Keldysh parameter, photoionization adiabaticity, and the tunneling time

A. M. Zheltikov

Phys. Rev. A **94**, 043412 — Published 17 October 2016

DOI: 10.1103/PhysRevA.94.043412

The Keldysh parameter, photoionization adiabaticity, and the tunneling time

A.M. Zheltikov

Physics Department, International Laser Center, M. V. Lomonosov Moscow State University, Moscow 119992, Russia

Department of Physics and Astronomy, Texas A&M University, College Station TX 77843, USA

Russian Quantum Center, Skolkovo, Moscow Region 143025, Russia

Ultrafast photoionization dynamics within a driver field cycle is shown to be the key to understand the multifaceted role of the Keldysh parameter γ as the borderline between multiphoton and tunneling ionization, ionization adiabaticity degree, and a measure for the electron tunneling time. We demonstrate that, when applied to subcycle ionization dynamics, the $\gamma << 1$ condition automatically translates into a criterion of both adiabaticity and photoionization via tunneling. We also show that the ratio of the Keldysh γ to the frequency of the driver field indeed defines an important time scale of photoionization. However, instead of connecting to the electron motion beneath the potential barrier, it relates to the time needed for an electron to acquire a pondermotive energy equal to the ionization potential. This time scale is shown to manifest itself in the experimentally measurable buildup time of the photoelectron yield.

Over five decades, the Keldysh theory of photoionization [1] has been pivotal to the research in strong-field laser science [2], providing a universal framework for a quantitative analysis of ionization in a remarkable diversity of light–matter interaction phenomena, including laser-induced breakdown [3, 4], high-order harmonic [5, 6] and terahertz [7, 8] generation, as well as filamentation of ultrashort light pulses [9, 10]. As one of its central results, the Keldysh theory elegantly shows that multiphoton ionization and electron tunneling are, in fact, two pathways that dominate photoionization in the weak- and strong-field regimes, respectively. Remarkably, transition from one regime to the other is controlled by a single physical parameter – the Keldysh gamma parameter $\gamma = \omega(2mI_0)^{1/2}/(eE_0)$, where ω and E_0 are the frequency and the amplitude of the driver field, I_0 is the ionization potential, and e and m are the electron charge and mass.

Besides defining a borderline between multiphoton and tunneling ionization, the Keldysh parameter quantifies the nonadiabaticity degree of the photoionization process, with adiabaticity understood in this context as no frequency dependence in the photoionization rate [1]. In the multiphoton regime, i.e., for $\gamma >> 1$, the photoionization rate is strongly dependent on the laser frequency. In the opposite limit of small γ, photoionization occurs via electron tunneling, and the photoionization rate becomes frequency-independent.

While the role of the Keldysh γ as the borderline between weak- and strong-field ionization regimes is easy to understand, the $\gamma << 1$ ionization adiabaticity criterion is much less intuitive. With the laser frequency dropping out of the photoionization rate in the $\gamma << 1$ regime, it is tempting to relate γ to the time of electron tunneling [1]. Indeed, the Keldysh parameter can be represented as $\gamma = 4\pi\tau_b/T_0$, where $T_0 = 2\pi/\omega$ is the cycle of the driver field, and $\tau_b = d/v = (mI_0/2)^{1/2}(eE_0)^{-1}$ is the time it takes for a classical particle with the velocity $v = (2I_0/m)^{1/2}$ to travel a distance $d = I_0/(eE_0)$ equal to the width of the potential barrier (Fig. 1) formed by the Coulomb potential and a dc driver field E_0.

While γ provides a central reference point for photoionization when understood as the borderline between the weak- and strong-field regimes, as well as the measure of nonadiabaticity of photoionization, it is this third interpretation of this parameter in its relation to the tunneling time that has been a subject of heated debate over decades (see, e.g., Refs. 11 – 13 for a review), as it is clearly at odds with canonical quantum mechanics, which does not provide much ground for a consensus with regard to the possibility of direct tunneling time measurements and the uncertainty of such measurements if possible at all. The opening paragraph of the seminal Keldysh paper [1] introduces γ as a measure of adiabaticity by putting it in the context of τ_b [14]. However, it does not necessarily identifies $\gamma/(2\omega)$ with $\tau_b = d/v$.

In the era of modern technologies, this debate goes way beyond purely methodological, interpretational matters, as a clear understanding of extremely fast electron tunneling dynamics would help maximize the speed of semiconductor electronic devices, achieve an ultimate accuracy in attosecond metrology [15], and identify the fundamental limitations of rapidly emerging petahertz optoelectronics [16, 17]. Experimental approaches developed in the past few years [18 – 24] enable the detection of electron tunneling dynamics using the methods of ultrafast laser science [25 – 29]. Physical interpretation of these measurements, however, faces fundamental difficulties as the available quantum approaches do not offer a simple recipe of defining the time of quantum tunneling. With this big picture of unique experimental opportunities facing fundamental conceptual difficulties in sight, the $\tau_b = d/v$ tunneling-time intuition seems to offer an appealing, physically transparent, albeit not perfectly rigorous perspective on electron tunneling, leading to real-valued predictions for the times of electron under-barrier motion, which seem to offer valuable insights into the results of numerical analysis of tunneling ionization [30, 31]. The question that remains to be addressed, however, is whether any sort of compromise between this tunneling-time perspective on γ and the canonical quantum picture of tunneling is possible.

Our main goal here is to address the questions as to why the Keldysh γ serves simultaneously as the adiabaticity parameter and a borderline between multiphoton and tunneling ionization, as well as whether the time γ/ω is in any way representative or in any way related to electron tunneling dynamics in general and beneath-the-barrier electron motion in particular. We show below in this paper that, to answer these questions, photoionization dynamics within a driver field cycle needs to be examined. When applied to such subcycle ionization dynamics, the $\gamma << 1$ condition will be shown to translate in a physically transparent way into a criterion of both photoionization via tunneling and frequency-independent photoelectron yield. Moreover, it will be demonstrated that the γ/ω time indeed defines an important time scale of photoionization. However, instead of connecting to the $\tau_b = d/v$ time and electron motion beneath the potential barrier (Fig. 1), it relates to the time needed for an electron to acquire a pondermotive energy equal to the ionization potential.

In the Keldysh theory of photoionization [1], the field $\mathbf{E}(t)$ couples an electron bound state with a wave function $\psi_0(\mathbf{r})$ to a free electron state with a wave function $\psi_\mathbf{p}(\mathbf{r}, t)$ (Fig. 1), inducing transitions between these states with the transition amplitude given by

$$w(\mathbf{p},t) = \int \psi_\mathbf{p}^*(\mathbf{r},t) V(\mathbf{r},t) \psi_0(\mathbf{r}) d\mathbf{r} \,, \qquad (1)$$

where $V(\mathbf{r},t)$ is the electron–field interaction part of the Hamiltonian. For a monochromatic laser field $\mathbf{E}(t) = \mathbf{E}_0 \cos(\omega t)$, we have $V(\mathbf{r},t) = e\mathbf{r}\mathbf{E}_0 \cos(\omega t)$. Photoionization from the ground state of a hydrohenlike atom is described by setting $\psi_0(\mathbf{r}) = (\pi a^3)^{-1/2} \exp(-r/a)$, where $a = \hbar^2/(me^2)$ is the Bohr radius. The free-electron wave function is taken in the form of the Volkov-type solution, $\psi_\mathbf{p}(\mathbf{r},t) = \exp\left\{i\,\mathbf{P}(t)\mathbf{r}/\hbar - i/(2m)\hbar \int_0^t [\mathbf{P}(\theta)]^2 d\theta\right\}$, with the generalized momentum $\mathbf{P}(t)$ in the case of a monochromatic driver field $\mathbf{E}(t)=\mathbf{E}_0\cos(\omega t)$ given by $\mathbf{P}(t)=\mathbf{p}+e\omega^{-1}\mathbf{E}_0\sin(\omega t)$.

With the transition amplitude $w(\mathbf{p}, t)$ averaged over the field cycle and integrated over the momentum, the photoionization rate is expressed as a series

$$w_K=\frac{2\pi}{\hbar}\int|L(\mathbf{p})|^2\sum_n\delta\left(I_0+\frac{p^2}{2m}+\frac{e^2E_0^2}{4m\omega^2}-n\hbar\omega\right)\frac{d^3p}{(2\pi\hbar)^3} \tag{2}$$

with

$$L(\mathbf{p})=\frac{1}{2\pi}\oint d\xi W\left(\mathbf{p}+\frac{e\mathbf{E_0}}{\omega}\xi\right)\exp\left\{\frac{i}{\hbar\omega}\int_0^{\xi}\left[I_0+\frac{1}{2m}\left(\mathbf{p}+\frac{e\mathbf{E_0}}{\omega}u\right)^2\right]\frac{du}{\left(1-u^2\right)^{1/2}}\right\}, \tag{3}$$

where $W(\mathbf{s})=e\pi^{-1/2}a^{-3/2}\int\exp(-i\,\mathbf{sr}/\hbar)\mathbf{Er}\exp(-r/a)d\mathbf{r}$ and integration is along a closed contour enclosing the [–1, 1] segment.

Since the exponential in $L(\mathbf{p})$ is rapidly oscillating, the integral in Eq. (2) is dominated by the saddle points, defined by the equation

$$I_0+\frac{1}{2m}\left(\mathbf{p}+\frac{e\mathbf{E_0}}{\omega}\sin\omega t\right)^2=0\,. \tag{4}$$

With the integral in $L(\mathbf{p})$ calculated using the saddle-point approximation, Eqs. (2) and (3) lead to the celebrated Keldysh formula for the rate of photoionization [1],

$$w_K=Q(\gamma,I_0,\omega)\exp\{-\xi(\gamma,I_0,\omega)\}, \tag{5}$$

where $Q(\gamma,I_0,\omega)$ is a preexponential factor and

$$\xi(\gamma,I_0,\omega)=\frac{2}{\hbar\omega}I_0\left(1+\frac{1}{2\gamma^2}\right)\left[\sinh^{-1}\gamma-\gamma\frac{\left(1+\gamma^2\right)^{1/2}}{1+2\gamma^2}\right]. \tag{6}$$

The beauty of this result is that it provides a uniform description of multiphoton and tunneling ionization, showing that these processes can be treated as two limiting regimes corresponding to photoionization with $\gamma \gg 1$ and $\gamma \ll 1$, respectively. Indeed, with $\gamma \gg 1$, Eqs. (5) and (6) recover the signature I^N scaling of multiphoton ionization rate as a function of the field intensity I and the minimum number of photons N needed for photoionization. With $\gamma \ll 1$, on the other hand, Eqs. (5) and (6) yield $w_K\propto\exp\left\{-(4/3)(2m)^{1/2}I_0^{3/2}(e\hbar E_0)^{-1}\left(1-\gamma^2/10\right)\right\}$. With the small γ^2 term omitted in the argument of the exponential, this expression recovers the canonical result for the transmission of an electron wave function tunneling through a triangular potential barrier formed by a rectangular potential step of height I_0 and a dc electric field E_0.

For γ<< 1, the driver frequency ω cancels out in Eq. (6), allowing γ to be interpreted as the degree of photoionization adiabaticity [1]. However, while the form of the exponential in Eq. (6) makes it easy to understand the role of the Keldysh γ as the borderline between the high- and weak-field ionization regimes, it offers no equally instructive physical insight into the frequency cancellation in the ionization rate in the γ<< 1 strong-field regime.

Although it cannot be directly verified with Eqs. (1) – (6), the $\gamma = 4\pi\tau_b/T_0$ interpretation of the gamma parameter in terms of the beneath-the-barrier passage time $\tau_b = d/v$ seems to offer a missing clue for the ω cancellation in the γ<< 1 regime [1, 14]. This rises expectations that the Keldysh parameter can also serve as a measure of the electron tunneling time, understood as the time electrons spend under the potential barrier in the process of tunneling. The solution of Eq. (4) for t_0 is, however, purely imaginary:

$$t_0 \approx \frac{i}{\omega}\left\{\sinh^{-1}\gamma + \frac{\gamma}{\left(1+\gamma^2\right)^{1/2}}\left[i\frac{p_{\parallel}}{\left(2mI_0\right)^{1/2}} + \frac{1}{4mI_0}\left(\frac{\gamma^2}{1+\gamma^2}p_{\parallel}^2 + p_{\perp}^2\right)\right]\right\}. \qquad (7)$$

The $\gamma = 4\pi\tau_b/T_0$ interpretation of γ is still in many ways useful, but only as a measure of the imaginary photoionization time (Fig. 1), which allows the stationary-phase points [Eq. (4)] dominating the integral in $L(\mathbf{p})$ to be described in a compact and appealing form [32]. The relation of t_0 to the real duration of the photoionization process is, however, anything but clear.

We are going to show now that, in the quantum-mechanical picture, where photoionization is described in terms of the generic transition amplitude of Eq. (1), tunneling instantly follows the driver field, with the electron wave packet leaking through the potential barrier formed by the Coulomb potential and the driver field (Fig. 1) without any time lag that could be related to under-barrier electron dynamics. To this end, we consider the dynamics of photoionization within the driver field cycle without averaging the probability amplitude $w(\mathbf{p}, t)$ over the field cycle. The photoelectron yield induced by the driver field by the time t is then given by

$$J(t) \propto \int_{-\infty}^{t} d\eta \int d\mathbf{r} \exp\left(-\frac{\mathbf{r}}{a_0}\right)\mathbf{E}_0\mathbf{r}\cos(\omega\eta)\exp\left\{\frac{i}{\hbar}\left[I_0\eta - \mathbf{P}(\theta)\mathbf{r} + \frac{1}{2m}\int_0^{\eta}[\mathbf{P}(\theta)]^2 d\theta\right]\right\}. \qquad (8)$$

Since $w(\mathbf{p}, t)$ is dominated by photoelectrons with small p [1, 32], the terms with $\mathbf{p}$ can be discarded, to give for a driver field linearly polarized along the z-axis, in the one-dimensional case,

$$J(t) \propto \int_{-\infty}^{t} d\eta \int_{-\infty}^{\infty} dz \exp\left(-\frac{z}{a_0}\right) E_0 z \cos(\omega t) \exp\left[\frac{i}{\hbar}\int_0^{\eta} \Phi(z,\theta) d\theta\right], \tag{9}$$

where

$$\Phi(z,\theta) = I_0 - eE_0 z \cos(\omega\theta) + \frac{1}{2m}\left(\frac{eE_0}{\omega}\right)^2 \sin^2(\omega\theta). \tag{10}$$

The exponential in Eq. (9) rapidly oscillates unless $\Phi(z,\theta) \approx 0$. Noting that the strong-field regime criterion $\gamma << 1$ is equivalent to $(eE_0)^2 / (2m\omega^2) >> I_0$, we see that, in this regime, an electron acquires the pondermotive energy equal to the ionization potential I_0 within a small time interval $[-\tau_0, \tau_0]$ around the peak of the driver field (Fig. 2) defined by the equation

$$\sin^2(\omega\tau_0) = \omega^2 \frac{2mI_0}{(eE_0)^2} = \gamma^2 << 1. \tag{11}$$

Since $\sin^2(\omega\tau_0) << 1$ within this interval, we can use Taylor-series expansions $\sin^2(\omega\theta) \approx (\omega\theta)^2$ and $\cos(\omega\theta) \approx 1 - (\omega\theta)^2/2$ in Eq. (10), to write the solution to the $\Phi(z,\theta)$ = 0 equation as

$$z_0 \approx \frac{I_0}{eE_0}\left[1 + \left(1 + \frac{2}{\gamma^2}\right)\frac{\omega^2\theta^2}{2}\right]. \tag{12}$$

As elegantly shown by Perelomov, Popov, and Terent'ev (PPT) [32], Eq. (4) for the saddle points of the integrand in Eq. (3) can be recovered from the Newtonian equations of motion for a classical electron that is allowed to have an imaginary momentum. The PPT theory also introduces a useful measure for the width of the potential barrier, d_{PPT}, defined as the distance that such an electron travels from the moment it enters the beneath-the-barrier domain, where both the time and the momentum are allowed to be imaginary, until the moment of time when the driver field reaches its peak value. Since the latter condition is expressed as $\theta = 0$, we find that, for $\gamma << 1$, z_0 defined by Eq. (12) is equal to d_{PPT}.

Using Eq. (12) to estimate $J(t)$, we derive

$$J(t) \propto \exp\left[-\frac{(2m)^{1/2} I_0^{3/2}}{e\hbar E_0}\right] \int_{-\infty}^{t} \exp\left\{-\frac{(2m)^{1/2} I_0^{3/2}}{e\hbar E_0}\left[\left(1 + \frac{2}{\gamma^2}\right)\frac{\omega^2\theta^2}{2}\right]\right\} d\theta. \tag{13}$$

Despite its approximate character, Eq. (13) correctly reproduces the $\omega \to 0$ limit of the photoionization rate in the $\gamma << 1$ regime, recovering, similar to the Keldysh theory of photoionization [Eqs. (5), (6)], the probability of dc-field-induced tunneling, $w_{dc} \propto \exp\left[-(4/3)\left(2mI_0^3\right)^{1/2}/\left(e\hbar E_0\right)\right]$, up to a factor of 4/3 in its argument.

Evaluating the integral in Eq. (13), we find

$$J(t) \propto \exp\left[-\frac{(2m)^{1/2} I_0^{3/2}}{e\hbar E_0}\right]\left[1+\operatorname{erf}\left(t/\tau_e\right)\right], \tag{14}$$

where $\operatorname{erf}(u) = 2(\pi)^{-1/2}\int_0^u \exp\left(-\xi^2\right)d\xi$ is the error function and

$\tau_e = (2)^{1/4}\left(e\hbar E_0\right)^{1/2}\left(mI_0^3\right)^{-1/4}\omega^{-1}\left(1+2/\gamma^2\right)^{-1/2}$.

It is straightforward to see from Eq. (14) that the profile of $J(t)$ has a shape of a step centered at $\theta = 0$, where the driver field reaches its maximum. Such steps are indeed observed in attosecond time-resolved studies of the photoelectron yield in the strong-field regime [18] The buildup rate of $J(t)$, that is, the steepness of the 1 + erf(t/τ_e) step in Eq. (14), is controlled by the τ_e time-scale parameter. Due to the $\tau_0 \propto E_0^{-1/2}$ scaling, stronger driver fields give rise to steeper steps in $J(t)$. In the $E_0 \to \infty$ limit, $J(t)$ tends to a Heaviside unit-step function, $J(t) \to \Theta(t)$, whose step is locked to the peak of the field at $t = 0$. In this sense, electron tunneling instantly follows the driver field without any time lag with respect to the driver field.

We are now in a position to address the question as to why the Keldysh gamma parameter serves simultaneously as the adiabaticity parameter and a borderline between weak- and strong-field ionization. To this end, we resort to the equation $\Phi(z,\theta) = 0$, which defines the effective thickness of the potential barrier formed by the Coulomb potential and the driver field. An important insight offered by this equation is that the pondermotive energy $U_p(\theta) = (1/2m)(eE_0/\omega)^2\sin^2(\omega\theta)$, acquired by the electron in the presence of a driver field, modulates the potential barrier, effectively increasing its height and width.

When the $U_p(\theta)$ term dominates over the ionization potential term I_0 everywhere within the field cycle except for a short time interval [$-\tau_0$, τ_0] around $\theta = 0$,

$$\frac{1}{2m}\left(\frac{eE_0}{\omega}\right)^2\sin^2(\omega\theta) >> I_0\,,\; -\tau_0 < \theta < \tau_0\,, \tag{15}$$

Eq. (12) for the effective width of the potential barrier is reduced to $z_0 \approx I_0\left(eE_0\right)^{-1}\left[1+(\omega/\gamma)^2\theta^2\right] = I_0\left(eE_0\right)^{-1} + (2m)^{-1}eE_0\theta^2$. The photoelectron yield $J(t)$ is then frequency-independent, with its buildup time given by

$\tau_e \approx \left(e\hbar E_0\right)^{1/2}\left(2mI_0^3\right)^{-1/4}(\gamma/\omega) = \left(2m/I_0\right)^{1/4}\left(\hbar/eE_0\right)^{1/2}$

Thus, the analysis of ultrafast ionization within the field cycle leads us to formulate the criterion of strong-field photoionization in the form of Eq. (15). When written in such a form, this criterion automatically implies, through Eqs. (11) – (13), that the photoionization rate is frequency-independent, or, for that matter, adiabatic. Notably, the term representing the pondermotive energy also leads to frequency cancellation in the Keldysh formula photoionization rate [Eqs. (2) – (6)] in the γ << 1 limit. However, since the Keldysh formula involves integration in time, the pondermotive energy enters into these equations in its averaged form – as the $e^2E_0^2/(4m\omega^2)$ term in Eq. (2) and as the $[1 + 1/(2\gamma^2)]$ factor in Eq. (6). The relation of ω cancellation in the photoionization rate to the subcycle dynamics of photoionization is thus lost.

Eqs. (8) – (15), on the other hand, clearly show that it is the photoionization occurring within a small fraction of the driver cycle, that is, within the time interval $[-\tau_0, \tau_0]$, that makes photoionization frequency-independent. The physics behind this involves ultrafast electron tunneling, which is strongly confined to a very short time gate $[-\tau_0, \tau_0]$. Within this interval, the pondermotive energy term, which dominates the width and height of the potential barrier, grows quadratically with time (Fig. 2), $U_p(\theta) \propto (\omega\theta)^2$. Its amplitude, on the other hand, scales as ω^{-2} with the driver frequency. As a consequence, the barrier width and, hence, the probability of photoionization become frequency-independent.

We can now confront another central question of field-induced ionization – the question as to whether the time γ/ω is in any way representative of beneath-the-barrier electron dynamics, as the $\tau_b = d/v = \gamma/(2\omega)$ intuition suggests. We note that Eq. (15) leads to $(eE_0)^2/(2m\omega^2) >> I_0$, which can be rewritten in the form γ^2 << 1 [see also Eq. (11)], equivalent to the criterion of both tunneling and adiabaticity in the Keldysh theory of photoionization. Then, solving Eq. (11) for τ_0, we find $\tau_0 = \omega^{-1} \arcsin\gamma$. Since γ<< 1, this reduces to $\tau_0 \approx \gamma/\omega = (2mI_0)^{1/2}/(eE_0)$. We see that the γ/ω ratio does indeed define an important time scale of photoionization. However, this time scale, τ_0, is not related to the time of electron motion beneath the potential barrier, $\tau_b = d/v$, but connects, via Eq. (11), to the time needed for an electron to acquire a pondermotive energy equal to the ionization potential (Fig. 2).

This result suggests a finite time lag of $\tau_0 \approx \gamma/\omega$ between the moment of time when electrons appear in the continuum, i.e., acquire an energy higher than I_0, and the instant when the electron wave packet is detected, in some thought experiment, right behind the potential barrier. Since for an electron in an excited bound state with ionization potential I_j, this time lag is $\tau_j \approx \gamma_j/\omega = (2mI_j)^{1/2}/(eE_0)$, the overall buildup time of continuous population can be expected to be strongly correlated with, but not necessarily equal to τ_0. Analysis of continuum population dynamics seems to be consistent with this intimation, showing that the continuum population buildup time is indeed strongly correlated with the Keldysh time [30].

Experiment-wise, the time τ_0 shows up in the photoelectron yield rise time as $\tau_e \approx (e\hbar E_0)^{1/2}(2mI_0^3)^{-1/4}\tau_0$. For a driver field intensity of 10^{14} W/cm^2, this gives $\tau_e \approx 0.14$ fs. Despite its brevity, the buildup time of the photoelectron yield can be measured with a reasonable accuracy in experiments by attosecond time-resolved studies [18] or by analyzing the spectra of optical harmonics [20, 21]. From the perspective of the fundamental principles of quantum mechanics, the photoelectron yield $J(t)$, whose definition [Eqs. (8) – (14)] involves integration over the wave functions of the initial and final states, is relevant as a macroscopic physical measurable. However, in real-life experiments performed in extended media, time-resolved photoelectron yield measurements are prone to distortions caused by the sensitivity of $J(t)$ to the carrier–envelope phase of the driver field, which may be a function of spatial coordinates. Additional precautions should be taken when the information on $J(t)$ is retrieved from the spectra of optical harmonics, e.g., using approaches developed in Refs. 21 and 21, rather from direct time-resolved measurements on the photoelectron yield [18], since optical harmonics are generated through wave-mixing processes with $J(t)$ playing the role of a source term, which inevitably involves propagation effects, such as phase matching.

The Keldysh time $\gamma/(2\omega) = \tau_b = (mI_0/2)^{1/2}(eE_0)^{-1}$ is sometimes viewed [33, 34] as a particular case of the Büttiker–Landauer (BL) tunneling time $\tau_{BL} = md/(\hbar\kappa)$, defined [35] for an electron of mass m tunneling through a rectangular barrier with a width d, $\hbar\kappa$ being the magnitude of the imaginary momentum of the electron under the barrier. This point of view is certainly justified in a sense that the Keldysh theory treats field-induced ionization in terms of a modulated potential barrier. In this regard, the Keldysh time τ_b can indeed be viewed, at least formally, as a BL time for a triangular barrier with a width $d = I_0/(eE_0)$. However, this does not mean that laser-induced ionization can be treated as a particular case of electron tunneling through the BL-type oscillating barrier. Indeed the BL oscillating barrier is not reduced to the time-dependent potential barrier encountered in photoionization induced by an ultrashort laser pulse. As a consequence, the transmission probability T_{BL} calculated for the

BL potential barrier does not recover the Keldysh-theory photoionization rate as its particular case. To make matters even more complicated, the interpretation of the BL time as the duration of the tunneling process has been a subject of debate because this interpretation is largely based on the sensitivity of T_{BL} to the electron energy [33, 34]. Since the electron energy in laser-induced ionization rapidly oscillates as a function of time within the field cycle, relating the times τ_0 and τ_e of this paper to the beneath-the-barrier passage time in the spirit of the BL treatment, beyond a simple observation that Eq. (12) recovers the $d = I_0/(eE_0)$ result at $\theta = 0$, is anything but trivial.

To summarize, when applied to subcycle ionization dynamics, the $\gamma << 1$ condition automatically translates into a criterion of both adiabaticity and photoionization via tunneling. We have demonstrated that the γ/ω ration can indeed be connected to a typical time scale of photoionization, τ_0. However, this time is not related to the beneath-the-barrier passage time $\tau_b = d/v$, which is defined as the time needed for a classical particle to cover a distance equal to the thickness of the potential barrier. Instead, the time τ_0 has been shown to be related to an ultrafast buildup of the electron pondermotive energy within a small fraction of the driver field cycle, manifesting itself in the experimentally measurable buildup of the photoelectron yield.

Inspiring and insightful discussions with Prof. L.V. Keldysh are gratefully acknowledged. This research was supported in part by the Russian Foundation for Basic Research (project nos. 14-29-07182 and 16-02-00843), the Russian Science Foundation (project no. 14-12-00772), the Welch Foundation (Grant No. A-1801), and ONR (Award No. 00014-16-1-2578).

References

[1] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1945 (1964) [Sov. Phys. JETP **20**, 1307 (1965)].

[2] L. DiMauro, M. Frolov, K.L. Ishikawa, and M. Ivanov, J. Phys. B **47**, 200301 (2014).

[3] N. Bloembergen, IEEE J. Quantum Electron. **10**, 375 (1974).

[4] M. Lenzner, J. Krüger, S. Sartania, Z. Cheng, C. Spielmann, G. Mourou, W. Kautek, and F. Krausz, Phys. Rev. Lett. **80**, 4076 (1998).

[5] T. Brabec and F. Krausz, Rev. Mod. Phys. **72**, 545 (2000)

[6] P. Colosimo, G. Doumy, C. I. Blaga, J. Wheeler, C. Hauri, F. Catoire, J. Tate, R. Chirla, A. M. March, G. G. Paulus, H. G. Muller, P. Agostini, and L. F. DiMauro, Nature Phys. **4**, 386 (2008).

[7] M. D. Thomson, M. Kress, T. Löffler, and H. G. Roskos, Laser Photon. Rev. **1**, 349 (2007).

[8] K. Y. Kim, A. J. Taylor, J. H. Glownia, and G. Rodriguez, Nature Photonics **2**, 605 (2008).

[9] A. Couairon and A. Mysyrowicz, Phys. Rep. **441**, 47 (2007).

[10] L. Bergé, S. Skupin, R. Nuter, J. Kasparian, and J.-P. Wolf, Rep. Prog. Phys. **70**, 16331713 (2007).

[11] R. Landauer and T. Martin, Rev. Mod. Phys. **66**, 217 (1994).

[12] E. H. Hauge and J. A. Støvneng, Rev. Mod. Phys. **61**, 917 (1989).

[13] A. S. Landsman and U. Keller, Physics Reports **547**, 1 (2015)

[14] In its English translation, the opening paragraph of Ref. 1 reads "An essential feature of the tunnel effect, a feature of importance in practical applications, is the practical absence of time lag. In other words, the probability of tunneling remains constant up to the highest frequencies of the radio band. The reason for this is that the tunneling time is determined essentially by the mean free time of the electron passing through a barrier of width $l = I/(eF)$, where I – ionization potential and F – electric field intensity. The average electron velocity is of the order of $(I/m)^{1/2}$ (m – electron mass). Therefore, up to frequencies on the order of $\omega_t = eF/(2mI)^{1/2}$ the tunnel effect is determined simply by the instantaneous value of the field intensity."

[15] P. B. Corkum and F. Krausz, Nature Phys. **3**, 381 (2007).

[16] M. Schultze, E. M. Bothschafter, A. Sommer, S. Holzner, W. Schweinberger, M. Fiess, M. Hofstetter, R. Kienberger, V. Apalkov, V. S. Yakovlev, M. I. Stockman, and F. Krausz, Nature **493**, 75 (2013).

[17] F. Krausz and M. I. Stockman, Nature Photonics **8**, 205 (2014).

[18] M. Uiberacker, T. Uphues, M. Schultze, A. Verhoef, V. Yakovlev, M. Kling, J. Rauschenberger, N. Kabachnik, H. Schröder, M. Lezius, K. Kompa, H.-G. Muller, M. Vrakking, S. Hendel, U. Kleineberg, U. Heinzmann, M. Drescher, and F. Krausz, Nature **446**, 627 (2007).

[19] P. Eckle, A. N. Pfeiffer, C. Cirelli, A. Staudte, R. D□orner, H. G. Muller, M. B□uttiker, and U. Keller, Science **322**, 1525 (2008).

[20] A. J. Verhoef, A. V. Mitrofanov, E. E. Serebryannikov, D. V. Kartashov, A. M. Zheltikov, and A. Baltuška, Phys. Rev. Lett. **104**, 163904 (2010).

[21] A. V. Mitrofanov, A. J. Verhoef, E. E. Serebryannikov, J. Lumeau, L. Glebov, A. M. Zheltikov, and A. Baltuška, Phys. Rev. Lett. **106**, 147401 (2011).

[22] A. N. Pfeiffer, C. Cirelli, M. Smolarski, D. Dimitrovski, M. Abu-samha, L. B. Madsen, and U. Keller, Nature Phys. **8**, 76 (2012).

[23] D. Shafir, H. Soifer, B. D. Bruner, M. Dagan, Y. Mairesse, S. Patchkovskii, M. Y. Ivanov, O. Smirnova, and N. Dudovich, Nature **485**, 343 (2012).

[24] A.S. Landsman, M. Weger, J. Maurer, R. Boge, A. Ludwig, S. Heuser, C. Cirelli, L. Gallmann, and U. Keller, Optica **1**, 343 (2014)

[25] N. Teeny, E. Yakaboylu, H. Bauke, and C.H. Keitel, Phys. Rev. Lett. **116**, 063003 (2016).

[26] T. Zimmermann, S. Mishra, B.R. Doran, D.F. Gordon, and A.S. Landsman, Phys. Rev. Lett. **116**, 233603 (2016).

[27] H. Ni, U. Saalmann, and J.-M. Rost Phys. Rev. Lett. **117**, 023002 (2016)

[28] A.S. Landsman and U. Keller, J. Phys. B, **47**, 204024, 2014

[29] R. Pazourek, S. Nagele, and J. Burgdörfer. Rev. Mod. Phys. **87**, 765 (2015)

[30] C. R. McDonald, G. Orlando, G. Vampa, and T. Brabec, Phys. Rev. Lett. **111**, 090405 (2013).

[31] G. Orlando, C.R. McDonald, N.H. Protik, and T. Brabec, Phys. Rev. A **89**, 014102 (2014)

[32] A.M. Perelomov, V.S. Popov, and M.V. Terent'ev, Zh. Eksp. Teor. Fiz. **50**, 1393 (1966) [JETP **23**, 924 (1966)].

[33] R. Landauer and Th. Martin, Rev. Mod. Phys. **66**, 217 (1994).

[34] E.H. Hauge and J.A. Støvneng, Rev. Mod. Phys. **61**, 917 (1989).

[35] M. Büttiker and R. Landauer, Phys. Rev. Lett. **23**, 1739 (1982).

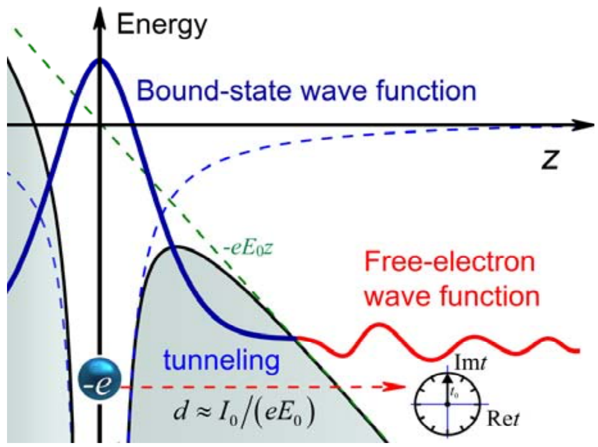

Fig. 1. Tunneling of an electron through the potential of an atomic nucleus (blue dashed line) modified by the electric field E_0 (the energy of interaction with this field is shown by the green dashed line). The wave function of the initial bound state of an electron is shown by the blue line. The wave function of a free-electron in an ac electric field is shown by the red line. Tunneling via beneath-the-barrier passage is shown by the dashed arrow. The dial shows the imaginary time related to this process.

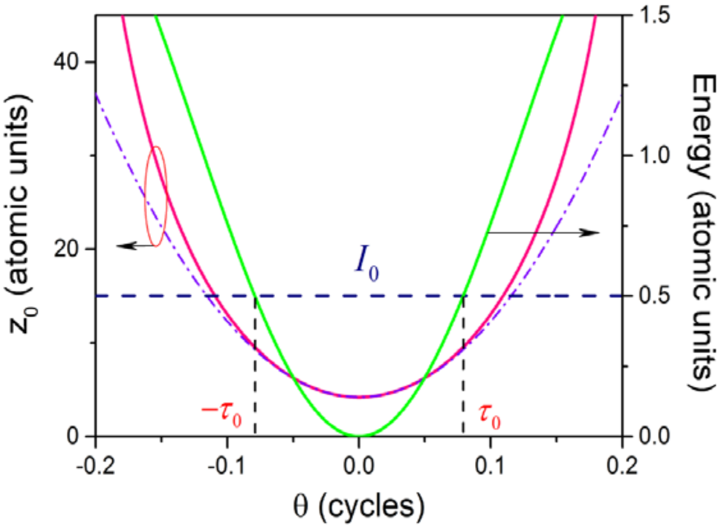

Fig. 2. The thickness z_0 of the potential barrier formed by the potential of an atomic nucleus and a laser field with a field intensity of 500 TW/cm^2 and a wavelength of 800 nm found as the solution of the $\Phi(z,\theta) = 0$ equation (red solid line) and using the approximate solution to this equation given by Eq. (12) (dash – dotted line). The green solid line shows the pondermotive energy U_p of the electron as a function of time θ, measured in field cycles. Also shown is the time τ_0 required for electron to gain the potential energy U_p equal to the ionization potential I_0 (shown by the horizontal dashed line).