Bell inequalities for falsifying mesoscopic local realism via amplification of quantum noise

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Bell inequalities for falsifying mesoscopic local realism via amplification of quantum noise

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Macroscopic realism (MR) per se specifies that where a system can be found in one of two macroscopically distinct states (like a cat being dead or alive), the system is predetermined to be in one or other of the two states. A minimal assumption of a macroscopic realistic theory therefore is the validity of a hidden variable λ_M that predetermines the outcome (whether dead or alive) of a measurement $\hat{M}$ distinguishing the two states. Proposals to test MR generally introduce a second premise to further qualify the meaning of MR. Thus, we consider a model, *macroscopic local realism* (MLR), where the second premise is that measurements at one location cannot cause an instantaneous macroscopic change δ to the predictions of a system at another. To provide a practical test, we define the intermediate concept of δ-scopic local realism (δ-LR), where $\delta \neq 0$ can be quantitified, but may not be macroscopic. By considering the amplification of quantum fluctuations, we show how negation of δ-LR is possible using fields violating a continuous variable Bell inequality. A Bell inequality is derived that tests δ-LR, and a quantitative proposal given for experiments based on polarisation entanglement. In the proposal, δ is the magnitude of the quantum noise scaled by an adjustable coherent amplitude α that can also be considered part of the measurement apparatus. Thus, δ is large in an absolute sense, but scales inversely to the square root of the system size, given as $|\alpha|^2$. We discuss how the proposed experiment gives a realisation of a type of Schrodinger-cat experiment without problems of decoherence.

I. INTRODUCTION

In his essay of 1935, Schrodinger considered the quantum interaction of a microscopic system with a macroscopic system [1]. After the interaction, the two systems become entangled. If the macroscopic system were likened to a cat, then according to the standard interpretation of quantum mechanics, it would seem possible for the "cat" to be in a state that is "neither dead nor alive". In a simplistic analogy to Schrodinger's system, the quantum state describing the microscopic and macroscopic systems after the interaction can be written

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|dead\rangle_C|\downarrow\rangle_S + |alive\rangle_C|\uparrow\rangle_S\Big) \qquad (1.1)$$

Here $|\uparrow\rangle$ and $|\downarrow\rangle$ represent two distinct states of the microscopic system S, and $|dead\rangle$ and $|alive\rangle$ symbolise two macroscopically distinct states for the macroscopic cat-system C. Different forms of cat-states have been realised experimentally [2–7]. The interpretation of the "cat" in the superposition state (1.1) is that it is "neither dead nor alive" prior to measurement [1, 8]. This apparent contradiction with macroscopic realism has motivated much research [9–19], including modifications of quantum mechanics that aim to resolve the measurement problem [20].

What constitutes a rigorous signature of a cat-state and how such signatures can be interpreted as a falsification of macroscopic realism is an important question. This question was addressed by Leggett and Garg, who proposed a model of macroscopic realism for dynamical systems [9]. The Leggett-Garg inequalities allow a falsification of the Leggett-Garg model and have motivated experiments and proposals to test macro-realism, including for superconducting flux quibits [21], solid state quibits [22], cold atoms and Bose-Einstein condensates [23–25], and mechanical oscillators [26].

Here, we consider an alternative approach for testing macroscopic realism (MR), that does not involve assumptions about dynamics. To address the need for a strict test of MR, Leggett and Garg defined macroscopic realism in terms of a macroscopic hidden variable [9]. In their model, the fundamental premise is as follows: where a system can be found in one of two macroscopically distinct states (like a cat being dead or alive), the system is predetermined to be in one or other of the two states. A minimal assumption of MR therefore is the validity of a hidden variable λ_M that predetermines the outcome (whether dead or alive) of a coarse-grained measurement $\hat{M}$, that distinguishes the two states. Leggett and Garg referred to this assumption as *macroscopic realism per se* (MRPS). The direct negation of the hidden variable λ_M proves difficult, and a second premise is normally introduced to qualify the meaning of MR. Leggett and Garg introduced the additional assumption of macroscopic noninvasive measurability, to define a model now called macro-realism. An analysis of the macroscopic realism models tested by the Leggett-Garg inequalities has been given recently by Maroney and Timpson [27].

In this paper, we consider a different model for macroscopic realism, where the second premise is the assumption of *macroscopic locality* (ML). In this way, we consider a model of MR called *macroscopic local realism* (MLR). ML asserts that measurements at one location cannot cause an instantaneous macroscopic change to the system at another. By a macroscopic change to the system, we mean a macroscopic change to the *predictions* of that system. The ML assumption is that defined originally by Bell [28], except that small nonlocal changes to the system may be permitted. The combined premises of MRPS and ML constitute the premise of MLR [29–31]. In many contexts, MLR cannot be expected to fail because of bounds placed on the predictions of quantum mechanics in accordance with the uncertainty relation [15, 19, 32].

To test a weaker hypothesis where the "dead" and "alive" states of the system are not necessarily macroscopically distinguishable, but may be mesoscopically so, it is useful to define the premise of *δ-scopic local realism* (δ-LR). We consider a coarse-grained measurement $\hat{M}$ that distinguishes the two states, by way of two distinct outcomes. We suppose that the two distinct outcomes have a difference of 2δ (Figure 1). The δ-LR premise asserts firstly the validity of a hidden variable λ_δ to describe that the outcome of the measurement $\hat{M}$ is predetermined, consistent with the system being predetermined in one state or the other. The second premise of δ-scopic locality asserts that a measurement made at a different location to the system cannot cause an instantaneous change of greater than (or equal to) 2δ to the outcome of the measurement $\hat{M}$.

The contribution of this paper is to derive modified Bell inequalities that may be used to test δ-LR where outcomes are not necessarily confined to a dichotomic spectrum (Figure 1), and to give a context in which negation of δ-LR is predicted by quantum mechanics. Certainly, violation of MLR occurs if one can demonstrate a violation of a Bell inequality for two spatially separated entangled cat-systems where the outcomes of all relevant measurements on the cat-systems are macroscopically distinct. Our main result is to give a potentially workable proposal for testing δ-LR, where δ is "macroscopic" in an *absolute* rather than a relative sense, for a scalable system. To do this, we consider polarisation squeezing experiments [30, 31]. These experiments give insight into the measurement process, by creating a transition from microscopic to macroscopic that is controlled by a coherent field α [30, 31, 33–35]. In the transition, the quantum fluctuations are amplified by the coherent field as part of the measurement process. In the proposed test, the meaning of "δ-scopically distinguishable" refers to particle number differences δ that are large in an absolute sense, but small ($\sim 1/\sqrt{N}$) compared to the total number N of particles of the system.

We give a firm proposal to test local realism at a quantifiable level, by deriving a Bell inequality, the violation of which falsifies a δ-scopic local realism. The Bell inequality is then applied to a definite proposal where experiments violate Bell inequalities for continuous variable measurements [36–38]. The experiment is in one sense "scalable", being feasible for large systems, because the usual severe limitations due to decoherence do not apply when δ is below the quantum noise level ($\sim \sqrt{N}$).

In the final section of this paper, we discuss the degree of analogy between the "cat" superposition state considered in this paper and the entangled cat-state (1.1). A state of type (1.1) is formed after a quantum measurement. In that case, the system C represents the macroscopic pointer of a measurement apparatus. The "dead" and "alive" outcomes for the cat-system (being correlated with the spin being "up" or "down") correspond to the two positions (N_+ and N_-) on the measurement dial that give the measurement outcome for spin. We emphasise that the δ-LR tests proposed in this paper are not sufficient to indicate that the measurement pointer is located "simultaneously at two distinct "positions" N_+ and N_- on the dial", or that there are nonlocal effects of the order $N_+ - N_-$. For the cat-state (1.1), the separation between N_+ and N_- is well beyond the quantum noise level given by $\alpha \sim 1$ (assuming $\alpha \to \infty$). The δ- LR tests however may indicate nonlocality between a macroscopic pointer and a second system, that manifests on the scale of "positions" with a spread $\sqrt{N_\pm}$.

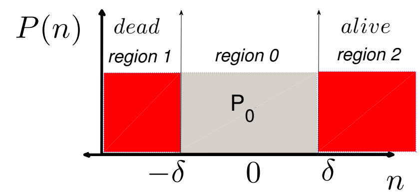

Figure 1. *Testing δ-scopic LR using a Bell inequality:* Here $P(n)$ is the probability of a obtaining a result n in a hypothetical experiment. P_0 is the probability of a result between $-\delta$ and $+\delta$. We consider experiments where the binned outcomes of a measurement $\hat{M}^A$ made on system A are distinct by an amount 2δ. Similarly, we consider that the outcomes of a measurement $\hat{M}^B$ made on a second system B (spatially separated from the first) are distinct by 2δ. This is described in the diagram with $P_0 = 0$. An example of such an experiment is illustrated in Figure 2, which introduces two adjustable coherent fields with amplitude α. The outcomes n for measurements J_θ^A, J_ϕ^B defined in Figure 2 are binned into one of the regions $1, 0, 2$. As $\alpha \to \infty$, $P_0 \to 0$ (for fixed δ). In that case, the two outcomes given by $n < -\delta$ and $n > \delta$ are said to be δ-scopically distinct, and are referred to as "dead" and "alive" as $\delta \to \infty$. A method accounting for nonzero P_0 is given in Section IV.

II. BELL INEQUALITIES FOR TESTING δ-SCOPIC LOCAL REALISM

We consider two spatially separated cat-systems A and B that at any given time can each be found (upon measurement) to be in one of two macroscopically distinguishable states ("dead" or "alive). We suppose that the two states can be distinguished by appropriate coarse-grained local measurements $\hat{M}^A$ and $\hat{M}^B$ made on each system (as discussed in the Introduction, with $\delta \to \infty$). In this section, we consider a Bell inequality to test macroscopic local realism.

Macroscopic local realism (MLR) asserts firstly the minimal assumption of macroscopic realism, called macroscopic realism per se (MRPS), that at any given time the result of the coarse-grained measurement on a particular cat-system is predetermined, prior to measurement. This implies the validity of a hidden variable that describes which state ("dead" or "alive") the system is in, prior to the measurement. The hidden variables will be denoted λ_M^A and λ_M^B for the cat-systems A and B respectively, and will be referred to as the "macroscopic hidden variables". In such a model, the result of the measurement $\hat{M}^A$ made on system A is determined by the value of the hidden variable λ_M^A; similarly the result of the measurement $\hat{M}^B$ at B is determined by the value of λ_M^B. In this paper we are careful to use only the *minimal* assumption of macroscopic realism (referred to as MRPS), in which the meaning of "state" refers only to the *macroscopic* state of the system, whether "dead" or "alive". This macroscopic meaning is defined with reference to the outcome of the coarse-grained measurements $\hat{M}_{A/B}$, and therefore does not distinguish states that have a microscopic difference in the predictions of an observable.

The second assertion of macroscopic local realism (MLR) is *macroscopic locality*, that the measurement $\hat{M}$ on one system cannot bring about an immediate macroscopic change to the system at the other location. By a macroscopic change in this context, we mean a transition of the macroscopic hidden variable λ_M being $+1$ to being -1 or vice versa i.e. a transition between "dead" and "alive" states. The premise of macroscopic locality asserts that a measurement cannot make a macroscopic change to the other system, but we cannot exclude that microscopic changes may occur.

We consider local measurements $\hat{M}_\theta^A$ and $\hat{M}_\phi^B$ that can be made on each system A and B. Here θ and ϕ are measurement settings and we consider two measurement choices θ, θ' and ϕ, ϕ' for each system. We suppose that the measurements $\hat{M}_\theta^A$, $\hat{M}_{\theta'}^A$ and $\hat{M}_{\phi'}^B$, $\hat{M}_\phi^B$ *each* give macroscopically distinct binary outcomes which are denoted $+1$ and -1 (corresponding to "alive" and "dead" regimes 2 and 1 shown in Figure 1). If we assume macroscopic local realism, the following Clauser-Horne-Shimony-Holt (CHSH) Bell inequality will hold [31, 39]

$$\langle \hat{M}_\theta^A \hat{M}_\phi^B \rangle - \langle \hat{M}_\theta^A \hat{M}_{\phi'}^B \rangle + \langle \hat{M}_{\theta'}^A \hat{M}_\phi^B \rangle + \langle \hat{M}_{\theta'}^A \hat{M}_{\phi'}^B \rangle \leq 2 \tag{2.1}$$

The MLR model is an example of an LHV model. The derivation of (2.1) follows as for the CHSH Bell inequality that applies to all LHV models where the measurements have binary outcomes [28, 39]. The violation of (2.1) will imply failure of MLR.

Violations of Bell inequalities for cat-states have been predicted and observed experimentally [6, 7, 38]. However these do not involve macroscopic distinct binary outcomes for all measurements θ, θ', ϕ and ϕ' and hence do not violate (2.1). We mention that similar hybrid inequalities based on the combined premises of macrorealism and Bell locality have been derived and tested experimentally for small systems [40]. These inequalities are however different to those above. Applying to a single experimental ensemble with fixed analyser settings and requiring weak measurements for their violation, they test a different model of macroscopic realism.

As might be expected, the possibility of violating the inequality (2.1) depends on how we interpret "*macroscopic*". To quantify the meaning of "macroscopic" in a given situation, we generalise the definition of MLR by defining δ-scopic local realism (δ-LR). The δ-LR is defined in the Introduction. The model of δ-scopic LR is falsified where the separation between the binary ("dead" and "alive") outcomes for the measurements $\hat{M}_\theta^A$, $\hat{M}_{\theta'}^A$ and $\hat{M}_{\phi'}$, $\hat{M}_\phi^B$ is greater than or equal to 2δ (Figure 1). We next examine scenarios where it is possible to falsify δ-scopic local realism for some quantifiable δ that can be made large by an amplification that occurs as part of a measurement process, in analogy to a Schrodinger-cat gedanken experiment.

III. AMPLIFICATION OF NONLOCAL CORRELATIONS

A. Amplification of the quantum noise

We now consider in detail proposals for violating δ-scopic local realism using field quadrature phase amplitude observables. Here, the measurement of the field amplitudes takes place via an amplification process that involves a second field, so that the final measurement is of a Schwinger spin [30, 31]. The relevant uncertainty principle for spin is

$$\Delta \hat{J}_X^A \Delta \hat{J}_Y^A \geq |\langle \hat{J}_Z^A \rangle|/2 \tag{3.1}$$

It is possible to create a situation where the quantum noise level given by $|\langle \hat{J}_Z^A \rangle|/2$ corresponds to a very large photon number difference (field intensity). This allows consideration of changes of order δ where δ is large in the absolute sense of particle number (intensity), but small compared to the quantum noise level $|\langle \hat{J}_Z^A \rangle|/2$. The highly non-classical mesoscopic effects that are predicted can then be understood as a property of amplified quantum fluctuations.

The system we consider comprises two spatially separated modes at A and B (Figure 2). We denote the modes initially prepared at A and B by the symbols a_1 and b_1, and define the boson operators, $\hat{a}_1$ and $\hat{b}_1$, respectively. A second mode pair a_2 and b_2 is defined similarly. These modes are prepared in an entangled state (see next section). At each location, the mode a_1 (or b_1) is combined with a second mode a_2 (or b_2) respectively, these second modes being prepared in independent coherent states $|\alpha\rangle$ (where $\alpha \to \infty$). The combination can occur through a 50/50 beam splitter (or equivalent). The outputs at each location are rotated modes with boson operators $\hat{a}_+ = (\hat{a}_1 + \hat{a}_2)\sqrt{2}$ and $\hat{a}_- = (-\hat{a}_1 + \hat{a}_2)/\sqrt{2}$ for A, and $\hat{b}_+ = (\hat{b}_1 + \hat{b}_2)/\sqrt{2}$ and $\hat{b}_- = (-\hat{b}_1 + \hat{b}_2)/\sqrt{2}$ for B. This amplification is similar to the homodyne detection used

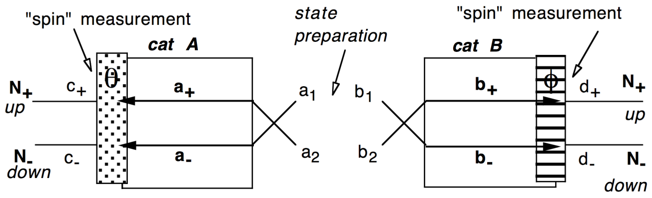

Figure 2. *Two entangled cat-systems are created as part of a measurement process:* The modes a_1 and b_1 are prepared in an microscopic entangled state $|\psi\rangle$. The modes a_2 and b_2 are independent intense coherent states $|\alpha\rangle$. The fields a_1, a_2 and b_1, b_2 are combined at each location A and B to create mode pairs $a_\pm$ and $b_\pm$. For $\alpha\rightarrow\infty$, these form two cat-systems at A and B. Final measurements of the number differences n, m given by $J_\theta^A=(N_+-N_-)/2$ and $J_\phi^B=(N_+-N_-)/2$ are made using a polariser beam splitter at each detector. This gives an amplified readout of the quadrature phase amplitudes x_θ and x_ϕ of the fields a_1 and b_1. Whether the cat-systems are considered "dead" or "alive" is determined by the sign of J_θ^A and J_ϕ^B. For a small fixed δ, we define the outcomes $J_{\theta,\phi}^{A/B}$ to be "dead" if $J_{\theta,\phi}^{A/B}<-\delta$ and "alive" if $J_{\theta,\phi}^{A/B}>\delta$. As $\alpha\rightarrow\infty$, the probability P_0 of a result $-\delta\leq J_{\theta,\phi}^{A/B}\leq\delta$ becomes zero (Figure 1). This is true for any large δ, provided $\alpha\gg\delta$. Thus, the dead and alive outcomes become macroscopically distinguishable as $\alpha\rightarrow\infty$.

in experiments that measure a squeezing of the quantum fluctuations of modes a_1 and a_2 [33].

The mode pairs that are created at A and B have a large total occupation number, because of the large coherent fields α. In Figure 2, these systems are called "cat A" and "cat B". At each location an experimentalist makes a measurement of a number difference $\hat{N}_+-\hat{N}_-$ that defines a Schwinger spin measurement. These measurements are made on the cat-systems, and (as we will see) enable a distinction to be made between "dead" and "alive" states of the mode-pairs. At location A, we define the outcome of the measurement $\hat{S}_\theta^A$ to be ±1 according to the sign of the outcome $J_\theta^A(\varphi)$ of the measurement $\hat{J}_\theta^A(\varphi)$, where

$$\hat{J}_\theta^A(\varphi)=(\hat{N}_+-\hat{N}_-)/2=(\hat{c}_+^\dagger\hat{c}_+-\hat{c}_-^\dagger\hat{c}_-)/2 \qquad (3.2)$$

Here $\hat{c}_+=\hat{a}_+\cos\theta+e^{i\varphi}\hat{a}_-\sin\theta$ and $\hat{c}_-=-\hat{a}_+\sin\theta+e^{i\varphi}\hat{a}_-\cos\theta$. This measurement $\hat{J}_\theta^A(\varphi)$ could be carried out using a phase shift φ and polarising beam splitters rotated to θ with the modes a_+ and a_- as inputs. With suitable choice of θ and φ, the measurement (3.2) corresponds to a measurement of a Schwinger spin observable for the operators $\hat{a}_1$ and $\hat{a}_2$. These are defined $\hat{J}_X^A=(\hat{a}_2^\dagger\hat{a}_1+\hat{a}_1^\dagger\hat{a}_2)/2$, $\hat{J}_Y^A=(\hat{a}_2^\dagger\hat{a}_1-\hat{a}_1^\dagger\hat{a}_2)/(2i)$ and $\hat{J}_Z=(\hat{a}_2^\dagger\hat{a}_2-\hat{a}_1^\dagger\hat{a}_1)/2$. We see that $\hat{J}_X^A=\hat{J}_0^A(\pi/2)$, $J_Y^A=\hat{J}_{\pi/4}(\pi/2)$ and $\hat{J}_Z^A=\hat{J}_{\pi/4}^A(0)$. The outcome of a measurement $\hat{S}_\phi^B$ on cat B is defined to be ±1 according to the sign of the spin observable

$$\hat{J}_\phi^B(\gamma)=(\hat{d}_+^\dagger\hat{d}_+-\hat{d}_-^\dagger\hat{d}_-)/2 \qquad (3.3)$$

where $\hat{d}_+=\hat{b}_+\cos\phi+e^{i\gamma}\hat{b}_-\sin\phi$ and $\hat{d}_-=-\hat{b}_+\sin\varphi+e^{i\gamma}\hat{b}_-\cos\varphi$. With suitable choice of ϕ and γ (as above), this measurement corresponds to a Schwinger observable at B, defined as $\hat{J}_X^B=(\hat{b}_2^\dagger\hat{b}_1+\hat{b}_1^\dagger\hat{b}_2)/2$, $\hat{J}_Y^B=(\hat{b}_2^\dagger\hat{b}_1-\hat{b}_1^\dagger\hat{b}_2)/(2i)$ and $\hat{J}_Z^B=(\hat{b}_2^\dagger\hat{b}_2-\hat{b}_1^\dagger\hat{b}_1)/2$.

It is well-known that in the limit of $\alpha\rightarrow\infty$, the measurements $\hat{J}_\theta^A(\varphi)$ and $\hat{J}_\phi^B(\gamma)$ are *also* measurements of the quadrature phase amplitudes $\hat{x},\hat{p}$ of the original modes a_1 and b_1 [30]. This is because the fields a_2 and b_2 are (to a good approximation) intense classical fields of amplitude α (which we take to be real) [33, 41]. In that limit, we can simplify: $\hat{J}_X^A=\alpha\sqrt{2}\hat{x}^A$, $\hat{J}_Y^A=\alpha\sqrt{2}\hat{p}^A$, $\langle\hat{J}_Z\rangle\rightarrow\alpha^2/2$ where $\hat{x}^A=(\hat{a}_1^\dagger+\hat{a}_1)/\sqrt{2}$ and $\hat{p}^A=i(a_1^\dagger-a_1)/\sqrt{2}$ are the quadrature phase amplitudes of a_1. The Heisenberg uncertainty relation (3.1) reduces to $\Delta\hat{x}^A\Delta\hat{p}^A\geq1/2$ for the quadratures. In fact, more generally, defining $\hat{J}_\theta^A=\hat{J}_\theta^A(\pi/2)$ we see that

$$\hat{J}_\theta^A=\alpha\sqrt{2}\hat{x}_{2\theta}^A \qquad (3.4)$$

where $\hat{x}_\theta=\hat{x}\cos\theta+\hat{p}\sin\theta$. The $\hat{J}_\theta^A(\pi/2)$ is thus a measurement of the amplified quadrature phase amplitude $\alpha\sqrt{2}\hat{x}_{2\theta}^A$. A similar result holds for the quadrature phase amplitudes $\hat{x}^B=(\hat{b}_1^\dagger+\hat{b}_1)/\sqrt{2}$ and $\hat{p}^B=i(\hat{b}_1^\dagger-\hat{b}_1)/\sqrt{2}$ defined at B. $\hat{J}_\phi^B=\hat{J}_\phi^B(\pi/2)$ gives the amplified quadrature amplitude $\alpha\sqrt{2}\hat{x}_{2\phi}^B$.

The increase in α also amplifies the total number of particles at each site. The nature of the amplification is evident by the uncertainty relation (3.1) for the spin measurements which reduces to

$$\Delta\hat{J}_X^A\Delta\hat{J}_Y^A\geq|\alpha|^2/4 \qquad (3.5)$$

since α is taken to be very large. The amplification that is crucial to creating the macroscopic states at the locations A and B is also an amplification of the quantum noise level, and there is no amplification *relative* to this level [29–31].

We envisage an experiment similar to the optical and atomic polarisation entanglement experiments reported in Refs. [33–35, 41]. In those experiments, at site A, the experimentalist can measure a particular $\hat{J}_\theta^A$. Each $\hat{J}_\theta^A$ is a measurement of a particle number difference according to (3.3), and is also a measurement of quadrature phase amplitude according to (3.4). Similar measurements are made at B. Different to other experiments that measure quadrature fluctuations however, the choice of measurement angle θ is made *after* the combination of the mode a_1 with the strong field a_2. With the definition of the "dead" and "alive" outcomes given in Figure 2 (see next Section), the binned measurements $\hat{S}_\theta^A$ and $\hat{S}_\phi^B$ of $\hat{J}_\theta^A$ and $\hat{J}_\phi^B$ defined above are analogous to the measurements $\hat{M}^A$ and $\hat{M}^B$ defined in the Introduction. The measurements $\hat{J}_\theta^A$ and $\hat{J}_\phi^B$ are macroscopic, in the sense that if one considers a change δ_θ in the quadrature phase amplitude $\hat{x_\theta}$, then one can define an amplified change $\delta=\alpha\sqrt{2}\delta_\theta$ for the particle number difference measured by $\hat{J}_\theta$. We will

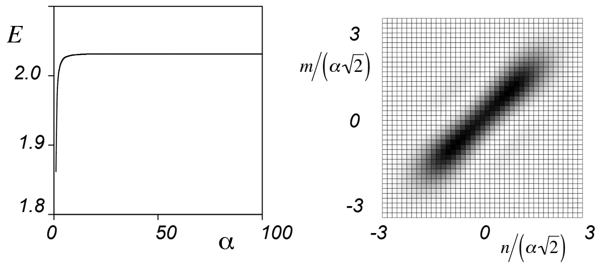

Figure 3. *Signature of the cat-state created by the apparatus of Figure 2:* The number differences $J_{\theta/\phi}^{A/B}=(N_+-N_-)/2$ at the sites A and B are denoted n and m respectively, and are binned to the values $S_{\theta,\phi}^{A/B}=1$ or -1 according to sign as described in text. Left: The expectation values $E=\langle S_\theta^A S_\phi^B\rangle-\langle S_\theta^A S_{\phi'}^B\rangle+\langle S_{\theta'}^A S_\phi^B\rangle+\langle S_{\theta'}^A S_{\phi'}^B\rangle$ generated when modes a_1 and b_1 are prepared in the state $|\psi\rangle$ given by eq. (3.7) will violate the Bell inequality (3.6) for all $\alpha\to\infty$. Right: A contour graph of the probability for joint outputs n and m (here $\theta=\phi$). The absolute values of the number difference outputs n, m increase with α.

see in the next Section that this allows us to define dead and alive outcomes for the cat-system that in the limit of $\alpha\to\infty$ are, in an *absolute* sense, mesoscopically or macroscopically distinguishable (Figures 1 and 2).

B. Violation of continuous variable Bell inequalities

We now discuss experiments to falsify a δ-scopic local realism. For some states, the correlations obtained for the quadrature phase amplitude measurements $\hat{x}_\theta^A$ and $\hat{x}_\phi^B$ at each site are predicted to violate a Bell inequality [36–38]. The outcome of the measurement $\hat{x}$ at each site can be binned into regions of non-negative and negative values. We define an observable $\hat{S}_\theta^A$ whose value is $+1$ if $x_\theta^A\geq 0$ and -1 otherwise. A similar observable $\hat{S}_\phi^B$ is defined at B, based on the quadrature phase amplitude $\hat{x}_\phi^B$. It has been shown that for certain states $|\psi\rangle$ and for certain angles ϕ, ϕ', θ' and θ, the following Bell inequality is violated [37, 38]

$$E=\langle S_\theta^A S_\phi^B\rangle-\langle S_\theta^A S_{\phi'}^B\rangle+\langle S_{\theta'}^A S_\phi^B\rangle+\langle S_{\theta'}^A S_{\phi'}^B\rangle\leq 2 \quad (3.6)$$

thus negating the possibility of an LHV model describing the results of those measurements.

Since we can also write $\hat{J}_\theta^A=\alpha\sqrt{2}\hat{x}_{2\theta}^A$ and $\hat{J}_\phi^B=\alpha\sqrt{2}\hat{x}_{2\phi}^B$, this inequality is also violated if we redefine $\hat{S}_\theta^A$ as the observable with value $+1$ if $J_\theta^A\geq 0$ and -1 otherwise; and $\hat{S}_\phi^B$ as the observable with value $+1$ if $J_\phi^B\geq 0$ and -1 otherwise. The violation implies that there is no predetermined (local) hidden variable description for the sign of the number differences J_θ^A, J_ϕ^B [30]. Because we can increase α, this gives a situation where one can falsify local hidden variables for noisy measurements of particle number difference. An example of $|\psi\rangle$ is the pair coherent state or "circle" state

$$|\psi\rangle=\frac{e^{r_0^2}}{2\pi\sqrt{I_0(2r_0^2)}}\int_0^{2\pi}|r_0e^{i\zeta}\rangle|r_0e^{-i\zeta}\rangle d\zeta \qquad (3.7)$$

(I_0 is the modifed Bessel function, $r_0=1.1$) that is generated near the threshold of nondegenerate parametric oscillation [37, 38]. These states are generated using nondegenerate parametric oscillation [42]. The predictions for the violation of the Bell inequality (3.6) for this state are given in Figure 3.

As α increases, it is argued that the $+1$ and -1 outcomes for $\hat{S}_\theta^A$ ultimately become "macroscopically" distinct (in the absolute sense discussed above). Similarly the $+1$, -1 outcomes for $\hat{S}_\phi^B$ become macroscopically distinct. The measurements $\hat{S}_\theta^A$ and $\hat{S}_\phi^B$ are then examples of macroscopic measurements $\hat{M}_\theta^A$ and $\hat{M}_\phi^B$ and the violation of (3.6) is a violation of (2.1). In this limit we would violate "macroscopic local realism".

To understand the argument, we define a region of measurement outcome x for $\hat{J}_\theta^A$ where the result falls between $-\delta$ and $+\delta$ for some $\delta\neq 0$ (see Figure 1). We call this region 0, and also define the region of outcomes $x\geq\delta$ as region 2, and the region of outcomes $x\leq-\delta$ as region 1. For any (*arbitrarily large*) fixed δ, the probability P_0 of a result in the region 0 becomes zero as $\alpha\to\infty$. Yet the violation of the Bell inequality is unchanged with α (Figure 3a). Hence, violation of the inequality (2.1) is possible for the two outcomes $+1$ and -1 that for sufficiently large α can be justified as separated by a region of width 2δ, with $P_0\to 0$. Hence, by taking δ large, there is a prediction for a violation of mesoscopic/ macroscopic local realism.

For a realisation of the experiment, however, δ is finite, and there will be a small nonzero probability $P_0\neq 0$ for a result in region 0. We would also prefer in an experiment to quantify *precisely by number* the level δ for which local realism is violated, since the labelling of "macroscopic" or "mesoscopic" is subjective. This is explained in the next Section.

IV. PRACTICAL QUANTIFIABLE δ-SCOPIC LOCAL REALISM TESTS

A. The modified Bell-CHSH inequalities

We now consider the case where there is a continuum of outcomes, meaning that $P_0\neq 0$, as defined in Figures 1 and 4. The meaning of macroscopic realism per se (MRPS) for the more general case where $P_0\neq 0$ is explained in the paper of Leggett and Garg [9] and in Refs. [24, 25, 43]. The MR premise for this generalised case is that the system be described as a probabilistic mixture of two *overlapping states*: the first gives outcomes in regions "1" or "0"; the second gives outcomes in regions "0" or "2". The MRPS assumption excludes the possibility that the system can be in a quantum superposition of

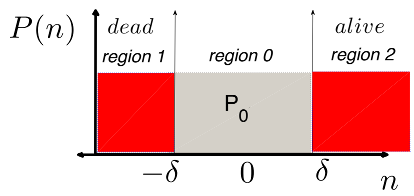

Figure 4. *Practical method for testing δ-scopic LR using a Bell inequality:* The outcomes of each measurement $\hat{J}_\theta^A$, $\hat{J}_\phi^B$ indicated in Figure 2 are binned into one of the regions $1,0,2$. As $\alpha\to\infty$, $P_0\to 0$. A method for testing δ-scopic LR where $P_0\neq 0$ is given in Section IV. This involves considering that the system is probabilistically in one of two states, that have overlapping outcomes. The first state is defined as producing an outcome in the combined regions 1 or 0; the second state is defined as producing an outcome in regions 0 or 2.

two states, one that gives outcomes in region 1 and the second that gives outcomes in 2. It does not however exclude quantum superpositions of states with outcomes in region 1 and 0, or quantum superpositions of states with outcomes in regions 0 and 2. Where δ is finite and not necessarily macroscopic, we use the term δ-scopic realism (per se) to describe the premise that is used.

We follow the approach of Refs. [24, 25, 43], and denote the hidden variable state associated with the outcomes in regions "1" or "0" for the system at A by the hidden variable value $\tilde{S}^A=-1$. Similarly, the hidden variable state that generates outcomes in regions "0" and "2" is denoted by the hidden variable value $\tilde{S}^A=1$. We define the variable $\tilde{S}^B$ similarly. The δ-scopic locality assumption asserts that the measurement at one location cannot change the result at the other in such a way that the system changes value of $\tilde{S}$ from $+1$ to -1, or vice versa. We define P_+ and P_- as the probabilities that the system is in the state with $\tilde{S}=+1$ or $\tilde{S}=-1$, respectively. Then we note that the δ-LR assumptions (which are δ -scopic realism per se and δ-scopic locality combined) predicts the Bell inequality

$$\langle\tilde{S}_\theta^A\tilde{S}_\phi^B\rangle-\langle\tilde{S}_\theta^A\tilde{S}_{\phi'}^B\rangle+\langle\tilde{S}_{\theta'}^A\tilde{S}_\phi^B\rangle+\langle\tilde{S}_{\theta'}^A\tilde{S}_{\phi'}^B\rangle\leq 2 \tag{4.1}$$

However, the moments $K_{\theta\phi}=\langle\tilde{S}_\theta^A\tilde{S}_\phi^B\rangle$ are no longer directly measurable, because an outcome between $-\delta$ and $+\delta$ could arise from either state, $\tilde{S}=-1$ or $+1$. Nonetheless, following a method similar to that given in Refs. [25, 43] we conclude that

$$P_1\leq P_-\leq P_1+P_0 \tag{4.2}$$

and

$$P_2\leq P_+\leq P_2+P_0 \tag{4.3}$$

where P_1, P_2 and P_0 are the probabilities of obtaining a result in regions 1, 2 and 0 respectively. These probabilities are experimentally measureable. The relations (4.2) and (4.3) are made clear on examining the depiction of the regions as given in Figure 4. From the relations, we establish bounds on the correlations assuming δ-LR, where $P_0\neq 0$. It is straightforward to see that δ-scopic local realism will imply

$$K_{\theta,\phi}^{lower}-K_{\theta\phi'}^{upper}+K_{\theta'\phi}^{lower}+K_{\theta'\phi'}^{lower}\leq 2 \tag{4.4}$$

where $K_{\theta\phi}^{lower}$ and $K_{\theta\phi}^{upper}$ are lower and upper bounds to $K_{\theta\phi}$ i.e.

$$K_{\theta\phi}^{lower}\leq K_{\theta\phi}\leq K_{\theta\phi}^{upper} \tag{4.5}$$

Correct upper and lower bounds are given by

$$\begin{aligned}K_{\theta\phi}^{lower}&=P_{2,2}(\theta,\phi)+P_{1,1}(\theta,\phi)\\&\quad-P_{10,20}(\theta,\phi)-P_{20,10}(\theta,\phi)\end{aligned} \tag{4.6}$$

and

$$\begin{aligned}K_{\theta\phi}^{upper}&=P_{20,20}(\theta,\phi)+P_{10,10}(\theta,\phi)\\&\quad-P_{1,2}(\theta,\phi)-P_{2,1}(\theta,\phi)\end{aligned} \tag{4.7}$$

Here we have introduced the notation that $P_{IJ,LM}$ is the joint probability for an outcome of J_θ^A in regions I or J and an outcome of J_ϕ^B in regions L or M (see Figure 4). Accordingly, $P_{I,L}$ is the joint probability for an outcome of J_θ^A in region I and an outcome of J_ϕ^B in regions L.

To emphasise the dependence on δ, the modified CHSH Bell-inequality (4.4) that follows from the assumption of δ-LR can be rewritten as $E_\delta\leq 2$ where

$$E_\delta=K_{\theta,\phi}^{lower}-K_{\theta\phi'}^{upper}+K_{\theta'\phi}^{lower}+K_{\theta'\phi'}^{lower} \tag{4.8}$$

Each term is measurable experimentally by dividing the regions of outcome into three binned regions as sketched in Figure 4, where the size of the middle region is determined by δ. The probabilites for obtaining outcomes in each region enables evaluation of the E_δ. Violation of the inequality $E_\delta\leq 2$ is sufficient to confirm a violation of δ-LR. This gives a practical means to demonstrate a violation of an δ-scopic local realism for a finite δ where there is a nonzero probability P_0 of an outcome in the region defined by $-\delta<x<\delta$. A similar inequality has been derived for Leggett-Garg experiments [25].

Rigorous Bell tests for continuous variable measurements are likely to be carried out in the future. The method we describe could be applied to any such experiment. For realistic tests based on current experiments, the shifts δ may not be macroscopic, but nonetheless offer a route to test local realism beyond the single particle level considered in most experimental tests of Bell nonlocality so far.

B. Analysis of feasibility

For an indication of what might be feasible in practice, let us assume the value of $E=2.2$ given in Figure 3 could be achieved as predicted by quantum mechanics, with a coherent field value α_0. Calculations given in Ref. [30] show that violations of the quadrature phase amplitude Bell inequality (3.6) are obtained with Gaussian noise added to the outcomes n, provided the noise standard deviation σ satisfies $\sigma \lesssim 0.2$. This suggests that a full calculation of E_δ may give violations ($E_\delta > 2$) for $\delta \sim 0.1\alpha_0$. The noise effectively adds a coarse-graining to the measurements.

The papers of Kofler and Brukner explain how ultimately macroscopic realism will be obtained through the coarse-grained measurements [16]. In the current paper, the violations of macroscopic local realism tolerate a "macroscopic" level of noise as $\alpha_0 \to \infty$, but the noise remains of order $1/\alpha_0$ relative to the overall system size given by $|\alpha_0|^2$.

We establish a conservative estimate for the value δ at which δ-LR might be falsifiable by deriving a less sensitive version of the modifed CHSH Bell inequality (4.4), given by $E_\delta \leq 2$. Suppose P_0^u is an upper bound on the value of P_0, for either site and for any of the relevant angle values. Then it is true that

$$P_{I,M} \leq P_{I0,M0} \leq P_{I,M} + 2P_0^u \tag{4.9}$$

Hence

$$\begin{aligned} K_{\theta\phi}^{lower} &\geq \langle S_\theta^A S_\phi^B \rangle - 4P_0^u \\ K_{\theta\phi}^{upper} &\leq \langle S_\theta^A S_\phi^B \rangle + 4P_0^u \end{aligned} \tag{4.10}$$

A modified CHSH-Bell inequality based on the premise of δ-scopic local realism is then seen to be

$$\langle S_\theta^A S_\phi^B \rangle - \langle S_\theta^A S_{\phi'}^B \rangle + \langle S_{\theta'}^A S_\phi^B \rangle + \langle S_{\theta'}^A S_{\phi'}^B \rangle \leq 2 + 16P_0^u \tag{4.11}$$

Let us assume the value of $E = 2.2$ given in Figure 3 is achieved in the experiment. Then from this inequality, we see that we will certainly obtain a violation of the δ-scopic realism for regions of width 2δ about the origin that correspond to a total probability of occupation of $P_0^u < 0.01$. One such region derived on the basis of the distributions for the current example (see Figure 3) has a width $\delta \sim 0.01\alpha_0$.

The value of α_0 is determined by the amplitude of the field modes indicated by a_2 and b_2 in Figure 2. These we sometimes refer to as the local oscillator fields. The treatment of these fields as well-defined modes is however simplistic. Examination of the experiments detecting polarisation and continuous variable entanglement reveals that the local oscillators are either the intense pulsed or continuous-wave output of a laser. Pulsed experiments are more in keeping with nonlocality tests. The experiment of Julsgaard et al [41] quotes 10^{13} photons in the 0.45ms 5mW pulse used to entangle two atomic ensembles. The fibre squeezing and EPR entanglement experiments cited in the review [44] use local oscillator pulses of a similar intensity. These figures suggest $\alpha_0 \sim 10^6$ to be possible, indicating values of $\delta \sim 10^4$ photons.

We stress that while the value of δ is amplified in absolute terms by this factor α_0, in this proposal the value δ is always *small* relative to the magnitude $|\alpha_0|$ which determines the quantum noise level in Eq. (3.5), and to $|\alpha_0|^2$ which determines the overall size of the system. In fact the ratio $\delta/|\alpha_0|^2$ decreases with increasing α_0. The argument can therefore be made that the value of δ be considered *microscopic*, in this sense. Certainly this is the viewpoint taken in standard squeezing and EPR entanglement experiments, where the local oscillator phase and hence the choice of measurement is selected prior the the amplification of system size that occurs as part of the measurement process.

In the proposed experiments, however, the system is amplified *prior* to the selection of the phase that determines the choice of measurement. This time order has been carried out in the polarisation squeezing and entanglement optical experiments [33], and in the atomic experiments of Julsgaard et al and Gross et al [41] where two large ensembles of atoms are entangled. Similar interpretations may be possible in other experiments [45]. It is these latter cases that give the arguable interpretation presented in this paper, that the fluctuation δ be regarded as mesoscopic (in fact δ-scopic) in an *absolute* sense.

V. THE MACROSCOPIC POINTER

The cat-system C of the entangled state (1.1) models the *pointer* of a measurement apparatus that measures the value "up" or "down" of the spin of system S. It is clear from (1.1) that the outcome of the measurement $\hat{M}$ on the pointer (whether dead or alive) is correlated with the value of spin.

An interesting question that we discuss in this section is whether falsification of the macrosopic realism (MR) test proposed in this paper gives evidence that the pointer is simultaneously in two "states" (locations) i.e. both "dead and alive" [8]? By definition, the macroscopic hidden variable λ_M predetermine*s* the "dead" or "alive" outcome for the macroscopic measurement $\hat{M}$, without further assumptions about underlying predetermined states [9]. Its direct negation could therefore potentially suggest the macroscopic paradox of the cat-system being "both dead and alive", analogous to the pointer being in two places at once. For the models of macro-realism considered or referenced in this paper, there is however a second premise also assumed. Therefore, logically, the falsification of these models need not suggest failure of the validity λ_M, but could be explained by failure of the second premise.

The cat-signature of this paper is based on the second premise being the assumption of a δ-scopic locality. There is thus a range of positions $\Delta m \sim \delta$ over which one pointer might potentially be interpreted as paradoxically "being simultaneously in both places", based also on the notion that nonlocality cannot be excluded over this range. This value of δ is however restricted to be *less than* $|\alpha|$, the level of quantum fluctuation according to eq. (3.5). This is *different* to the standard realisation of the entangled state (1.1) given by [2, 3, 7]

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|\alpha\rangle|\downarrow\rangle_S + |-\alpha\rangle_C|\uparrow\rangle_S\Big) \tag{5.1}$$

where the "dead" and "alive" states are considered distinct by several orders of α.

We also emphasise that the cat-signature of this paper does not imply a falsification of a macroscopic realism model for the pointer-state of (1.1). The differences are as follows: In the experiment of Figure 2, the two cat-states at A and B act as two "pointers" for the microscopic quadrature phase amplitudes of the original entangled field modes denoted a_1 and b_1. However the original modes are destroyed in the formation of the pointer states. We also see from Figure 3 [37] that the "positions" m and n of the two pointers are not well-correlated at the quantum noise level i.e. one pointer does not measure the "position" of the other to a precision beyond the quantum noise level. Put another way, the uncertainty in any such measurement is of order $\Delta m \sim |\alpha|$.

If the predictions of quantum mechanics were to be verified for the experiment proposed in this paper, the simplest interpretation of the pointers is consistent with a hybrid classical/ quantum model of a macroscopic pointer discussed in Ref. [19]. This model specifies that any lack of predetermination of the "position" of the pointer is constrained to be over "distances" of size δ bounded by the quantum noise level.

VI. CONCLUSION

In this paper, we ask how to test macroscopic or mesoscopic realism in a way that could *not* be explained by a theory allowing some degree of microscopic nonlocality. In this context, we define the distinction between "microscopic" and "macroscopic" in terms of a particle or photon number difference. To quantify the arguments, a Schwinger observable is introduced as the difference in occupation number for two modes. We show that tests of a δ-scopic local realism model may be possible, where δ can be regarded as macroscopic, provided by "macroscopically distinguishable outcomes" we mean outcomes that have a large absolute separation δ of the two-mode number difference. For the examples that we consider in this paper, however, the separation δ is very small relative to the total number of particles of the system. To allow quantifiable tests, we use the word δ-scopic and consider separations δ that allow a transition between microscopic and macroscopic.

Using this meaning of "δ-scopic", we outline a proposal to test *δ-scopic local realism* where two cat-systems are generated using two entangled field modes. The modes are prepared in a state that violates a continuous variable Bell inequality. The cat-systems are created using an amplification process brought about by local coherent fields. This amplification can be interpreted as part of a measurement process, in analogy to Schrodinger's original gedanken experiment. A practical method for testing δ-scopic local realism is developed that involves a quantifiable δ-scopic Bell inequality.

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