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There exists an analogy between Maxwell equations in a rotating frame and Schrödinger equation for a charged particle in the presence of a magnetic field. We exploit this analogy to point out that electromagnetic phenomena in the rotating frame, under appropriate conditions, can exhibit periodicity with respect to the angular velocity of rotation. In particular, in disordered ring cavities one finds the optical analog of the Al'tshuler-Aronov-Spivak effect well known in mesoscopic physics of disordered metals.

Introduction - When written in the four-dimensional covariant form the Maxwell equations (ME) look the same in any reference frame, inertial or not. Things change, however, when the equations are expressed in terms of the three-dimensional vectors designating the electric and magnetic fields. For instance, the equation for the electric field in a rotating frame acquires an additional term, as compared to its counterpart in an inertial frame (see Eq. (1) below) [1]. This additional term associated with a non-zero angular velocity of rotation $\Omega \neq 0$, resembles a magnetic flux Φ experienced by a quantum charged particle when it moves through a metallic ring [2]. The situation here is similar to that in mechanics, where the equation of motion for a particle in a uniformly rotating frame must be supplemented by the Coriolis and centrifugal forces - these forces are often called "fictitious", although for an observer in the rotating frame they have a very real, observable effect.

It is therefore natural to expect that the fictitious "magnetic flux" term appearing in the framework of electrodynamics of rotating media, can lead to important new fundamental insights of light propagation in such set-ups. Of particular interest to us is the case of rotating disordered optical ring cavities. The analogous quantum mechanical situation of an electron moving around a ring in the presence of magnetic flux results in Aharonov-Bohm (AB) oscillations [3] and their half-flux quantum oscillations of conductance [4–10]. The latter interference effect occurs only in the presence of disorder and has its origin in a coherent backscattering mechanism. These half-flux oscillations have been first predicted by Al'tshuler, Aronov and Spivak (AAS) [4] and attracted a lot of theoretical and experimental attention in the frame of mesoscopic quantum physics. A surprising finding was that these "anomalous" oscillations emerge only after an ensemble averaging which restores specific symmetries. We note that in the cylindrical geometry employed in the experiment [5] the averaging is automatic, since there are many independent channels, while in the 1D ring geometry considered in [6] averaging over an ensemble of disordered rings was needed to reveal the half-flux oscillations.

Do similar effects occur also in the frame of light propagation in rotating disordered ring cavities? As we will show below a direct inspection of the ME in the rotating frame uncovers many differences with the quantum mechanical Schrödinger equation in the presence of a magnetic field. The most noticeable one is that the fictitious "magnetic field", related to the rotation, depends not only on the angular rotation velocity Ω but also on the electromagnetic wave frequency ω. We demonstrate that, these differences notwithstanding, wave interference effects can lead under appropriate conditions to the optical analog of the AAS oscillations. Our theoretical results are supported by detail numerical calculations.

Model - We demonstrate the existence of half-period AB oscillations in the presence of rotations using the simple case of one-dimensional (1D) ring cavity of radius R [11]. We assume that the effective refraction index $n(s)$ of the 1D ring generally depends on the coordinate s defining the position on the circumference of the ring. The ring rotates uniformly about the axis going through its center perpendicular to its plane ($z-$direction), so that the vector angular velocity is $\vec{\Omega} = \Omega \hat{z}$. Assuming a monochromatic electric field, having only the $z-$ component, $\vec{E}(s,t) = \hat{z}\Psi(s)e^{-i\omega t}$, one can write the ME for Ψ *in the rotating frame* as

$$\frac{\partial^2 \Psi}{\partial s^2} + n^2(s)\frac{\omega^2}{c^2}\Psi - 2i\beta\frac{\omega}{c}\frac{\partial \Psi}{\partial s} = 0 \qquad (1)$$

where s increases from 0 to $L = 2\pi R$ (in the counter-clockwise direction) and $\beta \equiv \Omega R/c \ll 1$. This equation can be rewritten as

$$\left(\frac{\partial}{\partial s} - i\beta\frac{\omega}{c}\right)^2 \Psi + \left[n^2(s) + \beta^2\right]\left(\frac{\omega}{c}\right)^2 \Psi = 0 \quad (2)$$

which makes the analogy with quantum mechanics transparent [11]. The quantum analogue of the quantity $\beta\omega R/c$ is the magnetic flux Φ through the ring, measured in units of the fundamental quantum flux $\Phi_0 = hc/e$. This analogy is rather formal because the "vector potential" in Eq. (2) depends on the wave frequency ω which (for the isolated ring) is the quantity to be determined

from the same Eq. (2). As a result Eq. (2) is not periodic with respect to the "fundamental flux"– as opposed to its quantum mechanical equivalent and therefore it is not a-priori obvious that will demonstrate an AB periodicity in any observable including its own spectrum. Nevertheless the analogy is useful and it enables us to identify system parameters and physical conditions for which AB-type optical interference effects [12], similar to those known in mesoscopic physics of electronic systems, can emerge also in the framework of Eq. (2).

Let us consider, for example, a perfect ring cavity with a constant index of refraction $n(s) = n_0$. In the absence of any rotation the spectrum is degenerate with eigenfrequencies $\omega_0^{(N)} = Nc/Rn_0$, with $N = 1, 2, 3 \dots$. Drawing analogies from the quantum mesoscopic physics we expect that significant changes in the spectrum, will occur when the effective magnetic flux $\Phi = \beta \omega_0^{(N)} R/c = \beta N/n_0$ reaches values comparable to unity. It follows that, due to smallness of β, this can only happen for large mode numbers N. For example for a ring of radius $R = 3cm$ in the optical range of frequencies $\omega_0^{(N)} \sim 10^{15} sec^{-1}$, we have that $N = \omega_0^{(N)} Rn_0/c = 10^5$, corresponding to $\Omega \sim 10^5 sec^{-1}$. This number can be decreased further to $\Omega \sim 10^3 sec^{-1}$, which is easily achievable, if we consider a loop fiber of length $L \approx 1m$ coiled in many turns onto a mandrel.

Isolated ring cavity - To find the eigenvalues of an isolated ring one has to solve Eq. (2), with periodic boundary conditions. It is instructive to start with an ideal ring, i.e. $n(s) = n_0 =$const. For this case the eigenfrequencies, to first order in β, are $\omega_\pm^{(N)} = \omega_0^{(N)} \left(1 \mp \frac{\beta}{n_0}\right)$, where the subscript $+(-)$ denotes a wave propagating in (opposite to) the direction of increasing s. In the absence of rotation each eigenfrequency is doubly degenerate. Rotations remove this degeneracy, introducing frequency splitting $\Delta \omega = 2\beta \omega_0^{(N)}/n_0$. This lifting of degeneracy is similar to that in a metallic ring penetrated by flux Φ. There is, however, an important difference: in a metallic ring there is a strict universal periodicity, i.e. energy spectra for $\Phi = 0$ and $\Phi = \Phi_0$ are exactly the same. There is no such strict periodicity in a rotating optical ring cavity. Indeed, it follows from the above expression for $\omega_\pm^{(N)}$ that under increase of β the doubly degenerate eigenvalue $\omega_0^{(N)}$ is mapped onto $\omega_0^{(N-1)}$ and $\omega_0^{(N+1)}$. The point is that this mapping occurs at the value $\beta = n_0/N$ which depends on N itself, so there is no single period for mapping the entire spectrum onto itself.

The crucial observation is that in the limit of large N one can chose a large set of levels, in an interval ΔN near N, such that $1 \ll \Delta N \ll N$ for which the period is almost the same, see Fig. 1. Specifically we get a periodicity n_0/N - up to small corrections of the order of $\Delta N/N^2$. This period also follows from the aforementioned analogy between Φ/Φ_0 and $\beta \omega R/c$, since $\Phi = \Phi_0$ translates into $\beta = c/R\omega_0^{(N)} = n_0/N$. In addition, a degeneracy at $\beta = n_0/2N$ develops when the nearby levels

N and $N - 1$ cross, see Fig. 1.

Next we investigate the effect of one defect on the eigenfrequencies $\omega^{(N)}$ of a rotating ring cavity. The impurity splits the ring in two sections. The first one consists of the cavity formed by the defect which has an index of refraction n_1 and length l_1. The second one is associated with the rest of the ring and has an index of refraction n_0 and length $l_0 = L - l_1$ (L is the length of the ring circumference). We choose the origin $s = 0$ in the middle of the section with refraction index n_0. The interfaces between the two sections are at positions $s_0 = l_0/2$ and $s_1 = l_0/2 + l_1$. In each of the two uniform sections, the solution $\Psi_j(s)$ of Eq. (1) can be written as the superposition of two counter-propagating waves,

$$\Psi_j(s) = a_j(s) + b_j(s) = a_j e^{\imath k_+^j s} + b_j e^{-\imath k_-^j s}, \quad (3)$$

where $k_\pm^j = n_j \frac{\omega}{c}\left(1 \pm \frac{\beta}{n_j}\right)$ are the wavenumbers associated with a propagating wave in (+) or opposite (-) to the direction of increasing s while $j = 0, 1$ indicates the section associated with the index of refraction n_j.

At each position s, the field $\Psi(s)$ in Eq. (3) and its derivative are continuous. Implementation of these boundary conditions at the interfaces allow us to calculate the transmission $t_+ \equiv a_0(L)/a_0(0)$ and reflection $r_+ \equiv b_0(0)/a_0(0)$ amplitudes for a wave traveling in the direction of increasing s and experiencing scattering events due to the presence of the defect. Specifically:

$$t_+ = \frac{4n_0 n_1 e^{\imath \frac{\omega}{c} n_0 l_0}}{e^{-\imath \frac{\omega}{c} n_1 l_1}(n_0 + n_1)^2 - e^{\imath \frac{\omega}{c} n_1 l_1}(n_0 - n_1)^2}$$

$$r_+ = \frac{2\imath \sin\left(\frac{\omega}{c} n_1 l_1\right)\left(n_1^2 - n_0^2\right) e^{\imath \frac{\omega}{c} n_0 l_0}}{e^{-\imath \frac{\omega}{c} n_1 l_1}(n_0 + n_1)^2 - e^{\imath \frac{\omega}{c} n_1 l_1}(n_0 - n_1)^2}. \quad (4)$$

Moreover the fields at $s = 0$ and at $s = L$ (i.e. across the whole ring) are connected via the total transfer matrix T_{tot} [13] as

$$\begin{pmatrix} a_0(L) \\ b_0(L) \end{pmatrix} = T_{\text{tot}} \begin{pmatrix} a_0(0) \\ b_0(0) \end{pmatrix}; \; T_{tot} = \begin{bmatrix} \frac{1}{t_+^*} & -\frac{r_+^*}{t_+^*} \\ -\frac{r_+}{t_+} & \frac{1}{t_+} \end{bmatrix} e^{\imath \frac{\omega}{c} L\beta},$$

$$(5)$$

where $t_+ \equiv \sqrt{T_+}e^{\imath \phi_t}$ and $r_+ \equiv \sqrt{1 - T_+}e^{\imath \phi_r}$ can be further parametrized in terms of the transmission modulo T_+ and phase ϕ_t, and the reflection phase ϕ_r.

The eigenmodes and the associated eigenfrequencies are calculated by imposing periodic boundary conditions in Eq. (5) i.e. $a_0(L) = a_0(0)$ and $b_0(L) = b_0(0)$. The eigenfrequencies of the ring cavity are the solutions of the secular equation

$$\det\left(T_{tot}(\omega) - I_2\right) = 0, \quad (6)$$

where I_2 is the 2×2 identity matrix. Once the eigenvector $(a_0(0), b_0(0))^T$ (corresponding to unity eigenvalue) of T_{tot} is evaluated, the associated eigenfunctions are calculated via a straightforward implementation of the aforementioned boundary conditions for $\Psi(s)$ and Eq. (3).

The above formalism can be easily generalized to the case of multiple defects (disordered ring cavity) where now $T_{\text{tot}} = \prod_j T_j$ is a product of transfer matrices T_j [13]. The latter takes into consideration the free propagation within a uniform defect and the boundary conditions that the wavefunction has to satisfy at the interface between two consequent defects. The only β-dependence in the T_{tot}, Eq (5), is in the overall phase, irrespective of the number of defects. Therefore the eigenfrequencies (around a constant high ω_{avg} value) and the corresponding eigenmodes will respect a $\beta = 2\pi c/\omega_{\text{avg}}L$ periodicity.
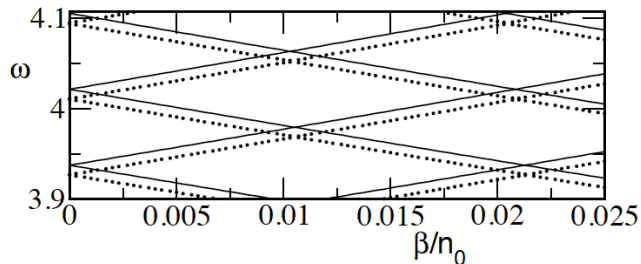

FIG. 1: Evolution of the spectrum $\omega \left[\frac{c}{l_1} \right]$ (we assume $l_1 = 1$) versus $\frac{\beta}{n_0}$ for (a) an ideal ring cavity with $n_0 = 1.5$ and circumference $L = 50l_1$, in the large N limit. Large portion of the spectrum, near $N \approx 48$, is accurately mapped onto itself when $\frac{\beta}{n_0} = \frac{1}{N} \approx 0.021$. (b) One defect with $n_1 = 1.7$ and length l_1. In this case the strict degeneracies turn into avoided crossings (dashed lines). Note the crossing for $\frac{\beta}{n_0} = \frac{1}{2N} \approx 0.01$ which play a major role in the disorder-related effect discussed in the text.

Substitution of Eq. (4) in Eqs. (5,6) allows us to calculate numerically the eigenfrequencies $\omega(\beta)$ of the rotating ring cavity with one defect. The results are shown in Fig. 1 together with the eigenfrequencies of the uniform ring. We find that the defect lifts the degeneracies at $\beta = 0, n_0/2N$ and n_0/N. The same conclusions apply also to disordered rings (not shown). The lift of these degeneracies due to the presence of the defects is crucial for the interference effects that we will discuss below.

Next we turn to the analysis of the eigenmodes and investigate the so-called participation ratio P_L

$$P_L(\beta) = \frac{1}{L} \left\langle \frac{\left(\int_0^L ds |\psi^{(N)}(s)|^2 \right)^2}{\int_0^L ds |\psi^{(N)}(s)|^4} \right\rangle \qquad (7)$$

where $\langle \cdots \rangle$ indicates an average over a small frequency window around a frequency ω_{avg} and over a number of disordered realizations of index of refraction $n(s) \in [n_0 - \Delta n, n_0 + \Delta n]$. The participation ratio allows us to determine the localization of the eigenmodes in the ring and how their spatial distribution within the ring is affected by rotations $\beta \neq 0$. Specifically $P_L(\beta)$ takes values between unity (uniformly extended states) and zero

(exponentially localized states). From the previous discussion it is clear that since the secular equation Eq. (6) is periodic with period $\beta = 2\pi c/\omega_{\text{avg}}L$, the same will apply also for $P_L(\beta)$. This AB-type periodicity occurs for *each* individual eigenmode, in the absence of any averaging, see Fig. 2a. However, we have found that when the ensemble averaging of the participation ratio (for large ω's) is taken, $P_L(\beta)$ demonstrates a half-flux periodicity, similar to the one found in mesoscopic quantum physics [6, 10], see Fig. 2b. Further, for strong disorder, such that $P_L \ll 1$, the oscillatory behavior of $P_L(\beta)$ completely disappears.
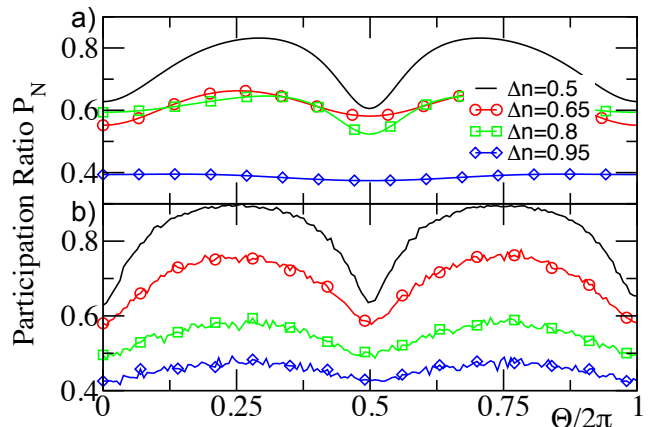

FIG. 2: Participation ratio P_L vs. $\frac{\theta}{2\pi} = \frac{\omega_{avg}}{2\pi c} \beta L$ for a disordered ring with the circumference $L = 50l_1$. All defects have the same length l_1 while their refraction index is given by a box distribution $n \in [n_0 - \Delta n, n_0 + \Delta n]$ with $n_0 = 2$. Different colors represent various disordered strengths Δn (indicated in the figure). (a) Single level around average angular frequency $\omega_{avg} \approx 4 \left[\frac{c}{l_1} \right]$ for one arbitrary realization. (b) An average over a small frequency window around $\omega_{avg} = 4 \left[\frac{c}{l_1} \right]$ and over disordered realizations is taken.

Scattering set-up - Since properties of the isolated ring eigenmodes are closely related to transmission through the ring, when the latter is connected to external leads, we expect oscillating behavior also in transmittances. The leads are realized by a straight fiber [14] coupled with the 1D ring cavity, see the inset of Fig. 3a. The coupler between the fiber and the ring cavity is modeled by a 4×4 scattering matrix Σ defined as [15]

$$\Sigma = \begin{bmatrix} O & K \\ K^T & O \end{bmatrix}; \quad K = \begin{bmatrix} \kappa & \tau \\ -\tau & \kappa \end{bmatrix}, \qquad (8)$$

where O is the null 2×2 matrix, $\tau \in (0,1)$, and $\kappa = \sqrt{1 - \tau^2}$. At the same time the scattering at the ring is described by the transfer matrix of Eq. (5). Combining Eqs. (5,8) together leads to the following expression for the transmittance $\mathcal{T}$

$$\mathcal{T} = 1 - \frac{(1-\tau^2)^2(1-T_+)}{1 + \tau^4 + 2\tau^2 T_+ \left(1 + \cos(2\theta) + \frac{\cos(2\phi_t)}{T_+}\right) + 4\tau(1+\tau^2)\sqrt{T_+}\cos(\theta)\cos(\phi_t)}, \tag{9}$$

where $\theta \equiv \frac{\omega}{c}\beta L$. The above expression applies equally well for both one scatterer and for a disordered ring. It follows from Eq. (9) that $\mathcal{T}(\theta+2\pi,\phi_t) = \mathcal{T}(\theta,\phi_t)$ i.e. for a single disorder configuration, the transmittance $\mathcal{T}$ is a periodic function of θ with period 2π. This expectation is confirmed by our numerical simulations for single disorder realization, see Fig. 3a. When, however, we average $\mathcal{T}$ over disorder realizations we obtain the half period oscillations in the transmittance, in complete analogy with our previous results for P_L. Our numerical results, for various disorder strengths, are shown in Fig. 3b.
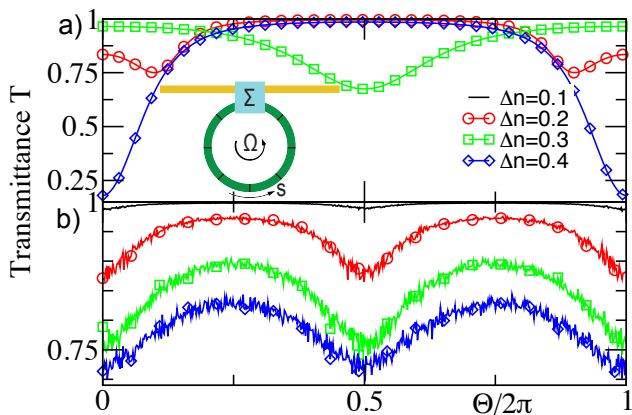

FIG. 3: Transmittance T vs. $\frac{\theta}{2\pi} = \frac{\omega}{2\pi c}\beta L$ for disordered rings described in Fig. 2 with $n_0 = 1.5$ and Δn indicated in the figure. The frequency of incoming wave is fixed at $\omega = 4\left[\frac{c}{l_1}\right]$. The coupling parameters between the fiber and the disordered ring are $\tau = 1/\sqrt{2}$ and $\kappa = 1/\sqrt{2}$. Different colors represent various disordered strengths Δn (indicated in the figure). (a) Single realization of disordered ring. (Inset: a schematic of the scattering set-up.) (b) An average over 800 disordered realizations is taken.

In the weak disorder limit, corresponding to fixed $T_+(\approx 1)$ and uniform random phase distribution $\phi_t \in [0, 2\pi]$, one can evaluate the average total transmittance as

$$\langle \mathcal{T}(\theta) \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\phi_t \mathcal{T}(\theta, \phi_t). \tag{10}$$

Direct substitution of Eq. (9) in the above equation lead to the AAS relation

$$\langle \mathcal{T}(\theta + \pi) \rangle = \langle \mathcal{T}(\theta) \rangle. \tag{11}$$

The AAS oscillations are due to interference between a pair of waves traversing the ring in opposite directions and recombining after making one (or more) full circle [10]. It is important to note that the "trivial" phase, not related to rotation (i.e. to the "flux") is exactly the same for the two waves and it cancels out in the interference pattern, which is governed by the relative phase equal to 2θ. This relative phase does not depend on the specific disorder realization which explains the robustness of the "half-flux" oscillations against averaging over disorder. The picture is different for the AB oscillations. For those the topological phase θ is always accompanied by the "trivial", sample specific phase ϕ_t. Therefore averaging over disorder leads to decrease of the amplitude of the AB oscillations until, for sufficiently strong disorder, they become negligible and only the AAS oscillations remain visible. The above argumentation is supported by a direct inspection of Eq. (9) where one sees that the $\cos(2\theta)$-term stands alone, not getting mixed with terms containing ϕ_t. In contrast, the $\cos\theta$-term is multiplied by $\cos\phi_t$, and therefore does not survive an averaging over ϕ_t. Finally as the disorder increases further, the amplitude of the oscillations decreases and eventually, in the strong localized regime, approaches zero. This is due to the fact that the total transmittance in Eq. (9) involves T_+ terms which control the amplitude of the oscillations and which in the strong disorder limit go to zero.

Conclusions - We have shown that the wave equation for light propagating in a rotating ring bears certain analogies with the Schrödinger equation describing a quantum particle moving in a ring in the presence of a magnetic flux. These analogies have, however, constraints – the most severe being the fact that the ME is not periodic with the angular velocity, which formally is associated with the magnetic flux in the quantum problem. Despite this distinction, we have found that in the optical high frequency domain the periodicity of the spectrum is (approximately) restored. In this frequency domain, AB oscillations emerge for single disorder realizations. When a disorder averaging is considered, the fundamental periodicity reduces to half of the AB oscillations. Our results are relevant for other classical waves as well. For example the propagation of sound in a randomly corrugated tube with a moving fluid is described by similar equations as Eq. (1) [16]. In this framework, AB and AAS oscillations must be easily observed for relatively small fluid velocities due to the low values of sound velocities.

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[11] In the general case where the index of refraction $n(x,y)$ depends on the coordinates x, y, the ME for Ψ takes the form $\nabla^2 \Psi + n^2(x,y)\frac{\omega^2}{c^2}\Psi - 2i\frac{\omega}{c^2}(\vec{v} \cdot \nabla)\Psi = 0$, where $\vec{v} = \vec{\Omega} \times \vec{r}$. This equation can be re-written in a form similar to Eq. (2) i.e. $\left(\vec{\nabla} - i\frac{\omega}{c}\frac{\vec{\Omega}\times\vec{r}}{c}\right)^2 \Psi + \left[n^2(x,y) + \left(\frac{\vec{\Omega}\times\vec{r}}{c}\right)^2\right]\frac{\omega^2}{c^2}\Psi = 0$. One can make use of "gauge freedom" in the last equation, in order to simplify treatment in certain geometries. For instance, if $n(x,y)$ depends only on one coordinate, then it is clearly advantageous to use Landau gauge rather than circular gauge. This gauge freedom also enables one to construct solutions of the "clean" problem (i.e. for $n(x,y) = n_0$=const.) which can be useful in defining the appropriate asymptotic states in a scattering set-up.

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