions of quasiparticles at low momentum. In both cases, namely, Beliaev damping of collective modes in a laser-free BEC, and Landau damping in a laser-driven condensate, the physical systems are characterized by a critical energy, below which collision of the collective modes and associated damping processes are completely suppressed. We conclude that even though the collisions between quasiparticles in the experiment were described by the Beliaev damping process, we expect that the same holds for Landau damping of collective modes in a laser-driven condensate.

V. CONCLUSIONS

We have investigated the quantum many-body physics of a driven two-component Bose-Einstein condensate in an optical lattice driven by a microwave field. We derived exact analytical results for the generalized Rabi-Bogoliubov spectrum and amplitudes of the condensate. We found a gap in the spectrum of elementary excitation in the BEC, which amounts to the effective Rabi frequency of the applied laser field. We discovered symmetries between the elementary excitations of a driven Bose gas, which generalize the underlying symmetries in the standard Bogoliubov amplitudes. The gapped spectrum and new symmetries of elementary excitations in a driven BEC dramatically modify dynamics of collective modes compared to the laser-free case. Specifically, we found that below the gap energy the collective mode are damping-free. Above the gap energy the damping rate is proportional to the density of the laser-induced roton mode. Thus, the Landau damping rate of the collective modes in a driven condensate is considerably reduced compared to the phonon-mediated damping processes in a laser-free condensate.

ACKNOWLEDGMENTS

This material is based in part upon work supported by the US National Science Foundation under grant numbers PHY-1306638, PHY-1207881, PHY-1520915, OAC-1740130, and the US Air Force Office of Scientific Research grant number FA9550-14-1-0287. This work was performed in part at the Aspen Center for Physics, which is supported by the US National Science Foundation grant PHY-1607611.

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