Optical surface breather in graphene

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Optical surface breather in graphene

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A theory of an optical breather of self-induced transparency for small area surface plasmon-polariton waves is constructed. The wave equation for an optical nonlinear electric field consisting of surface TM-modes, traveling along a two-dimensional layer of atomic systems (or semiconductor quantum dots), with a graphene monolayer (or graphene-like two-dimensional material), are shown to reduce to the nonlinear Schrödinger equation with damping. It is also shown that damped small intensity surface plasmon-polariton breathers can propagate in such a system and its characteristic parameters depends on the connected media, graphene conductivity, transition layer and transverse structures of the surface plasmon polariton. Explicit analytical expressions for the parameters of an optical surface breather are given. The breather and the soliton in graphene are compared with each other and the differences between their properties are contrasted.

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I. INTRODUCTION

One of the reasons for the strong interest in the interaction of light with graphene is the very unique optical properties of graphene [1, 2]. Graphene is a single carbon atomic layer consisting of a two-dimensional (2D) honeycomb lattice. It is the first attempt experimentally construct of 2D atomic crystals, whose features are significantly different from the three-dimensional graphene crystals. In recent years significant progress has also been made in the study of other two-dimensional systems. In particular, a wide class of graphene-like two-dimensional materials have been investigated, such as silicene, germanene, hafnene and several others (see, for instance,[3–6] and references therein). These materials are novel two-dimensional optical systems with unprecedented characteristics and they have been extensively investigated for use as next generation materials in applications for both nano-optics and nano-electronics [1, 4].

Two-dimensional systems, which can be created with one or a few crystalline monolayers of atoms, are of high interest, not only because of their unusual optical properties, but also due to their potential for applications in a new branch of nano-plasmonics (graphene-plasmonics), wherein one can create and use surface plasmon polaritons (SPPs) [7, 8]. SPP is a surface optical wave which is characterized by a strong enhancement of its wave power, due to the spatial confinement it undergoes near the interface of these two-dimensional layered structures. The amplitude of the SPP has a maximum at the interface and decays exponentially in the directions normal to the interface. The SPP is an electromagnetic wave, which can propagate along the boundary surface of different materials, provided that the permittivities of the two connected materials have opposite signs at the carrier wave frequency [9]. Graphene and other graphene-like two-dimensional materials are recognized as being promising materials for the investigation of potential applications for SPPs [10].

The graphene-plasmonics is currently a rapidly growing field of research which deals with the very high intensity SPP-graphene interactions found on the very short subwavelength scale, which is mainly determined by the remarkably small effective mass of the charge carriers in graphene. By means of an external gate, one can adjust the value of the Fermi energy of graphene. This is particularly interesting, because with an external gate, one can externally control the properties of an SPP in various ways. In the propagation of intense SPPs, the nonlinear effects are especially bright due to the nonlinear interaction between the SPP and the graphene layer. Nonlinear effects can provide a means for controlling the propagation of light on the nano-scale by the formation of surface optical solitons [8]. The large intrinsic nonlinearity of graphene at optical frequencies then enables the formation of optical solitons, whereby one could use the nonlinearity to compensate for a weaker dispersion. Such solitons are referred to as non-resonance solitons.

The optical response of graphene is characterized by its surface conductivity which is very closely related to its Fermi energy. Usually, the conductivity of graphene has a complex character which can be taken to be a sum of intraband and interband processes. To study surface nonlinear waves in graphene, one has to consider the influence of the optical conductivity of graphene on the parameters of the surface nonlinear waves.

The properties of an SPP are more varied, and consequently more interesting, when a transition layer(s) is sandwiched between the connected media. It is known that such transition layers can have an influence on the parameters of the SPP, especially when they are in resonance with the electronic excitations of the layer. In particular, a very attractive multi-layered system for the study of nonlinear SPPs, could be created by placing small concentrations of resonance optical active atoms or semiconductor quantum dots (SQDs) into the transition layer. In such structures, optical resonance solitons can be created under the condition of self-induced transparency (SIT) in graphene for SPPs [11] and also could create waveguide modes [12] as well. The investigation of SIT in graphene could definitely be expected to open up new applications for optoelectronic devices.

A natural extension of the study of the propagation of the optical nonlinear SPPs in graphene would be the investigation of small intensity resonance breathers (pulsing solitons) in a graphene nanostructure. Their presence could be expected to give rise to a variety of interesting nonlinear optical phenomena, as well as new applications.

Breathers arise in many physical situations where optical waves propagate at intensities too small to create solitons. However, the possibility of surface SIT breathers in two-dimensional materials has not been studied before. The purpose of the present work is to consider the conditions for the realization of resonance surface SIT breathers in a graphene monolayer, along with the resulting analytic expressions that would determine the breather parameters. At the same time, we will compare and contrast the propagations of a SPP soliton, and also that of a SPP breather, in graphene nanostructures.

II. BASIC EQUATIONS

We study the propagation of an optical resonant SPP SIT breather in a graphene monolayer (or some other similar two-dimensional graphene-like system), where the surface TM-mode optical pulse has some width, T, and frequency, $\omega >> T^{-1}$, and is orientated along the positive z axis. We will consider a four-layered system. The graphene monolayer and a thin transition resonance layer with thickness, h, which contains a small concentration of two-level optical active impurity atoms, or SQDs, of density n_0, which are sandwiched between the two semi-spaces: medium 1 ($x < 0$) and medium 2 ($x > h$), which have permittivities ε_1 and ε_2, respectively. The condition for the existence of the SPP is that the permittivities of these two media are the negative of each other, i.e., $\varepsilon_1 > 0$ and $\varepsilon_2 < 0$. For example, in a metallic medium or in a left-hand metamaterial, their permittivities can be negative for certain values of the carrier wave frequencies [13].

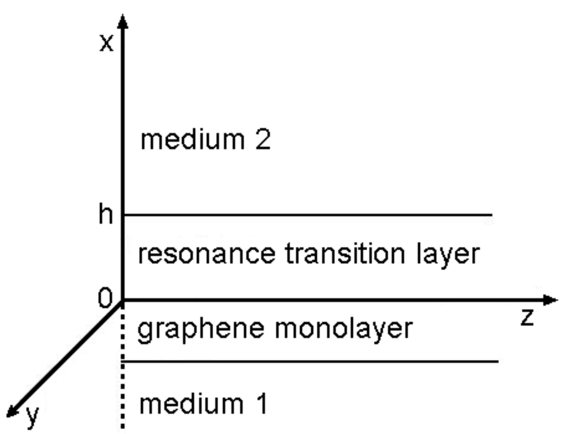

FIG. 1: The SPP is traveling along the z-axis. The vector of the electric field, $\vec{E}$, of the SPP TM-mode lies in the xz-plane. The vector of the associated magnetic field, $\vec{H}$, is parallel to the y-axis. The transition layer, of thickness h, contains the two-level atoms or SQDs. The graphene monolayer is sandwiched between the transition layer and medium 1.

For a surface TM-mode, the electric field, $\vec{E}(E_x, 0, E_z)$, lies in the xz-plane, while the magnetic field, $\vec{H}(0, H_y, 0)$ is directed along y-axis.

We will consider a Fourier-decomposition of the x- and z-components of the electric field, $\vec{E}$, and the y-component of the magnetic field, $\vec{H}$, of the SPP in the two connected semi-spaces, in the following form:

$$\mathfrak{E}_{1;i}(x,z,t)=\int\int\mathcal{E}_{1;i}(\tilde{\Omega},\tilde{Q})e^{\kappa_1(\tilde{\Omega},\tilde{Q})x}e^{i(\tilde{Q}z-\tilde{\Omega}t)}d\tilde{\Omega}d\tilde{Q},\qquad for\quad x<0,$$

$$\mathfrak{E}_{2;i}(x,z,t)=\int\int\mathcal{E}_{2;i}(\tilde{\Omega},\tilde{Q})e^{-\kappa_2(\tilde{\Omega},\tilde{Q})x}e^{i(\tilde{Q}z-\tilde{\Omega}t)}d\tilde{\Omega}d\tilde{Q},\qquad for\quad x>0, \tag{1}$$

where $\mathfrak{E}_{1;i}(x,z,t)$ is given in terms of the inverse Fourier transform for any one of the functions $E_{1;x}$, $E_{1;z}$ and $H_{1;y}$, while $\mathfrak{E}_{2;i}(x,z,t)$ is given in terms of the inverse Fourier transform for any one of the functions $E_{2;x}$, $E_{2;z}$ and $H_{2;y}$, where the subscripts 1 and 2 refer to the respective fields in medium "1" and in medium "2", $i=x,y,z$, with $\mathcal{E}_{1;x,z}$ and $\mathcal{E}_{2;x,z}$ being the respective Fourier amplitudes for the electric fields and $\mathcal{E}_{1;y}=\mathcal{H}_{1;y}$ and $\mathcal{E}_{2;y}=\mathcal{H}_{2;y}$ being the respective Fourier amplitudes for the magnetic fields.

Substituting equations (1) for $\mathfrak{E}_{1,2;y}=H_{1,2;y}$ into the wave equation for the magnetic field,

$$\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial z^2}-\frac{\varepsilon_i}{c^2}\frac{\partial^2 H}{\partial t^2}=0$$

we obtain, in the respective connected sami-spaces,

$$\kappa_i^2(\tilde{\Omega},\tilde{Q})=\tilde{Q}^2-\frac{\varepsilon_i}{c^2}\tilde{\Omega}^2,\qquad i=1,2. \tag{2}$$

where κ_1^2 and κ_2^2 are determined by the transverse structure of the surface TM-mode.

Let us imbue this transition layer with two-level optical active impurity atoms (or SQDs). Then, as the SPP pulse propagates along the flat surface of the separation (at $x=0$), between the two adjacent semi-spaces (the resonance transition layer and the graphene layer), SIT can occur provided the boundary conditions do take into account 1) the surface current caused by the presence of the two-level optical active impurity atoms (or SQDs) and also 2) the conductivity of the graphene monolayer.

Next we assume $h<<\lambda$, where λ is the wavelength of the surface mode. Then we approximate the transition resonance layer and the graphene monolayer layer, each to be infinitely thin, in which case we can approximate both by $\sim\delta(x)$. Thus we take that there would be no optical active atoms inside the transition resonance layer (all optical active atoms would be concentrated at $x=0$) while the graphene layer would have no internal conductivity (all conductivity centered at $x=0$). The polarization P_0 and conductivity σ would then contribute only through boundary conditions. In other words, we would have $P_0=\sigma=0$, outside the $x=0$ layer. We note that this has been the general approach for SPPs interacting with transition layers (see, for instance [11] and references therein).

Taking these points into account, the boundary conditions for SPPs at $x=0$ then become:

$$H_{2;y}-H_{1;y}=\frac{4\pi}{c}\left(\frac{\partial P_0}{\partial t}+\sigma E_{1;z}\right),\qquad E_{1;z}=E_{2;z},\qquad D_{1;x}=D_{2;x}, \tag{3}$$

where $\vec{D}$ is the electric induction and c is the velocity of light in vacuum. The polarization of the resonance transition layer, P_0, is determined by the ensemble of two-level optical active impurity atoms (or SQDs) and can be given by

$$\vec{P}_0(x,z,t)=\vec{e}_z\,p(z,t)\,\delta(x),$$

where $\vec{e}_z$ is the polarization unit vector along the z-axis. The electric current density of the graphene monolayer is $\sigma\,\vec{E}(z,t)\,\delta(x)$, where σ is the electrical conductivity of graphene.

Substituting Eqs. (1) and (2) into Eq.(3), we obtain for the z-component of the electrical field at $x=0$

$$E_z\,(x=0,z,t)=\int\int\mathcal{E}_z(\tilde{\Omega},\tilde{Q})e^{i(\tilde{Q}z-\tilde{\Omega}t)}d\tilde{\Omega}d\tilde{Q}\,,$$

the nonlinear wave equation at $x=0$ is found to be:

$$\int\int F(\tilde{\Omega},\tilde{Q})\mathcal{E}_z(\tilde{\Omega},\tilde{Q})e^{i(\tilde{Q}z-\tilde{\Omega}t)}d\tilde{\Omega}d\tilde{Q}+4\pi\left[p(z,t)+\sigma\int E_z(z,t)dt\right]=0, \tag{4}$$

where

$$F(\tilde{\Omega},\tilde{Q})=\frac{\varepsilon_1}{\kappa_1}+\frac{\varepsilon_2}{\kappa_2},$$

and $\mathcal{E}_z(\tilde{\Omega}, \tilde{Q})$ is given by

$$\mathcal{E}_z(\tilde{\Omega}, \tilde{Q}) = \mathcal{E}_{1;z}(\tilde{\Omega}, \tilde{Q}) = \mathcal{E}_{2;z}(\tilde{\Omega}, \tilde{Q})\,.$$

This equation is valid for any dependence of the polarization, $p(z,t)$, of the two-level optical active impurity atoms (or SQDs), on the strength of the electrical field E_z at $x = 0$.

III. SIT EQUATIONS IN GRAPHENE

We are interested in the case where the SPP pulse durations are much longer than the inverse frequency of the carrier wave. Following the standard procedure, we will transform the wave equation (4) into the slowly varying envelope case [14, 15], using the expansion:

$$E_z = \sum_{l=\pm 1} \hat{E}_l Z_{-l}, \tag{5}$$

where $\hat{E}_l$ is the slowly varying complex envelope of the electric field of the surface pulse and $Z_l = e^{il(kz-\omega t)}$ contains the rapidly varying phase of the carrier wave. We also assume that they satisfy the inequalities

$$|\frac{\partial \hat{E}_l}{\partial t}| << \omega |\hat{E}_l|, \quad |\frac{\partial \hat{E}_l}{\partial z}| << k|\hat{E}_l|. \tag{6}$$

We also take E_z to be real, in which case $\hat{E}_l = \hat{E}^*_{-l}$.

Since the function $F(\tilde{\Omega}, \tilde{Q})$ is slowly varying, we can expand it about ω and k in the form of the series

$$F(\tilde{\Omega}, \tilde{Q}) = F(\omega, k) + (\tilde{\Omega} - \omega)F'_{\Omega} + (\tilde{Q} - k)F'_{Q} + ... \tag{7}$$

where

$$F'_{\Omega} = \frac{\partial F}{\partial \tilde{\Omega}}|_{\tilde{\Omega}=\omega, \tilde{Q}=k}, \qquad F'_{Q} = \frac{\partial F}{\partial \tilde{Q}}|_{\tilde{\Omega}=\omega, \tilde{Q}=k},$$

and where ω and k are the frequency and the wave number of the carrier wave.

Substituting the expansions (5) and (7) into the wave equation (4), taking into account (6), and then after separating the real and imaginary parts of equation (4), we obtain the dispersion law for SPP

$$k^2 = \frac{\omega^2}{c^2}\,\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}\,, \tag{8}$$

and the nonlinear evolution equation for the SPP envelope (at $x = 0$)

$$\frac{\partial \hat{E}_{-1}}{\partial t} + V \frac{\partial \hat{E}_{-1}}{\partial z} = \frac{4\pi n_0 \mu}{F'_{\Omega}} \rho^- \int \frac{g(\Delta) d\Delta}{1 + T^2 \Delta^2} - \frac{4\pi\sigma}{\omega F'_{\Omega}} \hat{E}_{-1}, \tag{9}$$

where $\hat{E}_{-1} = \hat{E}^*_1$, $\Delta = \omega_0 - \omega$, ω_0 is the frequency of the atomic transitions in the transition region, $\rho^{\pm}$ are the slowly varying complex envelopes of the polarization [15], while the group velocity of the linear SPP is given by

$$V = \frac{\partial \omega}{\partial k} = \frac{kc^2}{\omega}\,\frac{\varepsilon_2 \tilde{\kappa}_1^3 + \varepsilon_1 \tilde{\kappa}_2^3}{\varepsilon_2^2 \tilde{\kappa}_1^3 + \varepsilon_1^2 \tilde{\kappa}_2^3}\,. \tag{10}$$

In the above, μ is the electric dipole moment of the two-level optical active impurity atoms (or SQDs), and $g(\Delta)$ is the inhomogeneous broadening function of the spectral line of the optical two-level atoms (or quantum dots). Also

$$F'_{\Omega} = \frac{\omega}{c^2}\left(\frac{\varepsilon_2^2}{\tilde{\kappa}_2^3} + \frac{\varepsilon_1^2}{\tilde{\kappa}_1^3}\right), \quad F'_{Q} = -k\left(\frac{\varepsilon_2}{\tilde{\kappa}_2^3} + \frac{\varepsilon_1}{\tilde{\kappa}_1^3}\right), \quad \text{where} \quad \tilde{\kappa}_i^2 = k^2 - \frac{\varepsilon_i}{c^2}\omega^2\,,$$

and $\tilde{\kappa}_1^2$ and $\tilde{\kappa}_2^2$ are determined by the transverse structure of the surface TM-mode.

For the determination of the polarization of the transition layer, we consider the average values of the Pauli operators $\hat{\sigma}_i$ which describe the induced dipole and the inverse of the probability for the state $|\Psi>$, where we take $s_i = Tr <\Psi|\hat{\sigma}_i|\Psi>$ and $(i = x, y, z)$ [14]. Here $s_x \pm i s_y = \pm 2i\rho^{\pm} Z_{\mp 1}$, where $(\rho^+)^* = \rho^-$.

Assuming that the envelopes $\rho^{\pm}$ vary sufficiently slowly in space and time as compared with the carrier wave parts, it follows that we may take:

$$|\frac{\partial \rho^{\pm}}{\partial t}| << \omega|\rho^{\pm}|, \quad |\frac{\partial \rho^{\pm}}{\partial z}| << k|\rho^{\pm}|\,.$$

The above together with the Eq.(6) is known as the slowly varying envelope approximation [15].

The quantity ρ^- is determined from the optical Bloch equations [11, 14]

$$\frac{\partial \rho^+}{\partial t} = i\Delta\rho^+ + \frac{\mu}{\hbar}\hat{E}_{+1} s_z, \qquad \frac{\partial s_z}{\partial t} = -\frac{2\mu}{\hbar}(\hat{E}_{+1}\rho^- + \hat{E}_{-1}\rho^+), \tag{11}$$

where $\hbar$ is Planck's constant. Equations (11) are exact only in the limit of infinite relaxation times.

Eqs.(9) and (11) are the general equations for the slowly varying complex amplitudes $\hat{E}_{\pm 1}$ and $\rho^{\pm}$ by means of which we can treat a rather wide class of coherent nonlinear phenomena in four-layer systems, when they have a graphene monolayer and a thin transition resonance layer, containing a small concentration of two-level optical active impurity atoms (or SQDs). The soliton solution of these equations has been developed and also expanded by an inverse scattering transform perturbation theory, in Ref. [11]. For the breather solution of these systems of equations, it will be necessary to detail some additional considerations regarding the inverse scattering transform and its perturbation theory.

IV. BREATHER SOLUTION OF SPP

In order to consider small pulse area $|\Theta_l| << 1$ breather solutions of the wave equation (9), we transform this equation into the following form:

$$\frac{\partial^2 \Theta_{-1}}{\partial t^2} + V\frac{\partial^2 \Theta_{-1}}{\partial z \partial t} = R^2\,\rho^- - \tilde{\sigma}^2 \frac{\partial \Theta_{-1}}{\partial t}, \tag{12}$$

where

$$R^2 = \frac{8\pi n_0 \mu^2}{\hbar F'_{\Omega}} \int \frac{g(\Delta)d\Delta}{1 + T^2\Delta^2}, \qquad \tilde{\sigma}^2 = \frac{4\pi\sigma}{\omega F'_{\Omega}},$$

and

$$\Theta_l(z,t) = \frac{2\mu}{\hbar}\int_{-\infty}^{t} \hat{E}_l(z,t')dt' \tag{13}$$

is the area of the optical pulse envelope at the interface $(x = 0)$.

To further analyze these equations, we make use of the perturbative reduction method [16], in the limit that Θ_l is $\mathcal{O}(\epsilon)$, with its scale-length being of order $\mathcal{O}(\epsilon^{-1})$. This is the typical scaling for the NLS equation. In this case Θ_l can be represented as:

$$\Theta_l(z,t) = \sum_{\alpha=1}^{\infty} \varepsilon^{\alpha} {\Theta_l}^{(\alpha)} = \sum_{\alpha=1}^{\infty}\sum_{n=-\infty}^{+\infty} \varepsilon^{\alpha} Y_n f_{l,n}^{(\alpha)}(\zeta,\tau), \tag{14}$$

where

$$Y_n = e^{in(Qz-\Omega t)}, \quad \zeta = \varepsilon Q(z - v_g t), \quad \tau = \varepsilon^2 t, \quad v_g = \frac{d\Omega}{dQ},$$

with ε being a small parameter.

Such a representation allows us to expand $\Theta_l(z,t)$ in the more slowly changing quantities $f_{l,n}^{(\alpha)}$. Consequently, it is assumed that the quantities Ω, Q, and $f_{l,n}^{(\alpha)}$ satisfy the inequalities

$$\left|\frac{\partial f_{l,n}^{(\alpha)}}{\partial t}\right| \ll \Omega \left|f_{l,n}^{(\alpha)}\right|, \quad \left|\frac{\partial f_{l,n}^{(\alpha)}}{\partial z}\right| \ll Q \left|f_{l,n}^{(\alpha)}\right|.$$

From the condition $\hat{E}_{-1} = \hat{E}_1^*$, it follows that ${f_{-l,-n}^{(\alpha)}}^* = f_{l,n}^{(\alpha)}$.

Substituting Eq.(14) into Eq.(12), to determine the values of $f_{l,n}^{(\alpha)}$, we collect the various terms in equation (12), according to their powers of ε, and setting each collection equal to zero. As a result, we obtain a chain of equations. Starting with first order in ε, the only component of $f_{l,n}^{(\alpha)}$ that differs from zero is $f_{l,n}^{(1)}$. The relations between the parameters Ω and Q also follows from (12) and has the form

$$(QV - \Omega)\Omega + \frac{R^2}{2} = 0. \tag{15}$$

From the Bloch equations (11) we can determine the quantity

$$\rho^- = -\frac{1}{2}[\varepsilon^1 {\Theta_{-1}}^{(1)} + \varepsilon^2 {\Theta_{-1}}^{(2)} + \varepsilon^3 {\Theta_{-1}}^{(3)} - \varepsilon^3 \frac{1}{2} \int \frac{\partial {\Theta_{-1}}^{(1)}}{\partial t} {\Theta_{+1}}^{(1)} {\Theta_{-1}}^{(1)} dt'] + ... \tag{16}$$

Substituting Eqs.(14) and (16) into Eq.(12), and taking into account (15), we obtain for the functions $f_{-1,\pm 1}^{(1)}$, the nonlinear Schrödinger (NLS) equations in the form

$$\mp i[\frac{V\Omega}{v_g}\frac{\partial f_{-1,\pm1}^{(1)}}{\partial \tau} + \Omega\Gamma^2 f_{-1,\pm1}^{(1)}] - Q^2 v_g (V - v_g)\frac{\partial^2 f_{-1,\pm1}^{(1)}}{\partial \zeta^2} - \frac{R^2}{4}|f_{-1,\pm1}^{(1)}|^2 f_{-1,\pm1}^{(1)} = 0, \tag{17}$$

where we have taken $\tilde{\sigma}^2$ to be of order ε^2, whence we take $\tilde{\sigma}^2 = \varepsilon^2\Gamma^2$, and thus defining Γ^2. We also find it convenient to define

$$v_g = \frac{V\Omega}{2\Omega - QV}\,. \tag{18}$$

Then upon defining the quantity $\Lambda_l = \sqrt{\tilde{q}}\varepsilon f_{-1,l}^{(1)}$, equation (17) becomes ($l = \pm 1$):

$$il\frac{\partial \Lambda_l}{\partial t} + \frac{\partial^2 \Lambda_l}{\partial y^2} + |\Lambda_l|^2 \Lambda_l = -il\gamma^2 \Lambda_l \tag{19}$$

which is a damped NLS equations, where

$$y = \frac{1}{\sqrt{\tilde{p}}}(z - v_g t), \quad t = t,$$

$$\tilde{p} = \frac{(V - v_g)v_g^2}{\Omega V}, \qquad \tilde{q} = \frac{2\pi n_0 \mu^2 v_g}{\Omega V \hbar F'_\Omega}\int \frac{g(\Delta) d\Delta}{1 + T^2\Delta^2},$$

$$\gamma^2 = \frac{4\pi\sigma c^2 \Omega}{\omega^2(2\Omega - QV)}\frac{\tilde{\kappa}_1^3\tilde{\kappa}_2^3}{\varepsilon_2^2\tilde{\kappa}_1^3 + \varepsilon_1^2\tilde{\kappa}_2^3}. \tag{20}$$

There are two phases in solving this equation. First, if we drop the damping term, then we have the standard NLS equation which is integrable and was originally solved by Zakharov and Shabat in 1972 [17]. In 1974, this solution method was expanded to include other nonlinear equations and was shown to be equivalent to a nonlinear Fourier transform [18] also. There are generally two types of solutions for these nonlinear equations. First, one generally has "soliton" solutions, which are localized solutions. There can also be continuous solutions which are wave-like, and are referred to as "radiation". There are also textbooks which explain in detail, how solutions of these equations may be constructed [19, 20].

Once one has obtained the undamped solutions of equation (19), one may then consider the effects of the damping term. To do this, one may use a perturbation expansion about the undamped solutions. This was first developed by Kaup in 1976 [14, 21–23].

V. ZAKHAROV-SHABAT EQUATIONS

Let us now summarize how to use the IST to obtain solutions of the NLS equation and present some well-known properties of the NLS equation [19, 21, 23]: Let us start with the NLS equation in the form $i\partial_t q + \partial_y^2 q + 2q(q^* q) = 0$, where $q(y,t)$ is a potential that vanishes sufficiently rapidly as $y \to \pm\infty$. One constructs the following linear differential Zakharov and Shabat equations (ZSE) for v_1 and v_2,

$$\frac{\partial v_1}{\partial y} = -i\zeta v_1 + q v_2, \qquad \frac{\partial v_2}{\partial y} = i\zeta v_2 + r v_1\,, \tag{21}$$

where $r = -q^*$ and ζ is a spectral parameter in the complex plane. There are two pairs of linearly independent solutions (Jost functions) of the ZSE: the first pair is denoted by Φ and $\bar{\Phi}$ and the second pair is Ψ and $\bar{\Psi}$. The first pair is Φ and $\bar{\Phi}$ and are defined by the asymptotic limit as $y \to -\infty$ to be $\Phi \to \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\zeta y}, \quad \bar{\Phi} \to \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{i\zeta y}$, and the second pair Ψ and $\bar{\Psi}$ is defined by the asymptotic limit as $y \to +\infty$ to be $\Psi \to \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\zeta y}, \quad \bar{\Psi} \to \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\zeta y}$. For real ζ, the scattering coefficients a, b, $\bar{a}$ and $\bar{b}$ are defined from the asymptotic limit as $y \to +\infty$, where $\Phi \to \begin{pmatrix} a e^{-i\zeta y} \\ b e^{i\zeta y} \end{pmatrix}, \quad \bar{\Phi} \to \begin{pmatrix} \bar{b} e^{-i\zeta y} \\ -\bar{a} e^{-i\zeta y} \end{pmatrix}$. On the real axis, one finds that $a\bar{a} + b\bar{b} = 1$. From the above definitions, one observes that in general the two pairs of solutions can be related as $\Phi = a\bar{\Psi} + b\Psi, \ \ \bar{\Phi} = -\bar{a}\Psi + \bar{b}\bar{\Psi}$. From the relation $r = -q^*$, it follows that $\bar{\Phi}$ and $\bar{\Psi}$ can be given in terms of Φ and Ψ: $\bar{\Phi} = \begin{pmatrix} \phi_2^* \\ -\phi_1^* \end{pmatrix}, \quad \bar{\Psi} = \begin{pmatrix} \psi_2^* \\ -\psi_1^* \end{pmatrix}$, and $\bar{a}(\zeta) = a^*(\zeta^*), \ \ \bar{b}(\zeta) = b^*(\zeta^*)$ where for real ζ we have $\bar{a}a + \bar{b}b = 1$.

In addition to the continuous spectra, ZSE (21) can also possess bound states. These occur whenever $a(\zeta)$ has a zero in the upper half complex ζ-plane. Here we shall consider the situation where a has only one zero. If we designate the zero of a by $\zeta_1 = \xi + i\eta$, with both ξ and η real, then since $a(\zeta_1) = 0$, $\Phi(\zeta_1) = b(\zeta_1)\Psi(\zeta_1)$, where $b(\zeta_1) = e^{i\beta} e^{2\eta y_0}$, which defines β and y_0.

Observe that what has occurred here is that the potential, $q(y,t)$, has been mapped into scattering data. The scattering data is contained in the coefficients $a\,, \bar{a}\,, b\,, \bar{b}$, each of which are functions of ζ. Given the scattering coefficients, one can reconstruct the potentials q and r.

We comment here that the scattering data is also time dependent, so that as time evolves, the scattering data also evolves in time; however the evolution of the scattering coefficients in time is governed by linear ordinary differential equations. Whence the power of the IST is to shift the action from nonlinear equations to linear equations, allowing a more rapid solution. The details of this are given in the references [19, 20].

VI. BREATHER SOLUTION

The main purpose of this paper is to obtain breather solutions of E_z. By using the IST to obtain the scattering data, one can then obtain the solution of Eq. (19). This gives us the soliton solution for the quantity Λ_l, but this solution is not a breather. To obtain a breather, we start with the expression for Λ_l, which is $\Lambda_l = \varepsilon\,\sqrt{\tilde{q}} f_{-1,l}^{(1)}$, which we then solve for $f_{-1,l}^{(1)}$, which we then insert into the solution for (14). Retaining only the ε terms, from Eq. (13) we then will obtain the breather solution of E_z.

First, we consider Eq. (19) without the damping term. In this case, this equation becomes:

$$il\frac{\partial \Lambda_l}{\partial t} + \frac{\partial^2 \Lambda_l}{\partial y_l^2} + |\Lambda_l|^2 \Lambda_l = 0, \tag{22}$$

which is the NLS equation (22), which is an exactly solvable equation. For the solution of interest, we require $\tilde{p}\tilde{q} > 0$, so that the soliton will be localize, vanishing as $y \to \pm\infty$ [19, 22]:

One then obtains the general soliton solution as

$$\Lambda_l = 2il\eta \frac{e^{-il\varphi_1}}{\cosh 2\eta\varphi_2}, \tag{23}$$

where

$$\varphi_1 = \frac{2\xi z}{\sqrt{\tilde{p}}} + 2[2(\xi^2 - \eta^2) - \frac{\xi v_g}{\sqrt{\tilde{p}}}]t - \varphi_0,$$

$$\varphi_2 = \frac{z}{\sqrt{\tilde{p}}} + (4\xi - \frac{v_g}{\sqrt{\tilde{p}}})t - y_0. \tag{24}$$

The quantities $\xi, \eta, \varphi_0 = \arg b(0)$ and $y_0 = \frac{1}{2\eta} ln|b(0)|$ are pieces of the scattering data, which are obtained when the NLS equation is solved by the IST. Substituting the soliton solution (23) into Eq. (14), we obtain the breather solution for Θ_{-1} , in the form

$$\Theta_{-1}(z,t) = \frac{4\eta}{\sqrt{\tilde{q}}} \frac{\sin(\varphi_1 - Qz + \Omega t)}{\cosh 2\eta\varphi_2} + \mathcal{O}(\varepsilon). \tag{25}$$

Combining equations (25) and (13), we have the breather solution for the envelope of the z-components of the electric field $\vec{E}$, where

$$\hat{E}_{-1} = \frac{2\eta\hbar\Omega}{\mu\sqrt{\tilde{q}}} \frac{\cos(Qz - \Omega t - \varphi_1)}{\cosh 2\eta\varphi_2} + \mathcal{O}(\varepsilon), \tag{26}$$

with $\frac{2\eta\hbar\Omega}{\mu\sqrt{\tilde{q}}}$ being the breather pulse height.

Soliton solutions of the NLS equation, which are not breathers, are given by Eq.(23). From this solution, we can construct the solution of Eq.(26), which is a breather solution. This is the solution that characterizes the propagation of the nonlinear SPP, which oscillates in time and space and propagates in space with the characteristic parameters, Ω and Q.

Lastly, to fully understand and describe the SPP breather in graphene, we need to take into account the first order corrections of the influence of the conductivity on the breather's oscillation and propagation. For this, we use the perturbation theory developed for the IST [14, 21, 23].

One can determine the perturbed evolution of the breather parameters ξ and η due to the influence of the conductivity of graphene on the pulse height $\frac{2\eta\hbar\Omega}{\mu\sqrt{\tilde{q}}}$ (or pulse width), by means of the equation [21]:

$$\zeta_{1_t} = \xi(0) + i\eta(0)e^{-2t\gamma^2}$$

The solution of this equation has the form:

$$\eta(t) = \eta(0)e^{-2t\gamma^2} \tag{27}$$

where the parameter ξ is constant. $\eta(0)$ is the initial value of η at t=0.

VII. CONCLUSION

This work has studied the propagation of TM-mode SPP waves, along the interface between two different media. Sandwiched between these media are a monolayer of graphene and a thin transition resonance layer. The latter has, as an impurity, optical active two-level atoms or quantum dots. Under the condition of SIT and providing that $\tilde{p}\tilde{q} > 0$, breathers can arise in the propagating SPP. As we have shown above, the amplitude of these breathers can be expected to decay exponentially, in the process of propagation. The explicit analytical expressions for the profile and parameters of the surface breather are contained in Eqs.(27), (26), (24), (20), (18) and (10). The dispersion equation for the surface TM-mode and the relations between the quantities Ω and Q are given by Eqs.(8) and (15), respectively. The transverse profile of the SPP is given by Eq.(2).

From these equations, it is obvious that the parameters of the optical SPP breather in graphene also depends on the parameters of the two-level optical active atoms or SQDs, through the quantity R^2 (which depends on the quantities μ, n_0, $g(\Delta)$), as well as the permittivities of the two connected media (ε_1 and ε_2). Also, the transverse structure of the surface TM-mode is influenced by the quantities($\tilde{\kappa}_1$ and $\tilde{\kappa}_2$). Lastly, the amplitude of a breather will decay exponentially according to the characteristic parameter γ^2 which is dependent also on the quantities: graphene conductivity σ, the two permittivites (ε_1 and ε_2), the transverse structure coefficients ($\tilde{\kappa}_1$ and $\tilde{\kappa}_2$), and also on the oscillate characteristic parameters Ω, Q and group velocity of the linear SPP, V.

SPP propagating in graphene nanostructure, under the condition of SIT, can produce solitons and breathers. Under the condition of SIT, SPP solitons were investigated earlier in Ref. [11], by the use of the IST perturbation theory. Comparing the SPP soliton and breather under the condition of SIT in graphene nanostructures, we note that the amplitudes of both nonlinear resonance waves, propagating through the multi-layered graphene nanostructure, will undergo exponentially damping. However, the damping coefficient of the breather, γ^2, is distinctly different from that of a soliton's damping coefficient, which is $4\pi\sigma/\omega F'_Q$. Consequently, the breather's damping coefficient depends additionally on the characteristic oscillation parameters Ω and Q, as well as the group velocity of the linear SPP, V.

In addition to the above, these two "damping coefficients" cannot be easily compared against each other, since the soliton and the breather in the graphene nanostructure undergo distinctly different evolutions. In particular, the damping for the soliton is along the coordinate "z", whereas for the breather, the damping coefficient is along the coordinate "t" (time). I.e. these damping coefficients act in different "directions".

The results of this theoretical study of resonant SPP breathers in graphene, along with the study of resonant SPP solitons in graphene, as treated in Ref.[11], give a more complete physical description of the propagation of resonant SPP solitons in graphene nanostructures.

These investigations are informative not only for further theoretically studies, but will also stimulate experimental investigations of the propagation of resonance nonlinear waves in graphene nanostructures, leading eventually to the development of graphene devices and their applications in the studies of nonlinear SPPs.

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