Quantum parameter estimation with the Landau-Zener transition

Jing Yang, Shengshi Pang, and Andrew N. Jordan

Phys. Rev. A **96**, 020301 — Published 1 August 2017

DOI: 10.1103/PhysRevA.96.020301

Quantum parameter estimation with the Landau-Zener transition

Jing Yang,[1, 2, *] Shengshi Pang,[1, 2, †] and Andrew N. Jordan[1, 2, 3, ‡]

[1] *Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627, USA*
[2] *Center for Coherence and Quantum Optics, University of Rochester, Rochester, NY 14627, USA*
[3] *Institute for Quantum Studies, Chapman University, 1 University Drive, Orange, CA 92866, USA*

(Dated: July 12, 2017)

We investigate the fundamental limits in precision allowed by quantum mechanics from Landau-Zener transitions, concerning Hamiltonian parameters. While the Landau-Zener transition probabilities depend sensitively on the system parameters, much more precision may be obtained using the acquired phase, quantified by the quantum Fisher information. This information scales with a power of the elapsed time for the quantum case, whereas it is time-independent if the transition probabilities alone are used. We add coherent control to the system, and increase the permitted maximum precision in this time-dependent quantum system. The case of multiple passes before measurement, "Landau-Zener-Stueckelberg interferometry", is considered, and we demonstrate that proper quantum control can cause the quantum Fisher information about the oscillation frequency to scale as T^4, where T is the elapsed time. This results are foundational for frequency standards and quantum clocks.

The Landau-Zener (LZ) transition is a classic example of exactly solvable, time-dependent quantum mechanics, whereby an effective two-level quantum system prepared in its ground state may either stay in the ground state, or transition to the excited state, depending on the speed of the energy separation of the levels [1–4]. LZ transitions have been extended to parabolic level crossing [5], finite time duration with various approximation regimes [6], multi-level transitions such as those encountered in cavity and circuit QED [7, 8], and have also been studied in the presence of noise [9–11, 39, 40]. In the context of quantum information, the LZ transition has been used as a qubit readout mechanism and for quantum control [12–14].

The LZ transition has been used as a way of estimating Hamiltonian parameters, such as the level splitting energy, or the speed of the transition through the avoided level crossing [15–17]. Going beyond the LZ transition probabilities, it is also possible to make multiple, coherent sweeps of the avoided level crossing to accumulate a phase, also known as Landau-Zener-Stueckelberg interferometry [18–23]; the acquired phase depends sensitively on the system parameters. The field of quantum metrology is concerned with the optimal precision quantum physics permits in the estimation of parameters [24, 25]. Recent interest in this field has moved beyond simple multiplicative parameters of the Hamiltonian and begun to examine general parameters [26], as well as the role of physical dynamics in the estimation process [27, 28], which may require coherent control to optimize the acquired information [29, 30].

The purpose of this paper is to apply the methods of quantum metrology to the LZ transition, and quantify the estimation precision of parameters in the LZ transition available by the various techniques aforementioned. We shall focus on the quantum Fisher information for the parameters of interest, as it determines the lower bound of the variance of the parameter estimates over all possible estimation strategies and all possible quantum measurements on the systems, giving the ultimate limits of precision allowed by quantum mechanics in the asymptotic data limit. We find that because of the time-dependent nature of the problem, with a proper control Hamiltonian applied, the time-scaling of the quantum Fisher information can be significantly improved, which demonstrates a fundamental metrological advantage of coherent quantum control on the level-crossing physics of the LZ transition.

The LZ Hamiltonian is given by

$$H(t) = \frac{vt}{2}\sigma_z + \frac{\Delta}{2}\sigma_x, \tag{1}$$

where v is the speed of the sweep assumed to be positive here, and Δ is the level splitting at the transition time $t = 0$. Denote $|0\rangle$,$|1\rangle$ as eigenvectors of σ_z corresponding to eigenvalues ±1 and the solution to the Schrödinger equation, $i\partial_t|\psi\rangle = H(t)|\psi\rangle$ as $|\psi(t)\rangle = C_0(t)\,|0\rangle + C_1(t)\,|1\rangle$, which gives two coupled differential equations for $C_{0,1}(t)$. We start for simplicity in the ground state $|1\rangle$ at an initial time $t = -T_0$ far away from the avoided level crossing time $t = 0$, i.e., $T_0 \gg \tau \equiv \max\{\frac{\Delta}{2v}, \frac{1}{\sqrt{v}}\}$. Sweeping through the Landau-Zener transition to a time $t = T \gg \tau$, which is also far away from the transition region (see Fig. 1 inset) gives (detailed calculations for this and subsequent results are provided in the Supplemental Material [31])

$$C_0(T) = \frac{\sqrt{2\pi i\gamma}}{\Gamma(1+\nu)}e^{-\pi\gamma/2-2i\phi},\ C_1(T) = e^{-\pi\gamma}, \tag{2}$$

where we define $\phi \equiv (vT^2 + \pi)/4 + \gamma/2\ln(vT^2)$; $\gamma \equiv \Delta^2/(4v)$ and Γ represents the gamma function. The absolute square of $C_{0,1}(T)$ recovers the celebrated (time-independent) LZ probabilities [2] to find the system in the (new) excited or ground states,

$$P_1 = 1 - P_0 = \left|C_1(T)\right|^2 = e^{-2\pi\gamma}. \quad (3)$$

Estimation using the LZ probabilities.—The simplest estimation scheme is to make a single pass starting from the ground state $|1\rangle$, and measure the system to be in the new excited or ground state, with the probabilities given in (3). Since the probabilities depend very sensitively on the system parameters Δ or v, we may use the measurement result to obtain an estimator $\hat{\Delta}$ or $\hat{v}$, respectively. The mean square error of the estimation is bounded from below by the inverse of the Classical Fisher information (CFI), which is referred to as the Cramér-Rao bound [32]. For a parameters $g = \Delta, v$ the CFI is given by $F_g = \sum_\xi \frac{1}{p(\xi|g)}[\frac{\partial p(\xi|g)}{\partial g}]^2$, where $\xi = 0, 1$ labels the two states. At time $t = T$, one obtains from Eq. (3)

$$F_\Delta(T) = \frac{16\pi^2\gamma^2}{(e^{2\pi\gamma}-1)\Delta^2}, \quad F_v(T) = \frac{4\pi^2\gamma^2}{(e^{2\pi\gamma}-1)v^2}. \quad (4)$$

Repeating the experiment N times from the same initial state will boost the information by a factor of N. When N is large, the Cramer-Rao bound can be saturated asymptotically by a maximum likelihood estimator [31, 32] and hence the Fisher information quantifies the ultimate precision limit of parameter estimation. The Fisher information Eq. (4) about either parameter limits to zero for either a diabatic transition $\gamma \ll 1$, or an adiabatic transition $\gamma \gg 1$. This is simply because in those extreme limits, the LZ probabilities become either 0 or 1, with little variation. Therefore, the estimation strategy is most sensitive in the intermediate range. For γ of order 1, the uncertainty of both parameters is of order of the parameter, which for tiny tunnel couplings can give rise to precise estimates with many measurements [15].

Estimation using any final quantum measurement.—We can generalize the above situation by rather than making a final measurement at time T in the $|0\rangle$, $|1\rangle$ basis, to measure in another basis (or equivalently, stopping the LZ sweep and applying a single qubit unitary). The maximum Classical Fisher Information over all possible generalized quantum measurements on a state $|\psi_g\rangle$ is defined as the Quantum Fisher Information (QFI) [33–35], $I_g = 4(\langle\partial_g\psi_g|\partial_g\psi_g\rangle - |\langle\psi_g|\partial_g\psi_g\rangle|^2)$. It has been shown [35] that the optimal projective measurements associated with the QFI are formed by the eigenvectors of the symmetric logarithm derivative L_g, defined as $L_g = 2\partial_g\rho_g = 2\left(|\partial_g\psi_g\rangle\langle\psi_g| + |\psi_g\rangle\langle\partial_g\psi_g|\right)$. As shown in [31], for a pure quantum state $|\psi_g\rangle = \sqrt{P_0}\,|0\rangle + \sqrt{P_1}e^{i\phi_g}\,|1\rangle$, where P_0 and P_1 are real and independent of g, the QFI is $I_g = 4P_0P_1\left(\partial_g\phi_g\right)^2$. So we can see from this expression, the decoherence will destroy the relative phase as well as the population of one of the two levels, degrading the QFI dramatically. Since we consider here the fundamental precision limit allowed by quantum mechanics, we assume the unitary dynamics for the system's evolution and ignore system-environment interactions throughout the paper. The QFI gives better precision since it is able to take advantage of the phase that is rapidly accumulating during the LZ sweep, $\varphi(T) \propto vT^2$, as predicted by Stueckelberg [3]. Still starting from the ground state at $t = -T_0$, as one can observe from Eq. (2,3), the state at time T can be rewritten as: $|\psi(T)\rangle = \sqrt{P_0}|0\rangle + \sqrt{P_1}e^{i\varphi(T)}|1\rangle$, where the relative phase is $\varphi(T) = \frac{vT^2}{2} + \gamma\ln(vT^2) + \arg\Gamma(1-\gamma) + \frac{\pi}{4}$. When T is sufficiently large, one can approximate the QFIs by keeping only the contributions due to highest order of T in the relative phase and neglecting the contributions from the transition probabilities,

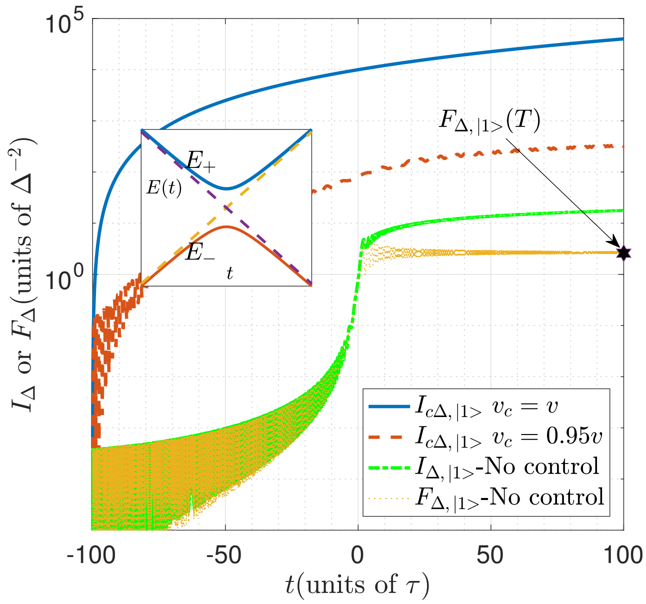

FIG. 1. The main figure is the QFIs/CFIs for estimating Δ versus time plotted in logarithmic graph(base 10). I_Δ or F_Δ denote the QFI or CFI without control while $I_{c\Delta}$ denotes the QFI with control. Best case Fisher information progressively increases by moving from classical estimation, to quantum estimation, to coherently controlled quantum estimation. The value of parameters in the LZ Hamiltonian are $v = 1$, $\Delta = 1$, and $\tau = 1$. The system starts to evolve at $-T_0 = -100\tau$. For cases with control Hamiltonians, we choose the initial state to be $\frac{1}{\sqrt{2}}[|+x\rangle - |-x\rangle] = |1\rangle$. The black marker is the result calculated from Eq. (4). The inset represents a single LZ transition.

$$I_\Delta(T) \sim \frac{\Delta^2}{v^2}P_0P_1\left[\ln\left(vT^2\right)\right]^2, \quad I_v(T) \sim P_0P_1T^4. \quad (5)$$

The above prefactor P_0P_1 attains its maximum $1/4$ when the transition probabilities are equal. Both of the QFIs exceed the CFIs, with I_v scaling as T^4 because the acquired phase difference *accelerates* as T^2. In general, we may further boost the QFIs by starting the system in a coherent superposition of $|0\rangle$ and $|1\rangle$, however, while this affects the prefactor of the QFIs, it does not change the time scaling. The two vectors forming the optimal projectors, either for estimation of Δ or v are an equal superposition of $|0\rangle$ and $|1\rangle$ with a relative phase of $\pm ie^{i\varphi(T)}$. The measurement basis can be adaptively shifted to maximize the acquired information [31].

Plots of the CFIs and QFIs are shown in Figs. (1,2) for $g=\Delta$, v respectively. Although the previous discussion assumes a positive sweeping velocity starting from the ground state, the discrete symmetries of the LZ Hamiltonian (1), relate this solution to the negative velocity case and to starting in the ground state; all these cases have the same CFIs or QFIs and the corresponding optimal measurements [31].

The T^4 scaling in this example originates from the linear time growth of the Hamiltonian (1) and does not require quantum control, different from the multiple-transition case below and that in [30] where the Hamiltonians are bounded. Nevertheless, we show below that proper quantum control can strengthen this T^4 scaling by increasing the prefactor.

Adding coherent control to boost precision.—Recently, Ref. [30] developed a comprehensive theory on quantum metrology with a general time dependent Hamiltonian $H_g(t)$ and coherent controls. There the authors found that the QFI at time t is bounded by $I_{cg}(t) \leq [\int_{t_0}^{t}(\mu_{\max}(t')-\mu_{\min}(t'))\,dt']^2$, where the subscript c denotes the QFI with coherent controls; t_0 is the initial time of the evolution of the system; and $\mu_{\max}(t)$ and $\mu_{\min}(t)$ are the maximum and minimum instantaneous eigenvalues of $\partial_g H_g(t)$. The equality can be saturated if the initial state is prepared in an equal superposition of the maximum eigenstate $|\mu_{\max}(t)\rangle$ and minimum eigenstate $|\mu_{\min}(t)\rangle$ of $\partial_g H_g(t)$, where the maximum (minimum) eigenstate denotes the eigenstate corresponding to the maximum (minimum) eigenvalue of $\partial_g H_g$, and an Optimal Control Hamiltonian (OCH) is applied. In this optimized regime, one can see that at any time t the state is still in an equal superposition between $|\mu_{\max}(t)\rangle$ and $|\mu_{\min}(t)\rangle$, with an the relative phase of the form $\exp[ig\int_{t_0}^{t}(\mu_{\max}(t')-\mu_{\min}(t'))\,dt'+\cdots]$, where the rest terms do not depend on the estimation parameter. This relative phase gives us the physical intuition of the scaling of the QFI with coherent controls discussed later. From this relative phase, it is clear that if $\mu_{\max}(t')$, $\mu_{\min}(t')$ do not cross with each other nor with other levels of $\partial_g H_g$ in $[t_0, t]$, then the Fisher information is always increasing with time and maximized. However, if $\mu_{\max}(t')$ or $\mu_{\min}(t')$ cross with each other or other eigenvalues of $\partial_g H_g$ at some points, then σ_x-like pulses are required at these points to interchange the crossing state amplitudes so that the Fisher information is maximally increasing with time (see Fig. 3 for pictorial interpretation). At the time of level crossings, we apply the Optimal Level Crossing Hamiltonian (OLCH) denoted as $H_{LC}(t)$. Examples for this case are the estimation of the transition velocity for single passage and transition frequency for multiple passages. The σ_x-like pulses become π-pulses for both examples.

In practice, optimal precision is achieved adaptively: one begins with an initial coarse estimate, g_0, about the parameter. This is followed by applying a control Hamiltonian and making a measurement basis with the estimate $g_c = g_0$, in order to obtain a next estimate, g_1. The procedure is iterated $g_j \rightarrow g_{j+1}$ [29–31]. Applying this theory of time-dependent quantum metrology to estimate Δ, we find $\partial_\Delta H = \sigma_x/2$, with eigenvalues $\pm 1/2$. Applying the above results for the QFI with respect to Δ, we find the upper bound

$$I_{c\Delta,\,|\Psi\rangle_{c\Delta}}(t) = \Big(\int_{-T}^{t} dt'\Big)^2 = (t+T)^2, \qquad (6)$$

for any time t, and giving a maximum of $4T^2$ at $t = T$, provided the initial state is prepared in $|\Psi\rangle_{c\Delta} = (1/\sqrt{2})[|+x\rangle + e^{i\beta}|-x\rangle]$, where β is an arbitrary initial relative phase. The corresponding OCH $H_{c\Delta} = -(vt/2)\sigma_z$ cancels the first term in Eq. (1), effectively turning off the LZ sweep in σ_z. No OLCH is required since $\partial_\Delta H$ and its eigenstates are time independent and no level crossing occurs. Note that if v is unknown, we should replace v in the OCH with an estimate v_c that can be updated based on further measurement data. Fig. 1 shows the comparison of the optimal case with the non-control and non-optimal cases.

The estimation of v with control is more complicated than Δ since the maximum and minimum eigenvalues of $\partial_v H = t\sigma_z/2$ have a crossing at $t = 0$. The QFIs for all time can be written in a uniform expression

$$I_{cv,\,|\Psi\rangle_{cv}}(t) = \Big(\int_{-T}^{t} |t'|dt'\Big)^2 = \frac{[t^2+\mathrm{sgn}(t)T^2]^2}{4}, \qquad (7)$$

where the value of $\mathrm{sgn}(t)$ is -1 for $t \leq 0$ and 1 for $t > 0$. We prepare the initial state in $|\Psi\rangle_{cv} = \left[|0\rangle + e^{i\beta}|1\rangle\right]/\sqrt{2}$ with an arbitrary chosen relative phase β. The OCH for estimating v is $H_{cv} = -(\Delta/2)\sigma_x$, which cancels the tunneling term. Since the maximum and minimum eigenstates of $\partial_v H$ have a level crossing at $t = 0$, an OLCH H_{LC} is required to avoid the level crossing of $\partial_v H$ at $t = 0$ [31]. This can be done simply by swapping the the state amplitudes with a π-pulse at time $t = 0$. The comparison of the optimal case with other cases are plotted in Fig. 2.

Optimal measurements.—In order to saturate the bounds with optimal controls, it is necessary to construct the optimal measurements. For estimating Δ, if the OCH is applied and the system is initially prepared in $|\Psi\rangle_{c\Delta}$, the system will evolve under the Hamiltonian $H + H_{c\Delta} = \frac{\Delta}{2}\sigma_x$. The vectors forming the optimal projectors performed at the final time T, expressed in the σ_x basis, are equal superposition of $|+x\rangle$ and $|-x\rangle$ with a relative phase $\pm ie^{2i\Delta_c T+\beta}$, where Δ_c represents our prior

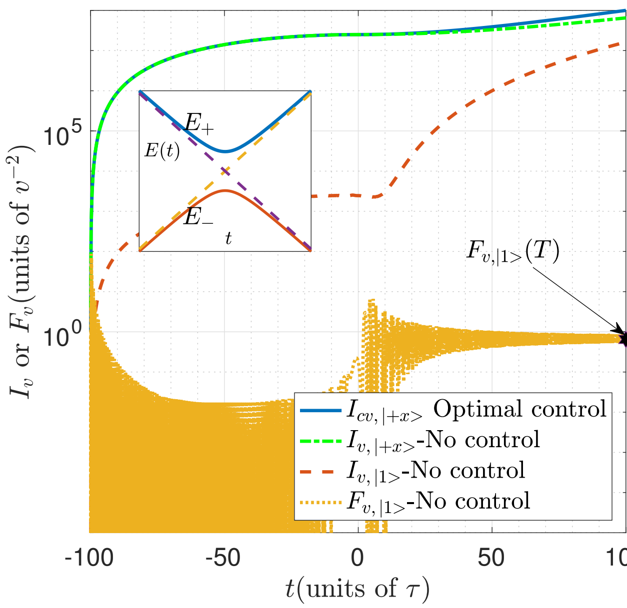

FIG. 2. The main figure is the QFIs/CFIs for estimating v versus time in a semi logarithmic plot (base 10). I_v or F_v denote the QFI or CFI without control while I_{cv} denotes the QFI with control. Starting the system in $|+x\rangle$ without control can nearly perform as well as the optimally controlled metrology. The parameter configurations are the same as FIG. 1. The solid line is the case with optimal controls, where $H_c = -\frac{\Delta}{2}\sigma_x$. The black marker is the result calculated from Eq. (4). The inset represents a single LZ transition.

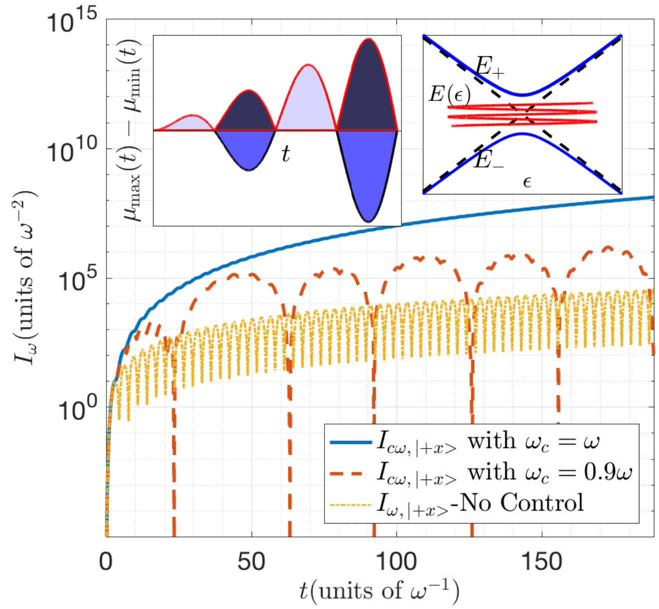

FIG. 3. The main figure is the QFIs for estimating ω versus time in semi-logarithmic graph (base 10). The subscript c in the notation of QFI implies it corresponds to the case with controls. As the control parameter approaches the true value, the ideal T^4 scaling of the QFI for frequency is obtained. The system starts to evolve at $t = 0$ and the value of parameters are $\epsilon_0 = 0$, $A = 1$, $\omega = 1$, $\Delta = 0.1$, $N = 60$, $T = \frac{N\pi}{\omega}$=60π. The two cases with controls have the same OCH $H_c = -\frac{\epsilon_0}{2}\sigma_z - \frac{\Delta}{2}\sigma_x$, whereas the additional control Hamiltonian is not optimal (yellow) and optimal (purple). The left inset shows the merit of the OLCHs: consider two cases both initially prepared in $(|0\rangle + |1\rangle)/\sqrt{2} = |+x\rangle$ and both with OCHs applied; the one without OLCHs has $\sqrt{I_{c\omega}} = \left|L - G\right|$; the other with OLCHs has $\sqrt{I_{c\omega}} = L + D$, where L, G, D represent the magnitudes of the light, intermediate and dark blue (grey) areas. The right inset represents oscillatory avoided level crossings.

knowledge of Δ. For estimating v with optimal controls applied, the vectors forming the optimal measuring projectors performed at the final time T are equal superposition of $|0\rangle$ and $|1\rangle$ with a relative phase $\pm ie^{i(v_c T^2 - \beta)}$, where v_c represents our prior knowledge of v.

Optimal estimation with controlled LZ interferometry.—Rather than take a single pass though the avoided level crossing, the concept of LZ interferometry is to make many passes, acquiring a phase shift given by a multiple of the phase shift acquired by a single cycle [18–20]. This leads to interference fringes in the occupation probability, known as "Stueckelberg oscillations" [36–38]. In contrast to past work, we will see that simply letting the phase accumulate does not give the optimal precision. Rather a series of control operations should be applied to optimize the information extraction and change the scaling law of the Fisher information with duration. This situation allows us to extend the time T of the experiment and gives an explicitly bounded Hamiltonian, in contrast to a single sweep, where the LZ Hamiltonian approximation (1) would otherwise break down at long time.

The Hamiltonian (1) is modified by replacing the coefficient of σ_z by an oscillating function of amplitude A and frequency ω, $vt \to \epsilon_0 + A\cos\omega t$, describing periodic LZ sweeps. We restart our clock from $t = 0$, beginning away from the transition region. We can now estimate four Hamiltonian parameters, but we focus on the frequency ω as the most interesting. We consider the weakly coupling and near resonance case, where the rotating wave approximation can be applied. The maximum QFI at $t = T$ over all possible initial pure states of estimating ω for this case is given in [30], and scales as T^2; $\max I_\omega(T) = A^2T^2/(A^2 + 4(\Delta - \omega)^2)$, plus oscillatory terms of sub-leading order. The dashed-dotted line in Fig. 3 is for the case of strong coupling and off resonance. By adding optimal controls, we can find the closed form of the QFI of estimating ω for general cases, regardless of the driven intensity and frequency. We find the parametric derivative of the Hamiltonian is $\partial_\omega H = -At\sin(\omega t)\sigma_z/2$, which has eigenvectors $|0\rangle, |1\rangle$, and eigenvalues $\mu_\mp = \mp At\sin(\omega t)/2$. An interesting feature arises in that there is a crossing of the eigenvalues of $\partial_\omega H$ at the *ends* of the LZ sweeps, not at the crossing of the energy eigenvalues. Additional OLCHs must be applied at each of these time points to swap the amplitudes of $|0\rangle$ and $|1\rangle$ in order to saturate the QFI bound. Since the oscillation frequency is not precisely known, generally the controls are applied with an estimated value ω_c, which is then iteratively updated in successive trials [31]. The left inset of Fig. 3 schematically shows the functionality of the OLCHs: when OCH is applied, the square root of the QFI is the integrated difference of the maximum and minimum eigenvalues of $\partial_\omega H$ over time, which in the absence of OLCHs is the magnitude of the difference of the light and intermediate blue (grey) areas, whereas in presence of the OLCHs is the sum of light and dark blue (grey) areas. With all optimal controls applied, the QFI is

$$I_{c\omega,|\Psi\rangle_{c\omega}} = \left(\frac{A\pi N^2}{\omega^2}\right)^2, \tag{8}$$

where we have considered $T = N\pi/\omega, (N \in \mathbb{N})$ for simplicity (the general solution is given in [31]) and the system is initially prepared in $|\Psi\rangle_{c\omega} = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\beta}|1\rangle\right)$ with an arbitrary initial relative phase β. We see that the QFI scales as T^4, giving a scaling law improvement in the estimation of ω. The optimal observable measured at the final time T to achieve this precision is σ_Y if we take $\beta = 0$ [31]. The required OCH is $H_c = -\epsilon_0\sigma_z/2 - \Delta\sigma_x/2$ applied in addition to the OLCHs. A comparison of the optimal case with both the non-control and non-optimal case is plotted in the main figure of Fig. 3.

Conclusions.— In summary, we quantified the ultimate estimation precision for the parameters which the Landau-Zener transition sensitively depends on, using the quantum Fisher information. With proper adaptive coherent quantum controls, the ultimate limits of estimation precision can be achieved, and we demonstrated a T^4 scaling of quantum Fisher information for the oscillation frequency, which can be achieved adaptively [31]. We gave an explicit measurement prescription to unlock the additional quantum advantages in the measurement time resource. Our numerical simulations confirmed the analytic results.

Acknowledgments.— This work was supported by by US Army Research Office Grants No. W911NF-15-1-0496, No. W911NF-13-1-0402, and by National Science Foundation grant DMR-1506081.

* jyang75@ur.rochester.edu
† pangshengshi@gmail.com
‡ jordan@pas.rochester.edu

[1] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-Relativistic Theory* (Elsevier, 1981).
[2] C. Zener, Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character **137**, 696 (1932).
[3] E. Stueckelberg, Helv. Phys. Acta,(Basel) **5**, 369 (1932).
[4] E. Majorana, Il Nuovo Cimento (1924-1942) **9**, 43 (1932).
[5] K.-A. Suominen, Optics Communications **93**, 126 (1992).
[6] N. V. Vitanov and B. M. Garraway, Physical Review A **53**, 4288 (1996).
[7] C. Sun and N. A. Sinitsyn, Physical Review A **94**, 033808 (2016).
[8] N. A. Sinitsyn and F. Li, Phys. Rev. A **93**, 063859 (2016).
[9] Y. Kayanuma, Journal of the Physical Society of Japan **54**, 2037 (1985).
[10] N. A. Sinitsyn and N. Prokof'ev, Phys. Rev. B **67**, 134403 (2003).
[11] V. L. Pokrovsky and D. Sun, Phys. Rev. B **76**, 024310 (2007).
[12] G. Ithier, E. Collin, P. Joyez, D. Vion, D. Esteve, J. Ankerhold, and H. Grabert, Phys. Rev. Lett. **94**, 057004 (2005).
[13] J. Petta, H. Lu, and A. Gossard, Science **327**, 669 (2010).
[14] C.M Quintana, K. D. Petersson, L.W. McFaul, S. Srinivasan, A. Houck, and J. R. Petta, Physical Review Letters **110**, 173603 (2013).
[15] W. Wernsdorfer, R. Sessoli, A. Caneschi, D. Gatteschi, A. Cornia, and D. Mailly, Journal of Applied Physics **87**, 5481 (2000).
[16] M. Urdampilleta, S. Klyatskaya, M. Ruben, and W. Wernsdorfer, Phys. Rev. B **87**, 195412 (2013).
[17] A. Izmalkov, M. Grajcar, E. Il'Ichev, N. Oukhanski, T. Wagner, H.-G. Meyer, W. Krech, M. Amin, A. M. van den Brink, and A. Zagoskin, EPL (Europhysics Letters) **65**, 844 (2004).
[18] A. Shytov, D. Ivanov, and M. Feigel'Man, The European Physical Journal B-Condensed Matter and Complex Systems **36**, 263 (2003).
[19] W. D. Oliver, Y. Yu, J. C. Lee, K. K. Berggren, L. S. Levitov, and T. P. Orlando, Science **310**, 1653 (2005).
[20] S. Shevchenko, S. Ashhab, and F. Nori, Physics Reports **492**, 1 (2010).
[21] G. Sun, X. Wen, B. Mao, J. Chen, Y. Yu, P. Wu, and S. Han, Nature communications **1**, 51 (2010).
[22] M. Sillanpää, T. Lehtinen, A. Paila, Y. Makhlin, and P. Hakonen, Phys. Rev. Lett. **96**, 187002 (2006).
[23] L. Gaudreau, G. Granger, A. Kam, G. Aers, S. Studenikin, P. Zawadzki, M. Pioro-Ladriere, Z. Wasilewski, and A. Sachrajda, Nature Physics **8**, 54 (2012).
[24] V. Giovannetti, S. Lloyd, and L. Maccone, Physical Review Letters **96**, 010401 (2006).
[25] H. M. Wiseman and G. J. Milburn, *Quantum measurement and control* (Cambridge University Press, 2009).
[26] S. Pang and T. A. Brun, Physical Review A **90**, 022117 (2014).
[27] J. Liu, X.-X. Jing, and X. Wang, Scientific reports **5** (2015).
[28] X.-X. Jing, J. Liu, H.-N. Xiong, and X. Wang, Phys. Rev. A **92**, 012312 (2015).
[29] H. Yuan and C.-H. F. Fung, Phys. Rev. Lett. **115**, 110401 (2015).
[30] S. Pang and A. N. Jordan, arXiv:1606.02166 [quant-ph] (2016), arXiv: 1606.02166.
[31] J. Yang, S. Pang, and A. N. Jordan, Supplemental Material.
[32] H. Cramer, *Mathematical methods of statistics* (Princeton University Press, USA, 1946).
[33] S. L. Braunstein and C. M. Caves, Physical Review Letters **72**, 3439 (1994).

[34] S. L. Braunstein, C. M. Caves, and G. Milburn, Annals of Physics **247**, 135 (1996).
[35] M. G. A. Paris, International Journal of Quantum Information **07**, 125 (2009).
[36] M. C. Baruch and T. F. Gallagher, Phys. Rev. Lett. **68**, 3515 (1992).
[37] S. Yoakum, L. Sirko, and P. M. Koch, Phys. Rev. Lett. **69**, 1919 (1992).
[38] H. Nakamura, *Nonadiabatic Transition:Concepts, Basic Theories and Applications, 2nd Edition* (World Scientific, 2002).
[39] R. S. Whitney, M. Clusel, and T. Ziman, Physical Review Letters **107**, 210402 (2011).
[40] Y. Avishai and Y. B. Band, Physical Review A **90**, 032116 (2014).