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Suppressed pair creation due to electron-positron attraction

Q.Z. Lv[1,2], J. Unger[2], Y.T. Li[1,3], Q. Su[1,2] and R. Grobe[2]

(1) Beijing National Laboratory for Condensed Matter Physics
Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China

(2) Intense Laser Physics Theory Unit
and Department of Physics
Illinois State University, Normal, IL 61790-4560 USA

(3) School of Physical Sciences
University of Chinese Academy of Sciences, Beijing, 100049, China

We propose a coupled-Dirac sea approach to examine numerically the electron-positron pair creation process with interfermionic interactions in a supercritical background field. In this approach the Hilbert space is doubled, leading to two separate Dirac equations for the dynamics of the electrons and the positrons. It permits us to remove the unphysical self-repulsion while the internal electric fields that couple different fermions are determined from the Maxwell equations. The time evolution of the yield, the spatial distribution, as well as the energy spectra of the particles show that the attractive force between the created particles decreases the pair-creation probability, independent of the spatial characteristics of the force-intermediating internal fields. This decrease is also accompanied by shifts in the kinetic energy spectrum of the emitted particles. These findings are also supported by an independent model that predicts the creation rate based on the quantum mechanical transmission coefficient.

Introduction

The possibility of creating an electron-positron pair from the vacuum triggered by a supercritical field has lead to wide theoretical interest [1]. In view of new laser-based experiments that are being planned at various laboratories around the world such as POLARIS [2], ELI [3], and XFEL [4], numerous theoretical investigations have been devoted to predict the ideal experimental parameters and optimal conditions for observing the breakdown process of the vacuum due to an external field. In addition to many technical problems that are inherent to these difficult calculations, there are still conceptual challenges that need to be addressed.

A good example of a conceptual problem that is potentially relevant for the pair-creation process is the theoretical framework to include the Coulombic interaction between the created particles. On a fundamental quantum field theoretical level, interfermionic interactions are facilitated by the photon field, which would have to be treated as an independent variable with its own dynamics [5]. However, solving the required coupled Maxwell-Dirac equations for the second-quantized operators is presently not feasible. Due to the lack of unambigous ab initio calculations, it is presently not even understood qualitatively, if these internal interactions actually increase or decrease the pair-creation yield. In general one could speculate that it should be more difficult to separate two particles that attract each other.

Due to these conceptual difficulties, only a limited number of studies that can address this question have been reported. There are counterintuitive speculations that the attractive Coulomb force between an electron and a positron could actually enhance the pair creation rate. If the created charges are separated by a sufficient amount, this field can actually break down the vacuum itself leading to string-breaking [6,7]. Another suggestion that the Coulombic force would increase the pair creation rate originates from an earlier work by Lebedev and Ritus [8]. These authors and others [9-11] included the (virtual) one-photon exchange between the electron and positron in the two-loop correction to the Euler-Heisenberg Lagrangian and were able to show that the original Schwinger (one-loop based) pair-production rate [12] needs to be multiplied by an exponential factor that is larger than one. The creation of electron-positron pairs was also predicted in

laser-plasmas where the emission of high-energy photon can increase the number of pairs produced. [13-14] Furthermore, several interesting QED effects such as electron-positron-photon cascades were predicted [15-20] for various field configurations.

An opposite conclusion can be derived by incorporating the spontaneous pair creation in the presence of back reactions into the quantum Vlasov equation. Here the simultaneous solutions of Maxwell's equation in the presence of the feedback yields an internal current and electric field that exhibit plasma oscillations [21]. It was predicted that the particles produced by the background field are accelerated by that field and generate a field that opposes and weakens it.

Each approach above employed the classical (mean field) approximation where the total charge and current density generate, via the (non-second quantized) Maxwell equation, the total field that is then coupled back to the Dirac equation. It was pointed out that - even in the absence of any pair creation - the classical field approximation leads unavoidably to an unphysical self-repulsion mechanism [22], where portions of the same electron wave function can react to their own electric field.

The present work was made possible by a recent proposal of how to eliminate the unwanted self-repulsion by partitioning the electron-positron field operator into two parts and by coupling the Dirac equation to two independent sets of Maxwell equations [23]. For example, the Maxwell equation for the electronic field has only the electronic charge and current densities as source terms and only this field (and not the total field) is coupled back to the positron in the Dirac equation. This approach was demonstrated for the case of relativistic two-particle scattering. However, it did not consider pair creation.

The present work has two purposes. The first one is to introduce three innovations with regard to the theoretical framework that permits us to examine the mutual interaction in the pair creation process. This includes the introduction of the doubled-Hilbert space approach, the partitioning of the internal electric fields, and a new rate-based theory. The second purpose is to provide three new insights with regard to the impact of interfermionic forces on the pair-creation process. Both the numerical simulations as well as the rate-based theory consistently predict that

the interfermionic forces reduce the pair creation yield and shift the kinetic energies spectra of the emitted particles. The spatial range of these interactions is also directly responsible for the long-time behavior, which can be either a steady state or an unstable growth.

The paper is organized as follows. In Sec. 2 we introduce the theoretical framework. In Sec. 3 we discuss the temporal behavior of the total yield, the spatial distribution of the created particles, their energy distribution and the structure of the internal electric fields considering influences due to self-repulsion, self-attraction and mutual attraction. In Sec. 4 we introduce a quantum mechanical transmission based model that can estimate the pair creation rate. In Sec. 5 we provide a brief summary and a discussion of possible new challenges that can now be addressed and hopefully motivate future work.

Theoretical framework for the numerical simulations

In this section we introduce the doubled-Hilbert space approach that permits us to calculate the time-dependent number of created electrons and positrons, their spatial charge and current densities and their kinetic energy spectra. The interaction between the particles is facilitated by internal electromagnetic fields that are obtained as solutions from the Maxwell equations with source terms obtained from the Dirac equation. An appropriate splitting of the internal field and their particular coupling to the particles permits us to quantify the relevance of interfermionic attraction and fermionic self-repulsion. While - in principle - this coupled Dirac-Maxwell approach can be applied to simulate the dynamics in three spatial dimensions if sufficient computer memory becomes available, we demonstrate its feasibility here for a spatially reduced system, for which the magnetic field is omitted. This has the advantage that in addition to computational simplifications, fundamental questions in the absence of complicated magnetic field effects can be addressed within a conceptually easier framework .

Evolution of the fermions

The interaction of a particle of charge q with a vector and scalar potential, A(z,t) and V(z,t), is

given in one spatial dimension by the Dirac Hamiltonian [24]

$$H(q) = c\,\sigma_1\,[p - q/c\,A(z,t)] + \sigma_3\,mc^2 + qV(z,t) \qquad (1)$$

where c is the speed of light and σ_1 and σ_3 are the two 2×2 Pauli matrices. In our numerical simulations below we use atomic and cgs units, where the four fundamental constants [amount of the electron's charge, its mass m, and Coulomb's and Planck's constants $1/(4\pi\varepsilon_0)$ and $\hbar$] are all unity by definition. As a result, the speed of light is c=137.036 a.u. and the atomic unit of time is 2.4×10^{-17} sec. For q=-1 [=1], the Hamiltonian $H(e^-)$ [$H(e^+)$] describes the evolution of an electron [positron].

The energy eigenstates of the force-free Hamiltonian (q=0) (denoted by H_0) with momentum p in the positive (*u*p) energy continuum are denoted by $H_0|u;p\rangle = w_p|u;p\rangle$ whereas those in the negative (*d*own) continuum are denoted by $H_0|d;p\rangle = -w_p|d;p\rangle$. The spatial representation of these states with energies $w_p \equiv [m^2c^4+c^2p^2]^{1/2}$ is given by

$$\langle z|u;p\rangle \equiv W_p(u;z) = \chi[1,\, c\,p/(mc^2+w_p)]\,\exp[ipz] \qquad (2)$$

$$\langle z|d;p\rangle \equiv W_p(d;z) = \chi[-c\,p/(mc^2+w_p),1]\,\exp[ipz] \qquad (3)$$

where $\chi \equiv (2\pi)^{-1/2}\,[1+c^2p^2/(w_p+mc^2)^2]^{-1/2}$ denotes the normalization factor.

For the special case for which the interfermionic forces were omitted, it is well-known that all aspects of the quantum field theoretical description can be obtained by evolving each (quantum mechanical) negative energy eigen state $|d;p\rangle$ in time under the Hamiltonian $H(e^-)$, corresponding to one (initially occupied) Dirac sea. As this traditional approach is based on a single Dirac equation for the electron-positron field operator (briefly summarized in the appendix), it would not permit us to remove the self-repulsion.

However, as it is necessary to eliminate the self-repulsion if one wants to study the interaction between the electrons and positrons, we have to generalize the traditional approach here to *two*

Dirac seas, given by two (initially equivalent) sets of eigen states $|d;p\rangle$ with negative energy. The first [second] set is evolved under the Hamiltonian $H(e^-)$ [$H(e^+)$], such that projection of the time evolved states on the positive energy manifold is associated with the negative [positive] charge density of the created electrons [positrons]. Exploiting the charge conjugation symmetry we show in the appendix that

$$\rho(e^-;z,t) \equiv -\Sigma_p \,|\, \Sigma_{p'} \langle u;p'| \, U(e^-;t) \, |d;p\rangle \, W_{p'}(u;z) \,|^2 \tag{4}$$

$$\rho(e^+;z,t) \equiv \Sigma_p \,|\, \Sigma_{p'} \langle u;p'| \, U(e^+;t) \, |d;p\rangle \, W_{p'}(u;z) \,|^2 \tag{5}$$

where $U(e^-;t)$ [$U(e^+;t)$] denotes the corresponding time evolution operator based on the coupling of the particle with negative [positive] charge to the external and internal field. In the propagator $U(q;t) \equiv T \exp[-i \int^t d\tau \, H(q)]$, T is the required time ordering operator as the potentials A and V are time dependent. In Figure 1 we have sketched this dynamics.

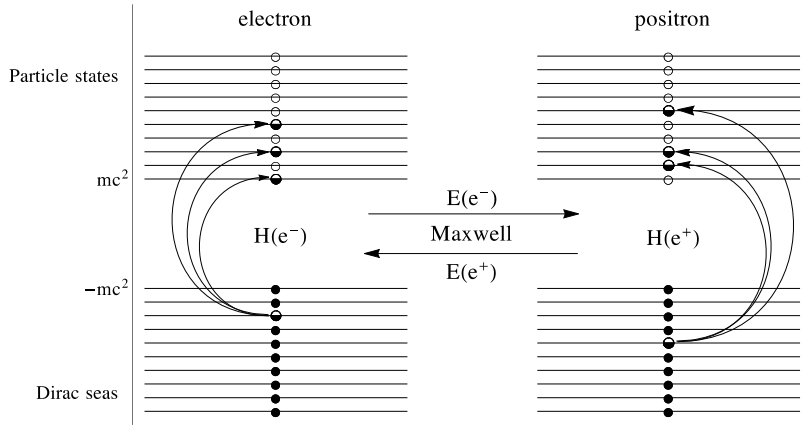

Figure 1. The two coupled Dirac seas. The left Dirac sea describes the creation process of the electrons where $H(e^-;t)$ is used for the time evolution. The coupling to the right sea is accomplished by the fact that $H(e^-;t)$ depends on the instantaneous electric field $E(e^+; z,t)$ that was generated by the positronic (right) manifold of states.

It is important to note that while the total charge and current densities can be obtained unambigously from quantum field theory even in the supercritical force zone, the electronic and positronic portions of them (Eq. 4, Eq. 5) can only be determined approximately. These partitioned

densities are based on the projection of the time-evolved states |d;p⟩ on the force-free eigen states ⟨u;p'|, which are hard to interpret inside the supercritical interaction zone. This well-known conceptual challenge is directly related to the difficulty to uniquely distinguish between electrons and positrons in those spatial regions where the energies are multiply degenerate.

We should point out that the proposed doubling of the Hilbert space per se is not an approximation to the field theoretical dynamics. For the traditional special case where the time-evolution of the internal electric fields is neglected, the two Hilbert spaces are decoupled from each other and mathematically identical, as they can be easily transformed into each other by the charge conjugation operator as we showed in the appendix. In fact, in our opinion this alternative approach can even be more intuitive as it puts the electrons and positrons on an equal footing. The electronic part of the dynamics is being evolved under the Dirac Hamiltonian $H(e^-)$ in which a negative charge is coupled to the external field while the positrons' time evolution is determined by the corresponding Hamiltonian $H(e^+)$ for a positive charge. In contrast, in the traditional (but mathematically equivalent) description the dynamics of the electron-positron field operator is governed by a single Hamiltonian, in which the coupling to the fields corresponds to that of a negative charge. Similarly, as none of the anticommutator relationships of the electronic or positronic operators are affected, the Hilbert space doubling does not affect the indistinguishability of the electrons or positrons. The mathematical equivalence of both approaches is even maintained if internal electric fields are involved whose time evolution is governed by the set of Maxwell equations.

Finally, we should also point out that the predictions of the space-time approach in the absence of interfermionic forces were compared with more established analytical approaches. In our case a direct comparison with the best known case of Schwinger's infinitely extended electric field is possible but rather non-trivial as it predicts non-vanishing pair creation rates for infinitesimally small fields, while in any finite geometry the associated potential has to exceed a threshold value. In several of our prior works, the suggested quantitative comparison of the Schwinger rate with numerical data was done and led to a good agreement [25-29].

Dynamical treatment of the internal fields

The potentials A(z,t) and V(z,t) in Eq. (1) can be written as a sum of external and internal potentials. The external potential represents the given supercritical force field that is responsible for the initial pair-creation process. Similar to many other studies [30-32], we model it here by two Sauter potentials [33], $V_{ext}(z) = V_0\{\tanh[(z+D/2)/W] + \tanh[(D/2-z)/W]\}/2$, with an amplitude V_0, a spatial extension D and spatial width W. It satisfies the required periodic boundary conditions and permits us to explore two pair creation zones centered at $z=\pm D/2$, where the external force is largest. From now on we will focus entirely on the left interaction zone and shift the coordinate axis by D/2, such that the force is maximum at z=0. In order to minimize the population burst due to a too abrupt turn-on, during the early time interval $0<t<T_{on} = 7.5\times10^{-4}$ a.u. = 0.1/c we have increased the amplitude smoothly to $V_0=3c^2$ with the function $\sin^2[\pi t/(2T_{on})]$.

In the two subsections below we outline two proposals to model the interaction between the electrons and positrons via the internal electric field. These two approaches differ by the way the electric field is determined from the charge and current densities in the Maxwell equation. They therefore predict different space-time structures of the internal fields that are then coupled back to the corresponding Dirac equations.

Coulomb fields for extended charge densities $\rho(r,t) = \rho(z,t)$

The internal parts of the potentials are generated by the particles and facilitate their mutual interaction. Their dynamical evolution can be modelled by the one-dimensional Maxwell equations for the electric field $\partial_z E = 4\pi\,\rho$ and $\partial_t E = -4\pi\,j$, where $\rho(z,t)$ and $j(z,t)$ are the charge and current densities, which fulfill automatically the continuity equation $\partial_t\,\rho + \partial_z\,j = 0$. If we introduce the potentials V_{int} and A_{int} such that $E_{int} = -\partial_z V_{int} - c^{-1}\partial_t A_{int}$, the two Maxwell equations read $c^{-1}\partial_z\partial_t A_{int} + \partial_z^{\,2} V_{int} = -4\pi\,\rho$ and $c^{-1}\partial_t^{\,2} A_{int} + \partial_z\partial_t V_{int} = 4\pi\,j$. Using the Lorenz gauge, $\partial_z A_{int} = -c^{-1}\,\partial_t V_{int}$, we obtain the two decoupled equations for the potentials

$$(c^{-2}\,\partial_t^{\,2} - \partial_z^{\,2})\,V_{int} = 4\pi\,\rho(z,t) \qquad (6)$$

$$(c^{-2}\partial_t^{\,2} - \partial_z^{\,2})\,A_{int} = 4\pi c^{-1}\,j(z,t) \qquad (7)$$

The proposal to model the electric field in one spatial dimension by the effective equation $\partial_z E(z,t) = 4\pi\,\rho(z,t)$ leads to some rather peculiar (but well-known) features. For example, the steady state electric field associated with a one-dimensional point charge $\rho(z,t) = q\,\delta(z)$ predicts a spatially constant field $E(z) = 2\pi\,q\,z/|z|$. However, this spatially non-decaying electric field is expected from the perspective of the analogeous three-dimensional situation, as our equation $\partial_z E(z,t) = 4\pi\,\rho(z,t)$ is formally identical to the corresponding 3d Maxwell equation for the z-component of the electric field vector $\mathbf{E}(\mathbf{r},t)$, given by $\nabla\bullet\mathbf{E}(\mathbf{r},t) = 4\pi\,\rho(\mathbf{r},t)$, where the 3d charge density varies only along the z-direction, $\rho(\mathbf{r},t) = \rho(z,t)$. For example, $\rho(\mathbf{r},t) = q\,\delta(z)$ is the 3d charge density of a charge sheet at z=0 that is infinitely extended along the x- and y-direction and indeed predicts an electric field that is spatially constant. As we will examine below, in the context of pair creation this leads to the (potentially undesirable) fact that the (infinite-range) fields of those electrons and positrons that were created long ago (and have therefore left the pair creation zone) can still affect the pair creation process.

Coulomb fields for localized charge densities $\rho(r,t) = \delta(x)\,\delta(y)\,\rho(z,t)$

In order to describe electric fields generated by a one-dimensional point charge $\rho(r,t) = q\,\delta(z)$ that can actually decay spatially, we could try to correct the very structure of the set of the 1d Maxwell equations itself. Alternatively, we could modify the functional form of the effective charge density, in other words, we could try to construct an effective 1d charge density, denoted by $\rho_{eff}(z)$, such that our 1d-equation $\partial_z E(z) = 4\pi\,\rho_{eff}(z)$ predicts an electric field that is identical to the z-component of the electric field vector $\mathbf{E}$, given by $\nabla\bullet\mathbf{E}(\mathbf{r}) = 4\pi\,\rho(\mathbf{r})$, with $\rho(\mathbf{r}) = \delta(x)\,\delta(y)\,\rho(z)$. Here the position vector $\mathbf{r}=(x,y,z)$ would be constrained to the cylindrical surface $x^2+y^2 = a^2$ with arbitrary radius a.

The usual Greens function solution $E_z(\mathbf{r}) = 4\pi \int d^3r'\ \delta(x')\delta(y')\rho(z')\ (z-z')/|\mathbf{r}-\mathbf{r}'|^3$, when evaluated at the surface $x^2+y^2 = a^2$ simplifies to $E_z(z) = 4\pi \int dz'\ \rho(z')\ (z-z')/[a^2+(z-z')^2]^{3/2}$. If we then take the spatial derivative, we obtain $\partial_z E(z) = 4\pi\ \rho_{eff}(z)$, where the effective charge density is given by $\rho_{eff}(z) = \int dz'\ F(z'-z)\ \rho(z')$ with $F(z) \equiv [a^2-2z^2] / [a^2+z^2]^{5/2}$.

To take a brief illustrative example of a positive 3d point-charge q located at $\mathbf{r}$=0, we would obtain $\rho_{eff}(z) = q\ [a^2-2z^2] / [a^2+z^2]^{5/2}$ and the resulting 1d Maxwell equation $\partial_z E(z) = 4\pi\ \rho_{eff}(z)$ would have the Coulomb solution $E(z) = 4\pi\ q\ z / [a^2+z^2]^{3/2}$. Contrary to the associated localized 3d density, the effective 1d charge density $\rho_{eff}(z)$ is extended and corresponds to a smeared out quadrupole distribution with two negative minima at $z=\pm a(3/2)^{1/2}$ and a maximum at z=0. While the total effective charge vanishes for any a as $\int dz\ [a^2-2z^2] / [a^2+z^2]^{5/2} = 0$, the effective charge between $z= \pm a(3/2)^{1/2}$ amounts to $4(3/5)^{1/2}/5$ q, being close to 0.62 q. In the interesting limit of $a\rightarrow 0$ where $E(z) = 4\pi\ q\ z / z^3$ reproduces the Coulomb field of the point charge, the effective charge density approaches $\rho_{eff}(z) = -\ 2\ q/z^3$ for $z\neq 0$ and $\rho_{eff}(z=0) = q/a^3$.

Following a similar approach for the current density j(z',t), we obtain the set of Maxwell equations for the 3d Coulomb internal fields given by

$$(c^{-2}\ \partial_t^2\ -\ \partial_z^2)\ V_{int} =\ 4\pi \int dz'\ F(z'-z)\ \rho(z',t) \qquad (8)$$

$$(c^{-2}\partial_t^2\ -\ \partial_z^2)\ A_{int} =\ 4\pi c^{-1} \int dz'\ F(z'-z)\ j(z',t) \qquad (9)$$

We note that the required continuity equation is also fully consistent with Eqs. (8), as $d/dt\ [\int dz'\ F(z'-z)\ \rho(z',t)] + d/dz\ [\int dz'\ F(z'-z)\ j(z',t)] = 0$. We should mention that this approach to construct effective charge densities for one-dimensional Maxwell theories to predict Coloumbic fields is actually known, though in a very different context. For example, if we solve Eq. 8 and Eq. 9 for the point charge density $\rho(z,t) = Z\ q\ \delta(z)$, we obtain $V = 4\pi\ Zq/(a^2+z^2)^{1/2}$, which is precisely the well-known Eberly potential that has been developed in the late eighties to model the strong-field multi-photon ionization process in one-dimensional hydrogenic atoms with nuclear charge Z [34-37]. Here a was interpreted as the screening parameter and the long-range Coulombic scaling

$V(z) \sim -1/|z|$ was essential to predict an infinite series of Ryberg levels from a 1d model atom.

Removal of the fermionic self-coupling

In order to be able to identify the impact of various inter-particle forces on the space-time evolution of the pair creation process, we can separate two source terms in the Maxwell equations according to their individual contributions. The total charge density $\rho(z,t) = \rho(e^{-};z,t) + \rho(e^{+};z,t)$ would generate a total internal electric field. In a prior work on the time evolution of a quantum mechanical wave packet [22] self-repulsion was predicted when the particle was coupled to the total electric field. This undesired mechanism leads to a very strong but unphysical enhancement in the spatial spreading behavior of the wave packet. It is caused by the fact that part of the total electric field that is coupled to the electron contains unavoidably portions that are generated by the same electron. As a result the electron would be able to respond to its own electric field.

In order to eliminate this unphysical self-coupling mechanism, a recent work [23] proposed a theoretical framework based on a partitioning of the total internal electric field E(z,t) into two portions $E(e^{-};z,t)$ and $E(e^{+};z,t)$, where each of which is generated by only one particle and is evolved independently of the other. For example, the Maxwell equation for the electronic field $E(e^{-};z,t)$ has only the electronic charge density $\rho(e^{-};z,t)$ as a source term and only this field is coupled to the positron's Dirac equation. By controlling which source terms contribute to the electronic field (that is coupled to the positron), we can now monitor and control the effects of the electron-electron and positron-positron repulsion as well as the relevant electron-positron attraction on the pair creation process.

For better computational analysis, we have included a dimensionless factor κ^2 to the coupling of the internal fields in the two Dirac equations (for e^{-} and e^{+}) to enhance the effect due to the inter-fermionic forces. We found that κ=300 was sufficiently large to permit us to differentiate between the mechanism due to the external and internal fields. The second way to treat the internals fields via Eqs. (8) & (9) introduces the screening parameter a. We have set this to $a=1.5\times10^{-3}$ a.u. We remark that assigning a value to the charge within a one-dimensional system is non-trivial and

in some sense even arbitrary fundamental point of view. For more details on this problem within a quantum electrodynamical context, see Ref. [38].

The computational simulations were obtained on a space-time lattice. While the Dirac equation for the states was solved with a FFT-based split-operator technique [39-43], we used a finite-difference algorithm to solve the Maxwell equations for the fields. The algorithmic details can be found in a prior work [44]. With regard to the physical parameters, we mainly used $V_0=3c^2$, W=0.5/c, D=L/2 for the external potential. For a total interaction time of $T=2\times10^{-3}$ a.u., a spatial grid of total length L=1 a.u. with 1024 grid points was sufficient such that a particle (or field) evolving with c would not reach the edges of our numerical box. We ran the simulations on 64 processors of a 2.6 GHz Del PowerEdge R815. To obtain numerically fully converged data a typical simulation takes about 2 hours.

The pair-creation process

The number of created electron-positron pairs as a function of time

Having computational access to the electronic and positronic charge density permits us also to examine the temporal growth of the total number of created pairs,

$$N(t) \equiv \int dz\, \rho(e^+;z,t) \equiv \Sigma_{p'}\, \Sigma_p \left| \langle u;p' | \, U(e^+,t) \, |d;p\rangle \right|^2 \qquad (10)$$

As this definition is based on the partitioned charge density, it only reflects the true particle number if the external fields were turned off abruptly. In other words, N(t) contains the modification of yield due to those particles that were created or annihilated due to the temporal turn off of the field [45].

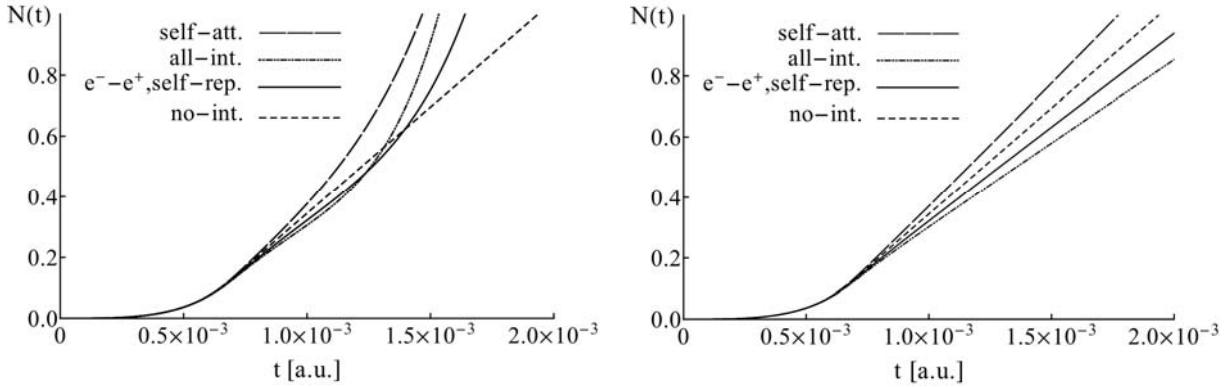

Figure 2 Left panel: 1d Coulomb field; Right panel: 3d Coulomb field. The effect of various interfermionic forces on the temporal growth of the total number of created particles N(t). The graphs are labeled

$N_{self\text{-}att}$: only the e^--e^- and e^+-e^+ interactions are coupled as attractive forces

$N_{all\text{-}int}$: both particles are coupled to the total electric field

$N_{e^-e^+}$: including the attractive e^- - e^+ force

$N_{self\text{-}rep}$: only the e^--e^- and e^+-e^+ repulsion is included

$N_{no\text{-}int}$: for comparison, no inter-fermionic forces

[L=1 a.u., 1024 spatial grid points, 8000 temporal grid points]

In Figure 2 we display the temporal growth of N(t). As a main reference line, we have shown the yield $N_{no\text{-}int}(t)$ for the simplest case without any interfermionic forces. After an initial creation burst of particles associated with the turn-on of the supercritical potential, the yield evolves into the linear growth regime, that can be characterized by the usual (Schwinger-like) rate $\Gamma_{no\text{-}int}$. In addition to determining $\Gamma_{no\text{-}int}$ directly from the slope of the $N_{no\text{-}int}(t)$ there are other approaches available to find this rate, one of which we review and generalize in Sec. 4.

The graph labeled $N_{e^-e^+}(t)$ is the most important one as it includes the attractive forces between the electron and positron, while the above partitioning scheme has permitted us to eliminate the unphysical self-repulsion. The (expected) agreement with $N_{no\text{-}int}(t)$ for very early times suggests that due to the limited particle growth during the turn on of the external field, the attractive field is too small to affect the pair creation process. At later times, however, we observe that the yield is less than $N_{no\text{-}int}(t)$, suggesting that the attractive interfermionic force decreases the

particle yield.

This suppression can be interpreted in terms of a simple picture. Due to the attraction the amount of particles that can leave the supercritical interaction zone is reduced, as a result more particles stay in this region of space and either Pauli-block [32] the further creation of pairs or simply annihilate. Both mechanisms would reduce the pair creation rate.

In order to test our interpretation for this spatial mechanism from a different perspective, we have computed the yield $N_{self\text{-}rep}(t)$ for an (unphysical) case, where the physical e^- - e^+ attraction has been excluded, but the repulsive e^-- e^- and e^+ - e^+ self-interactions were included. Here the repulsive electric force field generated by the portion of the electronic charge cloud (that was created at earlier times and has left the supercritical zone) would keep the newer created electrons from escaping the interaction zone as well. As this is a rather similar suppression mechanism as the one postulated for the e^- - e^+ attraction, we would predict that $N_{self\text{-}rep}(t)$ and $N_{e^-\text{-}e^+}(t)$ should reveal a similar growth behavior. The almost perfect match of $N_{self\text{-}rep}(t)$ and $N_{e^-\text{-}e^+}(t)$ displayed in the figure supports this interpretation.

To be completely consistent with our simplified explanation, we have also simulated the dynamics for another (unphysical) scenario where the (electronically generated) electric field was coupled back to the electron, but the effective charges in the Dirac equation were reversed such that the resulting effective force was attractive. Here two consecutively created portions of the electron cloud would attract each other, effectively pulling the later created electron out of the creation zone, thus predicting an increase of the yield [compared to $N_{no\text{-}int}(t)$]. The observed increase of the actual yield (denoted by $N_{self\text{-}att}(t)$ in the figure) relative to $N_{no\text{-}int}(t)$ confirms again that our simple spatial picture is qualitatively reasonable.

To make a connection with prior works [6,7,21,44] that have attempted to study the electron-positron attraction by coupling both particles to the total electric field, we include the yields due to this approach in the figure by the curve labeled $N_{all\text{-}int}(t)$. Following our spatial picture, we would expect that in this case the pair-creation yield should be suppressed due to the simultaneous action of the attractive e^- - e^+ forces as well as the repulsive e^- - e^- and e^+-e^+

self-interactions. As the graph for $N_{all\text{-}int}(t)$ is clearly below the ones for $N_{e^-\text{-}e^+}(t)$ and $N_{self\text{-}rep}(t)$, also this expectation is confirmed.

We believe that these mechanisms should be general and also representative of a three dimensional simulation. However, for longer times the growth behavior of $N_{e^-\text{-}e^+}$, $N_{self\text{-}rep}$, $N_{self\text{-}att}$ and $N_{all\text{-}int}$ is likely to be generic to quantum electrodynamics in only one-spatial dimension using the 1d electric fields as force-intermediators. The data show that all four yields begin to grow exponentially after a characteristic time. In our opinion, this long-time behavior might be due to the fact that here the internal electric field can propagate to both sides of the spatial axis without loosing its amplitude. For example, the one-dimensional Maxwell equations (Sect. 2.2) predict that the stationary electric field E(z) around a point charge q (located at z=0) is constant, $E(z) = 2\pi\, q\, z/|z|$. This means that any one-dimensional charge q would create an electric field, for which the potential difference between two points z=0 and z, given by $V(z) \equiv -\int_0^z E(z')dz' = -2\pi\, q\, z$ would grow ad infinitum with increasing z. In other words, qV(z) could exceed the threshold value $2c^2$ to become supercritical. To have a very rough estimate of the involved time scale for this to occur, we can assume that the potential energy [of a unit charge (positron)] that propagates away from a charge q (created at t=0 at z=0) would become supercritical after a time t_{sc}, defined as $2\pi\, q\, ct_{sc} = 2c^2$. As in our simulations the effective charge that couples to the internal field was multiplied by $\kappa^2=300^2$, this time scale would amount to $t_{sc} = c/(\pi\, \kappa^2\, q)$, which is about 3.2×10^{-3} a.u. for our charge q = N(t=7.5×10^{-4} a.u.) = 0.15 created after the completion of the turn on. As this time estimate for t_{sc} exceeds the time when the graphs begin to grow, we suspect that additional mechanisms might be responsible for this growth, for instance, the time-dependence of the internal field can also trigger pair creation.

Figure 2b shows the identical data, however, here the internal electric fields were obtained from the Maxwell equations (2.5) that lead to 3d Coulombic fields that decay spatially like $\sim |z|^{-2}$. The qualitative agreement with the data from Fig. 2a for shorter times shows the universality of the results with regard to the spatial structure of the internal fields. Furthermore, it seems that in this case the fields of the particles that have escaped from the interaction zone have only a minimal

impact on the pair creation process and as a result an actual steady state can develop, similarly to the one without any interfermionic interaction, just with a different pair creation rate.

The spatial distribution of the created electron-positron pairs

We will now re-examine the dynamics from a spatially resolved perspective. This will permit us to rather directly evaluate the dynamical picture based on the repulsive and attractive forces as proposed in the prior section.

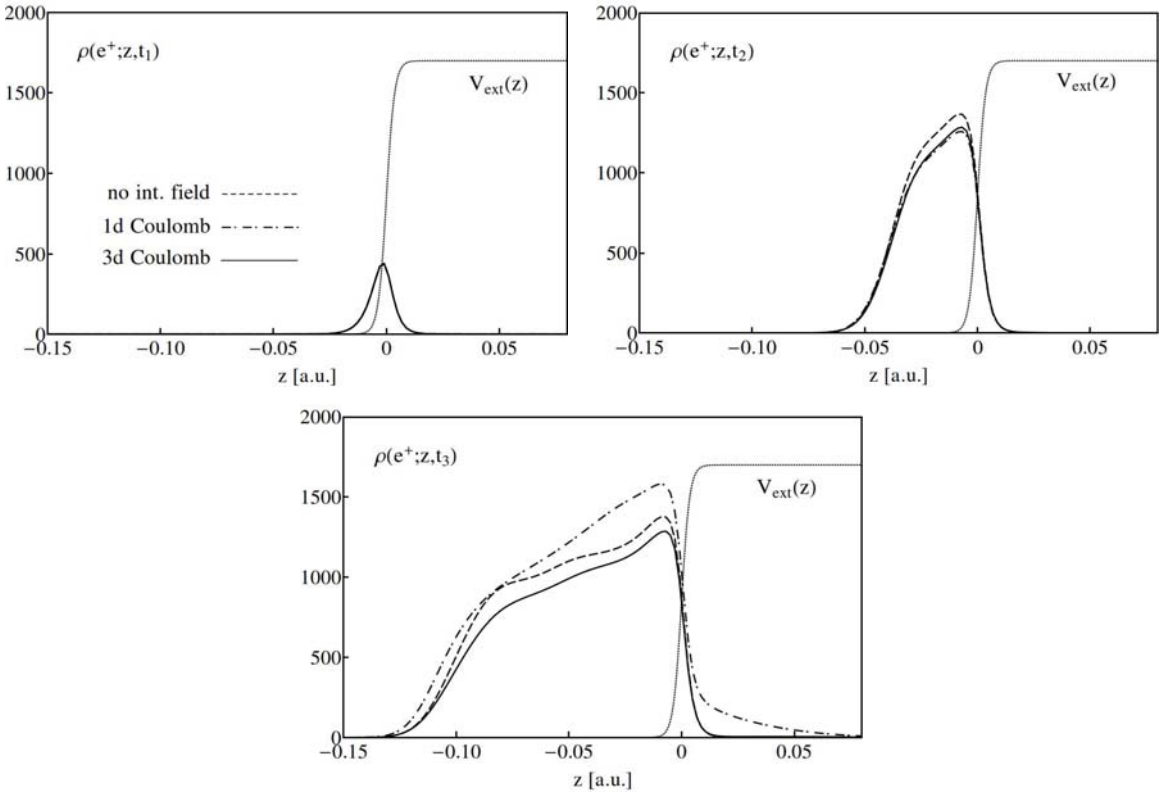

Figure 3. Temporal snapshots of the positron's charge density $\rho(e^{+};z,t)$ at three times $t_1 = 5\times10^{-4}$, $t_2 = 1\times10^{-3}$ and $t_3 = 1.6\times10^{-3}$ a.u. As a guidance for the spatial scale, the dotted line shows the external potential. [same parameters as in Fig. 2]

In Figure 3 we present spatial snapshots of the positron's charge density at three moments in time. There are three possible mechanisms that are responsible for the creation of particles for time before the temporal ramp up of the external potential is completed. The first one is associated with the fact that due to its temporal dependence, even a subcritical field can produce particles. The

second mechanism is unphysical and associated with the fact that due the projection of our states onto force-free states inside the interaction zone we have spurious contributions to the yield [46] that describe the annihilation or creation of particles only if the field was turned off [45]. The third contribution is due to the supercriticality. In our case this mechanism set in after time $t= 3.5\times10^{-4}$ a.u.. For $t= 5\times10^{-4}$ a.u. (directly before the turn on), the three densities are similar and confirm that for short times there were not too many particles created to make a sizable internal field.

For longer times (Figure 3b) we clearly see that these forces bind the particles inside the interaction zone. However, the two types of interactions do not show any major differences.

The (possibly) most interesting regime is the long-time regime (Fig. 3c), where the force-free as well as the Coulombic interaction reveals the formation of a steady state as the densities close to z=0 become independent of time. It is also characterized by the beginning of the development of a constant plateau $\rho(e^+;z,t)$ just outside the interaction zone. The growing pair-production rate for the 1d Coulomb fields clearly manifest itself not only in the increasing density inside the interaction zone but also outside. This supports our conjecture from above that the internal field might be sufficiently strong to produce particles even outside the region where the external force is located. Apparently the 1d internal attractive fields are even responsible for the occurrence of positrons for 0<z, a region that is usually only accessible to electrons.

The energy distribution of the created particles

In this section we will examine the effect of the 1d and 3d Coulombic interactions on the energy spectrum of the emitted particles. It is calculated from the time evolved states $|d;p\rangle$ as

$$S(e^-;E,t) \equiv \Sigma_p \, |\langle u;p'| \, U(e^-,t) \, |d;p\rangle|^2 \qquad (11)$$

$$S(e^+;E,t) \equiv \Sigma_p \, |\langle u;p'| \, U(e^+,t) \, |d;p\rangle|^2 \qquad (12)$$

where the energy is given by $E \equiv [m^2c^4+c^2p'^2]^{1/2}$. The two spectra are identical due to the symmetry of the total interaction energies $e^+ V(z) = - e^- V(-z)$.

As we have seen in Figure 2, there are several time scales involved that govern the dynamics, as a result, this is also reflected in the energy spectra. In Figures 4 we show the spectra at an earlier and a later time. For comparison, we accompany the spectra for the 1d and 3d Coulomb interactions again with the data for the dynamics where the interfermionic forces were omitted. During the very early times of the turn-on process (where the potential is not yet supercritical) the spectra (not shown) decrease monotonically with E as the particles are mainly created due to the temporal variation of the field. In addition, there is also a small amount of unphysical contribution to the spectrum associated with the projection on the force-free states inside the interaction zone.

In Figure 4a we display the spectrum at time $t = 10^{-3}$ a.u., corresponding to an accumulated population $N_{no\text{-}int}(t) = 0.345$. For reasons of consistency the particle yield is equal to the energy integral $\int dE\, S(e^-;E,t) = N(t)$. For the 1d internal Coulomb field the number of low-energy particles ($E<1.5\ c^2$) is less than the one for the case without any interaction. This could be related to the fact the electrons and positrons attract and partly bind each other, which slows down the escape velocity. For higher velocities, however, the interaction seems to have only a small impact on the particles. On the contrary, the 3d Coulomb field seems to suppress the particles for higher energy.

The reference line (no internal field) for the longer time spectra (Figure 4b) shows the formation of the semicircular-shaped distribution around the maximum energy $E_{max}= V_0/2$. We note that this shape is related to the energy dependence of the quantum mechanical transmission coefficient [32], see also Sec. 4. While slightly more oscillatory, the spectrum for the 3d Coulomb field is similar, except the overall shift to lower energies. This is consistent with the attraction between the positron and the electron and the lowering of the effective total potential, as we will discuss in more detail in Sec. 3.4. The data for the 1d interactions indicate a more complicated behavior. They suggest not only a spectral shift, but a redistribution of the energies. For lower energies the same mechanisms as in 3d increases the particle number. Then for $1.1c^2<E<1.7c^2$ the yield is decreased substantially, while the higher energy particles are increased due to the wide spatial range of the 1d forces. It seems to indicate that for 1d field the total potential is wider and therefore causing the particles to pick up higher speeds as they accelerate in a wider potential and

therefore less particles can stay in the medium energy range.

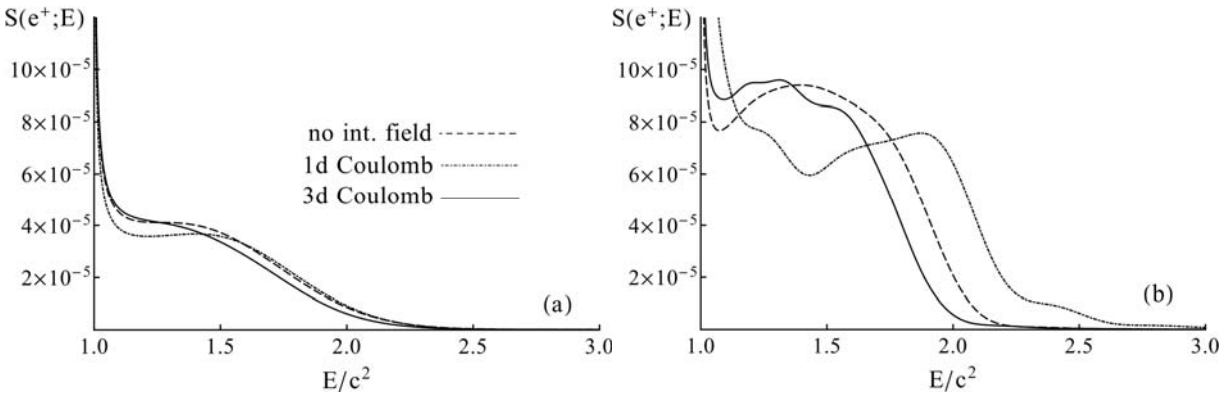

Figure 4 Energy distribution of emitted particles $S(e^-;E,t)$ for the same times (a) $t_2=1\times10^{-3}$ and (b) $t_3= 1.6\times10^{-3}$ a.u. as in Fig. 3. [same parameters as in Fig. 2]

The dynamics of the internal electric field

In Sec. 3.2 we have argued that the increase and decrease of the pair creation yield could be understood in terms of the attractive and repulsive fields generated by the *earlier created* charges. In this section we will show that similar conclusions can also be obtained from a purely energetic point of view that does not assume the prior existence of particles.

To do so we have to re-examine the dynamics from the perspective of the space-time evolution of the internal electric field created by the electron, denoted by $E(e^-;z,t)$. In the physically most important case (where the e^-- e^- and e^+ - e^+ self-repulsion forces were removed), this electric field, together with the external electric field, $E_{ext}(z) = – dV(z)/dz$ governs the pair creation process as well as the continued time evolution of the positrons after their creation. As the coupling to the positrons were amplified by κ^2, the effective electric field experienced by the positrons is $E(z,t) \equiv \kappa^2E(e^-,z,t) – dV(z)/dz$. For reasons of symmetry, the effective field experienced by the electrons is given by $\kappa^2E(e^+,z,t) + dV(z)/dz$.

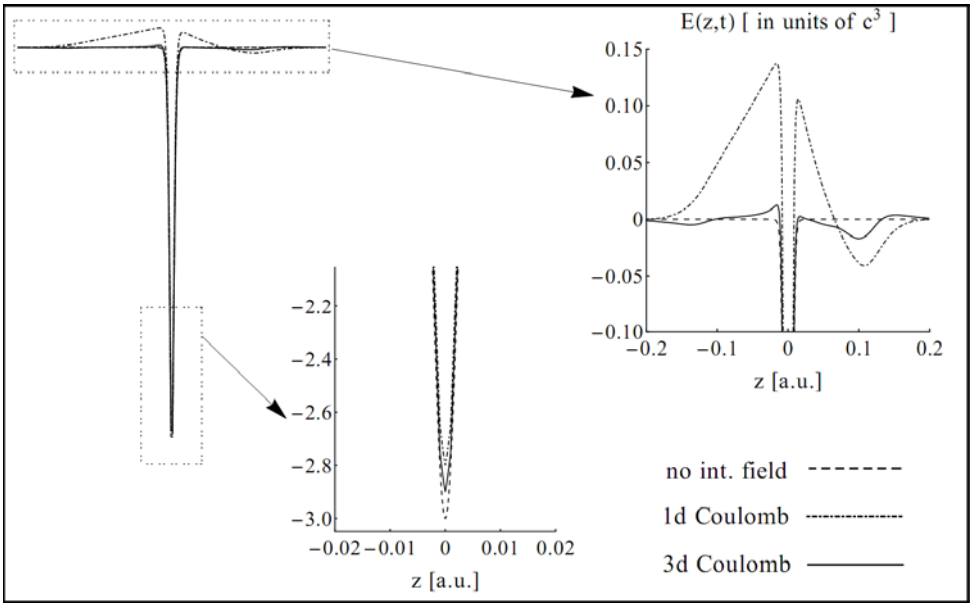

Figure 5. Snapshot of the total effective field E(z,t) experienced by the positrons at time $t_3 = 1.6\times10^{-3}$ a.u. It was generated by the Maxwell equation with the electronic charge and current density as the source terms. [same parameters as in Fig. 2]

In Figure 5 we display a snapshot of the effective field E(z,t) at time $t_3 = 1.6\times10^{-3}$ a.u. At early times (when the electron probability N(t) is small) the internal field is negligible and $E(z,t) \approx -dV(z)/dz$. Once the electron is born close to the interaction zone, its associated electric field $E(e^-;z,t)$ begins to propagate with the speed of light in both directions. It is positive (negative) to the left (right) side of the charge. As we showed in Sec. 3.2 the created electron is pushed by $V_{ext}(z)$ to the right. The total internal electric field is a linear sum of the (propagating) fields associated with each portion of the growing (and right moving) electron cloud. As a result, the location where the electric field changes its sign also moves to the right. This region might be close to the center of gravity of the charge cloud where the total force on any probe charge would vanish as it would be surrounded by the same number of particles on each side. More importantly, as the electrons are pushed to the right side, the internal electric field in the interaction zone itself grows. As a result, the magnitude of the effective force field that is coupled to positronic vaccum $|\kappa^2 E(e^-,z,t) - dV(z)/dz|$ decreases, as the individual fields $E(e^-,z,t)$ [positive] and $-dV(z)/dz$ [negative] have opposite signs. It is therefore intuitive to expect that the presence of the attractive $e^- - e^+$ force lowers the amplitude of the supercritical barrier, consistent with our prior findings. Similarly the

forces associated with the e^- - e^- and e^+ - e^+ self-repulsion would also lower the supercritical barrier.

As expected, the 1d Coulomb field is much wider than the corresponding spatially decaying 3d field. This wider field can create particles outside the interaction zone. It is interesting to note that in this domain the internal field dominates the creation process. In fact, the internal field becomes so large at some locations, that it effectively reverses the sign of the force field. In these regions the external field therefore reduces the yield.

Instantaneous potential theory for the rate

As the numerical simulations are converged with regard to the computational parameters such as the temporal and spatial grid spacings and the spatial box size, they represent exact solutions to the coupled Dirac-Maxwell equations. The price to pay for this advantage is an enourmous computational effort with regard to the required computer memory and cpu time. In this section we propose a model based approach to incorporate inter-fermionic forces into the pair creation yield. It is based on the early pioneering work by Hund [47], who speculated for the special case of no internal fields that the long-time limit of the pair creation rate Γ from the vacuum can be obtained from the (energy dependent) quantum mechanical transmission coefficient T(E) for an incoming electron that scatters off of the supercritical potential. In a prior work it was shown that this well-known relationship between the quantum mechanical single-particle scattering and pair creation approach remains accurate even if additional magnetic fields [48] are present or bosons [49] are described. The pair-creation rate is obtained from

$$\Gamma = (2\,\pi)^{-1} \int dE\; T(E) \qquad (13)$$

We will now examine, if this rather straighforward approach is even suitable to approximate the true rate dN(t)/dt for the more complicated dynamics that include the inter-fermionic forces. This, of course, requires several approximations. First, we assume that the required effective scattering potential is given by the sum of the external and the time-dependent internal vector and

scalar potential. Second, while Eq. (13) gives us only the (long-time) rate for a steady state for a time-independent potential, we interpret it here as a time-dependent instantaneous rate $\Gamma(t) \approx dN/dt$. We have used the quantum transmitting boundary method [28,50], which permits us to calculate numerically the energy dependence of the transmission coefficient T(E) for arbitrary scattering potentials.

In Figure 6 we present a comparison of the true instantaneous rate obtained from the numerical derivative of the exact yield dN/dt and the time-dependent rate Γ, calculated from the energy integral over the transmission coefficient T(E). This comparison is done for the three cases of no interfermionic interaction, the 1d and 3d Coulombic interactions. As expected, we see that the method is obviously not reliable to predict the true instantaneous rate for the early turn-on time period, where the particles are mainly created due to the time-dependence of the external field. Obviously, as the transmission coefficient T(E) vanishes identically when the potential is subcritical [$|V(z=-\infty) - V(\infty)| < 2c^2$], the rate Γ is zero here. However, once the potential exceeds this threshold value, Γ becomes accurate.

It is interesting to observe that this approach describes also the impact of the 1d and 3d inter-fermionic forces accurately for times up to $t=10^{-3}$a.u.. In fact, the steady state pair production rate for the 3d Coulomb field is accurate for all times. However, the long-time growth characteristic of the 1d interaction cannot be predicted. In this case, we have shown above the particle number does not grow linearly in time and does not evolve into a steady state as for the 3d case. We believe that this failure to predict dN/dt could also be related to the fact that the rate formula Eq. (13) above is intrisically asymptotic and assumes a time independent field.

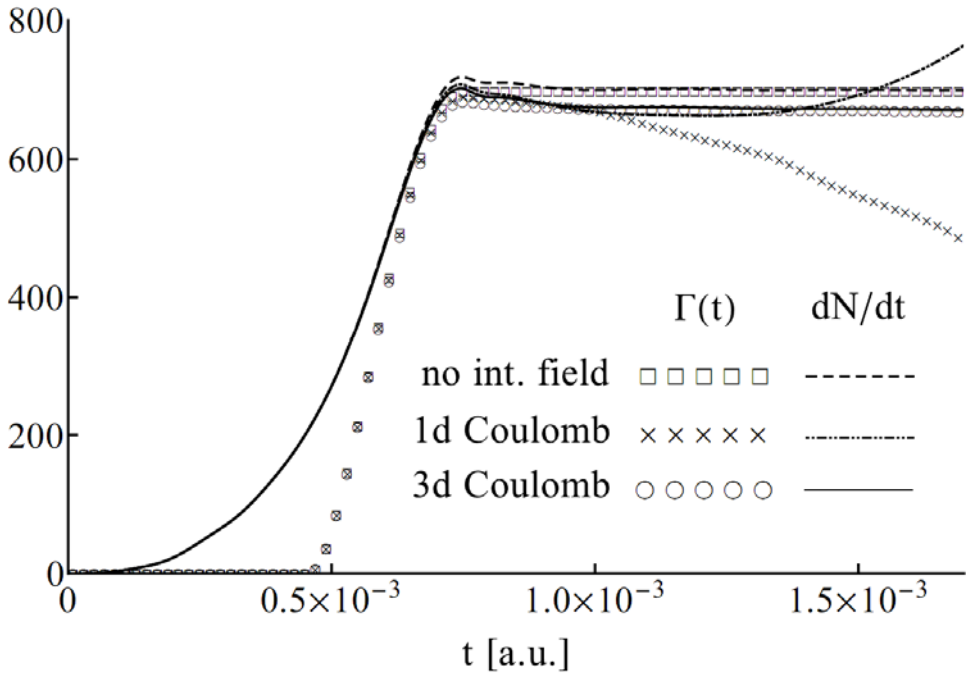

Figure 6. The discrete markers are the predicted rate Γ(t) from the instantaneous potential model according to the Eq. (13). For comparison, the curves are the exact rate obtained from the derivative dN/dt.

In addition to predicting the instantaneous pair creation rate, this approach permits us also to predict the distribution of the kinetic energy of the created particles, which according to Eq. (13) should be directly proportional to the transmission coefficient T(E). The trends indicated in the spectra of Figure 4 are also qualitatively reproduced by T(E) for both the 1d and 3d Coulombic interfermionic forces.

Summary and discussion of future challenges

In this work we have examined the impact of the electron-positron interaction on the pair creation process. In contrast to previous works where this interaction was modelled by a single total electric field, we have proposed here a new description, that is based on the partitioning of the fermionic Hilbert space into electronic and positronic states (including the negative energy continua) as well as a doubling of the Maxwell equations. This allowed us to eliminate the unphysical self-interaction that arises if the internal force is described as a classical field.

Furthermore we have compared two different Maxwell theories to model the electric fields and proposed an instantaneous-potential approach to predict the particle yield.

There are three main new insights from this work. First, and in possible contradiction to other studies [8], we found that the interaction reduces the pair-creation yield. Second, the (unphysical) effect of the self-repulsion by itself would also lower the pair creation yield, comparable to the reduction due to the physical electron-positron attraction. This finding seems to be rather universal and independent of the spatial structure of the electric field as it was predicted for 1d as well as 3d Coulombic internal fields. Third, this reduction is accompanied by an increase of low-energy particles while the number of particles emitted with larger energy is decreased. This trend is also in agreement with the predictions of the instantaneous potential approach.

In order to serve as an ultimate guidance to future laboratory experiments where the supercritical field configuration might be supplied by a highly focused laser pulse, there are unfortunately still conceptual challenges that must be addressed. In this work we have used a one-dimensional model system to obtain some first insight into the dynamical impact of the Coulombic forces between the created particles and we believe that the main finding and their qualitative explanations for the decreased pair-creation yield should generalize also to a three-dimensional system.

As our interaction was restricted to one spatial dimension, we were not able to incorporate any effect on the inter-particle interactions due to magnetic fields. Unfortunately, including realistic three-dimensional effects such as inter-particle magnetic field forces into an ab initio quantum field theoretical simulation is presently far beyond our present computational capabilities. While some 3D effects such as the coupling to purely *external* fields (such as external magnetic fields) might be possible one day within our approach, describing true inter-particle interactions by coupling the fermionic dynamics back to the complete set of the Maxwell equations with retardation is extremely difficult for us. Our approach is qualitative and mainly aimed at understanding fundamental principles, as a result it cannot necessarily lead to very accurate and quantitative predictions.

We should point out that even though this work proposed a theoretical framework (based on two coupled Dirac seas as well as two sets of Maxwell equations) that allows us for the first time to exclude the unphysical repulsive forces between portions of the same particle, it also points to the very fundamental limitations of the classical field approximation. In principle, the pair creation process from the vacuum should be triggered by the total electric field, and not just by the electric field created solely by the positron (in addition to the external field). So we have a possibly two conflicting requirements where on the one hand we would like the total electric field to determine the pair creation process, but once a positron is actually created, its time-evolution should not be subjected to its own repulsive self-field. It seems to us that only a truly quantum field theoretical treament of a (second-quantized) photon would be able to resolve this fundamental dilemma. Unfortunately, at the moment, computational simulations of this more accurate theoretical framework are presently too challenging.

Most likely, the present approach moves us also to the very frontier of our present capability to study interfermionic forces within the classical field approximation. We should point out that the very foundation of the present approach relies on the separation of the fermionic field operator into an electronic and positronic portions, which was based on the projection on force-free states and where it was possible to use the energy as an unambigous criterion to identify electrons and positrons. However, in the supercritcial field region this is not reliable. One could possibly improve this by using projections on a quasi-dressed basis of eigenstates of the Hamiltonian associated with external potentials that are just slightly below supercritical. Here the existence of an (albeit small) energy gap would still permit an unambigous distinction between positronic and electronic states.

An interesting and related problem occurs for those dynamical situations where the external field is so strong, such that more than just a single electron-positron pair can be created. In contrast to the self-repulsion, in this case is the electron-electron repulsion is an actual valid physical mechanism, as it acts between two different particles. It might be interesting to examine if it is possible – even in principle – to permit two different electrons to repel and simultaneously omit the

single-electronic self repulsion.

In contrast to several other quantum field theoretical studies of electrodynamical systems, it seems that the present approach (where the external field is described by a non-second quantized classical field) does not necessarily require the necessity for either a mass or charge renormalization procedure. As we choose the physical mass and charge from the very beginning and our numerical data are non-singular and fully converged, this procedure would provide at most a finite correction to the electron's mass or its charge –if necessary at all.

We are obviously just at the very first stage of understanding the role of interfermionic interactions in the context of strong-field induced pair creation. While the effect of the vacuum polarization on modifying the usual Coulomb force law for very short distances has been studied, the relevance of this mechanism in the context of pair creation is unknown. This is certainly another interesting question as particles are being created at sufficiently small spatial scales where these vacuum corrections could play a role. In a recent work on a 1d model system [37] it was observed that the inclusion of the vacuum polarization can actually increase the usual Coulomb force between like and also between unlike charges. But in our view, this is also an open area of study.

The inclusion of the interfermionic forces might also lead effectively to an increased threshold value for the potential to become supercritical. At early times, when the probability of finding particles in the interaction region is rather small, the original threshold of $2c^2$ might still be applicable. However, in order to maintain a continued particle creation at longer times, possibly a larger external field might be necessary. This conjecture might be tested by repeating the present simulations for external potentials that are barely supercritical. One could even speculate that this dynamics close to the threshold might lead to an interesting oscillatory behavior of the pair creation probabilty and therefore cannot even be described by a single (time-independent) rate.

As a last point we should mention that while the observed reduction of the yield as the result of the attraction between different charges might possibly not be too surprising, even though it is interesting that the works by Profs. Lebedev, Ritus and Schubert predict an increase for this case.

With regard to the mentioned effect due to particles of the same charge, we should point out that our present approach unfortunately does not permit us to examine exclusively the repulsive forces between two electrons (or equivalently between two positrons). One can only speculate and obtain some very first idea about a possible impact on the yield by examining the effect of the (though unphysical) electronic self-repulsion and assuming that this mechanism also includes somehow the true repulsion between two different electrons. But we presently do not have any means of separating the magnitudes of the purely single-electronic self-repulsion force from the physical repulsion force between two (different) electrons. Their strength ratio might also depend on the parameters. In any case, the observed reduction would suggest that the electron-electron repulsion might possibly also lead to a reduction of the rate.

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References

[1] For a recent review, see, e.g., A. Di Piazza, C. Müller, K.Z. Hatsagortsyan and C.H. Keitel, Rev. Mod. Phys. 84, 1177 (2012).

[2] For the Ultraphotonik – Polaris projects, see, http://www.physik.uni-jena.de/inst/polaris

[3] For review of the Extreme Light Infrastructure (ELI) projects, see http://www.extreme-light-infrastructure.eu/eli-home.php

[4] For review of the European X-Ray Laser Project XFEL, see http://xfel.desy.de

[5] I. Bialynicki-Birula and Z. Bialynickia-Birula, "Quantum electrodynamics" (Pergamon Press, Oxford, 1975).

[6] F. Hebenstreit, J. Berges and D. Gelfand, Phys. Rev. Lett. 111, 201601 (2013).

[7] F. Hebenstreit, J. Berges and D. Gelfand, Phys. Rev. D 87, 105006 (2013).

[8] S.L. Lebedev and V.I. Ritus, Sov. Physics JETP 59, 237 (1984).

[9] I.K. Affleck, O. Alvarez and N.S. Manton, Nucl. Phys. B 187, 509 (1982).

[10] For a nice review, see G.V. Dunne, "Heisenberg-Euler Effective Lagrangians: Basics and Extensions", in "From fields to strings: Circumnavigating theoretical physics" (ed. M. Shifman, A. Vainshtein, and J. Wheater), pp. 445-522. (World Scientific, Singapore, 2005) and arXiv:hep-th/0406216

[11] See transparency pages 30-33 and 38 in the talk by Prof. C. Schubert on July 31, 2014 at KITP, Santa Barbara, online.kitp.ucsb.edu/online/lasers14/schubert/rm/qtref.html

[12] J.S. Schwinger, Phys. Rev. 82, 664 (1951).

[13] C.P. Ridgers, C.S. Brady, R. Duclous, J.G. Kirk, K. Bennett, T.D. Arber and A.R. Bell, Physics of Plasmas 20, 056701 (2013).

[14] R. Duclous, J.G. Kirk and A.R. Bell, Plasma Physics and Controlled Fusion 53, 015009 (2011).

[15] E.N. Nerush, I.Yu. Kostyukov, A.M. Fedotov, N.B. Narozhny, N.V. Elkina and H. Ruhl, Phys. Rev. Lett. 106, 035001 (2011).

[16] A.R. Bell and J.G. Kirk, Phys. Rev. Lett. 101 200403 (2008).

[17] A. M. Fedotov, N. B. Narozhny, G. Mourou, and G. Korn, Phys. Rev. Lett. 105 080402 (2010); N.V. Elkina, A. M. Fedotov, I. Yu. Kostyukov, M. V. Legkov, N. B. Narozhny, E. N. Nerush, and H. Ruhl, Phys. Rev. ST Accel. Beams 14, 054401 (2011).

[18] E.N. Nerush, I.Yu. Kostyukov, A.M. Fedotov, N.B. Narozhny, N.V. Elkina and H. Ruhl. Phys. Rev. Lett. 106, 035001 (2011); A.M. Fedotov , N.V. Elkina , E.G. Gelfer , N.B. Narozhny and H. Ruhl , Phys. Rev. A 90, 053847 (2014).

[19] T. Grismayer, M. Vranic, J.L. Martins, R. Fonseca and L. O'Silva, "Seeded QED cascades in counter propagating laser pulses", arXiv:1511.07503 (2016).

[20] M. Tamburini, A. Di Piazza, C.H. Keitel, "Laser-pulse-shape control of seeded QED cascades", arXiv:1511.03987 (2016).

[21] J.C.R. Bloch, V.A. Mizerny, A.V. Prozorkevich, C.D. Roberts, S.M. Schmidt, S.A. Smolyansky and D.V. Vinnik, Phys. Rev. D 60, 116011 (1999).

[22] Q.Z. Lv, S. Norris, Q. Su and R. Grobe, Phys. Rev. A 90, 034101 (2014).

[23] S. Norris, J. Unger, Q.Z. Lv, Q. Su and R. Grobe, Phys. Rev. A 93, 032131 (2016).

[24] B. Thaller, "The Dirac Equation" (Springer, Berlin, 1992).

[25] P. Krekora, K. Cooley, Q. Su and R. Grobe, Las. Phys. 15, 282 (2005).

[26] T. Cheng, Q. Su and R. Grobe, Euro. Phys. Lett. 86, 13001 (2009).

[27] T. Cheng, Q. Su and R. Grobe, Phys. Rev. A 80, 013410 (2009).

[28] Q.Z. Lv, A.C. Su, M. Jiang, Y.J. Li, R. Grobe and Q. Su, Phys. Rev A 87, 023416 (2013).

[29] W.Y. Wu, F. He, R. Grobe and Q. Su, J. Opt. Soc. Am B 32, 2009 (2015).

[30] B.R. Holstein, Am. J. Phys. 66, 507 (1998).

[31] B.R. Holstein, Am. J. Phys. 67, 499 (1999).

[32] P. Krekora, Q. Su and R. Grobe, Phys. Rev. Lett. 93, 043004 (2004).

[33] F. Sauter, Z. Phys. 69, 742 (1931).

[34] J. Javanainen, J.H. Eberly and Q. Su, Phys. Rev. A 38, 3430 (1988).

[35] J.H. Eberly, Phys. Rev. A 42, 5750 (1990).

[36] Q. Su and J.H. Eberly, Phys. Rev. A 44, 5997 (1991).

[37] For a historical account of the soft core potential, see Q. Su and R. Grobe, Laser Phys. 15, 1381 (2005).

[38] Q.Z. Lv, N.D. Christensen, Q. Su and R. Grobe, Phys. Rev. A 92, 052115 (2015).

[39] J.A. Fleck, J.R. Morris and M.D. Feit, Appl. Phys. 10, 129 (1976).

[40] A.D. Bandrauk and H. Shen, J. Phys. A 27, 7147 (1994).

[41] J.W. Braun, Q. Su and R. Grobe, Phys. Rev. A 59, 604 (1999).

[42] G.R. Mocken and C.H. Keitel, Comp. Phys. Commun. 178, 868 (2008).

[43] M. Ruf, H. Bauke and C.H. Keitel, J. Comp. Phys. 228, 9092 (2009).

[44] A.T. Steinacher, R. Wagner, Q. Su and R. Grobe, Phys. Rev. A 89, 032119 (2014).

[45] C.C. Gerry, Q. Su and R. Grobe, Phys. Rev. A 74, 044103 (2006).

[46] P. Krekora, Q. Su and R. Grobe, Phys. Rev. A 73, 022114 (2006).

[47] F. Hund, Z. Phys. 117, 1 (1941).

[48] W. Su, M. Jiang, Z.Q. Lv, Y.J. Li, Z.M. Sheng, R. Grobe and Q. Su, Phys. Rev A 86, 013422 (2012).

[49] T. Cheng, M. Ware, Q. Su and R. Grobe, Phys. Rev. A 80, 062105 (2009).

[50] M. Leng and C. S. Lent, J. Appl. Phys. 76, 2240 (1994).

[51] For a review, see, e.g., W. Greiner, B. Müller and J. Rafelski, "Quantum Electrodynamics of Strong Fields" (Springer Verlag, Berlin, 1985).

[52] For review, see T. Cheng, Q. Su and R. Grobe, Contemp. Phys. 51, 315 (2010).

Appendix

The time-evolution of the electron-positron field operator $\Psi(z,t)$ is traditionally given [51] by the Dirac equation $i\,\partial\Psi(z,t)/dt = H(e^-;t)\,\Psi(z,t)$, where $H(e^-;t)$ is the *electronic* Hamiltonian of Eq. (1) with a negative charge $q=e^-$. A major assumption to this and also numerous past works [52] is to separate the total electron-positron field operator into a positronic and electronic portions $\Psi(z,t) = \Psi(e^-;z,t) + C\,\Psi(e^+;z,t)$ using the charge-conjugation operator $C \equiv \sigma_1 K$ (where K denotes the complex conjugation). This (anti-linear) charge conjugation operator transforms the electronic and positronic Hamiltonians to each other, $C\,H(q)\,C^{-1} = -H(-q)$. These definitions of $\Psi(e^-;t)$ and $\Psi(e^+;t)$ allow us to define the charge density of the created electrons and positrons as

$$\rho(e^-;z,t) = -\langle vac|\,\Psi(e^-;z,t)^\dagger\,\Psi(e^-;z,t)\,|vac\rangle \tag{14}$$

$$\rho(e^+;z,t) = \langle vac|\,\Psi(e^+;z,t)^\dagger\,\Psi(e^+;z,t)\,|vac\rangle \tag{15}$$

If we use the time evolved Hilbert space states and the matrix elements of the time evolution operator $U(e^-;t)$ between them, we can calculate these densities as

$$\rho(e^-;z,t) = -\Sigma_{p'}\,|\Sigma_p\,\langle u;p|\,U(e^-;t)\,|d;p'\rangle\,W_p(u;z)|^2 \tag{16}$$

$$\rho(e^+;z,t) = \Sigma_{p'}\,|\Sigma_p\,\langle d;p|\,U(e^-;t)\,|u;p'\rangle\,W_p(d;z)|^2 \tag{17}$$

All aspects of the quantum field theoretical description can therefore be obtained by evolving each (quantum mechanical) energy eigen state $|u;p\rangle$ and $|d;p\rangle$ in time under the same Hamiltonian $H(e^-)$. While this has been the traditional way to describe the pair-creation process, this formalism does not allow us to remove the self-repulsion mechanism as there is only a single time-evolution operator $U(e^-;t)$. However, if we use the charge conjugation operator with $U(e^-;t) = C\,U(e^+;t)\,C^{-1}$, the positrons' charge density in (A.2b) can be re-expressed as

$$\rho(e^+;z,t) = \Sigma_{p'}\,|\Sigma_p\,\langle d;p|\,C\,U(e^+;t)\,C^{-1}\,|u;p'\rangle\,W_p(d;z)|^2 \tag{18}$$

$$= \Sigma_{p'}\,|\Sigma_p\,\langle u;p|\,U(e^+;t)\,|d;p'\rangle\,W_p(d;z)|^2 \tag{19}$$

where we have used that $C^{-1}\,|u;p'\rangle = |d;p'\rangle$ and $C^{-1}\,|d;p'\rangle = |u;p'\rangle$, which follows directly from the symmetry $C\,H_0\,C^{-1} = -H_0$. As the notation $|\ldots|^2$ includes the spinor product of the wave function $W_p(d;z)$, we are permitted to include any anti-unitary operator such as C in it, $|\Sigma_p\, a_p\, W_p(d;z)|^2 = \Sigma_p\, \Sigma_{p'}\, a_p a_{p'}^{*}\, W^{\dagger}_{p'}(d;z)\, C\, C^{-1}\, W_p(d;z) = |\Sigma_p\, a_p\, C^{-1}\, W_p(d;z)|^2$. As a result, the density can also be written as

$$\rho(e^{+};z,t) \;\equiv\; \Sigma_p\, |\, \Sigma_{p'}\, \langle u;p'|\, U(e^{+},t)\, |d;p\rangle\, W_{p'}(u;z)\, |^2 \qquad (20)$$

As we will discuss in the main text, this will permit us to couple different internal fields to the electron and positron, which is the foundation for being able to remove the self-repulsion when both particles are coupled to the Maxwell equations.