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# Perfect-fluid behavior of a dilute Fermi gas near unitarity

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We present an *ab initio* calculation of the shear viscosity as a function of interaction strength in a two-component unpolarized Fermi gas near the unitary limit, within a finite temperature quantum Monte Carlo (QMC) framework and using the Kubo linear-response formalism. The shear viscosity decreases as we tune the interaction strength  $\frac{1}{ak_F}$  from the Bardeen-Cooper-Schrieffer (BCS) side of the Feshbach resonance towards Bose-Einstein condensation (BEC) limit and it acquires the smallest value for  $\frac{1}{ak_F} \approx 0.4$ , with a minimum value of  $\frac{\eta}{s}|_{min} \approx 0.2 \frac{\hbar}{k_B}$ , which is about twice as small than the value reported for experiments in quark-gluon plasma  $\frac{\eta}{s}|_{QGP} \lesssim 0.4 \frac{\hbar}{k_B}$ . The Fermi gas near unitarity thus emerges as the most “perfect fluid” observed so far in Nature. The clouds of dilute Fermi gas near unitarity exhibit the unusual attribute that, for the sizes realized so far in laboratory or larger (less than  $10^9$  atoms), can sustain quantum turbulence below the critical temperature, but at the same time the classical turbulence is suppressed in the normal phase.

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The ultracold atoms provide an ideal laboratory for very precise experimental and theoretical studies of an enormous range of quantum mechanical phenomena. A large set of studies focused on interacting Fermi gases at unitarity, where such systems exhibit remarkable properties. The average inter-particle separation  $\approx n^{-1/3}$  is large compared to the effective range of interaction  $r_{\text{eff}}$ , but small compared to the scattering length  $|a|$ , i.e.  $0 \leftarrow n^{1/3}r_{\text{eff}} \ll 1 \ll n^{1/3}|a| \rightarrow \infty$  (where  $n$  is particle density). In this limit these systems acquire universal properties and they have been widely studied over last dozen of years both experimentally and theoretically (see reviews [1–4]). A series of recent experiments [5–8] revealed the nearly ideal hydrodynamic behavior of the resonantly interacting Fermi gas, characterized by a very low shear viscosity coefficient to the entropy density ratio - very close to the conjectured bound originating from holographic duality methods [9, 10], called KSS bound. An “ideal fluid” which follows the laws of ideal hydrodynamics, is a fluid in which dissipative processes are absent [11]. The superfluid component of a quantum fluid below the critical temperature is treated as a physical realization of an ideal fluid [12]. The normal component is characterized by a finite viscosity, which in an infinite medium tends to infinity at absolute zero temperature, due to the contributions of phonons [12]. In a finite system however the longest phonon wavelength is of the order of the size of the container and with the number of excited phonons  $\propto T^3$  this has the consequence that the viscosity, while approaching the zero temperature, never formally diverges and a liquid is effectively in a collisionless regime.

A hypothetical physical system which saturates the KSS bound and has the lowest possible value of the shear viscosity is often referred to as “the perfect fluid”. A recent measurement of the shear viscosity of a unitary gas close to unitarity [8] provides the value  $\frac{\eta}{s} \approx 0.5 \frac{\hbar}{k_B}$ , while

KSS bound is  $\frac{1}{4\pi} \frac{\hbar}{k_B} \approx 0.08 \frac{\hbar}{k_B}$ . Thus the unitary Fermi gas (UFG) appears as the system being very close to “perfectness.” Another system is the quark-gluon plasma created in relativistic heavy ion collisions [13], where the value  $\frac{\eta}{s}|_{QGP} \lesssim 0.4 \frac{\hbar}{k_B}$  has been reported [14].

Simultaneously, an impressive effort has been made in order to theoretically determine transport coefficients of the UFG. A plethora of theoretical methods has been used to estimate these transport coefficients, both for homogeneous and trapped systems [15–24]. Typically these theoretical predictions differ both quantitatively and qualitatively. Among the methods used, only the works [23, 24] present *ab initio* calculations, obtained within the very powerful quantum Monte Carlo (QMC) framework, where the errors can be quantified quite accurately. In these papers however the shear viscosity has been evaluated only at unitarity. In Fig. 1 we compare these calculations with recent experimental data for a uniform system extracted from measurements for trapped system [8]. Agreement for absolute value of shear viscosity has not been obtained, which however does not rule out the QMC results. However, we observe a very good qualitative agreement. Experimentally, the results for uniform system [8] have been inferred from data for trapped systems under a number of assumptions (see appendix of Ref. [8]). To what extent these assumptions are valid, especially in the most interesting low temperature regime, it is not clear. In particular, viscous hydrodynamics has been used in Ref. [8] to describe the full dynamics of the cloud, even at the periphery of the cloud, where the density is very low and the collision rate is strongly suppressed and the system is in the collisionless regime [25]. In both cases we observe the rapid decrease of the shear viscosity well above critical temperature of superfluid-normal phase transition  $T_c$ . It is notable that this feature is present only in the QMC and the pseudo-gap  $T$ -matrix theory [19] predictions.

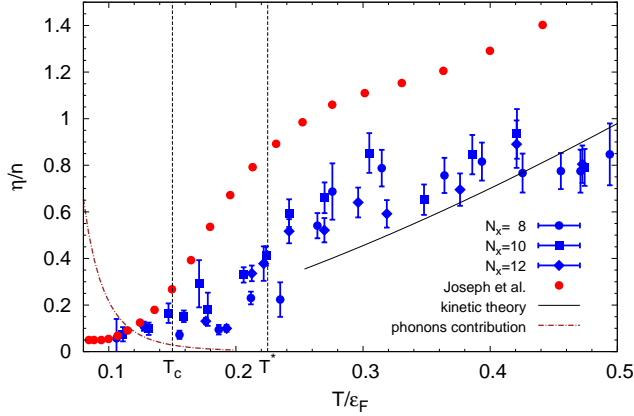


FIG. 1: (Color online) Comparison of the QMC predictions taken from Ref. [24] (blue points) with the results extracted from experimental data for trapped system (red points) [8]. Horizontal and vertical axis show the temperature in units of Fermi energy  $\varepsilon_F = \frac{\hbar^2 k_F^2}{2m}$  and the dimensionless shear viscosity  $\eta/n$  respectively. The theoretical results are provided for different lattice sizes  $N_x = 8, 10$  and  $12$ . The scatter of these points is a measure of the numerical accuracy of the available QMC predictions. The solid black line shows kinetic theory prediction  $\frac{\eta}{n} \cong 2.77 \left(\frac{T}{\varepsilon_F}\right)^{3/2}$ . The phonon contribution to the viscosity, which is not accounted for in QMC calculations in a finite volume [24] and evaluated in Ref. [16], is shown as a dot-dashed (brown) line. The vertical black dotted lines indicate the critical temperature of superfluid-to-normal phase transition for  $T_c = 0.15(1)\varepsilon_F$  and the onset of Cooper pair formation,  $T^* \approx 0.22\varepsilon_F$ , respectively - both extracted from QMC simulations in Refs [27] and [28].

Recently it has been reported that the system is even closer to the perfect fluid limit if it is slightly beyond the unitary point [26]. In this experiment it was revealed that the lowest viscosity coefficient can be obtained for an interaction strength corresponding to  $\frac{1}{ak_F} \approx 0.25$ , where  $k_F = (3\pi^2 n)^{1/3}$  is Fermi momentum. Since the QMC method provides results which are at least in very good qualitative agreement with experiment for the unitary limit (i.e.  $\frac{1}{ak_F} \rightarrow 0$ ), in this work we extend our studies beyond unitarity point.

In order to determine the shear viscosity coefficient of the ultracold atomic gas we employ the QMC technique with auxiliary fields on the lattice, which provides numerical results with controllable accuracy, up to quantifiable systematic uncertainties (for details see Ref. [27]). These simulations are very similar to those performed at unitarity when  $|a| = \infty$  Ref. [23, 24]. Here we briefly describe the main aspects of the computational process, focusing mainly on required modifications in order to study the system beyond the unitarity. Henceforth we use the system of units:  $\hbar = m = k_B = 1$ .

For simulations we employed a cubic lattice of size  $N_x = N_y = N_z = 10$ , with lattice spacing  $l = 1$  and

average number densities  $n \simeq 0.04$ . As shown in Supplemental Material of Refs. [23, 28] the systematic errors are no more than 10% for this lattice size and the errors are related mainly to corrections coming from the nonzero effective range  $r_{\text{eff}}$  and from the exclusion of the universal high momenta tail in the occupation probability due to the finite momentum cut-off  $k_{\text{max}} = \frac{\pi}{l}$ , where  $l$  is simulation box size. In the context of computation of transport coefficient it is important to have as small as possible statistical errors, so as to minimize the errors arising from the analytical continuation from the imaginary time to real frequencies as explained below. Thus we generate an ensemble containing about  $10^4$  uncorrelated samples in order to get statistical accuracy below 1%.

In order to extract the shear viscosity within a QMC framework one calculates the imaginary-time (Euclidean) stress tensor-stress tensor correlator

$$G_{\Pi}(\mathbf{q}, \tau) = \frac{1}{V} \langle \hat{\Pi}_{\mathbf{q}}^{(xy)}(\tau) \hat{\Pi}_{-\mathbf{q}}^{(xy)}(0) \rangle, \quad (1)$$

at zero momentum  $\mathbf{q} = 0$ . As shown in Ref. [18], for zero-range interaction it is sufficient to use only the kinetic part of the stress tensor:

$$\hat{\Pi}_{\mathbf{q}=0}^{(xy)} = \sum_{\mathbf{p}, \lambda=\uparrow, \downarrow} p_x p_y \hat{a}_{\lambda}^{\dagger}(\mathbf{p}) \hat{a}_{\lambda}(\mathbf{p}). \quad (2)$$

The average is performed in the grand canonical ensemble, at fixed temperature  $T = \frac{1}{\beta}$  and chemical potential,  $\hat{\Pi}_{\mathbf{q}}^{(xy)}(\tau) = \exp[\tau(\hat{H} - \mu\hat{N})] \hat{\Pi}_{\mathbf{q}}^{(xy)} \exp[-\tau(\hat{H} - \mu\hat{N})]$ , where  $\hat{H}$  is the Hamiltonian of the system,  $\mu$  is the chemical potential, and  $\hat{N}$  is the particle number operator. In order to capture the physics of a dilute fermionic gas it is sufficient to use a zero-range two-body interaction  $V(\mathbf{r}_1 - \mathbf{r}_2) = -g\delta(\mathbf{r}_1 - \mathbf{r}_2)$ , where the coupling constant  $g$  can be tuned to fix the value of the  $s$ -wave scattering length  $a$  [27] using a standard renormalization procedure of the coupling constant on the lattice. After this procedure the contact interaction acquires a finite effective range - for the parameters of presented simulation  $k_F r_{\text{eff}} \simeq 0.43$ . In this case the interaction part of the stress tensor also contributes to the correlator. However, we relegate all the corrections arising from the finite effective range to the systematic errors, which were estimated to be less than 10% for used lattice (see supplemental material of Ref. [28] for extensive discussion in context of lattice computation of transport coefficients). It is legitimate to drop this contribution, particularly since in the limit  $k_F r_{\text{eff}} \rightarrow 0$  it is vanishing anyway, a fact also consistent with previous QMC studies performed at unitarity for different densities. We emphasize that the dominant source of uncertainties is introduced by the analytic continuation procedure (described next). These are of the order 10%, as one can judge from Fig. 1, where the error bars show contribution to the uncertainties only from this source.

The frequency dependent shear viscosity  $\eta(\omega)$  is extracted via the analytic continuation of the imaginary-

time correlator to real frequencies. This procedure is equivalent to solving the integral equation:

$$G_{\Pi}(\mathbf{q} = 0, \tau) = \frac{1}{\pi} \int_0^{\infty} \eta(\omega) \omega \frac{\cosh[\omega(\tau - \beta/2)]}{\sinh(\omega\beta/2)} d\omega. \quad (3)$$

The static shear viscosity  $\eta$  is defined as  $\eta = \lim_{\omega \rightarrow 0} \eta(\omega)$ . The correlator  $G_{\Pi}$  is sampled only for a finite set of points and in a finite imaginary-time interval, and its evaluation is affected by the statistical noise, which we minimize by using a quite high statistics. This integral equation (3) belongs to a class of numerically ill-posed problems. Therefore, the use of special techniques is warranted in order to extract numerically stable results. We have employed an approach which combines two complementary methods: the Singular Value Decomposition (SVD) and the self-consistent Maximum Entropy Method (MEM), both described in great detail in Ref. [29] and in the Supplemental Material of Refs. [23, 28]. The stabilization procedure requires *a priori* information about the solution  $\eta(\omega)$ . The *a priori* information used is related to the known properties: the non-negativity of the shear viscosity  $\eta(\omega) \geq 0$ , the asymptotic tail behavior  $\eta(\omega \rightarrow \infty) = \frac{C}{15\pi\sqrt{\omega}}$ , and the sum rule (see [30] with subsequent corrections [18, 31])

$$\frac{1}{\pi} \int_0^{\infty} d\omega \left[ \eta(\omega) - \frac{C}{15\pi\sqrt{\omega}} \right] = \frac{\varepsilon}{3} - \frac{C}{12\pi a}, \quad (4)$$

where  $C$  is Tan's contact density [32], and  $\varepsilon$  is the energy density. Both quantities are obtained within the same QMC simulation. The contact density  $C$  was extracted from the analysis of the tail of the numerically generated momentum distribution, which for sufficiently large momenta decays as  $n(p) \sim \frac{C}{p^4}$ ; a similar technique was used in Ref. [33] and subsequent studies. Moreover, self-consistent MEM requires an appropriately chosen class of *a priori* models for the solution. Based on our past experience for the unitary limit [23, 24], we determined that the expected suitable models for the frequency dependent shear viscosity  $\eta(\omega)$  consist of Lorentzian-like structures at low frequencies, smoothly evolving into the asymptotic tail behavior:

$$M(\omega, \{\mu, \gamma, c, \alpha_1, \alpha_2\}) = f(\omega, \{\alpha_1, \alpha_2\}) \frac{C}{15\pi\sqrt{\omega}} + [1 - f(\omega, \{\alpha_1, \alpha_2\})] \mathcal{L}(\omega, \{\mu, \gamma, c\}), \quad (5)$$

where

$$f(\omega, \{\alpha_1, \alpha_2\}) = \exp(-\alpha_1 \alpha_2) \frac{\exp(\alpha_1 \omega) - 1}{1 + \exp(\alpha_1(\omega - \alpha_2))} \quad (6)$$

and

$$\mathcal{L}(\omega, \{\mu, \gamma, c\}) = c \frac{1}{\pi} \frac{\gamma}{(\omega - \mu)^2 + \gamma^2}. \quad (7)$$

The parameters  $\{\mu, \gamma, c, \alpha_1, \alpha_2\}$  describe admissible degrees of freedom of the model and are adjusted automatically in a self-consistent manner.

In Fig. 2, the dimensionless static shear viscosity  $\frac{\eta}{n}$  is shown for three selected temperatures  $\frac{T}{\varepsilon_F} = 0.26, 0.4$  and  $0.5$  as a function of interaction strength  $1/(ak_F)$ . Only one point, for  $\frac{T}{\varepsilon_F} = 0.26$  and  $\frac{1}{ak_F} = 0.2$ , corresponds to the system being in the superfluid phase, while all other points correspond to the system either in the normal phase or in the “pseudogap” regime. We emphasize that our QMC simulations are fully consistent with the existence of the “pseudogap” regime, i.e. a temperature regime above the critical temperature  $T_c$  where many Cooper pairs are present, even though the superfluidity is lost [28, 34]. The presence of pairs above the critical temperature is a property of the UFG well established in *ab initio* calculations. Naturally, this is also beyond controversy in the BEC limit as well, where the critical temperature is well below the dimer binding energy,  $T_c \ll \frac{\hbar^2}{ma^2}$ . The presence of a pseudogap regime is to some extent also confirmed by experiments [35–37]. The present QMC results clearly show that the static shear viscosity decreases as we enter into BEC regime. As we tune the system towards the BEC side of the Feshbach resonance the number of Monte Carlo samples required to get an acceptable signal-to-noise ratio increases rapidly. For this reason we were not able to perform reliable inversions of the Eq. (3) for interaction strengths beyond  $\frac{1}{ak_F} > 0.2$  for temperature  $\frac{T}{\varepsilon_F} = 0.26$  and beyond  $\frac{1}{ak_F} > 0.5$  for temperatures  $\frac{T}{\varepsilon_F} = 0.4$  and  $0.5$ . The ratio of the shear viscosity to the number density  $\frac{\eta}{n}$  appears to have a relatively weak temperature dependence above  $T_c$ . This behavior appears to be confirmed by the experiment [26] as well. Our results for temperatures  $\frac{T}{\varepsilon_F} = 0.4$  and  $0.5$  suggest that there exists a minimum for the static shear viscosity located on BEC side of resonance for an interaction strength corresponding to  $\frac{1}{ak_F} \approx 0.4$ . However, the present QMC uncertainties do not permit an accurate determination of the minimum position.

Comparing our results with experimental data [26] we find qualitative agreement. While the experiment reports existence of minimum for the shear viscosity for  $\frac{1}{ak_F} \simeq 0.25$ , the *ab initio* prediction sets it for slightly higher strengths. In general the minimum position can be temperature dependent, but experimental data suggest that this dependence is rather weak, in apparent agreement with our findings. Note, however that the measurements are performed in trapped systems and only the trap-averaged viscosity  $\langle \frac{\eta}{n} \rangle = \frac{1}{N\hbar} \int \eta(\mathbf{r}) d^3\mathbf{r}$  is reported in Ref. [26]. The reduced temperature  $\frac{T}{\varepsilon_F}$  as well as the interaction strength  $\frac{1}{ak_F}$  are position dependent in a trap and they diverge to infinity as one approaches the trap edges, where the gas enters the collisionless regime and where the hydrodynamical approach is inapplicable [25]. The trapped experiment probes all reduced temperatures and interaction strengths, starting from the values at the center of the cloud up to very high values. Moreover, the regions further from the center of the trap also contribute with increasingly higher weights in such averaged quantities, masking to a large extent the information about the

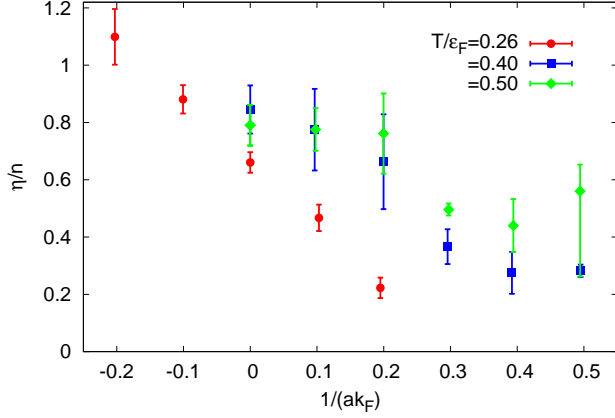


FIG. 2: (Color online) The dimensionless static shear viscosity  $\frac{\eta}{n}$  as a function of interaction strength  $\frac{1}{ak_F}$  for a  $10^3$  lattice for selected temperatures:  $T = 0.26\varepsilon_F$  - solid (red) circles,  $T = 0.4\varepsilon_F$  - (blue) squares and  $T = 0.5\varepsilon_F$  - (green) diamonds. The error bars show contribution to the computation uncertainty generated by the analytic continuation procedure only.

inner regions of the trap and making its evaluation very challenging. To what extent the averaging procedure affects the results for shear viscosity it is not clear at this time, and since a reliable validation of the theoretical predictions against the experimental data requires the knowledge of the shear viscosity for all temperatures and interactions strengths, such a comparison is beyond the scope of the present work. A similar disagreement with experimental values was noted in the analysis performed by Bluhm and Schäfer [38]. For example, at unitarity the values extracted in the experimental analysis of Ref. [8] of the ratio  $\frac{\eta}{n}$  exceed unity at temperatures  $\frac{T}{\varepsilon_F} > 0.3$ . The calculated ratio  $\frac{\eta}{n}$  within the kinetic theory [38] attains such values only for significantly larger temperatures  $\frac{T}{\varepsilon_F} \approx 1$  at unitarity, see Fig. 1, and even greater temperatures for positive values of the scattering length. The kinetic theory is appropriate for temperatures above  $T^*$ , where pairs have completely dissociated and where the collision integral is more or less well defined. Below  $T^*$  a more complex kinetic approach is required, which should include dimer-dimer, fermion-fermion, and dimer-fermion collisions as well as a dimer-to-two-fermions and its time-reverse processes. On the other hand, when extrapolated, the kinetic theory and the present QMC results appear, surprisingly to some extent, to be in agreement.

In order to confront the QMC results with the KSS conjecture one has to have information about the entropy density  $s = \frac{S}{V}$ . This can be extracted from static observables including the energy  $E$ , the chemical potential  $\mu$ , and the contact  $C$ , which are easily obtained within the QMC framework. Combining the basic thermodynamic relation (where  $T$ ,  $P$  and  $N$  are respectively temperature, pressure and particle number):

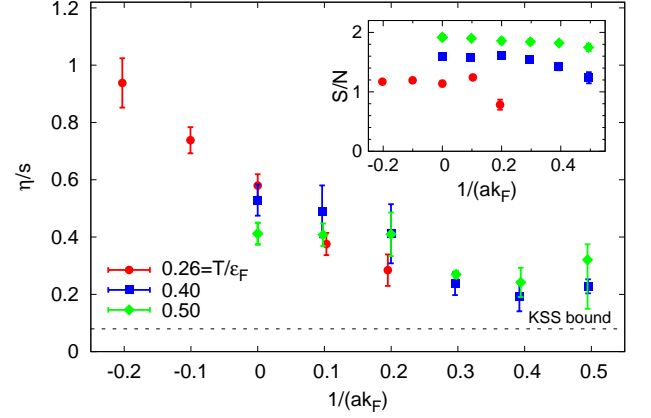


FIG. 3: (Color online) The ratio of the shear viscosity to entropy density  $\frac{\eta}{s}$  as a function of interaction strength  $\frac{1}{ak_F}$  for a  $10^3$  lattice. The notation for theoretical series is identical to Fig. 2. The black dashed line indicates KSS bound:  $\frac{1}{4\pi}$ . The error bars encapsulate uncertainty originating from the analytic continuation procedure and statistical uncertainty of the entropy density determination. The inset shows corresponding values of entropy per particle.

$$E = TS - PV + \mu N \quad (8)$$

together with the Tan's pressure relation [39]:

$$P - \frac{2}{3} \frac{E}{V} = \frac{C}{12\pi a V}, \quad (9)$$

one can show that

$$\frac{S(x, y)}{N} = \frac{\xi(x, y) - \zeta(x, y) + \frac{1}{6\pi} \tilde{C}(x, y)y}{x}, \quad (10)$$

where we introduced the following dimensionless quantities: the reduced temperature  $x = \frac{T}{\varepsilon_F}$ , the strength of the interaction  $y = (ak_F)^{-1}$ , the Bertsch parameter  $\xi = \frac{5E}{3N\varepsilon_F}$ , the reduced chemical potential  $\zeta = \frac{\mu}{\varepsilon_F}$  and the reduced contact parameter  $\tilde{C} = \frac{C}{Nk_F}$ .

In Fig. 3 we show the ratio of the shear viscosity to the entropy density  $\frac{\eta}{s}$ . These results suggest that there is minimum for this ratio located at  $\frac{1}{ak_F} \approx 0.4$ . However the present QMC uncertainties do not permit to make a very precise determination of its location. Moreover, the QMC data also reveal a rather weak temperature dependence, similar to experimental findings.

The  $\frac{\eta}{s}$  ratio at the minimum  $\frac{\eta}{s}|_{min} \approx 0.2$ , is about 2.5 times smaller than its value at the unitary limit, and only 2.5 times larger than the KSS holographic bound  $\frac{1}{4\pi}$ . In the inset we also provide the entropy dependence on the coupling constant for our selected temperatures. For the temperature  $\frac{T}{\varepsilon_F} = 0.26$  the value of the entropy for  $\frac{1}{ak_F} = 0.2$ , which visually appears to deviate from the smooth pattern, reflects the fact that there is a phase

transition from the normal to the superfluid state as we increase interaction strength at a fixed temperature.

The viscosity is at the root of classical turbulence. In an ideal fluid turbulence does not exist, while it can develop in either a normal fluid or in a superfluid. In superfluids viscosity was expected to play a minor role (only in the normal component) and there was no clear route to turbulence when the temperature tends to zero and the viscosity therefore decreases. However, in 1955 Feynman [40] conjectured that in superfluids the crossing and reconnection of quantized vortices could lead to “quantum turbulence,” a field which since then is one of the most active areas of research in the physics of liquid helium 3 and 4 [41–43]. Feynman’s conjecture was demonstrated to be the correct theoretical mechanism in a dilute Bose superfluid [44] and in a dilute Fermi superfluid as well [45, 46]. The theoretical framework of classical turbulence laid by Kolomogorov [47] appears to explain many features (though not all) of quantum turbulence in liquid helium [42]. At the same time many characteristics, such as the non-Gaussian velocity distributions, are drastically different [43] in classical and quantum turbulence. The non-Gaussian velocity distributions have been predicted to emerge also in the UFG [46]. The quark-gluon plasma and the Fermi gas in the unitary regime above the critical temperature are two physical systems, which are not superfluid, but in which the shear viscosity attains extremely low values. Quantized vortices do not exist either in the UFG above  $T_c$  or in the quark-gluon plasma and the dynamics of these systems can be very close to that of a hypothetical classical fluid with zero viscosity, called the ideal fluid, if existence of classical turbulence is prohibited. Indeed, this is a case of ultracold fermionic gases produced experimentally. Classical fluid hydrodynamics is governed by dimensionless numbers, where the most important is the Reynolds number  $Re = \frac{umvL}{\eta}$ ,  $v$  and  $L$  are characteristic velocity and linear dimension describing flow. Classical turbulence in 3D systems is achieved for values of the Reynolds number of the order of  $10^4$ .

In a UFG quantized vortices, and therefore quantum turbulence, can exist in clouds with as little as 500-1000 fermions [45, 46] and for flow velocities  $v \approx 0.7v_F$ , which are larger than the Landau’s critical velocity  $v_c \approx 0.4v_F$ . The largest cold atomic clouds created so far in the laboratory have  $\mathcal{O}(10^6)$  atoms.

One can estimate the Reynolds number for a UFG using as characteristic scales: number of atoms  $N$  in a cloud that defines density  $n = N/L^3$ , the critical velocity  $v_c = 0.4v_F = 0.4 \frac{\hbar(3\pi^2N)^{1/3}}{mL} \approx 1.2N^{1/3}\hbar/mL$  and the minimal value of the shear viscosity  $\frac{\eta}{n\hbar} \approx 0.2$ . With this one obtains for the Reynolds number the values  $Re \lesssim \frac{n\hbar}{\eta} \frac{mLv_c}{\hbar} \approx 60$  for  $N = 1000$  and  $Re \approx 620$  for  $N = 10^6$ . One can argue that one can attain higher

values of the Reynolds number by increasing the flow velocity by a factor  $\approx 10$  so as to reach  $Re \approx 10^4$ . In a dilute Fermi gas near the unitary point the scattering cross section is on average  $\sigma \approx 4\pi/k_F^2$ . If the flow velocity is increased to  $v \approx 10v_F$ , the cross section decreases by a factor of  $k^2/k_F^2 \approx 100$  and the mean free path  $1/n\sigma$  becomes very large, comparable or exceeding the size of any atomic cloud created so far in the laboratory and the system enters the collisionless regime. One could alternatively contemplate an increase in the linear size of a cloud by a factor of 10, thus up to cloud particle numbers  $\mathcal{O}(10^9)$  (a size likely difficult to achieve for condensates), in order to increase the Reynolds number by an order of magnitude.

The Fermi gas in the unitary regime is thus a rather unique physical system; below the critical temperature the system is superfluid and can sustain quantum turbulence in rather small clouds, while above the critical temperature the turbulent dynamics is strongly suppressed for any current experimental realizations for a very wide range of flow velocities and cloud sizes. The almost “death” of classical turbulence above  $T_c$  and its revival into a new “body,” the quantum turbulence below  $T_c$ , makes the unitary Fermi gas the unitary regime a quite unique physical system. The small value of the shear viscosity in a dilute Fermi gas near unitarity, which is attained in the normal phase, requires that the onset of classical turbulence be achieved in relatively large clouds, so far not realized experimentally.

In summary, we have presented *ab initio* results for interaction strength dependence of the static shear viscosity and the shear viscosity to the entropy density ratio. Both quantities decrease as we tune interaction strength from the BCS side of the unitarity point towards the BEC limit. The results suggest that the Fermi gas in the unitary regime is the closest physical system known to being a “perfect fluid,” for an interaction strength corresponding to  $\frac{1}{ak_F} \approx 0.4$ , with a minimum value of shear viscosity to entropy density  $\frac{\eta}{s}|_{min} \approx 0.2 \frac{\hbar}{k_B}$  about twice as small than the value reported for the quark-gluon plasma  $\frac{\eta}{s}|_{QGP} \lesssim 0.4 \frac{\hbar}{k_B}$  [14]. Our simulations qualitatively confirm the experimental observation of Ref. [26], that shear viscosity attains a minimum on the BEC side of the unitary point, albeit for a stronger value of the coupling constant.

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