Reply to ``Comment on `Ramsey spectroscopy, matter-wave interferometry, and the microwave-lensing frequency shift' ''

Kurt Gibble

Reply to 'Comment on "Ramsey Spectroscopy, Matter-Wave Interferometry, and the Microwave Lensing Frequency Shift"'

Kurt Gibble

Department of Physics, The Pennsylvania State University, University Park, PA 16802

The Comment by [Jefferts *et al*. PRA] discusses the microwave lensing frequency shift's possible dependence on the initial wave packet size and two effects of wall interactions, the frequency shifts that they produce and the nature of how dressed states are clipped by apertures. I identify conceptual errors in their criticisms, some of which are related to fundamental problems in their lensing treatment [Ashby *et al*. Phys. Rev. A **91**, 033624 (2015)] for the NIST-F1 and F2 atomic clocks. Aside from typesetting errors that they note, the criticisms in the Comment are shown to be incorrect.

The preceding Comment from NIST [1] addresses two issues related to the microwave lensing frequency shift, the initial wave packet size and the clipping by apertures in clocks, in addition to pointing out typesetting errors in [2]. This Reply demonstrates that the criticisms in [1] are incorrect and based on conceptual errors and fundamental problems [3] in the NIST treatment of this frequency shift [4].

The NIST treatment of the microwave lensing frequency shift by Ashby *et al.* in [4] has several errors [4]. First, an established error is their use of a microwave field that neglects the significant perturbations of the holes in the cavity, which reduces their calculated dipole forces and microwave lensing frequency shift by a factor of 2.3 [5]. Second, the NIST treatment yields a specious dependence of the frequency shift on weak microwave fields, b^2 or $\phi\sin(\phi)$, "just like the microwave leakage shift" [6,7] and yields a non-sinusoidal Ramsey fringe (Fig. 3 in [4]). In contrast, a key point of [2] was that the frequency shift is non-zero in the limit of zero

microwave amplitude and that the Ramsey fringe is purely sinusoidal. The third problem is directly related to one of the two principal criticisms in [1]. Regarding the dependence on the initial wave packet size, Ashby *et al.* [4] find "the magnitude of the effect is strongly dependent on this parameter and there are a number of equally physically reasonable choices which predict very different magnitudes." Although Jefferts *et al.* claim that "Never in [1] or previous references [2-4,8], does the author ever explain what atom wave packet size he is using for his calculations of corrections as used in [2-4]" [1], in fact I showed in my original treatment [8] that the microwave lensing frequency shift depends on the velocity distribution of the cold atoms, but not on the initial wave packet size "because, in essence, the frequency shift from Eq. (3) is proportional to the difference of the dressed state populations, whether the dressed state populations are for one atom or the ensemble" [8].

The incorrect dependence on the initial wave packet size that NIST predicts can be expected [3] because they only treat the gradient of the dipole energy at the center of a wave packet and neglect its curvature across the wave packet [4]. Minimum-uncertainty initial wave-packets that yield the experimentally observed velocity distribution spread quickly after atoms are launched and have a significant spatial extent at the first microwave interaction in fountain clocks [6,9-11]. The dipole force therefore varies importantly across the wave packet, weakly focusing and defocusing the two dressed states [2,8]. However, NIST considers that the dipole force for these atoms is everywhere 0, incorrectly yielding no lensing shift for a small initial atom cloud. This shortcoming of [4] leads to the conceptual error that the lensing shift decreases significantly for small initial wave packets and that value of this parameter is "crucial" [1].

NIST also cites the thermal deBroglie wavelength as a scale for the size of the initial wave packets and writes "in later parts of [1], the atom size is given as 30μm, which is not a typographical error" [1]. The relevance of the deBroglie wavelength is unclear since the laser-cooled atoms have not thermalized. Having shown in [8] that the lensing shift is independent of the initial wave packet size allows us in [5,9-11] to "semiclassically treat the atom propagation, originating from a point source and having classical trajectories with deflections" [2] given by

the dipole forces. While not essential, this "semiclassical approximation, which neglects the atomic diffraction of the apertures, simplifies the calculations and yields valuable clarity" [2]. Here, 30μm is the minimum waist size, $4(\hbar T/m)^{\frac{1}{2}}$, of a wave packet with a Rayleigh range that corresponds to a typical interrogation time T≈0.5 s, providing a scale for treating wave packets semiclassically.

The second major objection raised by NIST regards the clipping of dressed states by the surfaces of the cavity apertures. There are two distinct effects that I address in turn: 1) clipping by apertures and 2) phase shifts of atomic coherences travelling near surfaces in a clock. Foremost, I emphasize that clipping by surfaces is not fundamental to the microwave lensing frequency shift and therefore is a separate effect. Atom interferometers and clocks can be constructed so that atoms never interact significantly with surfaces and instead, the detection laser and fluorescence imaging aperture the atoms, as discussed and treated in [2,8-11]. For example, near 5π/2 pulses for NIST-F2, the effective clipping by the variation of the tipping angle in the clock cavity [5] produces the microwave lensing shift and the clipping by the surface of the final aperture does not contribute [9]. Therefore, effects from surface interactions during clipping are an additional effect that can be considered, including in clock accuracy evaluations.

In the course of preparing [8], I nonetheless considered surface interactions and discuss some aspects here. The associated conceptual error in [1] is: 'the two atomic states used in [1] as dressed-states are themselves hyperfine superposition states and that the accuracy of the clock depends critically on the phase of exactly those superpositions, the nature of the clipping must be investigated. Any differential phase shift of the wave functions occurring during the "clipping" could easily and grossly change the size of the frequency shift or even its sign.' The clipping by surfaces considered in [2,8] is by apertures after (and before) the two Ramsey interactions, as occurs in existing fountain clocks, including [6,9-11]. While the atoms can always be treated as superpositions of dressed states, here it is more transparent to use the bare basis of ground and excited states. Note that the problematic phase shifts between dressed states that Jefferts *et al.* describe are due to fields oscillating at the clock frequency of 9.2 GHz and the (dominant)

3

surface interaction is static. Although static surface interactions do produce phase shifts of the bare states, they do not change the bare state populations, the detected observable. Most importantly, a clock frequency shift arises due to different excited-state transition probabilities for temporal phase shifts $\Phi=\pm\pi/2$ of the second Ramsey pulse [8]. Because the surface interactions are incoherent with the applied Ramsey field, the effective size of the aperture for clipping the excited state population is the same on either side of the Ramsey fringe ($\Phi=\pm\pi/2$), and therefore the error due to surface interactions proposed in [1] is unfounded.

NIST suggests that atom-surface interactions between the Ramsey interactions could produce a comparable frequency shift to microwave lensing in clocks. While this shift is distinct from the microwave lensing frequency shifts that are treated in [2,4,5-11], it might nonetheless be useful to offer some quantitative analysis. Jefferts' *et al.* statement that "radian level phase shifts are observed in similar systems as a result of wall interactions" at distances of 25 nm [12] is injudicious. The wall interactions are governed by the static polarizability $\alpha(0) =401$ a.u. of the cesium ground state. Unlike the situation in [13] that is cited by Jefferts *et al.*, the cesium atoms are free and the wall interactions of the two clock states are very nearly identical, with a small differential polarizability $\delta\alpha(0)=0.01825$ a.u. [14]. Therefore, cold atoms that travel within 25 nm of a single wall almost immediately collide with that wall, within $\approx$10 ns. To survive the T=0.5s interrogation time, the atoms have to be further than 10 microns from the walls, nominally in the thermal Lifshitz regime. We can consider a uniform atom cloud illuminating the cavity, with no transverse velocity distribution, undeflected trajectories, and detect all atoms that would not be clipped by surface interactions in the fountain. This gives a highly conservative estimate of a trivial frequency shift $\delta\nu/\nu$ of less than 10^{-18}. While a classical treatment of 30 μm full-width wavepackets that are 10 μm from a surface is not valid, it would be surprising if shifts of order 10^{-16} emerged from a quantum treatment. Other considerations such as the thermal velocity distribution, a small initial cloud and the usual aperture below the Ramsey cavity [9], surface roughness, and small tilts of the cavity axis, will reduce this estimated shift by several orders of magnitude.

Regarding other criticisms of Jefferts *et al.*, I regrettably missed several typesetting errors in intermediate equations in [2]. The second subscript of the first line of (4) should be $\chi=-\pi/2$ and the w_{dx} denominators in the $\varepsilon_{\alpha\beta X}(r)$ terms should be squared in (6) & (7). While Jefferts *et al.* write that these typesetting errors "make it difficult to confirm the validity of the conclusions," note that [2] expanded on the treatment in [8], where the corresponding intermediate equations were correctly typeset and subsequently considered in Refs. [4,5-11], all of which were prepared before [2] appeared. I am grateful to Jefferts *et al.* for bringing these errors to my attention.

Jefferts *et al.* call for a "thorough and convincing theoretical considerations and/or careful and repeatable experimental demonstrations" before corrections are applied to International Atomic Time. To have had sufficient confidence to apply corrections, it was crucial for our treatment [2,8,9] to give an intuitive and physical picture of the microwave lensing frequency shift – the focusing of dressed state by well-established dipole forces. Furthermore, in contrast to the NIST treatment [4], we have derived verifiable concise and analytic expressions [2,5,8,9] and our treatment reproduces the well-established photon recoil shift in the limit of large minimum-uncertainty initial wave-packets [3]. Because the lensing shift has not yet been experimentally observed, previous evaluations have assigned conservative uncertainties of 50% [9,10] or 100% [11] of the calculated microwave-lensing frequency shifts of $\delta\nu/\nu=6\times10^{-17}$ to 9×10^{-17}. Given the shortcomings of the NIST treatment [3] in the context of the significantly earlier work [8-11], it is unfortunate that the NIST-F2 standard has already contributed to TAI using their non-peer-reviewed treatment of microwave lensing [7], recently published as [4], and experimental measurements, whose reanalysis has yielded a significantly larger uncertainty [5] that is greater than the total uncertainty reported by NIST [6]. Additionally, there is some inconsistency since none of the criticisms that NIST expressed in [1] were included in their evaluation of the uncertainty of the microwave-lensing frequency shift of NIST F2 [6]. In summary, all of NIST's substantive criticisms in [1] are shown to be incorrect and based on conceptual errors and fundamental problems in [4].

References

1. S.R. Jefferts, T.P. Heavner, S.E. Barlow and N. Ashby, Phys. Rev. A xx xxxx (2015).

2 K. Gibble, Phys. Rev. A **90**, 015601 (2014).

3 K. Gibble, submitted (2015).

4 N. Ashby, S. Barlow, T.P. Heavner, S.R. Jefferts, Phys. Rev. A **91**, 033624 (2015).

5 K. Gibble, Metrologia **52**, 163 (2015).

6 T. P. Heavner, E. A. Donley, F. Levi, G. Costanzo, T. E. Parker, J. H. Shirley, N. Ashby, S. Barlow, and S. R. Jefferts, Metrologia **51**, 174 (2014).

7 N. Ashby, S. Barlow, T.P. Heavner, S.R. Jefferts, arXiv1404.4101v2.

8 K. Gibble, Phys. Rev. Lett. **97**, 073002 (2006).

9 R. Li, K. Gibble and K. Szymaniec, Metrologia **48**, 283 (2011).

10 S. Weyers, V. Gerginov, N. Nemitz, R. Li and K. Gibble, Metrologia **49**, 82 (2011).

11 J. Guéna, M. Abgrall, D. Rovera, Ph. Laurent, B. Chupin, M. Lours, G. Santarelli, P. Rosenbusch, M. E. Tobar, R. Li, K. Gibble, A. Clairon, and S. Bize, IEEE Trans. Ultrasonics, Ferroelectrics, and Freq. Contr. **59**, 391 (2012).

12 A. D. Cronin and J. D. Perreault, J. of Phys.: Conf. Series **19**, 48(2005).

13 A. Derevianko, B. Obreshkov, and V. A. Dzuba, Phys. Rev. Lett. **103**, 133201 (2009).

14 K. Beloy, U. I. Safronova, and A. Derevianko, Phys. Rev. Lett. **97**, 040801 (2006).