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Phys. Rev. A 95, 043637 — Published 26 April 2017
DOI: 10.1103/PhysRevA.95.043637
Far-field interference of a neutron white beam and the applications to non-invasive phase contrast imaging

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(Dated: March 31, 2017)

The phenomenon of interference plays a crucial role in the field of precision measurement science. Wave-particle duality has expanded the well-known interference effects of electromagnetic waves to massive particles. The majority of the wave-particle interference experiments require a near monochromatic beam which limits its applications due to the resulting low intensity. Here we demonstrate white beam interference in the far-field regime using a two phase-grating neutron interferometer and its application to phase-contrast imaging. The functionality of this interferometer is based on the universal moiré effect that allows us to improve upon the standard Lau setup. Interference fringes were observed with monochromatic and polychromatic neutron beams; for both continuous and pulsed beams. Far-field neutron interferometry allows for the full utilization of intense neutron sources for precision measurements of gradient fields. It also overcomes the alignment, stability, and fabrication challenges associated with the more familiar perfect-crystal neutron interferometer as well as avoids the loss of intensity due to the absorption grating requirement in Talbot-Lau interferometer.

I. INTRODUCTION

The discovery of the neutron [1] led to the construction of a variety of phase sensitive neutron interferometers. Thermal and cold neutrons are a particularly convenient probe of matter and quantum mechanics postulates given their relatively large mass, nanometer-sized wavelengths, and zero electric charge. One of the first neutron interferometers [2] was based on wave-front division using a Fresnel biprism setup. The perfect crystal neutron interferometer (NI), based on amplitude division, has achieved the most success due to its size and modest path separation of a few centimeters. Numerous perfect crystal NI experiments have been performed exploring the nature of the neutron and its interactions [3–9]. However, perfect crystal neutron interferometry requires extreme forms of environmental isolation [10, 11], which significantly limits its expansion and development.

Advances in micro- and nano-fabrication of periodic structures with features ranging between 1 μm to 100 μm enables employing absorption and phase gratings as practical optical components for neutron beams. The first demonstration of a Mach-Zehnder based grating NI in 1985 [12] used 21 μm periodic reflection gratings as beam splitters for monochromatic (λ = 0.315 nm) neutrons. A few years later, a three transmission phase grating Mach-Zehnder NI was demonstrated for cold neutrons (0.2 nm < λ < 50 nm) [13–16]. The need for cold- or very cold- neutrons with a high degree of collimation limits the use of such interferometers in material science and condensed matter research. An alternative approach was the Talbot-Lau interferometer (TLI) proposed by Clauser and Li for cold potassium ions and x-ray interferometry [17], and implemented by Pfieffer et al. for neutrons [18]. The TLI is based on the near-field Talbot effect [19] and uses a combination of absorption and phase gratings. In this setup the sample, introduced in front of the phase grating (middle grating), modifies the phase and amplitude of the Talbot self-image. While the previously mentioned Mach-Zehnder type grating interferometers are sensitive to phase shifts induced by a sample located in one arm of the interferometer, this near-field TLI is sensitive to phase gradients caused by a sample. Even though chromatic sensitivity of the TLI is reduced, thus leading to a gain in neutron intensity, in this setup the absorption gratings are challenging to manufacture and limits the incident flux reaching the detector; while the neutron wavelength spread will cause contrast loss as the distance to interference fringes (fractional Talbot distance) is inversely proportional to the neutron wave-
length.

It has been previously demonstrated for atoms and electromagnetic waves that the two grating TLI setup produces fringes in the far field [20–23]. This “Lau effect” [24] requires the first grating to be an intensity mask which serves as an array of mutually incoherent sources, and the second grating is either an intensity or a phase mask which through Fresnel diffraction results in the interference pattern on the screen. When the distance between the two gratings is at multiples of half Talbot length, an image of the first grating is produced at the plane at the second grating, and that image beats with the second grating to produce the beat pattern in the far field downstream.

Here we implement a far-field regime interferometer and report the first demonstration of a multi-beam, broadband NI using exclusively phase gratings. Both Lau and Talbot effects turned out to be special cases of a more universal moiré effect [25]. The conceptual step forward of our neutron interferometer is to go to the other end of the moiré effect spectrum for the matter-waves, where two pure phase masks produce a beat pattern when both the source and the detector are in the far-field range. This interference would be observed even if the two phase gratings are in contact, provided that their periods are appropriately different. The demonstration includes the use of a continuous monochromatic beam (λ = 0.44 nm), a continuous bichromatic beam (1/3 intensity λ₁ = 0.22 nm and 2/3 intensity λ₂ = 0.44 nm), a continuous polychromatic beam (approximately given by a Maxwell-Boltzmann distribution with Tn = 40 K or λc = 0.5 nm), and a pulsed neutron beam (λ = 0.5 nm to λ = 3.5 nm). The advantages of this setup include the use of widely available thermal and cold neutron beams, relaxed grating fabrication and alignment requirements, and broad wavelength acceptance.

II. TWO PHASE-GRATING INTERFEROMETER

As the neutron can be described as a matter-wave with a de Broglie wavelength \( \lambda = 2\pi\hbar/(m_nv_n) \), where \( \hbar \) is the reduced Planck’s constant, \( m_n \) is the neutron mass, and \( v_n \) is the neutron velocity, the problem could be treated similar to the X-ray case. The full mathematical treatment of the general situation of polychromatic beam passing through double phase grating setup (see Fig. 1) is described by Miao et al. [25]. Here we give a brief description and the key points of the universal moiré effect in a far-field regime for neutrons.

The experimental setup, shown in Fig. 1, consists of a slit, two identical linear phase gratings of silicon combs with a period of \( P_{G_1} = P_{G_2} = 2.4 \, \mu\text{m} \), and a neutron imaging detector (neutron camera). Although \( \pi/2 \)-phase gratings for a specific neutron wavelength give optimal fringe visibility, available to us were five different gratings with various depths. We had used gratings corresponding
to thermal energies and then further cooled using a liquid hydrogen cold source [26] before traversing a neutron guide, the end of which is a slit. After exiting the slit and propagating in free space, the neutron acquires a transverse coherence length of \[ \ell_c = \frac{\lambda \Delta}{s_w}, \] where \( s_w \) is the width of the slit and \( \Delta \) is the distance between the slit and the point of interest.

In order for the neutron to diffract from the first grating \( G_1 \) at the distance \( L_1 \), the neutron’s coherence length (along the \( y \)-axis in Fig. 1), should be at least equal to the period of the grating:

\[ \ell_c = \frac{\lambda L_1}{s_w} \geq P_{G_1}. \]  

(1)

The second grating \( G_2 \) is placed at a distance \( D \) from the first grating, and a distance \( L_2 \) from the neutron camera. As neutron cameras have limited spatial resolution \( \eta \), the fringe period \( P_d \) at the camera should be bigger than neutron camera resolution [25]:

\[ P_d = \frac{L P_{G_2}}{D} > \eta, \]  

(2)

where \( L = L_1 + D + L_2 \) is the distance between the slit and the camera. Similarly the phase of the fringe pattern on the detector is a periodic function of the slit position, with the period (often called source period) given by [25]:

\[ P_s = \frac{L P_{G_1}}{D}. \]  

(3)

Therefore, in order to observe a fringe pattern on the detector the slit width should be smaller than the source period, i.e. \( P_s < s_w \).

To verify that we are indeed in a far-field regime we consider the Fraunhofer distance when the coherence length is used as the source dimension:

\[ d_F = 2\frac{\ell_c^2}{\lambda} = 2\lambda \left( \frac{L_1}{s_w} \right)^2. \]  

(4)

We consider the coherence length because it is always equal or greater than the grating period in the setup. To satisfy the far-field regime \( L_2 \) should be greater than the Fraunhofer distance:

\[ \frac{d_F}{L_2} \approx \frac{\lambda L}{s_w^2} \ll 1. \]  

(5)

Given the experimental parameters of the monochromatic beamline \( d_F/L_2 \approx 0.04 \), polychromatic beamline \( d_F/L_2 \approx 0.02 \) for \( \lambda = 0.5 \) nm, and the beamline at the pulsed source \( d_F/L_2 \approx 0.08 \) for \( \lambda = 0.35 \) nm. The other two conditions for far-field regime are:

\[ L_2 \gg \lambda, \quad L_2 \gg s_w. \]  

(6)

In our cases they are the least strict conditions.

If we consider equal period \( P_{G_1} = P_{G_2} \equiv P_g \) and \( \pi/2 \)-phase gratings, with 50 % comb fraction, then the maximum contrast is optimized for the condition \( \delta_1(\lambda) = \)
FIG. 4. (Top) The contrast as a function of the separation distance between the gratings for the monochromatic, bichromatic, and polychromatic neutron beams. The fits are given by Eq. 12 in [25], and the contrast for the monochromatic, bichromatic and polychromatic beamlines is optimized at $D = 12$ mm, $12$ mm, and $10$ mm respectively, agreeing well with theoretical predictions [25]. (Bottom) The frequency of the oscillation fringes varies linearly as a function of the distance between the gratings. The linear fit is according to $(D−D_0)asterslash(L*P_g)$ which for the monochromatic setup gives $D_0 = −0.75$ mm and $L = 3.04$ m, the bichromatic setup gives $D_0 = −0.8$ mm and $L = 3.51$ m, and polychromatic setup gives $D_0 = −3.8$ mm and $L = 8.36$ m. The exposure time was 2 s.

$\delta_2(\lambda) = 0.5$ [25], where:

$$\delta_1(\lambda) = \frac{\lambda L_1 D}{L P_g^2}, \quad \delta_2(\lambda) = \frac{\lambda L_2 D}{L P_g^2}. \quad (7)$$

The closed-form expression of the contrast is given by Eq. 12 in [25], and is computed numerically.

To align the gratings the setup is initially arranged with theoretically calculated optimal slit width and lengths $L_1$, $D$, and $L_2$. Then one of the gratings is rotated around the neutron propagation axis ($z$-axis) until the fringe pattern is observed at the camera. The contrast with the monochromatic setup as a function of the first grating rotation around the $z$-axis is shown in the top plot of Fig. 2.

The slit height $s_h$ (slit length along the $x$-axis direction in Fig. 1) can be larger than the slit width $s_w$ in order to increase neutron intensity, provided that the gratings are well aligned to be parallel to that direction. The angular range of appreciable contrast is inversely related to the slit height $P_g/(2P_g s_h/L)$. The expected range of $±0.007°$ agrees with the range depicted on Fig. 2.

FIG. 5. Linear phase stepping is achieved with parallel translation of the first grating $G_1$ by increments smaller than the period of the gratings. (Top) Data from the monochromatic beamline (Bottom) Data from the polychromatic beamline. In both cases linear dependence between the phase and grating translation is observed, while the contrast is preserved. Data was collected for the translation range of $0$ µm to $5$ µm, in increments of $0.2$ µm ($0.5$ µm) for monochromatic (polychromatic) plot.

The intensity of the fringe pattern recorded by the camera can be fit by a cosine function

$$I = A + B \cos(f x + \phi). \quad (8)$$

where $x$ is the pixel location on the camera. Thus the mean $A$, the amplitude $B$, the frequency $f$, and the differential phase $\phi$ can be extracted from the fit. The contrast or fringe visibility is given by:

$$C = \frac{\max\{I\} − \min\{I\}}{\max\{I\} + \min\{I\}} = \frac{B}{A}. \quad (9)$$

Due to the generally low neutron flux with monochromatic beamlines, the slit widths are optimized for intensity vs contrast. The contrast as a function of the slit width for the bichromatic setup is shown in the bottom plot of Fig. 2. Variation of the contrast vs. slit width could be described by the sinc function:

$$C = C_0 \left| \sin\left(\frac{\pi s}{P_s}\right) \right|. \quad (10)$$

where $C_0$ is the maximum achievable contrast with a given setup. Thus, given a slit width of one third of the source period, in our case $281$ µm, would give an upper bound of $83\%$ contrast. The fit in the bottom plot of Fig. 2 gives a source period of $P_s = 843 ± 43$ µm, which is in good agreement with $P_s = 845$ µm obtained with Eq. 3 where $D = 10$ mm.
The imaging detector was an Andor sCMOS NEO camera viewing a 150 \( \mu \text{m} \) thick LiF:ZnS scintillator with a Nikon 85 mm lens with a PK12 extension tube for a resolving power of about 3.7, yielding a spatial resolution of \( \eta = 150 \mu \text{m} \) [30]. To reduce noise in the sCMOS system, the median of three images were used for analysis.

The fourth configuration used a pulsed neutron beam produced at the Energy-Resolved Neutron Imaging System (RADEN) [31], located at beam line BL22 of the Japan Proton Accelerator Research Complex (J-PARC) Materials and Life Science Experimental Facility (MLF). The wavelength range that was used was from 0.05 nm to 0.35 nm. The slit to detector length is \( L = 8.6 \text{ m} \) and slit to G1 distance is \( L_1 = 4.24 \text{ m} \). The slit width was set to 200 \( \mu \text{m} \) and slit height to 4 cm. The neutron imaging system employed a micro-pixel chamber (\( \mu \text{PIC} \)), a type of micro-pattern gaseous detector with a two-dimensional strip readout, coupled with an all-digital, high-speed FPGA-based data acquisition system [32]. This event-type detector records the time-of-arrival of each neutron event relative to the pulse start time for precise measurement of neutron energy, and it has a spatial resolution of 280 \( \mu \text{m} \) (FWHM). The readout of the \( \mu \text{PIC} \) detector introduces a fixed-pattern noise structure which is completely removed by normalizing to empty beam measurements. Thus, the visibility measurements are from open-beam normalized images of the moiré pattern. The average number of detected neutron events was about 80 per 160 micron pixel with a 4 h integration time.

By observing the interference fringes with and without a sample that has been placed either upstream or downstream of the gratings we can extract the conventional beam attenuation, dark-field contrast, and differential phase contrast. By varying the grating spacing, this interferometer may also be employed to measure small angle scattering of microstructures in the range of \( \text{nm} \) to several \( \mu \text{m} \) [33].

Fig. 3 (a)-(d) show examples of typical images of the interference pattern obtained in optimized configurations for different beamlines: (a) bichromatic beam with \( \lambda_1 = 0.22 \text{ nm} \) and \( \lambda_2 = 0.44 \text{ nm} \) (b) same beamline as (a) but with a Be-filter to completely eliminate the \( \lambda_1 = 0.22 \text{ nm} \) component (c) polychromatic neutron beam with peak wavelength \( \lambda_c = 0.5 \text{ nm} \) and (d) a pulsed source for \( \lambda = 0.25 \text{ nm} \). In Fig. 3 (a) the middle dark region corresponds to the collimator which was placed at the front in the setup, and not the grating pattern. As the Be-filter adds divergence to the beam it can be seen that the dark middle region gets washed out in Fig. 3 (b). The box on each image represents a region of integration along the vertical axis and the integral curve is shown under each image. Such integral curves were used to fit with Eq. [8] to extract phase, frequency, and compute the contrast via Eq. [9].

The top plot in Fig. 4 shows contrast (fringe visibility) change versus grating separation, \( D \). The data ob-
tained at NIST for the monochromatic, bichromatic and polychromatic beamlines is plotted on the same figure for comparison. The theoretically calculated contrast curves for the three conditions are also plotted, which were based on estimates of $0.27\pi$ phase shift gratings at 0.44 mm wavelength for the mono- and bi-chromatic setups and $0.2\pi$ phase shift at 0.5 nm wavelength for the polychromatic setup. The maximum contrast for the monochromatic, bichromatic and polychromatic beamlines are achieved at $D = 12$ mm, 12 mm, and 10 mm respectively, agreeing well with theoretical predictions [25]. Theoretical estimates indicate that there is room for improvement of contrast by improving grating profile and detector resolution. The bottom plot in Fig. 4 shows the linear dependence of the fringe frequency at the camera on the grating separation. As the distance between the gratings is increased the period of the fringes at the camera is decreased. The linear fit is according to $(D - D_0)/(L * P_0)$ which for the monochromatic setup gives $D_0 = -0.75$ mm and $L = 3.04$ m, the bichromatic setup gives $D_0 = -0.8$ mm and $L = 3.51$ m, and polychromatic setup gives $D_0 = -3.8$ mm and $L = 8.36$ m.

To implement phase stepping of the fringe pattern at the camera one grating needs to be translated in-plane in the perpendicular direction to the grating lines (along the y-axis in Fig. 1 or along the grating vector). The translation step size needs to be smaller than the grating period. The top plot in Fig. 5 shows the two-dimensional plot of phase stepping for the monochromatic beamline setup; and the bottom plot in Fig. 5 shows the phase stepping for the polychromatic beamline setup. In both cases linear dependence between the phase and grating translation is observed, while the contrast is preserved.

Similar aligning procedures and measurements were performed at J-PARC spallation pulsed source. Fig. 6 shows the contrast as a function of the wavelength for various grating separations. Due to the nature of the pulse source we were able to extract contrast as a function of wavelength. Note that at the time of the experiments J-PARC was running at 200 KW as opposed to 1 MW due to technical problems. This lowered the neutron flux to 1/5 of the standard flux and the low intensity proved to be a significant challenge in terms of optimizing the setup for each independent wavelength.

The beam attenuation, decoherence, and phase gradient images shown in Fig. 7 (b)-(d) are of an aluminum sample shown in Fig. 7 (a). The approximate imaged area is depicted by the rectangular box. The images in Fig. 7 (b)-(d) were obtained by the Fourier transform method described in [34]. At the described polychromatic beamline at NIST three images with 20 $s$ exposure time were taken at each step in the phase-stepping method [35]. A median filter was then applied to every set of three images. The phase step size was $0.24$ $\mu$m ranging from 0 to 2.4 $\mu$m of the G$_2$ transverse translation. The G$_1$ - G$_2$ grating separation was $D = 11.5$ mm.

Fig. 7 (b) shows the conventional attenuation-contrast radiography of the sample, where white color represents full transmission (no attenuation). The shape and features of the sample are well defined in the image. Fig. 7 (c) shows the decoherence of the fringe contrast due to the sample, $-\log(C_{\text{sample}}/C_{\text{empty}})$, where the white color represents loss of contrast and the black color represents no contrast reduction. As expected the areas which caused the largest attenuation also caused the largest loss of contrast. Fig. 7 (d) shows the phase shift in the moiré pattern at the detector due to the sample. The white and black patterns represent highest phase gradient that the neutrons acquire when passing through the sample.

![Figure 7](image)

**FIG. 7.** Phase-contrast imaging with a polychromatic beam (a) The aluminum sample in front of the G$_1$ grating. The yellow box roughly outlines what is being imaged. The sample has a step profile in the middle region and threaded screw holes in the corners. (b) Neutron attenuation image due to absorption and scattering. Grayscale bar represents transmission through the sample. (c) Spatial variation of the contrast attenuation due to the sample, $-\log(C_{\text{sample}}/C_{\text{empty}})$. In this case white represents a loss in contrast and black represents no loss in contrast. (d) Phase shift in the moiré pattern at the detector due to the sample. Grayscale bar represents radians. Here, the white and black patterns represent the highest phase gradient that the neutrons acquire when passing through the sample.

**V. CONCLUSION**

For the first time we have demonstrated a functioning two phase-grating based, moiré effect neutron interferometer. The design has a broad wavelength acceptance and requires non-rigorous alignment. The interferometer operates in the far-field regime and can potentially circumvent many limitations of the single crystal and grating based Mach-Zehnder type interferometers, and the near field Talbot-Laue type interferometers that are in operation today. Mach-Zehnder type interferometers may provide the most precise and sensitive mode of measurements but a successful implementation requires highly collimated and low energy neutron beams. On the other hand, a near-field Talbot-Laue interferometer requires absorption analyzer gratings which curtails flux and interference fringe contrast. These constraints can
be significant in a variety of applications.

The performance of our demonstration interferometer was limited primarily by grating imperfections and detector resolution. However, the design is simple and robust. We expect that the next generation of interferometers based on the far-field design will open new opportunities in high precision phase based measurements in materials science, condensed matter physics, and bioscience research. In particular, because of the moiré fringe exploitation in this type of interferometers, the uses may be highly suitable for the studies of biological membranes, polymer thin films, and materials structure. Also, the modest cost and the simplicity of assembly and operation will allow this type of interferometers to have wide acceptance in small to modest research reactor facilities worldwide.

VI. AKNOWLEDGEMENTS

This work was supported by the U.S. Department of Commerce, the NIST Radiation and Physics Division, the Director’s office of NIST, the NIST Center for Neutron Research, and the National Institute of Standards and Technology (NIST) Quantum Information Program. This work was also supported by the Canadian Excellence Research Chairs (CERC) program, the Natural Sciences and Engineering Research Council of Canada (NSERC) Discovery program, and the Collaborative Research and Training Experience (CREATE) program. D.A.P. is grateful for discussions with Michael Slutsky.


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