# Quantum speed meter based on dissipative coupling

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# Quantum Speed Meter Based on Dissipative Coupling

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We show that generalized dissipative opto-mechanical coupling enables a direct quantum measurement of speed of a free test mass. An optical detection of a weak classical mechanical force based on this interaction is proposed. The sensitivity of the force measurement can be better than the standard quantum limit.

## I. INTRODUCTION

Resonant opto-mechanics [1] involves interaction of an optical cavity and a mechanical oscillator or a free mass. The simplest interaction of this kind is based on radiation pressure effect in which a coordinate of a 1D mechanical system experiences a force proportional to optical power or number of optical quanta circulating in the 1D optical cavity, so that the size of the optical cavity increases with increase of number of optical quanta localized in there. Opto-mechanical interaction of systems having several degrees of freedom enable more complex interactions ranging from radiation puling (negative radiation pressure) [2, 3] and opto-mechanical interaction proportional to the quadrature of electromagnetic field [4–7] to the interaction depending on the speed and not the coordinate of the mechanical system [8, 9].

Opto-mechanics plays an important role in precision measurements proving an efficient quantum transduction mechanism between the mechanical and optical degrees of freedom enabling various sensors, like gravitational wave detectors [10–12], torque sensors [13], and magnetometers [14].

The sensitivity of the mechanical coordinate measurement in an opto-mechanical system usually is limited due to quantum back action by so called standard quantum limit (SQL) [15, 16]. The SQL was studied in various configurations ranging from macroscopic kilometre-sized gravitational wave detectors [7] to microcavities [17, 18]. Sensitivity of other types of measurements being derivatives of the coordinate detection is also limited by SQL. Detection of a classical force acting on a mechanical degree of freedom of an opto-mechanical system is an example of such a measurement. SQL of force measurement is not a fundamentally unavoidable limit. It can be overcome with several approaches including variational measurement [4, 7, 19], opto-mechanical velocity measurement [8, 9], and measurements in opto-mechanical systems with ponderomotive rigidity [20, 21].

In this paper we study a possibility of improvement of sensitivity of a classical force detector using a particular type of an opto-mechanical transducer based on dissipative coupling of optical and mechanical degrees of freedom. It was shown recently that the dissipative coupling allows obtaining a significantly better position resolution at low power levels in comparison with the conventional dispersive case, however, the measurement sensitivity is still limited by SQL [22]. In this paper we show that usage of dissipative coupling for detection of small signal force acting on a free mechanical test mass provides a direct possibility to realize speed meter and to beat the SQL.

Among the variety of the opto-mechanical processes dissipative opto-mechanical coupling takes a special place. The dissipative coupling is characterized by dependence of relaxation of an optic cavity on coordinate of the mechanical oscillator, whereas dispersive coupling is characterized by the dependence on eigen frequency of the optical cavity on the coordinate. Dissipative coupling allows observing quantum effects in an optical resonator with ring down time comparable to the frequency of the mechanical system. The system cannot be considered lossless any more. The trick is that the dissipation here does not lead to decoherence or absorption of light. Instead, it results in lossless coupling between a continuous optical wave and a mode of an optical cavity. The optical cavity performs as a perfect transducer between the continuous optical wave and the mechanical degree of freedom, enabling efficient cooling of the mechanical oscillator [13, 23–26], exchange of the quantum states between the optical and mechanical degrees of freedom, mechanical squeezing [27–30], as well as a combination of the cooling and squeezing [31, 32]. A combination of conventional, dispersive, and dissipative coupling adds more complexity to the interaction and leads to new effects [33, 34].

Dissipative coupling was proposed theoretically [23] and implemented experimentally [13, 24, 25, 35] nearly a decade ago. It was studied in a variety of opto-mechanical systems, including Fabry-Perot interferometer [13, 24, 25, 35], Michelson-Sagnac interferometer [22, 26, 36], and ring resonators [33, 34].

We found another important feature of dissipative coupling in an opto-mechanical system. The optical wave reflected from an optical cavity interacting with a mechanical oscillator via dissipative coupling contains information about speed of the mechanical degree of freedom, giving a direct possibility to realize opto-mechanical speed meter. This type of interaction is optimal for detection of a classical force because the speed meter is not impacted by the initial coordinate of the mechanical system and allows better than SQL force detection when operating in both a narrow-band and a wide-band regimes [8, 9].

In what follows we solve the problem of detection of a classical force acting on a free test mass, position of which determines attenuation of an optical interferometer. Any mechanical oscillator can be considered as a free mass at the time scale much smaller than the oscillation period. Motion of a free mass is completely characterized by energy and momentum conservation. Both energy and momentum can be measured using the quantum nondemolition (QND) technique, which does not disturb the variables, but increases uncertainty of their quantum conjugated ones (phase and coordinate, respectively) [16].

The idea of speed meter enabling the force detection is that output optical field contains information about difference $\sim \big[x(t) - x(t-\tau)\big] \simeq \tau \dot{x}(t)$ (where $x$ is the mechanical coordinate, $\tau$ is a delay time). In frequency domain it corresponds to $\sim x(\Omega)\left(1 - e^{i\Omega\tau}\right) \simeq -i\Omega\tau\, x(\Omega)$. In proposed earlier speed meters such subtraction is realized by interaction of a mechanical degree of freedom with two coupled optical modes. Presence of several optical modes in the system increases complexity of experimental realization [8, 9] if compared with a conventional position meter. We here show that the dissipative coupling without the dispersive one [22] provides this kind of subtraction automatically, enabling a simple way of measurement of speed of a free test mass, which is a quantum nondemolition (QND) variable, suitable for a sensitive detection of a classical force.

## II. MODEL

To describe the force detection we consider a 1D optomechanical configuration (Fig. 1) involving an optical mode characterized with eigen frequency $\omega_0$ pumped with resonant light (the pump frequency coincides with the mode frequency, $\omega_\mathrm{p} = \omega_0$). The optical mode is dissipatively coupled with the mechanical system represented by a free mass $m$. Relaxation rate $\kappa$ of the optical mode depends on test mass displacement $x$. The force of interest, $F_s$, acts on the free mass and changes its position.

We use the Hamiltonian approach to describe the dissipative coupling [37]:

$$H = \hbar\omega_0 \hat{a}_c^\dagger \hat{a}_c + \frac{\hat{p}^2}{2m} + H_T + H_\kappa - F_s \hat{x}, \tag{2.1a}$$

$$H_T = \int \hbar\omega\, \hat{b}^\dagger(\omega)\hat{b}(\omega)\, d\omega, \tag{2.1b}$$

$$H_\kappa = -i\hbar\sqrt{\kappa}\left[\hat{a}_c^\dagger \hat{a}_{in} - \hat{a}_{in}^\dagger \hat{a}_c\right], \tag{2.1c}$$

$$\kappa = \kappa_0(1+\eta\hat{x}),\ \sqrt{\kappa} \simeq \sqrt{\kappa_0}\left(1 + \frac{\eta}{2}\hat{x}\right), \tag{2.1d}$$

where $\hat{p}$ is the momentum of the test mass, $H_T$ describes electromagnetic continuum, $H_\kappa$ stands for attenuation of the pump photons and associated quantum noise, and $\eta$ is a constant of dissipative coupling.

From this point we present the annihilation operators of the input and intracavity optical field throught slow amplitudes as

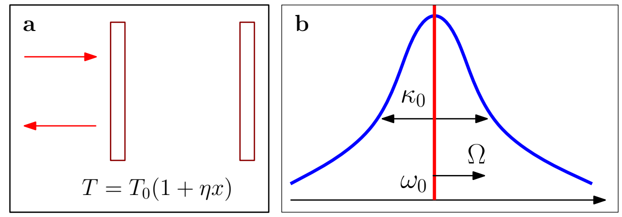


FIG. 1: *a)* Schematic of the speed meter. A mode of an optical Fabry-Perot cavity is dissipatively coupled with a mechanical degree of freedom represented by a free mass $m$ a force of interest, $F_s$, acts upon. The coupling changes the power transmission coefficient of the cavity front mirror $T(x)$. In linear approximation this change can be presented as $T(x) = T_0(1+\eta x)$, where $x$ is the displacement of free mass, $\eta$ is a coupling coefficient, $T_0$ is the unperturbed power transmission coefficient related to full width at the half maximum of the mode as $\kappa_0 = T_0\tau$, where $\tau$ is the round trip time of light in the cavity. *b)* The optical mode is resonantly pumped by coherent light ($\omega_\mathrm{p} = \omega_0$). The light reflected from the cavity carries information about the force $F_s$.

$$\hat{a}_c \Rightarrow \hat{a}_c e^{-i\omega_0 t}, \quad \hat{a}_{in} \Rightarrow \hat{a}_{in} e^{-i\omega_\mathrm{p} t}, \quad \omega_0 = \omega_p. \tag{2.2}$$

The set of corresponding equations describing time evolution of the opto-mechanical system is

$$\dot{\hat{a}}_c = -\frac{\kappa}{2}\hat{a}_c + \sqrt{\kappa}\hat{a}_{in}, \tag{2.3}$$

$$\ddot{\hat{x}} = i\hbar\frac{\sqrt{\kappa_0}\eta}{2m}\left[\hat{a}_c^\dagger \hat{a}_{in} - \hat{a}_{in}^\dagger \hat{a}_c\right] + F_s. \tag{2.4}$$

These equations have to be supplied with an expression for the output field, which can be presented in the case of $T_0 \ll 1$ as

$$\hat{d}_{out} \simeq -\hat{a}_{in} + \sqrt{\kappa}\hat{a}_c. \tag{2.5}$$

Below we present amplitudes as large mean values plus small addition:

$$\hat{a}_c = A + \hat{a},\ \hat{a}_{in} = A_0 + \hat{a}_{fl},\ \hat{d}_{out} = A_{out} + \hat{a}_{out} \tag{2.6a}$$

$$\hat{a}_{fl} = -\int \hat{b}(\omega) e^{-i(\omega-\omega_\mathrm{p})t}\frac{d\omega}{2\pi}, \tag{2.6b}$$

$$\big[\hat{b}(\omega), \hat{b}^\dagger(\omega')\big] = 2\pi\delta(\omega-\omega'), \tag{2.6c}$$

$$\big[\hat{a}_{fl}(t), \hat{a}_{fl}^\dagger(t')\big] = \delta(t-t'), \tag{2.6d}$$

We assume that the expectation values exceed the fluctuation parts of the operators and use the method of successive approximations to derive a set of equations describing the system. We select $A_0 = A_0^*$ and find in steady state (zero order of approximation)

$$A = \frac{2}{\sqrt{\kappa_0}} A_0,\ A_{out} = A_0, \tag{2.7}$$

where $A_{out}$ is the expectation value of the output field.

The fluctuation part of the field and the deviation of the test mass are described by equations (first order os approximation)

$$\dot{\hat{a}} + \frac{\kappa_0}{2} a = -\frac{\eta\kappa_0}{4} A\hat{x} + \sqrt{\kappa_0}\hat{a}_{fl}, \tag{2.8}$$

$$\hat{a}_{out} = -\hat{a}_{fl} + \sqrt{\kappa_0}\hat{a} + \frac{\eta\sqrt{\kappa_0}}{2} A\hat{x}, \tag{2.9}$$

$$\ddot{\hat{x}} = i\hbar A \frac{\eta\sqrt{\kappa_0}}{2m}\left[(\hat{a}_{fl} - \hat{a}_{fl}^\dagger) - \frac{\sqrt{\kappa_0}}{2}(\hat{a} - \hat{a}^\dagger)\right] + \frac{F_s}{m}.$$

that can be solved in frequency domain as

$$\hat{x}_\Omega = -\frac{\hbar\eta A\sqrt{\kappa_0}}{m\Omega(\kappa_0 - 2i\Omega)}(\hat{b}_+ - \hat{b}_-^\dagger) - \frac{F_\Omega}{m\Omega^2}, \tag{2.10}$$

$$a_{out+} = \frac{\kappa_0 + 2i\Omega}{\kappa_0 - 2i\Omega}\hat{b}_+ + \eta A\sqrt{\kappa_0}\frac{-i\Omega x_\Omega}{\kappa_0 - 2i\Omega}, \tag{2.11}$$

where $\hat{x}_\Omega$, $a_{out+}$, $F_\Omega$, and $\hat{b}_\pm \equiv \hat{b}(\omega_0 \pm \Omega)$ are Fourier amplitudes of corresponding operators.

From last equation (2.11) we see that the output field provides information on speed of the probe mass $-i\Omega x$ if $\kappa_0 \gg \Omega$, but not the displacement. This is one of the major results of the paper. We have shown that the opto-mechanical sensor becomes a speed meter if the coordinate of the probe mass modifies transparency of the front mirror of the optical interferometer, or, in other words, changes the dissipative coupling with the probe light.

## III. DETECTION OF THE CLASSICAL FORCE

Let us find sensitivity of detection of the classical force acting on the test mass. We assume that the output of the interferometer (Eq. 2.11) is optimally processed for this purpose. The processing is achieved using a homodyne detector that is able to measure arbitrary quadrature amplitudes of the output light, which can be presented as a linear combination of the amplitude and phase quadratures

$$q_a = \frac{a_{out+} + a_{out-}^\dagger}{\sqrt{2}}, \quad q_p = \frac{a_{out+} - a_{out-}^\dagger}{i\sqrt{2}}. \tag{3.1}$$

These quadrature amplitudes can be expressed through the quadratures of input light

$$d_a = \frac{\hat{b}_+ + \hat{b}_-^\dagger}{\sqrt{2}}, \quad d_p = \frac{\hat{b}_+ - \hat{b}_-^\dagger}{i\sqrt{2}}, \tag{3.2}$$

in the following way

$$q_p = \frac{\kappa_0 + 2i\Omega}{\kappa_0 - 2i\Omega} d_p \tag{3.3a}$$

$$q_a = \frac{\kappa_0 + 2i\Omega}{\kappa_0 - 2i\Omega}\left\{d_a - \mathcal{Q}\cdot d_p + \sqrt{2\mathcal{Q}}\cdot f_s\right\}, \tag{3.3b}$$

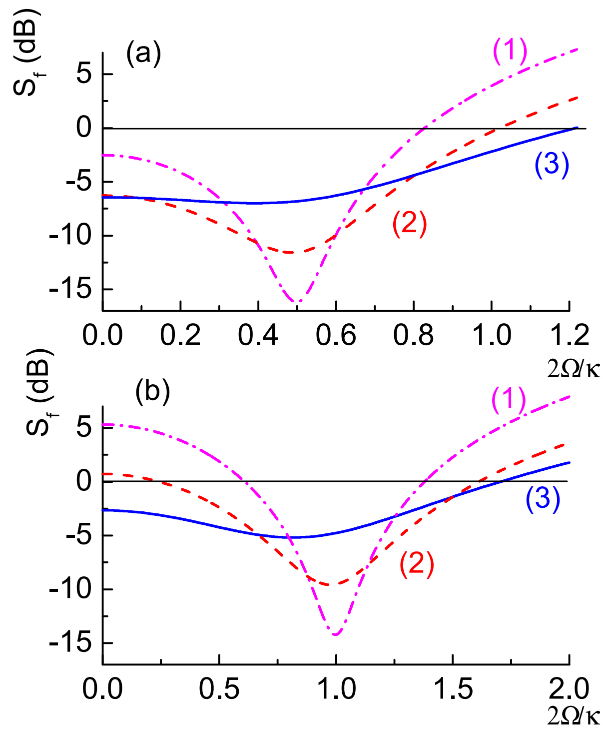


FIG. 2: Normalized spectral density of the noise $f$ for the optimal procedure of the force measurement using the speed meter technique evaluated for the optimal homodyne angle (3.6) at frequencies $2\Omega_0/\kappa_0 = 0.5$ (a) and for $2\Omega_0/\kappa_0 = 1$ (b) for different power parameters $P$ (3.4): curve (1) on (a) and (b) corresponds to $P = 27$, (2) — $P = 9$ and (3) — $P = 3$. The horizontal line describes SQL.

where

$$\mathcal{Q} \equiv \frac{P\kappa_0^2}{\kappa_0^2 + 4\Omega^2}, \quad P \equiv \frac{2\hbar|A|^2\eta^2}{m\kappa_0}, \tag{3.4a}$$

$$f_s \equiv e^{i\beta}\frac{F_s(\Omega)}{F_{\rm SQL}}, \quad F_{\rm SQL} \equiv \sqrt{2\hbar m\Omega^2}. \tag{3.4b}$$

Here $f_s$ is the signal force normalized by Standard Quantum Limit (SQL). We see that the amplitude quadrature of the output field $q_a$ contains shot noise term ($\sim d_a$), back action term ($\sim \mathcal{Q}d_p$) and signal term ($\sim \sqrt{2\mathcal{Q}}f_s$).

Inferring the force from the measurement of the amplitude quadrature $q_a$ results in the maximum measurement sensitivity limited by SQL. This is not the optimal detection strategy. It is possible to achieve better measurement sensitivity by observing quadrature $q = d_a\cos\theta + d_p\sin\theta$, where $\theta$ is an angle that can be optimized. In order to be registered Fourier component of normalized signal force $f_s(\Omega)$ should be larger than Fourier component of recalculated noise $f(\Omega)$ (c.f. [7])

$$f \equiv \frac{e^{-i\beta}}{\sqrt{2}}\left\{\frac{d_a}{\sqrt{\mathcal{Q}}} + \left(-\sqrt{\mathcal{Q}} + \frac{\tan\theta}{\sqrt{\mathcal{Q}}}\right)d_p\right\}. \tag{3.5}$$

Single-sided spectral density $S_f(\Omega)$ of noise $f$ may be expressed through spectral densities $S_a$ and $S_p$ of corre-

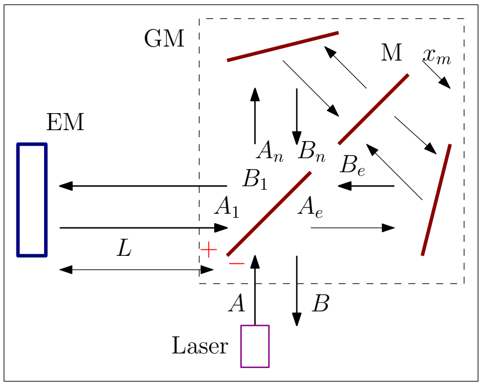


FIG. 3: Michelson-Sagnac Interferometer as a input generalized mirror (GM) of a Fabry-Perot cavity. Perfectly reflecting mirror $M$ represents a free test mass. Distance $L$ between the generalized mirror and the end mirror (EM) is much larger than the size of interferometer.

sponding quadratures $d_a$ and $d_p$. Below we assume that the input light is prepared in coherent state, corresponding to $S_a = S_d = 1$ and no correlation between $d_a$ and $d_p$ [7].

Homodyne angle $\theta_{\rm opt}$ determined by equation

$$\tan\theta_{\rm opt} = \frac{P\kappa_0^2}{\kappa_0^2 + 4\Omega_0^2} \tag{3.6}$$

has to be selected in order to minimize $S_f$ at frequency $\Omega_0$. The noise limiting the measurement sensitivity expressed in terms of spectral density $S_f$ is presented in Fig. 2 .

These dependencies show that the sensitivity of the force detection better than SQL can be achieved in a relatively large frequency band. This is another important result of this paper. It worth noting, though, that this is also a known feature of any speed meter used for a classical force detection [8, 9].

## IV. EXAMPLE OF REALIZATION OF DISSIPATIVE COUPLING

Realization of the dissipative coupling without dispersive one is not straightforward. To the best of our knowledge, the only proven example of pure dissipative coupling is suggested in [22]. The system is based on a Michelson-Sagnac interferometer, simplified version of which is shown in Fig. 3. The interferometer contains the generalized mirror, being an interferometer itself, that provides the dissipative coupling for the Fabry-Perot interferometer. We assume that the size of the generalized mirror is much smaller than distance $L$ between the beam splitter and the end mirror, so both transmittance $\mathbb{T}$ and reflectivity $\mathbb{R}$ of the generalized mirror are the constants and has no dependence on spectral frequency.

Taking advantage of [22] we find

$$\mathbb{T} = i\sin 2kx_m, \quad \mathbb{R} = \cos 2kx_m, \tag{4.1}$$

$$B_1 = \mathbb{T}A + \mathbb{R}A_1, \quad B = \mathbb{T}A_1 + \mathbb{R}A\,, \tag{4.2}$$

where $k = \omega_p/c$ is the wave vector of the input light wave. We assume that the light travelling between the beam splitter and mirror $M$ through east arm accumulates phase $\phi_e$ and the light travelling between the beam splitter and the north arm accumulates phase $\phi_n$. In the case of zero mechanical deviation ($x_m = 0$) we select the phases to be

$$e^{2i\phi_e} = -1, \quad e^{2i\phi_n} = 1, \quad e^{i(\phi_e+\phi_n)} = i. \tag{4.3}$$

The cavity has high finesse, $|\mathbb{T}| \ll 1$, so the magnitude of the mechanical deviation has to be small enough to allow the interferometer operate near the resonance $kx_m \ll \mathbb{T}$.

Small displacements $x$ of mirror $M$ from the mean position $x_m$ provides modulation of the relaxation rate of the Fabry-Perot interferometer:

$$\kappa = \kappa_0(1+\eta x), \tag{4.4a}$$

$$\kappa_0 = \frac{|\mathbb{T}|^2}{2\tau} = \frac{\sin^2 2kx_m}{2\tau}, \quad \tau = \frac{L}{c} \tag{4.4b}$$

$$\eta = 4k\cot 2kx_m \simeq \frac{4k}{|\mathbb{T}|} = \frac{4k}{\sqrt{2\kappa_0\tau}} \tag{4.4c}$$

We see that the considered generalized mirror demonstrates pure dissipative coupling because the spatial shift of mirror $M$ changes only the relaxation rate of Fabry-Perot cavity, not its eigen frequency.

## V. DISCUSSION

To understand the advantage of the speed meter if compared with the standard interferometric technique of detection of a classical force based on pure dispersive opto-mechanical coupling let us consider a system described by Hamiltonian

$$H = \hbar\omega_0\left(1+\xi\hat{x}\right)\hat{a}_c^\dagger\hat{a}_c + \frac{\hat{p}^2}{2m} + H_T + H_\kappa - F_s\hat{x}, \tag{5.1}$$

where $\xi$ is a constant of dispersive coupling. The other terms are the same as in (2.1). The relaxation rate is constant in the case of the dispersive coupling, $\kappa = \kappa_0$, while the information about mechanical displacement is carried by the frequency of the optical cavity.

It is straightforward to obtain for the amplitude and phase quadrature amplitudes of the output light (c.f. [7])

$$q_a^{\rm disp} = e^{-2i\alpha}\, d_a\,, \tag{5.2a}$$

$$q_p^{\rm disp} = e^{-2i\alpha}\left\{d_p - \mathcal{K}\, d_a + \sqrt{2\mathcal{K}}\, f_s^{\rm disp}\right\}, \tag{5.2b}$$

$$\mathcal{K} \equiv \frac{8\hbar\kappa_0\omega_0^2\xi^2A^2}{m\Omega^2\left(\kappa_0^2+4\Omega^2\right)}\,, \quad e^{-2i\alpha} \equiv \frac{\kappa_0+2i\Omega}{\kappa_0-2i\Omega}. \tag{5.2c}$$

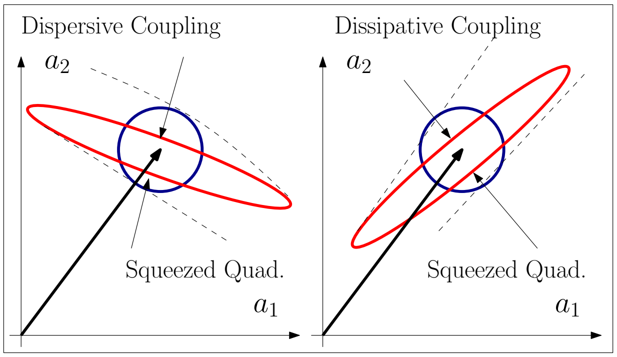


FIG. 4: Illustration of the difference between types of ponderomotive squeezing achieved in the cases of dispersive and dissipative coupling. The blue circles on the both diagrams represent distribution of quantum fluctuations of the input wave prepared in the coherent state, red ellipses describe quantum squeezing of the fluctuations of the output waves.

Comparing output quadratures (3.3) for dissipative coupling with (5.2) for dispersive one we see that formulas are similar with exception of four features.

Firstly, amplitude quadrature $q_a$ in (3.3) is replaced with phase quadrature amplitude $q_p^{\rm disp}$ in (5.2) as well as $q_p \Rightarrow q_a^{\rm disp}$. Ponderomotive squeezing takes place for the both cases of coupling at large enough pump power ($|\mathcal{Q}|,\ |\mathcal{K}| \gg 1$). However, squeezed quadratures are different (Fig. 4).

Secondly, power parameters $\mathcal{Q}$ in (3.4) and $\mathcal{K}$ in (V) have different spectral dependence. In particular, $\mathcal{K}$ infinitely increases at the limit of $\Omega \to 0$, whereas $\mathcal{Q}$ stays limited. The steep frequency dependence of $\mathcal{K}$ complicates the detection procedure.

Thirdly, the ponderomotive squeezing for dissipative coupling practically does not depend on spectral frequency at low frequency offsets, $\Omega \ll \kappa_0$. In contrast, the same squeezing for dispersive coupling is strongly frequency dependent [5].

It is interesting to compare the magnitude of the power parameters for the dispersive and dissipative coupling:

$$\frac{\mathcal{Q}}{\mathcal{K}} = \left(\frac{\Omega\eta}{2\omega_0\xi}\right)^2 = \frac{4\Omega^2\tau}{\kappa_0} = \tag{5.3}$$
$$= 0.04 \cdot \left[\frac{\Omega}{1000\,{\rm s}^{-1}}\right]^2 \left[\frac{\tau}{10^{-5}\,{\rm s}}\right] \left[\frac{1000\,{\rm s}^{-1}}{\kappa_0}\right].$$

In (5.3) we used (4.4) and $\xi = 1/L$. The parameters are comparable at large enough spectral frequencies. This is an important conclusion since it seems to be obvious that the dispersive coupling may be stronger as compared with the dissipative coupling. Really, the dispersive opto-mechanical coupling corresponds to the change of the optical frequency, while the dissipative coupling corresponds to change of the optical bandwidth. The optical frequency is larger than the bandwidth since the system has a large quality factor. Fortunately, the coupling coefficient $\eta$ is much larger than the coefficient $\xi = 1/L$. It means that the sensor of a classical force based on a dissipative opto-mechanical system has sensitivity comparable with the sensitivity of the force measurement by means of a dispersive opto-mechanical system but has much broader measurement bandwidth.

## Conclusion

We have shown that the generally defined purely dissipative opto-mechanical coupling naturally provides a realization of a quantum speed meter. Such a device can be used for measurements of a classical force with sensitivity better than the standard quantum limit. The force should act on the mechanical degree of freedom of the opto-mechanical system that modifies the coupling of an optical cavity with the external world. Such a sensor has a broad measurement bandwidth needed for detection of mechanical forces characterized with complex wave forms and broad spectrum. The device can be used in various metrology applications ranging from magnetometry to gravity wave detection.

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