Topological Aharonov-Bohm suppression of optical tunneling in twisted nonlinear multicore fibers

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Topological Aharonov-Bohm Suppression of Optical Tunneling in Twisted Nonlinear Multicore Fibers

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We show that the Aharonov-Bohm-like suppression of optical tunneling in twisted multicore fibers can persist even under highly nonlinear conditions. Our analysis indicates that the topological phase is robust and remains intact in the presence of nonlinearity. The energy exchange dynamics are theoretically analyzed via closed-form solutions in four-core ring systems. Effects arising from asymmetry are also investigated. A possible arrangement to experimentally observe this effect is suggested.

I. INTRODUCTION

Electrons interacting with a magnetic field can display an array of interesting and counterintuitive effects. These include for example the emergence of Landau levels [1], quantum Hall [2] and topological insulator effects [3], as well as quantization of magnetic flux through a superconductor [4], to mention a few. In recent years, the possibility of observing processes akin to those expected from magnetic fields has also been intensely explored in bosonic settings. These include for example photon and cold atom dynamics under the influence of synthetic magnetic fields [5–12], photonic topological insulators [13–16] and nonreciprocal optical elements [17]. In such arrangements, an artificial magnetic field can be effectively introduced by exploiting the intimate connection between Berry's phase in parameter space and the Aharonov-Bohm phase [18–21]. An intriguing phenomenon arising from the presence of a magnetic field is a possible inhibition of electron tunneling in degenerate quantum channels-a process never been observed before in any physical system [22]. This latter effect is a direct byproduct of an Aharonov-Bohm (AB) phase [23, 24] that in turn leads to a complete elimination of tunneling, a process resulting from the destructive interference of the eigenfunctions involved. A possible optical realization of this effect has also been suggested in a twisted annular or multicore fiber configuration in [6]. In addition, similar systems have also been studied in parity-time-symmetric configurations, where it was found that the exact $\mathcal{PT}$ phase can be broken in a quantized fashion [25, 26]. Apart from being fundamental in nature, this effect can be potentially utilized for applications, such as torsion sensors [27], mode management [28], and dispersion and polarization control [29]. At this point we emphasize that this topological phenomenon has so far been considered only in the linear regime. In this respect, one may ask whether this Aharonov-Bohm tunneling suppression will still persist even under nonlinear conditions. In other words, is this process robust enough to withstand nonlinear effects?

In this work, we show that the topological suppression of light tunneling in a twisted ring waveguide array can be maintained completely intact in spite of the presence of optical nonlinearity. This holds true in any ring multicore system irrespective of dimensionality. Analytical results pertaining to four-core twisted nonlinear fiber structures indicate that the Aharonov-Bohm phase remains invariant and has no dependence whatsoever on the power levels. At higher intensities, a discrete spatial soliton is formed that further suppresses the energy exchange or tunneling process. The effect of the twist rate on the onset of these mechanisms is also investigated. Moreover, the aforementioned effect can manifest itself even when the waveguide channels are asymmetrically detuned. Beam propagation simulations further corroborate our results - as obtained from nonlinear coupled mode theory.

II. LIGHT PROPAGATION IN TWISTED MULTICORE RING-FIBER SYSTEMS

A. Linear regime

In order to elucidate the mechanism behind the Aharonov-Bohm suppression of optical tunneling, perhaps it is best to explore this effect under linear conditions. In this respect, consider a circular $2N$-core waveguide arrangement as shown in Fig. 1 (a). Each waveguide channel is supposed to be single-moded, while it is evanescently coupled to its nearest neighbors. In addition, the structure is twisted along the propagation axis with a spatial period Λ. Under these conditions, one can show that in the rotating frame, the evolution of the modal field amplitudes E_n obey the following set of differential equations [11]:

$$i\frac{dE_n}{dz} + \beta_n E_n + \kappa(E_{n+1}e^{-i\phi} + E_{n-1}e^{i\phi}) = 0, \qquad (1)$$

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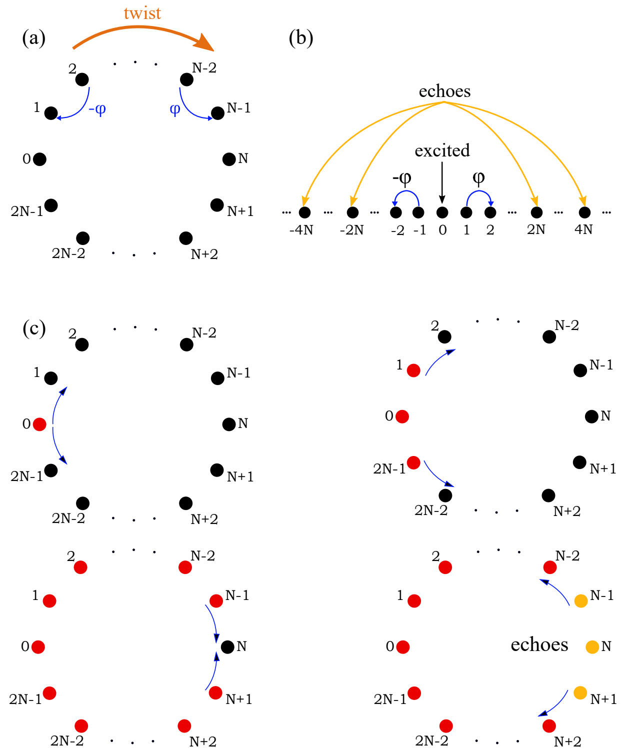

FIG. 1. (a) Schematic of a 2N-core twisted optical fiber, and (b) its equivalent one-dimensional lattice. (c) Light coupling dynamics in the twisted structure, illustrating the formation of periodic echoes circulating inside the array.

where the index $n = 0, 1, ..., 2N - 1$ indicates the site number (modulo $2N$), β_n represents the propagation constant of each core, and κ is the coupling coefficient among nearest neighbors. In Eq. (1), $\phi = k_0 n_0 \epsilon R^2 \sin(\pi/N)$ is the tunneling phase introduced by the twist, $k_0 = 2\pi/\lambda_0$, R is the radius of the circle around which the waveguide elements are located, and $\epsilon = 2\pi/\Lambda$ is the angular twist rate. Equation (1) clearly shows that in such a setting, the coupling coefficients are in fact complex, having equal and opposite phases depending on whether the tunneling direction is clockwise or counter-clockwise. In what follows we show that for specific twist rates satisfying the phase condition

$$N\phi = \pi/2 + p\pi, \tag{2}$$

where p is any integer number, the energy exchange between sites #0 and #N is totally eliminated - in other words, these two channels become effectively decoupled. To analytically prove this assertion, let us consider an infinite version (unfolded) of this same lattice, as shown in Fig. 1 (b). In this system the field dynamics are governed by the same equation, only this time $n \in (-\infty, +\infty)$. If the central site is the only one initially excited, the field distribution in this infinite array is given by [30, 31]:

$$E_n(z) = i^n J_n(2\kappa z) e^{in\phi}. \tag{3}$$

In the $2N$ circular array, the field amplitude at site n can then be obtained by summing up all the echoes resulting from the periodicity of the circular array (Fig. 1 (c)), and hence one now finds that:

$$E_n(z) = \sum_{m=-\infty}^{\infty} i^{n+2mN} J_{n+2mN}(2\kappa z) e^{i(n+2mN)\phi}, \tag{4}$$

where $n = 0, 1, ..., 2N - 1$. From here, it is straightforward to see that the optical field in waveguide #N is always zero:

$$E_N(z) = \sum_{m=0}^{\infty} \Big[i^{(2m+1)N} J_{(2m+1)N}(2\kappa z) e^{i(2m+1)N\phi} + i^{-(2m+1)N} J_{-(2m+1)N}(2\kappa z) e^{-i(2m+1)N\phi} \Big] = 0. \tag{5}$$

This completes the proof.

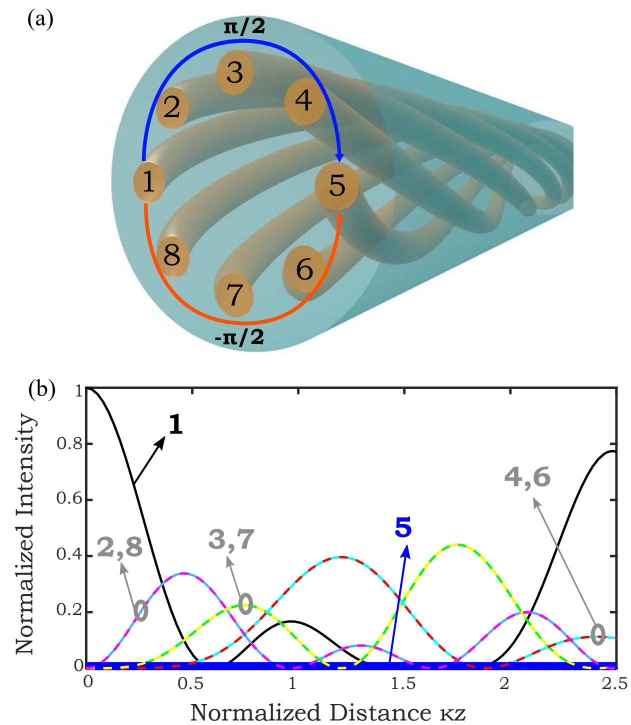

FIG. 2. (a) An eight-core twisted optical fiber where destructive interference between the two possible tunneling paths is manifested. (b) Linear light intensity evolution within the individual cores of the structure.

To demonstrate these dynamics, let us consider linear light evolution in an eight-core twisted fiber as depicted in Fig. 2 (a). In order to achieve topological Aharonov-Bohm suppression, we set $4\phi = \pi/2$. Hence, in this case the twist pitch is given by $\Lambda = 8\sqrt{2} k_0 n_0 R^2$. The results obtained after solving the coupled mode Eqs. (1) for this structure are shown in Fig. 2 (b). It is clear that core #5 remains dark, confirming the results predicted by the above general linear analysis. Because of this topological effect, any cross-talk between sites #1 and #5 is totally prohibited. This effect can be intuitively explained by noticing the fact that the two phase paths from core #1 to #5 (upper and lower) in Fig. 2 (a) differ from each other by a phase factor of $\pm\pi$. Subsequently, light transport along these two paths results in destructive interference, thus leaving core #5 completely dark.

B. Nonlinear regime

From the previous discussion it is clear that the topological phenomenon under consideration is by nature linear. In this respect, one may ask whether this AB suppression can still persist under nonlinear conditions. In this section we address this question by numerically and analytically solving the underlying equations of motion. In the case of a four-core structure, the dynamical system is fully integrable in terms of Jacobi-elliptic functions. Beam propagation methods are also employed to corroborate these results.

In the presence of an optical Kerr nonlinearity, the modal fields in a twisted $2N$ circular array are now described by:

$$i\frac{dE_n}{dz} + \beta_n E_n + \kappa(E_{n+1}e^{-i\phi} + E_{n-1}e^{i\phi}) + \gamma|E_n|^2 E_n = 0, \qquad (6)$$

where γ is proportional to the nonlinear Kerr coefficient. As an example, we numerically investigate the wave dynamics in a nonlinear eight-core system, similar to that of Fig. 2 (a), when $E_1(0) = E_0 = \sqrt{\kappa/\gamma}$. These results, depicted in Fig. 3, clearly indicate a complete AB suppression of coupling between cores #1 and #5. This suppression still persists even at higher power levels.

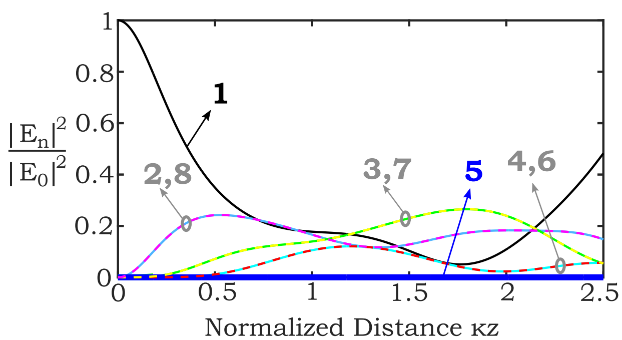

FIG. 3. Modal intensity evolution in a nonlinear twisted eight-core fiber.

To gain further insight into the nonlinear AB dynamics, we analytically solve the case of a four-core twisted fiber ($N = 2$), as shown in Fig. 4 (a). To do so, Eqs. (6) are scaled based on the following normalizations, $Z = \kappa z$ and $E_n = E_0 a_n \exp(i\beta_1 z)$:

$$\begin{aligned} &i\frac{da_1}{dZ} + a_2 e^{-i\phi} + a_4 e^{i\phi} + |a_1|^2 a_1 = 0 \\ &i\frac{da_2}{dZ} + a_1 e^{i\phi} + a_3 e^{-i\phi} + \delta a_2 + |a_2|^2 a_2 = 0 \\ &i\frac{da_3}{dZ} + a_2 e^{i\phi} + a_4 e^{-i\phi} + |a_3|^2 a_3 = 0 \\ &i\frac{da_4}{dZ} + a_1 e^{-i\phi} + a_3 e^{i\phi} + \delta a_4 + |a_4|^2 a_4 = 0, \end{aligned} \qquad (7)$$

where $\delta = \Delta/\kappa$. For purposes of generality, we allow the two auxiliary cores 2,4 to have a wavenumber detuning Δ with respect to sites 1,3. In this scenario Eq. (2) demands that $\phi = \pi/4$. Numerical simulations carried out on Eqs. (7) reveal that site #3 remains completely dark even under highly nonlinear conditions. In other words, the manifestation of AB suppression is not affected by the presence of Kerr nonlinearity. In this particular case $a_3(Z) = 0$ and a_2, a_4 are phase related via $a_4 = \exp(-i\pi/2)a_2$ (because of symmetry). In view of this, Eqs. (7) can now be effectively described by a reduced coupled system:

$$\begin{aligned} &i\frac{da_1}{dZ} + 2e^{-i\pi/4}a_2 + |a_1|^2 a_1 = 0 \\ &i\frac{da_2}{dZ} + e^{i\pi/4}a_1 + \delta a_2 + |a_2|^2 a_2 = 0. \end{aligned} \qquad (8)$$

These equations can be further simplified using the new variables $u = a_1 \exp(i\pi/4)$ and $v = \sqrt{2}a_2$:

$$\begin{aligned} &i\frac{du}{dZ} + \sqrt{2}v + |u|^2 u = 0 \\ &i\frac{dv}{dZ} + \sqrt{2}u + \delta v + \frac{1}{2}|v|^2 v = 0. \end{aligned} \qquad (9)$$

In turn, equations (9) are equivalent to a system of four real differential equations:

$$\dot{U}_0 = 0, \qquad (10a)$$

$$\dot{U}_1 = 2\sqrt{2}U_3, \qquad (10b)$$

$$\dot{U}_2 = \left(-\frac{1}{4}U_0 - \frac{3}{4}U_1 + \delta\right)U_3, \qquad (10c)$$

$$\dot{U}_3 = -2\sqrt{2}U_1 + \left(\frac{1}{4}U_0 + \frac{3}{4}U_1 - \delta\right)U_2, \qquad (10d)$$

where for convenience we have used the Stokes parameters,

$$U_0 = |u|^2 + |v|^2, \qquad (11a)$$

$$U_1 = |u|^2 - |v|^2, \qquad (11b)$$

$$U_2 = uv^* + u^*v, \qquad (11c)$$

$$U_3 = i(u^*v - uv^*). \qquad (11d)$$

From Eqs. (10) one can directly obtain the following two conservation laws:

$$U_0 = C_1, \qquad (12a)$$

$$U_2 = \frac{1}{2\sqrt{2}}\left(-\frac{C_1}{4}U_1 - \frac{3}{8}U_1^2 + \delta U_1\right) + C_2, \qquad (12b)$$

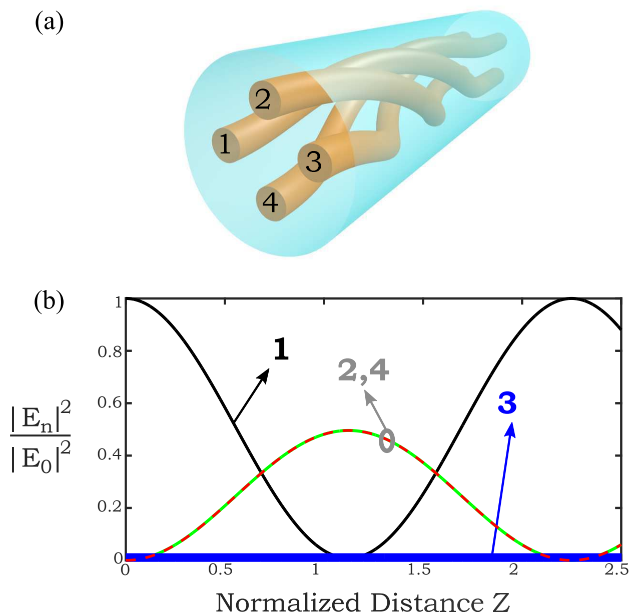

FIG. 4. (a)A nonlinear four-core twisted fiber. (b) Intensity evolution within the four cores as obtained from Eq. (15), when $a_1(0) = 1$.

where C_1 and C_2 are constants determined by the initial conditions, given by $a_1(0) = a_0$ while $a_i(0) = 0$ for $i = 2, 3, 4$. Hence,

$$C_1 = |a_0|^2, \tag{13a}$$

$$C_2 = \frac{1}{2\sqrt{2}}\left(\frac{5}{8}|a_0|^4 - \delta|a_0|^2\right). \tag{13b}$$

Substituting these latter results into Eqs. (10), we then obtain the following differential equation for U_1:

$$\dot{U}_1^2 = -\frac{9}{64}U_1^4 - \frac{2}{3}B_1U_1^3 + B_2U_1^2 + 2B_3U_1 + 2B_4, \tag{14}$$

where the constants B_i are defined as follows: $B_1 = (9/32)C_1 - (9/8)\delta$, $B_2 = -C_1^2/16 + (C_1/2)\delta + (3\sqrt{2}/2)C_2 - \delta^2 - 8$, $B_3 = (\sqrt{2}/2)C_1C_2 - (2\sqrt{2})\delta C_2$, $B_4 = (9/128)C_1^4 + (B_1/3)C_1^3 - (B_2/2)C_1^2 - B_3C_1$. From here, one can show that Eq. (14) can be solved analytically in terms of Jacobi-elliptic functions [32–34]:

$$U_1(Z) = \frac{r_1B + r_2A - (r_1B - r_2A)cn(x,k)}{A + B + (A - B)cn(x,k)}, \tag{15}$$

where r_1 and r_2 are the two real roots corresponding to the fourth order polynomial on the right hand side of Eq. (14). Meanwhile, r_3 and r_3^* are the complex conjugate roots of this same polynomial. The two constants A, B in Eq. (15) can be obtained from:

$$A^2 = \left(r_1 - \frac{r_3 + r_3^*}{2}\right)^2 - \frac{(r_3 - r_3^*)^2}{4},$$
$$B^2 = \left(r_2 - \frac{r_3 + r_3^*}{2}\right)^2 - \frac{(r_3 - r_3^*)^2}{4}. \tag{16}$$

The argument x in the Jacobi-elliptic functions is related to the elliptic integral of the first kind $F(\varphi, k)$ and the normalized propagation distance Z via $x = F(\pi, k) - 3Z/8g$. Moreover, $k^2 = ((r_1 - r_2)^2 - (A - B)^2)/(4AB)$ provides the elliptic modulus, and $g = (AB)^{-1/2}$. These analytical results are corroborated by numerical simulations of Eqs. (7), as illustrated in Fig. 4 (b). The actual intensities in the four cores can then be directly obtained from $|u|^2 = (C_1 + U_1)/2$ and $|v|^2 = (C_1 - U_1)/2$.

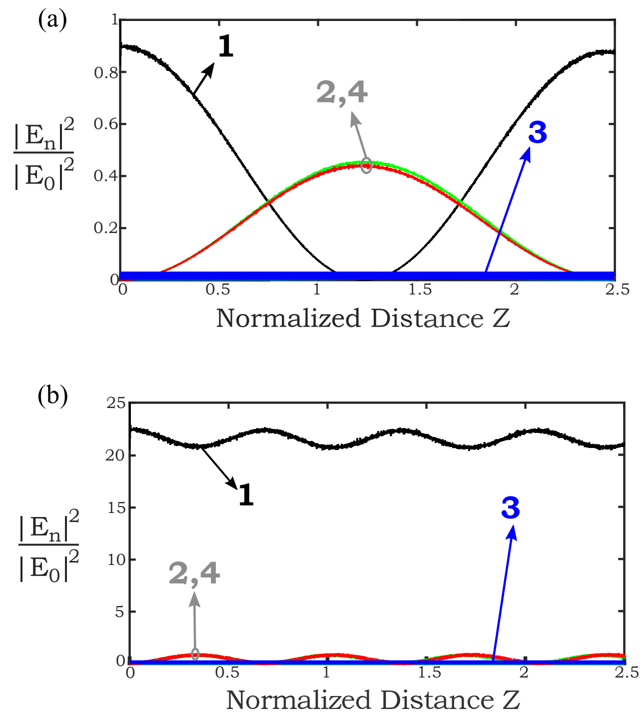

FIG. 5. Beam propagation analysis of a nonlinear twisted four-core fiber structure: (a) and (b) depict intensity evolution at low and high input powers, respectively.

A possible silica-based four-core arrangement where one can observe the aforementioned Aharonov-Bohm tunneling suppression can be designed based on the following parameters. We assume that the core radii are $r = 4.5\ \mu m$ while their center-to-center distance is $D = 24\ \mu m$. The operating wavelength is taken here to be $\lambda_0 = 1550\ nm$ and the numerical aperture of each waveguide element is $NA = 0.1$. The structure is twisted around its central axis with a pitch of $\Lambda = 1.4\ cm$, corresponding to $\phi = \pi/4$. In order to validate the coupled mode results obtained before, we use beam propagation methods to monitor the intensity evolution in each core along the propagation axis when core #1 is excited at different power levels. The results are summarized in Fig. 5. These dynamics clearly indicate that the differential phase between the two light channels is left unchanged even under highly nonlinear conditions. Consequently, the quenching of the coupling can be preserved. At considerable higher power levels ($\sim 10\ kW$), the nonlinearity starts to dominate the coupling effects (Fig. 5 (b)). As a result, a discrete soliton is established on site #1 [35–37], which further reduces energy transfer to the nearest cores (#2, #4). It is observed that even in this highly localized regime, the topological phases are left intact. Finally, we examined the robustness of this AB effect in the presence of an asymmetric detuning δ' between cores #2 and #4. Figure 6 shows the intensity dynamics in the four channels when $\delta' = 0.05$. Although a deviation from the ideal case of Fig. 5(a) is observed, it is evident that the nonlinear AB effect can in principle withstand such a perturbation. This is attributed to the topological nature of the Aharonov-Bohm phase.

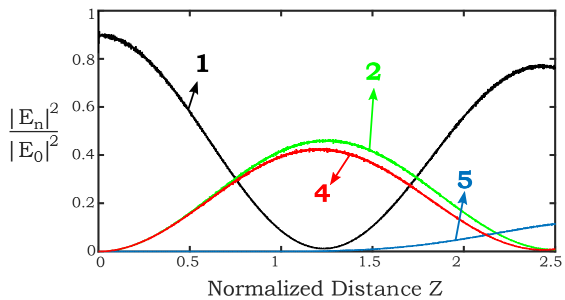

FIG. 6. Robustness of the nonlinear AB effect against asymmetric detuning perturbations, when core #2 is detuned from the rest of the structure by $\delta' = 0.05$.

III. CONCLUSION

In conclusion, we studied the Aharonov-Bohm topological suppression of light tunneling in a nonlinear multicore fiber structure. Our analytical and numerical results indicate that the Aharonov-Bohm phase remains invariant and has no dependence whatsoever on the power levels. Our results present a promising platform to observe this effect in the context of photonics, especially considering the topological robustness of this process against nonuniformities.

ACKNOWLEDGMENTS

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