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# Ubiquitous diffraction resonances in positronium formation from fullerenes

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Due to the dominant electron capture by positrons from the molecular shell and the spatial dephasing across the shell-width, a powerful diffraction effect universally underlies the positronium (Ps) formation from fullerenes. This results into trains of resonances in the Ps formation cross section as a function of the positron beam energy, producing structures in recoil momenta in analogy with classical single-slit diffraction fringes in the configuration space. The work opens a hitherto unknown avenue of Ps spectroscopy with nanomaterials and motivates level-differential measurements.

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## I. INTRODUCTION

Following the impact of positrons with matter the formation of exotic electron-positron bound-pair, the positronium (Ps), is a vital process in nature. This channel accounts for as large as half of the positron scattering cross section from simple atoms and molecules [1], as well as an even higher success rate of Ps formation on surfaces and thin films [2]. Other than probing structure and reaction mechanism of matters, the Ps formation is a unique pathway to the electron-positron annihilation process [3, 4] with both astrophysical [5] and applied [6] interests. Possible production of Bose-Einstein condensate of Ps has also been predicted [7, 8], besides the importance of Ps in diagnosing porous materials [9] as well as in probing bound-state QED effects [10]. Moreover, efficient Ps formation is the precursor of the production of dipositronium molecules [11] and antihydrogen atoms [12, 13] required to study the effect of gravitational force on antimatter [14, 15].

Theoretical investigations to calculate Ps formation cross sections from atomic hydrogen [16–18], noble gases [19–21], and alkali metals [22, 23] are aplenty. For some of these systems, *ab initio* close-coupling calculations, pioneered by Walters and collaborators [24], have been very successful [25]. Calculations with molecular targets, although relatively limited, include the molecular hydrogen [25, 26], polyatomic molecules [27], and the water molecule [28]. Simultaneously, precision experimental techniques to measure Ps formation signals have also been achieved by impinging positrons into varieties of materials, such as, atomic and molecular gases [29, 30], molecular solids [31], liquids and polymers [32], zeolites [33], metal surfaces and films [34], metal-organic-frameworks [35, 36], and embedded mesostructures [37]. Very recently high yields of laser assisted production of low-energy excited Ps is achieved in the interac-

tion of cold-trapped positrons with Rydberg excited Cs atom [12].

However, in spite of such broad landscape of Ps research, little attempt of Ps formation by implanting positrons in nanoparticles, in gas or solid phase, has so far been made, except for a single theoretical study using Na clusters [38]. On the other hand, straddling the line between atoms and condensed matters are clusters and nanostructures that not only have hybrid properties of the two extremes, but also exhibit outstanding behaviors with unusual spectroscopic effects [39]. Formation of a quasi-free electron gas within a finite nanoscopic region of well-defined boundary, as opposed to a longer-range, highly diffused Coulomb-type boundary characteristic of atoms and molecules, is a property of nanosystems which ensures predominant electron capture from localized regions in space. This may lead to diffraction in the capture amplitude, particularly at positron energies that cannot excite plasmon modes. The Ps formation from fullerenes can be singularly attractive to access this diffraction phenomenon due to fullerene’s eminent symmetry and stability, and its previous track record of success in spectroscopic experiments [40]. In this communication, we show that the Ps formation amplitude from the positron colliding with C<sub>60</sub> does include a strong, observable diffraction effect, resulting in a system of peaks in the form of broad shape resonances, in the total ground and excited Ps formation cross section for state-selected captures.

## II. A BRIEF DESCRIPTION OF THE METHOD

The C<sub>60</sub> molecule is modeled by smearing sixty C<sup>4+</sup> ions in a spherical jellium shell, fixed in space, with an experimentally known C<sub>60</sub> mean radius  $r_c = 6.7$  a.u. and a width determined *ab initio* [41]. Inclusion of molecular orientations will have minimal effect on the result due to the C<sub>60</sub> symmetry [42]. The delocalized system of total 240 valence electrons from sixty carbon atoms constructs the ground state in the Kohn-Sham local-density approximation (LDA) [41] improved by the gradient-corrected Leeuwen and Baerends exchange-correlation functional

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Ps formations in the forward direction, which was found to be the most dominant direction in both earlier [50] and recent experiments [51], so that  $\hat{r}_-$  and  $\hat{r}_+$  are identical and thus  $\vec{\rho}$  is independent of  $\hat{r}_+$ . Adopting the same simplification for  $F_{\vec{k}_-}^{(-)*}(\vec{r}_-)$  in Eq. (1) to apply in Eq. (3) and then combining with Eq. (4), we can rewrite the full amplitude (1) for a capture from  $C_{60} n_i \ell_i$  state as,

$$T(\vec{k}_i) \sim S(\vec{k}_i) + \frac{1}{k-q} \iint dr_- dr_+ r_+ R_{n_f s}(\rho) V_i^{sc}(r_+) \times [r_- R_{n_i \ell_i}(r_-)] \sin(k_- r_- - \ell_i \pi/2) \sin(q r_+) \times Y_{0,0}^*(\hat{q}) Y_{\ell_i, m_i}^*(\hat{k}_-), \quad (5)$$

where we used asymptotic forms of the spherical Bessel function  $j_\ell(r) \sim \sin(r - \ell\pi/2)/r$ . We will ignore the spherical harmonics in Eq. (5), since the forward direction is considered.

Before moving further let us note the following in the amplitude (5) in conjunction with Fig. 1: (i) Large values of  $V_i^{sc}$  only at the  $C_{60}$  shell indicates the shell to be the window of appreciable repulsive interactions of the positron with  $C_{60}$ . (ii) The shape of the Ps radial wavefunction  $R_{n_f s}(\rho)$  identifies the region of success of Ps formation as a function of electron-positron separation  $\rho = |\vec{r}_- - \vec{r}_+|$  where a Ps s-state has the maximum probability density at  $r_- = r_+$ . (iii) The radial wave function of  $C_{60} i$ -th level  $r_- R_{n_i \ell_i}$  ensures that electrons must be available for the Ps to form. Considering Ps(1s) formation from the capture off  $C_{60}$  HOMO level, we took eleven electron-positions in Fig. 1 within the interaction window around  $C_{60}$  radius ( $r_c$ ) and plot the product  $R_{1s}(\rho) V_{7h}^{sc}(r_+) [r_+ R_{7h}(r_+)]$  of the three quantities mentioned above; the  $r_-$  values considered are shown by dots in the same color of that of the curve it generates. Since Ps will have a propensity to successfully form toward  $\rho = 0$ , the curves show maxima and minima at various locations symmetrically on the left and right side, respectively, of  $r_c$ . This opposite orientation is because of the shape of  $C_{60}$  7h wavefunction which is asymmetric about its node. It is fair to approximate that the dominant behavior comes from the optimum positions at  $r_p$ . Thus, choosing  $r_- = r_+ = r_p$  in Eq. (5) these contributions indicate trains of peaks given by  $\sin(k_- r_- - \ell_i \pi/2) \sin(q r_+) \sim \sin(Q r_p - \ell_i \pi/2)$  where the recoil momentum  $Q = k_i - 2k_\pm$  for the Ps formation in the forward direction. The amplitude  $A(r_p)$  of the oscillation is proportional to the area ( $r_+$  integral) under the corresponding bump. Hence, the remaining  $r_-$  integral in Eq. (5) can approximate to a simpler form

$$T(\vec{k}_i) \sim S(\vec{k}_i) + \frac{1}{k-q} \int dr_- A(r_p) \sin(Q r_p - \ell_i \pi/2). \quad (6)$$

Noticing that the bumps appear on either side of  $r_c$  at similar distances  $\alpha$  (Fig. 1), Eq. (6) can be rewritten as

$$T(\vec{k}_i) \sim S(\vec{k}_i) + \frac{1}{2k-q} \int dr_- A(r_p) [\sin\{Q(r_c - \alpha(r_p)) - \ell_i \pi/2\} - \sin\{Q(r_c + \alpha(r_p)) - \ell_i \pi/2\}], \quad (7)$$

where the negative sign between the two terms justifies the maxima and minima about  $r_c$  (Fig. 1). We finally obtain

$$T(\vec{k}_i) \sim S(\vec{k}_i) - \frac{1}{k-q} \cos(Q r_c - \ell_i \pi/2) \times \int dr_- A(r_p) \sin(Q \alpha(r_p)). \quad (8)$$

Obviously, the integral in the above equation spatially dephase the  $\sin(Q \alpha)$  modulation, since  $\alpha$  varies with  $r_-$ , retaining peaks in the amplitude only via  $\cos(Q r_c - \ell_i \pi/2)$ . Here is the essence of what is going on. Within about the fullerene width region, a strong constructive and destructive interference take place as a function of  $Q$ . When the odd integer multiple of the half-wavelength of *effective* continuum wave as a function of  $Q$  fits the distance  $r_p$ , systems of peaks (bright-spots) in the energy domain are formed which subsequently results into a single *centroid* fringe pattern via a dephasing mechanism in the integration over the electron position as described above.

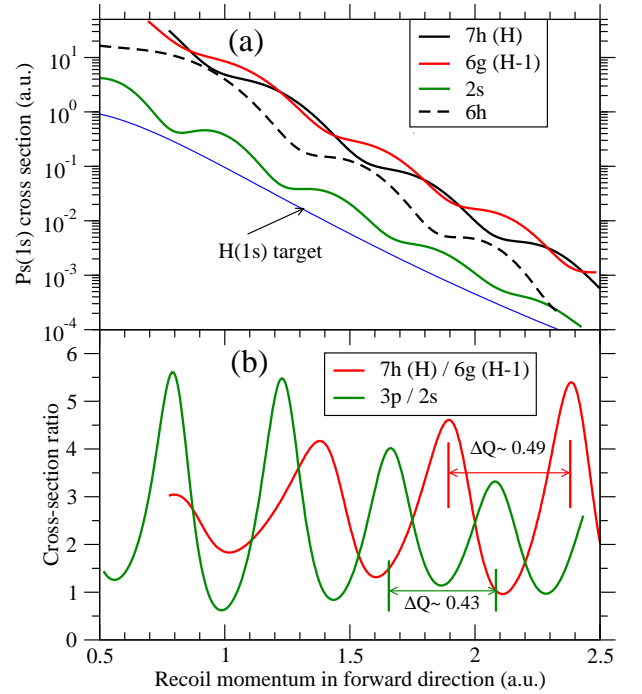


FIG. 2. (Color online) (a) Ps(1s) formation cross sections for captures from HOMO (H), HOMO-1 (H-1), 2s (bottom most  $\pi$  level) and 6h (a  $\sigma$  level) as a function of recoil momentum ( $Q$ ) in the forward direction. The result of hydrogen 1s capture is also shown for comparisons. (b) The ratios of two pairs of these cross sections illustrate strong resonances. Separations ( $\Delta Q$ ) between some resonances are marked.

Since these diffraction peaks appear in the energy (momentum) domain, we call them diffraction resonances which must also show up in the Ps formation cross section proportional to the squared modulus of Eq. (8). Fur-

ther, this squaring operation modifies the functional form of the structures as  $\cos(Qd_c - \ell_i\pi)$  in the cross section, where  $d_c$  is the  $C_{60}$  diameter. Broad resonances are seen in Fig. 2(a) for the numerical Ps(1s) formation cross section for the captures off four  $C_{60}$  levels as a function of the forward-emission recoil momentum  $Q$  corresponding to the electron excitation energy from roughly 50 eV (above the plasmon excitation) to 270 eV (below the  $K$ -shell of atomic carbon). For a direct comparison, also presented in Fig. 2(a) is the steady result for the atomic hydrogen target showing no such resonances. The offsets between the peaks for the HOMO versus HOMO-1 versus  $2s$  (the bottom most level of  $C_{60}$   $\pi$  band) results connect to the phase-shift  $\ell_i\pi/2$  in Eq. (8) that depends on the angular symmetry of the initial state. The shape and strength of these resonances are best illustrated by considering the cross section ratios, shown in Fig. 2(b), which neutralizes the non-resonant background decays, and therefore can be accessed in experiments by the Ps formation spectroscopy with better accuracy much freer of the experimental noise. It is important to note that we used plane wave descriptions in our analysis to interpret the key results. But the actual character of the resonances are more diverse than this simple account, as seen in Fig. 2(b).

If the capture is from a  $C_{60}$   $\sigma$  state, which has no radial node, we still also find Eq. (7) but with a positive sign between the two terms in the integration. This will produce a sine function in Eq. (8) leading to  $\cos(Qd_c - \ell_i\pi + \pi)$  structures in the cross section for a  $C_{60}$   $\sigma$  electron capture. This suggests that for the same angular momentum the  $\pi$  electron capture cross section will produce  $180^\circ$  out-of-phase resonances compared to a  $\sigma$  electron capture, as clearly seen between the HOMO and  $6h$  results in Fig. 2(a). This indicates that electronic structural information can also be accessed spectroscopically by Ps formation from fullerenes.

An elegant way to bring out the connection of diffraction resonances with the fullerene diameter is to evaluate the Fourier spectra of the cross sections as a function of  $Q$ . To generate the input signals with only the resonances on a flat, non-decaying background we considered ratios of the results of two consecutive angular levels of  $\pi$  electrons. Calculated Fourier magnitudes of these ratios in the reciprocal (radial) coordinate are displayed in Fig. 3(b). All the curves exhibit strong peaks located close to the the diameter  $d_c$ , as expected. Note that the small, systematic offset of the peaks towards lower values with the increasing angular momentum is another signature of the fact that the continuum waves are Coulomb distorted and so are more complicated than simple plane waves used in the analysis. In fact, we anticipate this variation by noting in Fig. 2(b) the separations  $\Delta Q$  of 0.49 a.u. and 0.43 a.u. respectively for HOMO/HOMO-1 and 3p/2s ratio and then determining their reciprocal ( $2\pi/\Delta Q$ ) values as 12.8 a.u. and 14.6 a.u. being somewhat different. In any case, these Fourier reciprocal spectra unequivocally supports the theme that

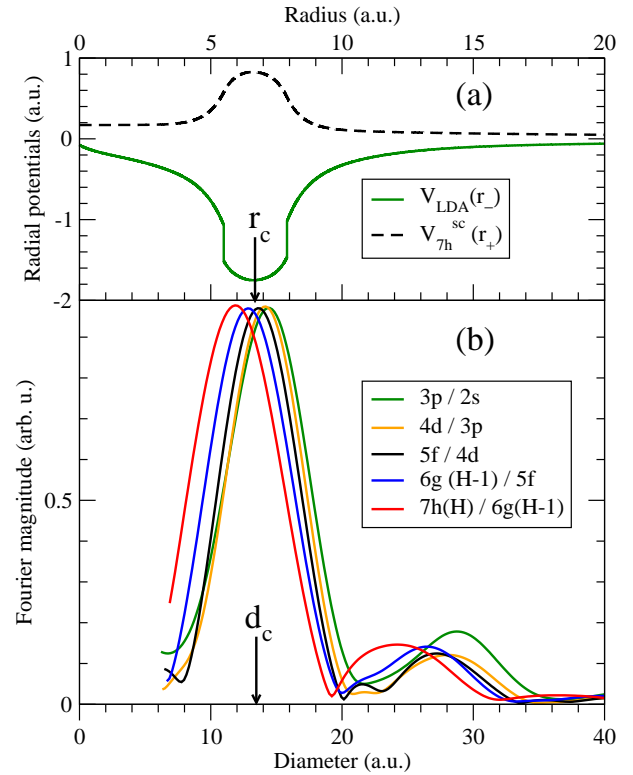


FIG. 3. (Color online) (a)  $C_{60}$  LDA radial potential and the positron scattering potential for the HOMO capture. (b) The Fourier transform magnitudes of Ps(1s) cross section ratios for captures from various  $C_{60}$   $\pi$  levels.

the host of broad resonances are indeed the fringe patterns in the energy domain for a Ps formation channel where the forward Ps-emission diffracts in energy off the shell – a spherical slit. Furthermore, our calculations (not shown) for the formation of excited Ps(2s) produced similar resonance structures. **The Ps(2p) channel is attractive too**, since it can be monitored optically [52]. Also, using the  $C_{240}$  fullerene we found (not shown) the resonances to be more compact in energy than  $C_{60}$ , since  $C_{240}$  is a larger diffractor.

#### IV. FINAL COMMENTS

Fullerenes [40, 47] and metallic nanoparticles [39, 53] are nowadays available in gas-phase. The challenge, however, is to probe the target-state differential Ps-signals which current techniques can not readily access [54]. But accessing this will be beneficial in general and in particular, since the predicted resonances, having a target angular-state dependent shift [Eq. (8)], will wash out in the total Ps measurement. The technique to measure the recoil momentum of the cations can be improved by using a supersonic gas jet to increase the overlap with the positron beam. Resolving the Ps level is not so critical (although may still be done by laser spectroscopy of a



dense Ps gas), since Ps(1s) signal should largely dominate. We believe that the current work will provide a much needed motivation to break the barrier to measure differential Ps production cross sections at least within an acute forward angle.

In conclusion, we calculate the Ps formation cross section in the CDW-FS method for electron captures from various C<sub>60</sub> levels. The C<sub>60</sub> ground state structure is modeled by a simple but successful LDA frame. Hosts of strong and broad shape resonances in the Ps formation are predicted. The resonances are engendered from a diffraction effect in the Ps formation process localized on the fullerene shell. The effect should be universal for

Ps formation from nanosystems, including metal clusters, carbon nanotubes, or even quantum dots that confine finite-sized electron gas. The work ushers a new research direction to apply Ps formation spectroscopy to gas-phase nanosystems which we hope can begin with fullerenes, since they currently enjoy significant attraction in precision measurements.

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