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Engineering squeezed phonon reservoir with a bichromatic driving of a quantum dot

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We demonstrate how an acoustic phonon bath when coupled to a quantum dot with the help of a bichromatic laser field may effectively form a quantum squeezed reservoir. This approach allows to achieve an arbitrary degree of squeezing of the effective reservoir and it incorporates the properties of the reservoir into two parameters, which can be controlled by varying the ratio of the Rabi frequencies of the bichromatic field. It is found that for unequal Rabi frequencies, the effective reservoir may appear as a quantum squeezed field of ordinary or inverted harmonic oscillators. When the Rabi frequencies are equal the effective reservoir appears as a perfectly squeezed field in which the decay of one of the polarization quadratures of the quantum dot dipole moment is inhibited. The decay of the quantum dot to a stationary state which depends on the initial coherence is predicted. This unusual result is shown to be a consequence of a quantum-nondemolition type coupling of the quantum dot to the engineered squeezed reservoir. The effect of the initial coherence on the steady-state dressed-state population distribution and the fluorescence spectrum is discussed in details. The complete polarization of the dressed state population and asymmetric spectra composed of only a single Rabi sideband peak are obtained under strictly resonant excitation.

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I. INTRODUCTION

The study of the effect of phonons on the dynamics and coherent excitation of a quantum dot has been a subject of considerable interest in recent years. A number of different situations have been investigated [1–15]. It includes experimental studies of the effect of phonons on the Rabi oscillations, Autler-Townes splitting, and the Mollow triplet of the fluorescence field emitted by a driven quantum dot [16–24]. In particular, it has been observed that the linewidths of the Rabi sidebands of the Mollow triplet increase linearly with temperature and with the square of the driving field strength [25, 26]. These properties of the spectrum have been explained as arising from the coupling of the exciton transition of the quantum dot to longitudinal acoustic phonons [27]. Moreover, an interesting phenomenon of population inversions between the excitonic states of a quantum dot located inside an optical cavity and interacting with a phonon bath has been demonstrated both theoretically and experimentally [28–32]. The investigation of this interaction in a quantum dot-cavity system has led to the prediction of single photon sources and the realization of single-photon devices [33–36]. The influence of a phonon bath on the photon blockade effect in a driven dot-cavity system and the emission of correlated and entangled photons has also been treated [37–39].

It is well known that the decay of a quantum system can be controlled and significantly modified by coupling the system to a squeezed vacuum field, which is characterized by the

noise in one of the field quadrature components reduced below the usual vacuum level [40–44]. Particularly interesting effects of the squeezed vacuum field on the atomic radiative processes are the inhibition of the atomic decay and its dependence on the squeezing phase. However, there are many practical problems with the application of the squeezed field produced by an external sources [45–50]. The main obstacle is due to the requirement that nearly all the modes to which the quantum system is coupled must be squeezed. In addition, there is a general lack of squeezed light sources coinciding with convenient atomic transitions [51]. It has been proposed that these difficulties could be circumvented by engineering a squeezed-reservoir-type interaction of a quantum system rather than coupling the system to a squeezed field produced by an external source [52, 53]. For example, Lütkenhaus *et al.* [54] have studied the dynamics of a four-level system driven by two laser fields and have shown that the system may effectively behave as a two-level system coupled to a squeezed reservoir.

In this paper we propose a method to construct a squeezed-vacuum type multimode reservoir from a phonon bath based on a suitable engineering of the coupling of the phonon bath to a quantum dot. The quantum dot is modeled as a two-level system and experiences fluctuations and decay of its excitation that are due to the dissipative interaction with the phonon bath. We derive the master equation for the reduced density operator of the quantum dot and show that the phonon bath combined together with a bichromatic laser field tuned close to the dot's transition frequency can result in a squeezed reservoir type interaction of the phonon bath with the quantum dot. We find that the squeezing properties of the effective reservoir and then the quantum dot's relaxation dynamics can be con-

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trolled through variation of certain tunable system parameters e.g., the Rabi frequencies of the bichromatic field. By varying the ratio between the Rabi frequencies of the bichromatic field the effective squeezed reservoir displays interesting differences in its properties. In particular, the reservoir may appear as a quantum squeezed field of ordinary or inverted harmonic oscillators, or can behave as a perfectly squeezed field. When in addition to the interaction with the squeezed reservoir, the quantum dot is driven by a resonant laser field we find that the steady-state dressed state population distribution and in the properties of the fluorescence field can be governed by the initial coherence between the ground and excited states of the quantum dots. The fluorescence spectrum can be asymmetric and its structure varied with the initial coherence. We show that the asymmetries are manifestation of the complete polarization of the dressed state populations, and thus the spectrum offers a method of observing the polarization.

The paper is organized as follows. In Sec. II, we describe the model and derive the master equation for the reduced density operator of a quantum dot interacting with a low frequency phonon bath and driven by a bichromatic laser field. In Sec. III, we examine the conditions for quantum features of the engineered reservoir and their dependence on the number of phonons. We distinguish between different forms of the squeezed phonon reservoir which can be engineered, including a perfectly squeezed reservoir and a squeezed reservoir of inverted harmonic oscillators. In Sec. IV, we concentrate on the dynamics of the quantum dot which in addition to the interaction with the engineered reservoir is driven by a resonant laser field. We are particularly interested in the stationary state and its dependence on the form of the engineered squeezed reservoir. Section V is devoted to the discussion of the fluorescence spectrum. The dependence of the stationary spectrum on the initial coherence is exhibited and explained in terms of the dressed states of the driven quantum dot. The results are summarized in Sec. VI.

II. DESCRIPTION OF THE SYSTEM

We consider a single quantum dot (QD) coupled to a low frequency phonon bath and driven by a bichromatic laser field. The quantum dot is modeled as a two-level system with the upper state $|e\rangle$, the ground state $|g\rangle$, transition frequency ω_0 , and transition dipole moment $\vec{\mu}$. The driving field is characterized by two frequencies ω_1 and ω_2 and the amplitudes $\vec{\mathcal{E}}_1$ and $\vec{\mathcal{E}}_2$, respectively. The components of the laser field are tuned near the atomic resonance, at detunings $\Delta_1 = \omega_1 - \omega_0$ and $\Delta_2 = \omega_0 - \omega_2$, as illustrated in Fig. 1. The phonon bath is treated as a quantized multi-mode reservoir. In practice this scheme could be realized by the bichromatic driving of an exciton transition between the semiconductor ground state and single exciton state of an InAs/GaAs quantum dot. Typical parameters of experimental samples of quantum dots [17, 20] are shapes with heights of 3 – 5 nm, base diameters of 25 – 30 nm, the exciton transition wavelength $\lambda_0 = 950$ nm ($\omega_0/2\pi = 300$ THz).

The total Hamiltonian of the system can be written in the

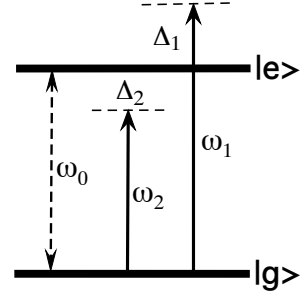


FIG. 1: Two-level system driven by a bichromatic laser field of frequencies ω_1 and ω_2 tuned close to the atomic transition frequency ω_0 at detunings Δ_1 and Δ_2 , respectively.

form

$$H = H_0 + H_1 + H_2, \quad (1)$$

where H_0 is the Hamiltonian of the phonon field (setting $\hbar = 1$ throughout the paper)

$$H_0 = \sum_p \omega_p b_p^\dagger b_p, \quad (2)$$

H_1 is the Hamiltonian of the quantum dot plus the interaction with the bichromatic laser field

$$H_1 = \omega_0 S_z + \left[\Omega_1 e^{-i(\omega_1 t - \phi_1)} + \Omega_2 e^{-i(\omega_2 t - \phi_2)} \right] S^+ + \text{H.c.}, \quad (3)$$

and H_2 is the interaction Hamiltonian of the quantum dot with the phonon reservoir

$$H_2 = \sum_p g_p S_z (b_p + b_p^\dagger). \quad (4)$$

Here b_p^\dagger and b_p are the creation and annihilation operators of mode p of frequency ω_p of the phonon bath, $S^+(S^-)$ is the raising (lowering) operator and S_z is the population difference operator of the quantum dot, and g_p is the coupling strength of the mode p of the phonon reservoir to the quantum dot. The parameters Ω_1 and Ω_2 are the Rabi frequencies between the quantum dot and the components of the laser field, which are given by the product of the atomic transition dipole moment $\vec{\mu}$ and the laser field amplitudes $\vec{\mathcal{E}}_1$ and $\vec{\mathcal{E}}_2$, respectively.

To remove the fast oscillating terms in Eq. (3), we transform the Hamiltonian into a frame rotating with the frequency ω_0 and obtain

$$\tilde{H}_1 = \left(\Omega_1 e^{-i(\Delta_1 t - \phi_1)} + \Omega_2 e^{i(\Delta_2 t + \phi_2)} \right) S^+ + \text{H.c.} \quad (5)$$

We now derive the master equation for the reduced density operator ρ of the quantum dot subject of the driving bichromatic field and the low frequency phonon reservoir. In the treatment, we derive the effective interaction Hamiltonian between the driven QD and the phonon bath, and the derivation closely follows the approach previously used in Refs. [6, 7, 22, 25].

We assume that the bichromatic field is weak so that the dynamics of the QD are mostly affected by the interaction with the phonon reservoir. In order to analyze the effect of the phonon reservoir on the QD, we define a unitary operator

$$U = i \sum_p \frac{g_p}{\omega_p} (b_p^\dagger - b_p) S_z, \quad (6)$$

and make the unitary transformation of the Hamiltonian of the system. Hence, we obtain

$$H_T = e^{-iU} H e^{iU} = H_R + H_I, \quad (7)$$

where

$$\begin{aligned} H_R &= e^{-iU} (H_0 + H_2) e^{iU} \\ &= \sum_p \hbar \omega_p b_p^\dagger b_p - \sum_p \frac{g_p^2}{4\omega_p}, \end{aligned} \quad (8)$$

and

$$\begin{aligned} H_I &= e^{-iU} \tilde{H}_1 e^{iU} = [\Omega(t) S^+ + \text{H.c.}] \\ &+ \left[\sum_p \frac{g_p \Omega(t)}{\omega_p} (b_p^\dagger - b_p) S^+ + \text{H.c.} \right] + \dots, \end{aligned} \quad (9)$$

in which

$$\Omega(t) = (\tilde{\Omega}_1 e^{-i\Delta_1 t} + \tilde{\Omega}_2 e^{i\Delta_2 t}), \quad (10)$$

with $\tilde{\Omega}_i = \langle B \rangle \Omega_i \exp(i\phi_i)$, and

$$\langle B \rangle = \exp \left[-\frac{1}{2} \sum_p (g_p/\omega_p)^2 (2\bar{n}_p + 1) \right]. \quad (11)$$

Here, $\Omega(t)$ is the total time-dependent Rabi frequency of the driving laser and $\bar{n}_p \equiv \langle b_p^\dagger b_p \rangle = [\exp(\omega_p/k_B T_p) - 1]^{-1}$ is the average occupation phonon number of a mode p , where k_B is the Boltzmann constant and T_p corresponds to the temperature of the reservoir. The first term in Eq. (8) represents the energy of the phonon reservoir, while the second term represents a shift of the energy levels of the QD due to the interaction with the phonon reservoir. The shift is known in the literature as the Lamb shift. This term is usually considered to be absorbed into the atomic transition frequency and is not included explicitly in the dynamics of the system. Therefore, the Hamiltonian (8) can be simply considered as the energy of the phonon reservoir.

The first term on the right-hand side of Eq. (9) contains the interaction of the QD with the driving laser field. The second term represents the interaction of the QD with the phonon reservoir. It is in a form of the electric dipole interaction in which the phonon reservoir couples to the dipole transition of the QD. In the derivation of Eq. (9), we have performed a Taylor expansion and have kept only the terms up to first-order in g_p . With the higher-order terms ignored, we simply limit the interaction of the QD with the phonon reservoir to one-phonon processes only.

It is worthwhile noting at this point that the main result of the unitary transformation of the Hamiltonian of the system is the coupling of the phonon reservoir to the atomic dipole moment. Thus, with the help of the driving laser, the low frequency phonon reservoir effectively couples to the atomic dipole transition $|g\rangle \leftrightarrow |e\rangle$.

We may transform the Hamiltonian (7) into the interaction picture with the unitary operator $U(t) = e^{iH_R t}$, and find

$$\tilde{H}_T = e^{-iH_R t} H_T e^{iH_R t} = V_L(t) + V_R(t), \quad (12)$$

where $V_L(t)$ is the interaction of the laser with the QD, and

$$V_R(t) = \sum_p \frac{g_p \Omega(t)}{\omega_p} (b_p^\dagger e^{i\omega_p t} - b_p e^{-i\omega_p t}) S^+ + \text{H.c.} \quad (13)$$

is the interaction of the QD with the phonon reservoir. The interaction $V_R(t)$ can be written explicitly as

$$\begin{aligned} V_R(t) &= \sum_p \frac{g_p}{\omega_p} \left\{ \left[b_p^\dagger \left(\tilde{\Omega}_1 e^{i(\omega_p - \Delta)t} + \tilde{\Omega}_2 e^{i(\omega_p + \Delta)t} \right) \right. \right. \\ &\quad \left. \left. - b_p \left(\tilde{\Omega}_1 e^{-i(\omega_p + \Delta)t} + \tilde{\Omega}_2 e^{-i(\omega_p - \Delta)t} \right) \right] S^+ + \text{H.c.} \right\}, \end{aligned} \quad (14)$$

where we have assumed that the components of the bichromatic field are equally detuned from the atomic transition frequency, i.e. $\Delta_1 = \Delta_2 \equiv \Delta$.

We see from Eq. (14) that the interaction contains terms which oscillate at frequencies $\omega_p - \Delta$ and $\omega_p + \Delta$. If the density of modes of the phonon reservoir is large only in the vicinity of the laser field detuning Δ , then $\omega_p \approx \Delta$. In such a case, the interaction Hamiltonian reduces to resonant, non-oscillating terms, and nonresonant terms oscillating at frequency 2Δ . We can make the rotating-wave approximation in which the resonant terms play a dominant role whereas the nonresonant terms make much smaller contributions and can be omitted. The interaction Hamiltonian $V_R(t)$ then simplifies to

$$\begin{aligned} V_R(t) &= \sum_p \frac{g_p}{\omega_p} \left\{ \left[\tilde{\Omega}_1 b_p^\dagger e^{i(\omega_p - \Delta)t} \right. \right. \\ &\quad \left. \left. - \tilde{\Omega}_2 b_p e^{-i(\omega_p - \Delta)t} \right] S^+ + \text{H.c.} \right\}. \end{aligned} \quad (15)$$

Having derived the effective interaction Hamiltonian of the driven QD with the phonon reservoir, we now turn to the derivation of the master equation for the reduced density operator of the quantum dot

$$\rho(t) = \text{Tr}_F W(t), \quad (16)$$

where $W(t)$ is the density operator of the total system, the QD plus the phonon bath. We choose an initial state with no correlations between the QD and the phonon bath modes

$$W(0) = \rho_F(0) \otimes \rho(0), \quad (17)$$

and specify the phonon bath as a vacuum thermal bath with the following correlations

$$\begin{aligned} \langle b_p \rangle &= \langle b_p^\dagger \rangle = 0, \quad \langle b_p^\dagger b_{p'} \rangle = \bar{n} \delta(p - p'), \\ \langle b_p b_{p'}^\dagger \rangle &= (\bar{n} + 1) \delta(p - p'), \end{aligned} \quad (18)$$

where \bar{n} is the average number of phonons.

After tracing over the phonon bath operators, and using the standard Born-Markov approximations, we arrive at the master equation

$$\frac{\partial}{\partial t}\rho = -i[V_L(t), \rho] + \mathcal{L}_b\rho + \mathcal{L}_p\rho, \quad (19)$$

in which

$$V_L(t) = \Omega(t)S^+ + \text{H.c.} \quad (20)$$

is the interaction of the quantum dot with the bichromatic field,

$$\mathcal{L}_b\rho = \frac{1}{2}\Gamma(2S_-\rho S_+ - S_+S_-\rho - \rho S_+S_-) \quad (21)$$

represents the damping of the quantum dot at the rate Γ by spontaneous emission to vacuum radiation modes, other than the phonon modes, and

$$\begin{aligned} \mathcal{L}_p\rho = & \gamma_s(2S^-\rho S^+ - S^+S^-\rho - \rho S^+S^-) \\ & + \gamma_n(2S^+\rho S^- - S^-S^+\rho - \rho S^-S^+) \\ & - \gamma_m(2S^+\rho S^+e^{2i\phi} + 2S^-\rho S^-e^{-2i\phi}), \end{aligned} \quad (22)$$

represents the damping of the quantum dot by a reservoir mediated by the phonon bath modes and the bichromatic field. Here, the parameters are defined as

$$\begin{aligned} \gamma_s &= \gamma_1\bar{n} + \gamma_2(\bar{n} + 1), & \gamma_n &= \gamma_1(\bar{n} + 1) + \gamma_2\bar{n}, \\ \gamma_m &= (2\bar{n} + 1)\sqrt{\gamma_1\gamma_2}, & 2\phi &= \phi_1 + \phi_2, \end{aligned} \quad (23)$$

with

$$\gamma_i = 2\pi\tilde{\Omega}_i^2 \sum_p \left(\frac{g_p}{\omega_p}\right)^2 \delta(\omega_p - \Delta), \quad i = 1, 2. \quad (24)$$

The assumption of the Born approximation, valid to second orders in the quantum dot environment coupling strengths, $|g_p|^2$ and $|f_k|^2$, where f_k is the coupling strength of the k th radiation mode to the quantum dot, restricts the master equation to weak-system reservoir coupling regimes. The assumption of the Markov approximation restricts the master equation to times t longer than the time Δt required for a phonon to traverse the quantum dot, $t \gg \Delta t = l/u$, where l is the size of a quantum dot and u is the speed of sound. Based on the typical sizes of experimental samples of an InAs/GaAs quantum dot of $l = 5$ nm, $\Delta t \approx 1$ ps. For the Markov approximation to be valid, the time Δt should be shorter than any relaxation time in the system. In practice this may well be a reasonable assumption. For example, the radiative recombination time of the exciton, determined by $1/\Gamma$, is usually taken 500 – 800 ps [11, 21].

The Liouvillian (22) has a structure analogous to the damping of a two-level system by a squeezed reservoir. The parameters γ_s and γ_n correspond to incoherent damping and incoherent pumping rates, respectively, and γ_m corresponds to the strength of two-photon correlations. A close look at the parameters in Eq. (23) reveals that not always γ_s constitutes

the incoherent rate at which a population is damped by the reservoir. We can have $\gamma_s > \gamma_n$ as well as $\gamma_n > \gamma_s$. Which of these takes place depends principally on whether $\gamma_2 > \gamma_1$ or $\gamma_1 > \gamma_2$. For $\gamma_2 > \gamma_1$, we have $\gamma_s > \gamma_n$. In this case, γ_s can be viewed as the incoherent damping rate. Otherwise, when $\gamma_2 < \gamma_1$, we have $\gamma_n > \gamma_s$ that the incoherent pumping rate exceeds the damping rate. In this case, the reservoir is formed from a bath of inverted harmonic oscillators, and the rate of transferring the population from the ground state $|1\rangle$ to the upper state $|2\rangle$ is larger than the rate of transferring the population from $|2\rangle$ to $|1\rangle$. It is easy to see from Eq. (23) that the condition of $\gamma_1 > \gamma_2$ can be achieved when the Rabi frequency $\tilde{\Omega}_1$ of the bichromatic field component tuned above the resonance exceeds the Rabi frequency $\tilde{\Omega}_2$ of the component tune below the resonance. Figure 2 illustrates the role of the parameters γ_s and γ_n in the dynamics of the quantum dot. It is seen that the roles of the parameters reverse when $\gamma_2 > \gamma_1$ reverses to $\gamma_1 > \gamma_2$.

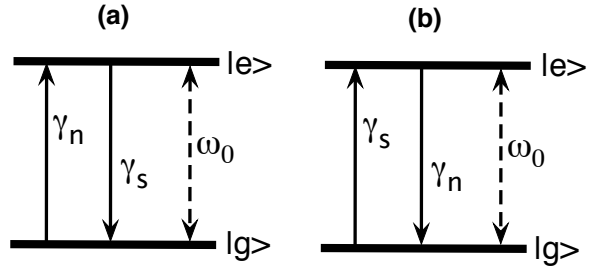


FIG. 2: Illustration of the role of the parameters γ_s and γ_n in the dynamics of the quantum dot for (a) $\gamma_2 > \gamma_1$ and (b) $\gamma_1 > \gamma_2$.

Apart from the damping by the squeezed reservoir there is also a contribution from the damping by the radiation field modes, Eq. (21). If the phonon bath modes occupy all modes to which the quantum dot is coupled, $\Gamma \sim |f_k|^2 = 0$ and then the quantum dot is damped solely by the squeezed reservoir. However, if there is a small fraction of modes not occupied by the phonon bath modes, $\Gamma \neq 0$, and then the quantum dot is damped by both the squeezed reservoir and the "unsqueezed" radiation modes.

To investigate how efficient the phonon bath together with the bichromatic field is in the creation of a squeezing type reservoir to the quantum dot, we compare the master equation (22) with the equation when a two-level system is illuminated by a squeezed vacuum field produced by an external squeezing source, such as an optical parametric oscillator [47–49]. The squeezed vacuum field is characterized by the correlation functions [40–44]

$$\begin{aligned} \langle a_k a_{k'}^\dagger \rangle &= (N + 1)\delta_{k,k'}, \\ \langle a_k^\dagger a_{k'} \rangle &= N\delta_{k,k'}, \\ \langle a_k a_{k'} \rangle &= |M|e^{-2i\Phi}\delta_{2k_s - k, k'}, \\ \langle a_k^\dagger a_{k'}^\dagger \rangle &= |M|e^{2i\Phi}\delta_{2k_s - k, k'}, \end{aligned} \quad (25)$$

where $a_k(a_k^\dagger)$ is the annihilation (creation) operator for mode k of the squeezed field, N is the number of photons in the

field, the parameter $|M|$ determines the degree of two-photon (squeezing) correlations between modes symmetrically located about the squeezing carrier mode $2k_s$, and Φ is the phase of the field. The parameter $|M|$ may fall into one of the two separate regions

$$|M| - N < 0 \quad \text{or} \quad N < |M| \leq \sqrt{N(N+1)}. \quad (26)$$

If $|M|$ falls into the region of $|M| < N$, the field corresponds to the so-called classically squeezed field in the sense that fluctuations in one of the quadratures of the field amplitudes are reduced but not below the shot-noise level. If $|M|$ falls into the region of $N < |M| \leq \sqrt{N(N+1)}$, the field is then a quantum squeezed field in the sense that the fluctuations of one of the quadratures are suppressed below the shot-noise level. The equality $|M| = \sqrt{N(N+1)}$ corresponds to maximal correlations, an ideal squeezed field. Thus, there are lower and upper limits, $|M| = N$ and $|M| = \sqrt{N(N+1)}$, respectively, for the quantum correlations of the squeezed field.

The interaction of a two-level system with a reservoir characterized by the correlation functions (25) leads to the following master equation [40–44]

$$\begin{aligned} \mathcal{L}_p \rho = & \frac{1}{2} \gamma (N+1) (2S^- \rho S^+ - S^+ S^- \rho - \rho S^+ S^-) \\ & + \frac{1}{2} \gamma N (2S^+ \rho S^- - S^- S^+ \rho - \rho S^- S^+) \\ & - \frac{1}{2} \gamma |M| (2S^+ \rho S^+ e^{2i\Phi} + 2S^- \rho S^- e^{-2i\Phi}), \end{aligned} \quad (27)$$

where γ is the spontaneous emission rate of the atomic transition.

Matching coefficients in Eqs. (22) and (27), we find that for $\gamma_2 > \gamma_1$:

$$\gamma_s - \gamma_n \rightarrow \frac{1}{2} \gamma, \quad \gamma_n \rightarrow \frac{1}{2} \gamma N, \quad \gamma_m \rightarrow \frac{1}{2} \gamma |M|. \quad (28)$$

Similarly, for $\gamma_1 > \gamma_2$:

$$\gamma_n - \gamma_s \rightarrow \frac{1}{2} \gamma_I, \quad \gamma_s \rightarrow \frac{1}{2} \gamma_I N, \quad \gamma_m \rightarrow \frac{1}{2} \gamma_I |M|, \quad (29)$$

where the subscript "I" stands for inverted harmonic oscillator. Thus, the effective squeezing type reservoir can be described by field operators b_k and b_k^\dagger satisfying the following correlation functions ($\gamma_2 > \gamma_1$):

$$\begin{aligned} \langle b_k b_{k'}^\dagger \rangle &= \frac{\gamma_s}{\gamma_2 - \gamma_1} \delta_{k,k'}, \\ \langle b_k^\dagger b_{k'} \rangle &= \frac{\gamma_n}{\gamma_2 - \gamma_1} \delta_{k,k'}, \\ \langle b_k b_{k'} \rangle &= \frac{\gamma_m}{\gamma_2 - \gamma_1} e^{-2i\phi} \delta_{2k_s - k, k'}, \\ \langle b_k^\dagger b_{k'}^\dagger \rangle &= \frac{\gamma_m}{\gamma_2 - \gamma_1} e^{2i\phi} \delta_{2k_s - k, k'}. \end{aligned} \quad (30)$$

As we have mentioned above, there are lower and upper limits for the quantum correlations of the squeezed field. If

we evaluate the lower limit $|M| = N$ according to Eq. (28), we find the result

$$|M| - N = \frac{\sqrt{\gamma_1} - \bar{n} (\sqrt{\gamma_2} - \sqrt{\gamma_1})}{\sqrt{\gamma_1} + \sqrt{\gamma_2}}, \quad (31)$$

and

$$|M| - N = \frac{\sqrt{\gamma_2} - \bar{n} (\sqrt{\gamma_1} - \sqrt{\gamma_2})}{\sqrt{\gamma_1} + \sqrt{\gamma_2}}, \quad (32)$$

if we evaluate the limit according to Eq. (29). It is seen from Eqs. (31) and (32) that for $|M|$ to fall into the region of quantum squeezing, $|M| - N > 0$, it is necessary that

$$\begin{aligned} \bar{n} &< \frac{\sqrt{\gamma_1}}{\sqrt{\gamma_2} - \sqrt{\gamma_1}}, \quad \gamma_2 > \gamma_1, \\ \bar{n} &< \frac{\sqrt{\gamma_2}}{\sqrt{\gamma_1} - \sqrt{\gamma_2}}, \quad \gamma_1 > \gamma_2, \end{aligned} \quad (33)$$

and if these conditions hold, then $N < |M| \leq \sqrt{N(N+1)}$. Thus, the phonon bath with the help of the bichromatic field creates a squeezed reservoir which can be unique to the quantum field.

If we evaluate the upper limit for the correlations, $|M|^2 - N(N+1) = 0$, which determines the maximal two-photon (squeezing) correlations in the field, we find

$$|M|^2 - N(N+1) = -\bar{n}(\bar{n}+1), \quad (34)$$

for both $\gamma_2 > \gamma_1$ and $\gamma_1 > \gamma_2$ cases. We see that maximal correlations are achieved only at $\bar{n} = 0$. For $\bar{n} \neq 0$ the reservoir appears as an imperfectly squeezed reservoir with the correlations decreasing with an increasing \bar{n} . It is interesting to note that the upper limit for the correlations is independent of γ_1 and γ_2 , whereas the lower limit, as seen from Eqs. (31) and (32), varies with γ_1 and γ_2 .

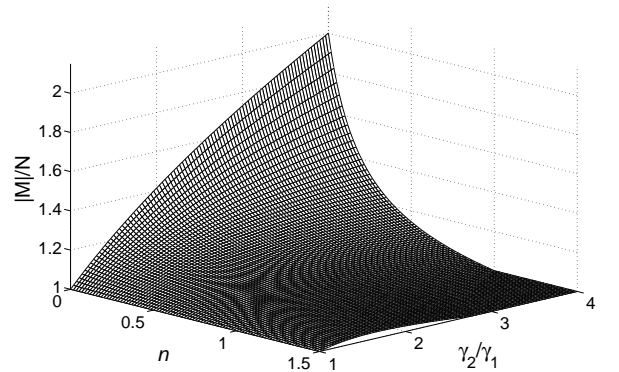


FIG. 3: The ratio $|M|/N$ plotted as a function of \bar{n} and γ_2/γ_1 . The quantum nature of the correlations ($|M|/N > 1$) occurs for $\bar{n} < 1/(\sqrt{\gamma_2/\gamma_1} - 1)$.

Figure 3 shows the ratio $|M|/N$ as a function of \bar{n} and γ_2/γ_1 for the case of $\gamma_2 > \gamma_1$. Values $|M|/N > 1$ signal

the quantum nature of the correlations. The correlations increase with an increasing γ_2/γ_1 and can reach large values but the largest increase of the correlations above the classical limit occurs for weak squeezed fields, i.e., for large γ_2/γ_1 at which N is small. The correlations are very sensitive to \bar{n} . For not too large γ_2/γ_1 , the ratio decreases slowly with \bar{n} so that the quantum nature of the correlations persists at large \bar{n} . However, for large γ_2/γ_1 , the decrease of the ratio with \bar{n} is considerably more rapid than it is for small γ_2/γ_1 that the quantum correlations decrease rapidly with \bar{n} . Beyond $\bar{n} = 1/(\sqrt{\gamma_2/\gamma_1} - 1)$, the ratio falls below the lower limit for quantum correlations.

The case $\gamma_1 = \gamma_2$ has to be treated separately. In the limit $\gamma_1 = \gamma_2 \equiv \gamma_0$, the coefficients in Eq. (22) are

$$\gamma_s = \gamma_n = \gamma_m = (2\bar{n} + 1)\gamma_0. \quad (35)$$

When compared with the coefficients of Eq. (27), we find

$$\begin{aligned} \gamma_s &\rightarrow \frac{1}{2}\gamma(N+1), & \gamma_n &\rightarrow \frac{1}{2}\gamma N, \\ \gamma_m &\rightarrow \frac{1}{2}\gamma|M| = \sqrt{\gamma_n\gamma_s} \rightarrow \frac{1}{2}\gamma\sqrt{N(N+1)}. \end{aligned} \quad (36)$$

Since $\gamma_s = \gamma_n$, this limit can be regarded as corresponding to a very strong squeezed field, $N \gg 1$, with maximal correlations $|M| = \sqrt{N(N+1)}$. We see that in this case the upper limit of the squeezing correlations is achieved for any values of \bar{n} .

III. DYNAMICS OF THE QUANTUM DOT

Let us now apply these considerations explicitly to the dynamics of the quantum dot interacting with the engineered squeezed reservoir. The dynamical response of the quantum dot interacting with the squeezed reservoir is best described in terms of the expectation values of the dipole components which obey the following optical Bloch equations

$$\begin{aligned} \langle \dot{S}_x \rangle &= -\left(\frac{1}{2}\Gamma + \gamma_s + \gamma_n + 2\gamma_m \cos 2\phi\right) \langle S_x \rangle \\ &\quad + 2\gamma_m \sin 2\phi \langle S_y \rangle, \\ \langle \dot{S}_y \rangle &= -\left(\frac{1}{2}\Gamma + \gamma_s + \gamma_n - 2\gamma_m \cos 2\phi\right) \langle S_y \rangle \\ &\quad + 2\gamma_m \sin 2\phi \langle S_x \rangle, \\ \langle \dot{S}_z \rangle &= -\left(\gamma_s - \gamma_n + \frac{1}{2}\Gamma\right) \langle S_z \rangle - 2\left(\gamma_s + \gamma_n + \frac{1}{2}\Gamma\right) \langle S_z \rangle, \end{aligned} \quad (37)$$

where $S_x = (S^- + S^+)/2$ and $S_y = i(S^- - S^+)/2$ are the dipole polarization components. The effect of squeezing is best seen through the quadrature phase components

$$\begin{aligned} S_\phi(t) &= S_x(t) \sin \phi + S_y(t) \cos \phi, \\ S_{\phi+\frac{\pi}{2}}(t) &= S_x(t) \cos \phi - S_y(t) \sin \phi. \end{aligned} \quad (38)$$

When the equations (37) are integrated, we obtain

$$\begin{aligned} \langle S_\phi(t) \rangle &= \langle S_\phi(0) \rangle e^{-(\frac{1}{2}\Gamma + \gamma_s + \gamma_n - 2\gamma_m)t}, \\ \langle S_{\phi+\frac{\pi}{2}}(t) \rangle &= \langle S_{\phi+\frac{\pi}{2}}(0) \rangle e^{-(\frac{1}{2}\Gamma + \gamma_s + \gamma_n + 2\gamma_m)t}, \\ \langle S_z(t) \rangle &= -\frac{\gamma_s - \gamma_n + \frac{1}{2}\Gamma}{2(\gamma_s + \gamma_n + \frac{1}{2}\Gamma)} \\ &\quad + \left[\langle S_z(0) \rangle + \frac{\gamma_s - \gamma_n + \frac{1}{2}\Gamma}{2(\gamma_s + \gamma_n + \frac{1}{2}\Gamma)} \right] e^{-2(\frac{1}{2}\Gamma + \gamma_s + \gamma_n)t}. \end{aligned} \quad (39)$$

The components display simple exponential decays, the component $\langle S_\phi(t) \rangle$ is damped at a reduced rate $\gamma_\phi = \frac{1}{2}\Gamma + \gamma_s + \gamma_n - 2\gamma_m$, while $\langle S_{\phi+\frac{\pi}{2}}(t) \rangle$ is damped at an enhanced rate $\gamma_{\phi+\frac{\pi}{2}} = \frac{1}{2}\Gamma + \gamma_s + \gamma_n + 2\gamma_m$. The population inversion $\langle S_z(t) \rangle$ decays to a steady-state value which depends strongly on the relation between γ_s and γ_n .

The decay rates depend also on Γ . Because it is precisely the effect of the phonon bath on the dynamics of the quantum dot that interests us most here, in what follows, we shall assume $\gamma_s, \gamma_n \gg \Gamma$ and set $\Gamma = 0$. This is justified if one notices from Eq. (24) that γ_s and γ_n increase with an increasing $\bar{\Omega}_i$. Thus, we may increase the Rabi frequencies of the bichromatic field such that $\gamma_{s,n} \gg \Gamma$. This situation can be achieved in current experiments since the radiative lifetime 500 – 800 ps corresponds to $\Gamma \sim 1.2$ GHz. Using the definition $J(\omega) = \sum_p g_p^2 \delta(\omega - \omega_p)$, which is equivalent to $J(\omega) = \alpha \omega^3 \exp[-(\omega/\omega_c)^2]$, where $\omega_c = \sqrt{2}u/l$ is the cut-off frequency [37], the damping parameters γ_i can be estimated by writing [26]

$$\gamma_i = 2\pi \tilde{\Omega}_i^2 \sum_p \left(\frac{g_p}{\omega_p} \right)^2 \delta(\omega_p - \Delta) = 2\pi \tilde{\Omega}_i^2 \alpha \Delta. \quad (40)$$

For the Rabi frequencies of the bichromatic field we choose the average value $\Omega_i = 70$ GHz [26]. For the phonon bath we assume $\bar{n} = 0.5$ and a temperature $T = 2.35$ K, and take $\omega_c = 1500$ GHz [37]. These give $\Delta (= \omega_p) = 490$ GHz. Taking $\alpha = 2.535 \times 10^{-7} (\text{GHz})^{-2}$ [7, 26], the damping parameters γ_i are then $\gamma_i = 4$ GHz, which are larger than $\Gamma = 1.2$ GHz.

As discussed in Sec. II, the manner in which the squeezed reservoir can affect the dynamics of the quantum dot depends on the relation between γ_1 and γ_2 . There are three distinct regimes of the parameters at which the effective squeezed reservoir can have significantly different properties, A. $\gamma_2 > \gamma_1$, B. $\gamma_1 > \gamma_2$, and C. $\gamma_1 = \gamma_2$.

A. The case, $\gamma_2 > \gamma_1$

This limit corresponds to $\gamma_s > \gamma_n$ that the effective reservoir is formed from a phonon bath of the ordinary harmonic oscillators. As showed in Sec. II, there is in this case a direct correspondence between the engineered squeezed reservoir and that produced by an external source of the squeezed vacuum field. Therefore, the effects of the engineered squeezed

reservoir on the dynamics of the quantum dot are expected to be analogous to those which are well known for a two-level atom damped by a squeezed vacuum field produced by an external source. However, there are some subtle differences. For example, if an external source produces a squeezed field with maximal correlations, $|M| = \sqrt{N(N+1)}$, the correlations remain maximal independent of the value of N . However, in the squeezed reservoir engineered from a phonon bath, an increase of the number of phonons \bar{n} is accompanied by a decrease of the two-photon correlations $|M|$, as seen from Eq. (34). As a consequence, ideally squeezed reservoir at $\bar{n} = 0$ becomes an imperfectly squeezed reservoir when $\bar{n} \neq 0$. Thus, we lose the option of having an ideally squeezed reservoir when $\bar{n} \neq 0$. Under this circumstance, many effects unique to the quantum nature of the squeezed field may not be observed due to the presence of thermal phonons.

In order to determine the range of the parameters, \bar{n} in particular, at which the effects unique to the quantum nature of the squeezed field still could be observed in the presence of thermal phonons, we write $N = N_s + N_b$ such that $N_s(N_s + 1) = |M|^2$. Thus, N_s corresponds to the number of phonons in the maximally squeezed field and N_b is the number of excess phonons, which are not correlated, and therefore correspond to a thermal (background) field. When this division of N is applied to Eq. (22), it is straightforward to find from Eq. (23) that

$$\begin{aligned} N_s &= \sqrt{4\bar{n}(\bar{n}+1)u^2 + w^2} - \frac{1}{2}, \\ N_b &= (2\bar{n}+1)w - \sqrt{4\bar{n}(\bar{n}+1)u^2 + w^2}, \end{aligned} \quad (41)$$

where $u = 2\sqrt{\gamma_1\gamma_2}/\gamma$ and $w = (\gamma_1 + \gamma_2)/\gamma$. The Liouvillian (22) can then be written in the form

$$\begin{aligned} \mathcal{L}_p\rho &= \frac{1}{2}\gamma(2\Upsilon\rho\Upsilon^\dagger - \Upsilon^\dagger\Upsilon\rho - \rho\Upsilon^\dagger\Upsilon) \\ &+ \frac{1}{2}\gamma N_b(2S^-\rho S^+ - S^+S^-\rho - \rho S^+S^-) \\ &+ \frac{1}{2}\gamma N_b(2S^+\rho S^- - S^-S^+\rho - \rho S^-S^+), \end{aligned} \quad (42)$$

where

$$\Upsilon = \sqrt{N_s+1}S^-e^{-i\phi} + \sqrt{N_s}S^+e^{i\phi}. \quad (43)$$

Thus, the Liouvillian (22) describing the damping of the quantum dot by an imperfectly squeezed reservoir has been divided into two parts, one describing damping by the maximally squeezed reservoir, the first term in Eq. (42), and the other describing damping by the background thermal reservoir, the second and third terms in (42). In other words, the interaction of the quantum dot with an imperfect squeezed reservoir may be viewed as the interaction with two separate reservoirs, a maximally correlated squeezed reservoir and a thermal reservoir. Which reservoir dominates in the interaction depends on the ratio N_b/N_s . If N_s is much larger than N_b , then significant effects of the squeezed reservoir should be observable. For example, both N_b and N_s contribute to the damping rate of the quantum dot. Therefore, the reduction of γ_ϕ below the

standard quantum limit is possible only for $N_b < |M| - N_s$. Viewed as a function of \bar{n} , the inequality $N_b < |M| - N_s$ becomes $\bar{n} < 1/(\sqrt{\gamma_2/\gamma_1} - 1)$ which is, as one could expect, the same as the condition (33) for quantum squeezing.

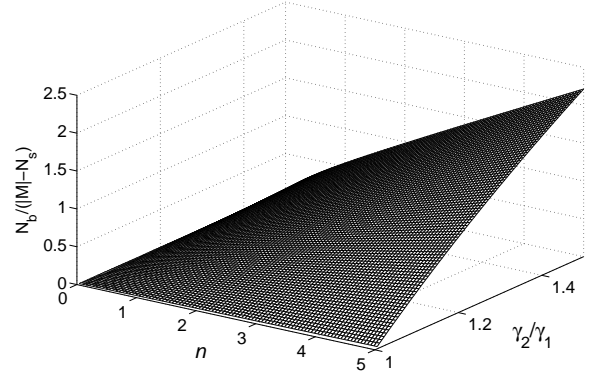


FIG. 4: The ratio of the number of the background thermal phonons N_b to the degree of quantum squeezing correlations $|M| - N_s$ plotted as a function of \bar{n} and γ_2/γ_1 .

Figure 4 shows the ratio $N_b/(|M| - N_s)$ as a function of \bar{n} and γ_2/γ_1 . The grow of the ratio with \bar{n} depends strongly on γ_2/γ_1 and the effect of increasing γ_2/γ_1 is to decrease the range of \bar{n} over which the ratio is smaller than 1.

B. The case, $\gamma_1 > \gamma_2$

The exchange $\gamma_1 \leftrightarrow \gamma_2$ leads to an exchange $\gamma_s \leftrightarrow \gamma_n$. Therefore, the damping rates of the quadrature components in Eq. (37) are formally identical with the corresponding damping rates for $\gamma_2 > \gamma_1$, and the interpretation of the properties of the Liouvillian (22) follows the same lines as for $\gamma_2 > \gamma_1$.

There is, however, an important difference in the evolution of the average inversion $\langle S_z(t) \rangle$. In the parameter regime, $\gamma_n > \gamma_s$, an incoherent excitation of the quantum dot is more likely to be followed with a further excitation than with a spontaneous decay to the ground level. The net effect of these processes is to accumulate more population in the excited state rather than in the ground state. In physical terms, the system behaves as an inverted harmonic oscillator. It is clearly seen from Eq. (37) that in the steady-state the population inversion is

$$\langle S_z \rangle_s = \frac{\gamma_n - \gamma_s}{2(\gamma_s + \gamma_n)} = \frac{\gamma_1 - \gamma_2}{2(2\bar{n} + 1)(\gamma_1 + \gamma_2)}. \quad (44)$$

Clearly, the population between the bare states of the quantum dots is inverted. The population inversion increases with an increasing γ_1/γ_2 and for $\gamma_1/\gamma_2 \gg 1$ can reach the total inversion. The effect of the squeezed reservoir on the dynamics of the quantum dot is therefore much more drastic when $\gamma_1 > \gamma_2$ than when $\gamma_2 > \gamma_1$. The result (44) implies that the engineered squeezed reservoir can be employed to maintain a large population inversion necessary for laser generation in a two-level quantum dot.

C. The case, $\gamma_1 = \gamma_2$

In this limit $\gamma_s = \gamma_n = \gamma_m$, and then the damping rate of the $\langle S_\phi(t) \rangle$ component

$$\gamma_\phi = \gamma_s + \gamma_n - 2\gamma_m, \quad (45)$$

reduces to zero. Consequently, the decay of $\langle S_\phi(t) \rangle$ is completely inhibited that the squeezing of the fluctuations of the S_ϕ component is perfect. This shows that by a proper engineering of the coupling of a phonon bath to a quantum dot one can produce a coherent atomic dipole without the accompanying quantum fluctuations and incoherent excitations associated with the presence of phonons. Note that $\gamma_\phi = 0$ is obtained independent of the number of phonons \bar{n} . It follows that $\langle S_\phi(t) \rangle$ can be locked at its initial value for an arbitrary long time independent of the temperature of the phonon bath.

On the other hand, the component $S_{\phi+\frac{\pi}{2}}(t)$ decays at the rate $\gamma_{\phi+\frac{\pi}{2}} = 4(2\bar{n} + 1)\gamma_0$, which is enhanced and dependent on \bar{n} but is not infinite. Moreover, the two-level transition in the quantum dot becomes saturated in the steady-state, that $\lim_{t \rightarrow \infty} \langle S_z(t) \rangle \equiv \langle S_z \rangle_s = 0$. Thus, independent of \bar{n} the population is evenly distributed between the ground and excited levels of the quantum dot. Therefore, the interaction of the quantum dot with the squeezed reservoir (22) offers the possibility of both inhibiting the phase decay and an alignment of the spin vector along the x axis.

These features are significantly different from those produced by the decay of a two-level system to a squeezed reservoir generated by an external source. When the Liouvillian (27) is used instead of (22), one can easily find that the components display the following exponential decays

$$\begin{aligned} \langle S_\phi(t) \rangle &= \langle S_\phi(0) \rangle e^{-\gamma(\frac{1}{2} + N - |M|)t}, \\ \langle S_{\phi+\frac{\pi}{2}}(t) \rangle &= \langle S_{\phi+\frac{\pi}{2}}(0) \rangle e^{-\gamma(\frac{1}{2} + N + |M|)t}, \\ \langle S_z(t) \rangle &= -\frac{1}{2N+1} \\ &\quad + \left[\langle S_z(0) \rangle + \frac{1}{2N+1} \right] e^{-\gamma(2N+1)t}. \end{aligned} \quad (46)$$

Clearly, for the inhibition of the decay of the component $\langle S_\phi(t) \rangle$ one evidently requires a very strong squeezed field, $N \rightarrow \infty$ at which $N - |M| \rightarrow -\frac{1}{2}$. In this limit, the decay rate of the $\langle S_{\phi+\frac{\pi}{2}}(t) \rangle$ goes to infinity. Moreover, $\langle S_z \rangle_s < 0$ and the population inversion approaches zero only in the limit of $N \rightarrow \infty$.

The physical reason for the changed decay behavior in the engineered squeezed reservoir is most clearly understood by considering the expectation value of the spin vector of the quantum dot and its fluctuations. In the steady-state, we have

$$\langle S_\phi \rangle_s = \langle S_\phi(0) \rangle, \quad \langle S_{\phi+\frac{\pi}{2}} \rangle_s = 0, \quad \langle S_z \rangle_s = 0. \quad (47)$$

Thus, the expectation value of the spin vector $\langle \vec{S} \rangle$ lies in the $x - y$ plane such that it forms an angle ϕ with the y axis

$$\langle S_x \rangle_s = S \sin \phi, \quad \langle S_y \rangle_s = S \cos \phi, \quad \langle S_z \rangle_s = 0, \quad (48)$$

where $S = \sqrt{\langle S_x \rangle_s^2 + \langle S_y \rangle_s^2}$ and $\tan \phi = \langle S_x \rangle_s / \langle S_y \rangle_s$.

Assume for simplicity that $\phi = 0$. In this case, the following Heisenberg uncertainty principles are obeyed

$$\begin{aligned} \sqrt{\langle (\Delta S_x)^2 \rangle \langle (\Delta S_y)^2 \rangle} &\geq 0, \quad x - y \text{ plane}, \\ \sqrt{\langle (\Delta S_z)^2 \rangle \langle (\Delta S_x)^2 \rangle} &\geq \frac{1}{2} |\langle S_y \rangle_s|, \quad z - x \text{ plane}, \\ \sqrt{\langle (\Delta S_y)^2 \rangle \langle (\Delta S_z)^2 \rangle} &\geq 0, \quad y - z \text{ plane}. \end{aligned} \quad (49)$$

The form of the uncertainty relations resembles very much of that occurring in a planar squeezing situation where one can independently change fluctuations in two quadrature components which lie in the plane of the spin vector [55–57].

It is not difficult to show that in the case considered here the Liouvillian (22) can be written as

$$\mathcal{L}_p \rho = 4(2\bar{n} + 1)\gamma_0 (2S_\phi \rho S_\phi - S_\phi^2 \rho - \rho S_\phi^2), \quad (50)$$

from which we see that only the quadrature phase S_ϕ is coupled to the reservoir. Thus, we conclude that the case $\gamma_1 = \gamma_2$ corresponds to a quantum-nondemolition type coupling of the quantum dot to the effective squeezed reservoir [52, 58].

IV. STATIONARY STATE OF A DRIVEN QUANTUM DOT

Suppose that in addition to the bichromatic field, which couples the quantum dot to the phonon bath, the dot is subjected to an exciting laser field of frequency ω_L which is on resonance with the transition frequency of the quantum dot, i.e., detuning $\Delta_L = \omega_L - \omega_0 = 0$. With the addition of the exciting field, the Bloch equations (37) take the form

$$\begin{aligned} \dot{\langle S_x \rangle} &= -\gamma_x \langle S_x \rangle, \\ \dot{\langle S_y \rangle} &= -\gamma_y \langle S_y \rangle - \Omega \langle S_z \rangle, \\ \dot{\langle S_z \rangle} &= -(\gamma_s - \gamma_n) - \gamma_z \langle S_z \rangle + \Omega \langle S_y \rangle, \end{aligned} \quad (51)$$

where

$$\begin{aligned} \gamma_x &= \gamma_s + \gamma_n \pm 2\gamma_m, \quad \gamma_y = \gamma_s + \gamma_n \mp 2\gamma_m, \\ \gamma_z &= 2(\gamma_s + \gamma_n), \end{aligned} \quad (52)$$

and Ω is the Rabi frequency of the exciting field. In writing Eq. (51) we have chosen the phase of the laser $\phi_L = 0$ and have made the choices of the squeezing phase $\phi = 0$ and $\phi = \pi/2$ corresponding to the limits of the variation of the damping rates.

Our purpose is to determine the steady-state values of the average values of the spin components. It is clear from Eq. (51) that the polarization component $\langle S_x \rangle$ is decoupled from the exciting field and the other components. The equation of motion for $\langle S_x \rangle$ can be integrated immediately to give

$$\langle S_x(t) \rangle = \langle S_x(0) \rangle e^{-\gamma_x t}. \quad (53)$$

It is a simple exponential decay with the rate γ_x , so in order to determine the steady-state value of $\langle S_x \rangle$ we have to look at the properties of the damping rate γ_x . According to Eqs. (52) and

(23), the rate depends strongly on the relation between γ_1 and γ_2 and the phase ϕ . When $\gamma_1 > \gamma_2$ or $\gamma_1 < \gamma_2$, at which $\gamma_s \neq \gamma_n \neq \gamma_m$, we see that $\langle S_x(t) \rangle$ decays to zero independent of the phase. However, in the case $\gamma_1 = \gamma_2$, that is when $\gamma_s = \gamma_n = \gamma_m$, the decay rate γ_x depends on the phase. It follows that if $\phi = 0$ then in the steady state $\langle S_x \rangle_s = 0$, whereas the component decays to a nonzero steady-state value $\langle S_x \rangle_s = \langle S_x(0) \rangle$ if $\phi = \pi/2$. This implies that the coherence between the ground and excited states of the quantum dot is locked at its initial value. Hence, the two choices of phase lead to widely different behavior of the $\langle S_x \rangle$ component.

The steady-state values of the remaining two components $\langle S_y \rangle$ and $\langle S_z \rangle$ are found to be

$$\langle S_y \rangle_s = \frac{(\gamma_s - \gamma_n)\Omega}{\gamma_y\gamma_z + \Omega^2}, \quad \langle S_z \rangle_s = \frac{-(\gamma_s - \gamma_n)\gamma_y}{\gamma_y\gamma_z + \Omega^2}. \quad (54)$$

Provided that $\gamma_s \neq \gamma_n$, the steady-state values are different from zero. However, if $\gamma_1 = \gamma_2$ we have $\gamma_s = \gamma_n$ and then $\langle S_y \rangle_s = \langle S_z \rangle_s = 0$ regardless of the phase ϕ . Thus, for $\gamma_1 = \gamma_2$, the steady-state value of the total spin of the system depends solely on the initial value of $\langle S_x \rangle$. For $\langle S_x(0) \rangle \neq 0$ the polarization is locked at its initial value due to the interaction with the perfectly squeezed field.

In this way, we may modify the steady-state population distribution between dressed states of the driven quantum dot. In order to show it, we introduce the semiclassical dressed states, which are the eigenstates of the two-level system and the classical driving field. Since the driving laser is on resonance with the two-level transition, the dressed states are

$$|+\rangle = \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle), \quad |-\rangle = \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle). \quad (55)$$

It is easily verified that the populations of the dressed states are related to the populations and coherences of the bare states through the relations

$$\begin{aligned} \rho_{++} &= \frac{1}{2}(\rho_{11} + \rho_{22} + \rho_{12} + \rho_{21}) = \frac{1}{2}(1 + 2\langle S_x \rangle_s), \\ \rho_{--} &= \frac{1}{2}(\rho_{11} + \rho_{22} - \rho_{12} - \rho_{21}) = \frac{1}{2}(1 - 2\langle S_x \rangle_s). \end{aligned} \quad (56)$$

We see that only the component $\langle S_x \rangle$ contributes to the populations of the dressed states. Since in the steady-state $\langle S_x \rangle_s = \langle S_x(0) \rangle$, we see that one can polarize the dressed state populations, i.e., create an asymmetry in populations within dressed-state doublets simply by choosing an initial state at $t = 0$ such that $\langle S_x(0) \rangle \neq 0$. Particularly interesting is the phenomenon of dressed state population trapping or complete polarization of the dressed state populations which happens when $\langle S_x \rangle = \pm \frac{1}{2}$. From the foregoing discussion on the possible steady-state values of $\langle S_x \rangle$, we see that complete polarization of the dressed state populations occurs when $\phi = \pi/2$ and initially at time $t = 0$, $\langle S_x(0) \rangle = \pm \frac{1}{2}$. In practice, arbitrary initial values of $\langle S_x \rangle$ could be prepared using the standard technique of a $\pi/2$ pulse excitation. For example, the quantum dot could be prepared in one of the dressed states ($\langle S_x(0) \rangle = \pm \frac{1}{2}$) using a $\pi/2$ pulse laser field that is $\pm\pi/2$ out of phase with the exciting field [59].

Note that the polarization effect in the system considered here is obtained in the resonant case ($\Delta_L = 0$). This is in contrast to the polarization effect found for the steady-state of a two-level atom damped by an externally generated squeezed vacuum field, where it was shown [60, 61] that the dressed-state polarization is possible only at a non-zero laser detuning, $\Delta_L \neq 0$.

V. FLUORESCENCE SPECTRUM

We now consider the spectrum of the fluorescence field, which can be written as a sum of two parts

$$S(\omega) = S_{coh}(\omega) + S_{in}(\omega), \quad (57)$$

where

$$S_{coh}(\omega) = 2\pi \langle S^+ \rangle_s \langle S^- \rangle_s \delta(\omega - \omega_0) \quad (58)$$

is the coherent (elastic) part of the spectrum, and

$$\begin{aligned} S_{in}(\omega) &= 2\text{Re} \left\{ \int_0^\infty d\tau e^{i(\omega - \omega_0)\tau} \right. \\ &\quad \left. \times \lim_{t \rightarrow \infty} \langle \delta S^+(t) \delta S^-(t + \tau) \rangle \right\} \end{aligned} \quad (59)$$

is the incoherent (noise) part of the spectrum. Here, $\delta S^\pm(t) = S^\pm(t) - \langle S^\pm(t) \rangle$ are the fluctuation parts of the dipole operators.

The two-time correlation function appearing in Eq. (59) can be written as

$$\begin{aligned} \langle \delta S^+(t) \delta S^-(t + \tau) \rangle &= \langle \delta S^+(t) \delta S_x(t + \tau) \rangle \\ &\quad - i \langle \delta S^+(t) \delta S_y(t + \tau) \rangle, \end{aligned} \quad (60)$$

and the correlation functions $\langle \delta S^+(t) \delta S_x(t + \tau) \rangle$ and $\langle \delta S^+(t) \delta S_y(t + \tau) \rangle$ may in turn be evaluated from Eq. (51) with the help of the quantum regression theorem [62]. By Laplace transforming of the resulting equations of motion for the two-time correlation functions, we obtain

$$\begin{aligned} \Lambda(z) \equiv \lim_{t \rightarrow \infty} \mathcal{L}\{\langle \delta S^+(t) \delta S^-(t + \tau) \rangle\} &= \frac{\langle \delta S^+ \delta S_x \rangle_s}{z + \gamma_x} \\ &\quad - i \frac{\langle \delta S^+ \delta S_y \rangle_s (z + \gamma_z) - \Omega \langle \delta S^+ \delta S_z \rangle_s}{z^2 + (\gamma_y + \gamma_z)z + \gamma_y\gamma_z + \Omega^2}, \end{aligned} \quad (61)$$

where z is the Laplace transform parameter and the steady-state averages of the various operator products arising are

$$\begin{aligned} \langle \delta S^+ \delta S_x \rangle_s &= \frac{1}{2} \left(\langle S_z \rangle_s + \frac{1}{2} \right) - \langle S_x \rangle_s (\langle S_x \rangle_s + i \langle S_y \rangle_s), \\ \langle \delta S^+ \delta S_y \rangle_s &= \frac{i}{2} \left(\langle S_z \rangle_s + \frac{1}{2} \right) - \langle S_y \rangle_s (\langle S_x \rangle_s + i \langle S_y \rangle_s), \\ \langle \delta S^+ \delta S_z \rangle_s &= -\frac{1}{2} (\langle S_x \rangle_s + i \langle S_y \rangle_s) (1 + 2 \langle S_z \rangle_s). \end{aligned} \quad (62)$$

To illustrate the analytic structure of the spectrum in a simple form as possible, we focus on the case $\gamma_1 = \gamma_2$ and

the strong-field limit $\Omega \gg \gamma_1, \gamma_2$. The cases, $\gamma_1 > \gamma_2$ and $\gamma_2 > \gamma_1$ lead to the spectrum which is essentially the same as that of the fluorescence field emitted by a two-level system whose relaxation is determined through coupling to a squeezed vacuum field produced by an external source [41–43, 50]. In the strong-field limit, we readily find that the two roots of the quadratic equation in the denominator of Eq. (61) are approximately given by

$$z_{1,2} = -\frac{1}{2}(\gamma_y + \gamma_z) \pm i\Omega, \quad (63)$$

and then the spectral components take the form

$$S_{coh}(\omega) = 2\pi \langle S_x \rangle_s^2 \delta(\omega - \omega_0), \quad (64)$$

and

$$\begin{aligned} S_{in}(\omega) &= 2\text{Re}\{\Lambda(z)\}_{z=-i(\omega-\omega_0)} \\ &= \frac{1}{2}(1 - 4\langle S_x \rangle_s^2) \frac{\gamma_x}{\gamma_x^2 + (\omega - \omega_0)^2} \\ &\quad + \frac{\frac{1}{8}(1 + 2\langle S_x \rangle_s)(\gamma_y + \gamma_z) + \frac{\gamma_z - \gamma_y}{8\Omega}(\omega - \omega_0 - \Omega)}{\frac{1}{4}(\gamma_y + \gamma_z)^2 + (\omega - \omega_0 - \Omega)^2} \\ &\quad + \frac{\frac{1}{8}(1 - 2\langle S_x \rangle_s)(\gamma_y + \gamma_z) + \frac{\gamma_z - \gamma_y}{8\Omega}(\omega - \omega_0 + \Omega)}{\frac{1}{4}(\gamma_y + \gamma_z)^2 + (\omega - \omega_0 + \Omega)^2}. \end{aligned} \quad (65)$$

where we have used the fact that $\langle S_y \rangle_s = \langle S_z \rangle_s = 0$.

One can see from Eqs. (64) and (65) that the amplitudes of the spectral components are solely dependent on the polarization (coherence) component $\langle S_x \rangle_s$. We first note that the coherent part of the spectrum is present only if $\langle S_x \rangle_s \neq 0$. Otherwise when $\langle S_x \rangle_s = 0$ the spectrum consists only of the incoherent part, which is always present. In general, the incoherent part of the spectrum is composed of three Lorentzian peaks of the widths and magnitudes varying with the phase ϕ . The most interesting feature of the incoherent spectrum is its asymmetry related to $\langle S_x \rangle_s \neq 0$, because this feature is not encountered at all under the damping of the quantum dot by an externally produced squeezed vacuum field. If we consider the variation of the spectrum with the phase, we find for $\phi = 0$,

$$\gamma_x = \gamma_z = 4(2\bar{n} + 1)\gamma_0, \quad \gamma_y = 0, \quad \langle S_x \rangle_s = 0, \quad (66)$$

while for $\phi = \pi/2$,

$$\gamma_x = 0, \quad \gamma_y = \gamma_z = 4(2\bar{n} + 1)\gamma_0, \quad \langle S_x \rangle_s = \langle S_x(0) \rangle. \quad (67)$$

For $\phi = 0$, the coherent part of the spectrum is suppressed, whereas the incoherent part is composed of three peaks of equal amplitudes. The width of the central peak is $4(2\bar{n} + 1)\gamma_0$ and it is twice as wide as the width of the Rabi sidebands. Thus, the spectrum is symmetric about the laser frequency and entirely composed of the incoherent part.

The spectrum changes dramatically when the phase is varied to $\phi = \pi/2$. The coherent part appears and the central peak of the incoherent part becomes a δ -type peak. The widths

of the Rabi sidebands as twice as wide as for $\phi = 0$. Thus, for $\phi = \pi/2$, the central peak contributes to a coherent (elastic) part of the spectrum leading to an enhancement of the coherent scattering. The incoherent part is then effectively composed of two peaks located at the Rabi sidebands. Depending on $\langle S_x(0) \rangle$ the number of peaks in the incoherent part can vary from three to a single side peak located at $\omega - \omega_0 = \pm\Omega$. The disappearance of two peaks is a consequence of the complete polarization of the dressed state population. For example, when $\langle S_x(0) \rangle = \frac{1}{2}$, the population is entirely in the upper dressed state $|+\rangle$. Consequently, the central and the lower frequency Rabi sideband peaks disappear. On the other hand, when $\langle S_x(0) \rangle = -\frac{1}{2}$, the population is entirely in the lower dressed state $|-\rangle$ resulting in the absence of the central peak and the higher frequency Rabi sideband. The disappearance of the central peak is accompanied by an increase of the amplitude of the coherent part of the spectrum. On the other words, the energy contained in the central peak is coherently scattered by the quantum dot. The disappearance of one of the Rabi sidebands is accompanied by an increase of the amplitude of the opposite Rabi sideband, which after the complete transfer of the population is twice as high as for $\langle S_x(0) \rangle = 0$.

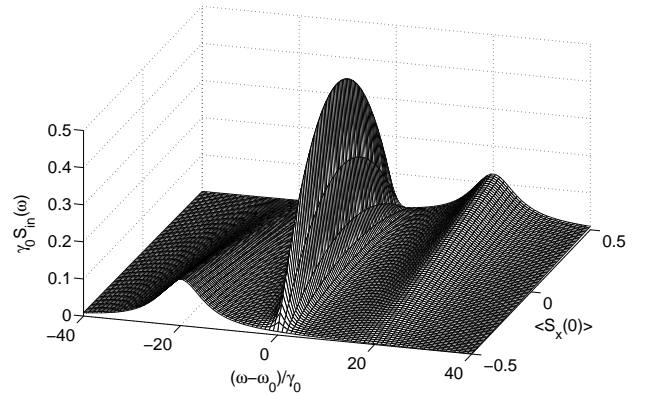


FIG. 5: The incoherent part of the fluorescence spectrum as a function of $\langle S_x(0) \rangle$ for $\phi = \pi/2$, $\Omega = 20\gamma_0$ and $\bar{n} = 0.5$. In order to make the central component visible in the plot, the delta function has been replaced by a Lorentzian of a width γ_0 .

The features described above are easily seen in Fig. 5 which shows the incoherent fluorescence spectrum plotted as a function of $\langle S_x(0) \rangle$ for fixed Ω and \bar{n} . We see the disappearance of the central peak and one of the Rabi sidebands when $\langle S_x(0) \rangle = \pm\frac{1}{2}$, and simultaneously the increase in the height of the opposite Rabi sideband. We again point out that the asymmetric spectrum and its variation with the initial coherence are obtained under strictly resonant excitation. The variation of the fluorescence spectrum with the initial coherence shows clearly that the phase relationships between the irradiating field and the initial dipole moment of the radiating quantum dot are important even in the steady-state fluorescence.

VI. SUMMARY

We have shown that the combined effect of a phonon bath and a bichromatic field can result in an effective squeezed-vacuum-type reservoir to a two-level quantum dot. It has been found that depending on the ratio of the Rabi frequencies of the components of the bichromatic field, one can engineer a squeezed reservoir of ordinary or inverted harmonic oscillators. We have shown that in the case of the inverted harmonic oscillators the steady-state population of the bare states of the quantum dot can be highly inverted. We have examined the conditions for quantum two-photon correlations characteristic of a squeezed field and have distinguished between the quantum correlations and a form of classical two-photon correlations which may exist at high temperatures of the phonon bath.

When in addition to the squeezed reservoir, the quantum dot interacts with a strong laser field, the dynamics and the stationary state could depend critically on whether or not a coherence between the ground and excited states is initially present. With the coherence present, the steady-state population distribution between the dressed states of the driven quantum dot could be completely polarized (trapped) in one of the dressed states. We have calculated the steady-state spectrum of the fluorescence field and have found that the structure spectrum depends on the initial value of the coherence. In particular,

with a nonzero initial coherence the spectrum is asymmetric even if the quantum dot is exposed to a resonant laser field. We have found that the asymmetric features are the same as those exhibited by the spectrum of a two-level system excited off resonance and damped by a squeezed vacuum field produced by an external squeezing source. The appearance of the asymmetric features have been interpreted as a direct consequence of locking the coherence at its initial value, resulted from the coupling of the quantum dot to perfectly squeezed field. In the absence of the initial coherence, the spectrum was found to be composed of only the incoherent part displaying the symmetric triplet spectrum. With the coherence present, both coherent and incoherent parts are present and the number of peaks in the incoherent part becomes strongly dependent on phase. By varying the phase, the central peak can become a δ -type peak and one of the Rabi sidebands could be suppressed.

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