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Comment on “Breakdown of the expansion of finite-size corrections to the hydrogen Lamb shift in moments of charge distribution”

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# Comment on “Breakdown of the expansion of finite-size corrections to the hydrogen Lamb shift in moments of charge distribution”

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(Dated: February 2, 2016)

In a recent paper, Hagelstein and Pascalutsa examine the error associated with an expansion of proton structure corrections to the Lamb shift in terms of moments of the charge distribution. They propose a small modification to a conventional parameterization of the proton’s charge form factor and show that this can resolve the proton radius puzzle. However, while the size of the “bump” they add to the form factor is small, it is large compared to the total proton structure effects in the initial parameterization, yielding a final form factor that is unphysical. Reducing their modification to the point where the resulting form factor is physical does not allow for a resolution of the radius puzzle.

Ref. [1] proposes a possible explanation to the proton radius puzzle [2, 3], noting that the error associated with the expansion of the Lamb shift in terms of the moments of the charge radius,  $\langle r^2 \rangle$  and  $\langle r^3 \rangle$ , can be large in the presence of sharp structures in the form factors. They demonstrate that a small, narrow contribution to the proton’s charge form factor at very low  $Q^2$  could explain the discrepancy in the extracted RMS charge radius from the muonic hydrogen Lamb shift measurements [4, 5]. Their example involved a narrow peak added to a standard parameterization of the charge form factor,  $G_E(Q^2)$ , at  $Q^2$  values which significantly impact the Lamb shift in muonic hydrogen [4, 5]. The modification is too high in  $Q^2$  to significantly modify the Lamb shift in electronic hydrogen [6, 7], but below the  $Q^2$  region where electron scattering data exist and can be used to extract the charge radius [8–12].

Their proposed modification to  $G_E$  is very small, with a peak contribution to  $G_E$  of  $3 \times 10^{-5}$ , narrowly localized around  $Q^2 \approx 10^{-6} \text{ GeV}^2$ . However, while the change in  $G_E$  is extremely small, that does not mean that this is a minor modification to the proton form factor. This modification should not be compared to  $G_E$ , which is close to unity at low  $Q^2$ , but should be compared to  $G_E - 1$  which represents the deviation of the form factor from that of a point proton:  $G_E(Q^2) = 1$ . For the form factor parameterization [13] used in [1],  $|G_E - 1| = 3.5 \times 10^{-6}$  for  $Q^2 \approx 10^{-6} \text{ GeV}^2$ . The proposed modification, while small compared to  $G_E$ , is roughly ten times larger than the total finite-size effect in [13]. Because this bump is added to the form factor, their modified form factor is unphysical, yielding  $G_E > 1$  as shown in Figure 1.

Based on Fig. 3 of Ref. [1], reducing the size of the modification by an order of magnitude to avoid  $G_E > 1$  would not provide a significant improvement in the agreement between eH and  $\mu$ H Lamb shift results. Similar features in the region of the eH sensitivity peak,  $Q^2 \approx 10^{-10} \text{ GeV}^2$ , would have to be  $10^5$  times smaller to avoid exceeding the full finite-size correction from [13]. Even if a smaller (or negative) modification were made,

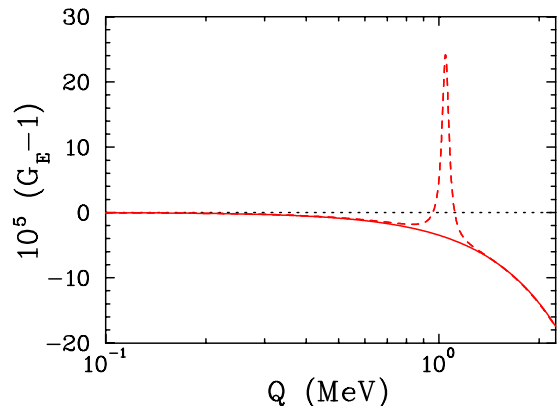


FIG. 1: The finite structure correction to the  $G_E$ ,  $G_E(Q^2) - 1$ , vs  $Q$  for the parameterization of Ref. [13] (solid line), and including the modification of Ref. [1] (dashed line).

such that the resulting  $G_E$  would not be unphysical, it would most likely be inconsistent with the constraints from analyticity of the form factors [14].

While the bump added to  $G_E$  in [1] brings the Lamb shift extractions into agreement after correcting for the error made when expanding in moments of the charge radius, the resulting form factor is unphysical. Simply reducing or broadening the bump near the peak of sensitivity for the  $\mu$ H Lamb shift measurements cannot provide a resolution to the discrepancy. It seems unlikely that it is possible to find another such modification which resolves the discrepancy and is consistent with the constraints from the analyticity of the form factors.

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