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# Excess optical quantum noise in atomic sensors

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Enhanced nonlinear optical response of a coherent atomic medium is the basis for many atomic sensors, and their performance is ultimately limited by the quantum fluctuations of the optical read-out. Here we demonstrate a novel type of fundamental noise in atomic sensors. The off-resonant interactions, with the aid of the near-resonant process, can significantly modify the quantum noise of a coherent light field, even when its effect on the mean signal is negligible. The altered quantum optical noise distribution results in excess noise in the measurement quantity. We illustrate this concept by using an atomic magnetometer based on the nonlinear Faraday effect. These results show the existence of previously unnoticed factors in fundamental limitations in atomic magnetometry and could have impacts in a wide range of atom-light based precision measurements.

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Atoms are nature's most sensitive measurement devices, and their strong optical responses to external fields enable both precision measurements of fundamental constants as well as realization of practical devices for detection of electric and magnetic fields [1]. Since rotation of optical polarization often serves as the measurable output [2], the sensitivity of such sensors is ultimately limited by quantum polarization fluctuations (the photon shot noise), that can be surpassed only when squeezed input optical probe is used [1, 3, 4].

In this paper, we identify a novel fundamental source of excess quantum noise in atomic sensors, due to the quadrature noise modification resulting from coupling of light simultaneously to both resonant and off-resonant atomic levels. This noise is distinct from previously identified noise types such as photon shot noise, atomic shot noise, or dissipation induced noises [2, 5]. The noise mechanism also differs from the amplification of the polarization fluctuations of an optical field due to their coupling into atomic spin noise [2, 6]. Since the presented excess noise occurs in a relatively general atom-light interaction process, our result should have an impact on a wide range of atomic optical sensors.

Fig. 1 illustrates the origin and impact of the excess quantum noise in an atomic magnetometer, based on the nonlinear magneto-optical rotation (NMOR) in Faraday configuration [8]. In such a device, the polarization direction of an input field is rotated by the angle  $\phi_{\text{NMOR}}$ , proportional to the longitudinal magnetic field, due to the resonant light-induced alignment of atomic spins. Such interaction leaves the quantum fluctuations of the optical field unchanged, so the output optical field is expected to be in a coherent state. At the same time, the coupling of light to off-resonant excited levels modifies the quantum fluctuations of the optical field [6, 9]. Such nonlinear interaction has been used to produce squeezed vacuum in orthogonal polarization of the optical field, known as polarization self-rotation (PSR) squeezing [9–12] or po-

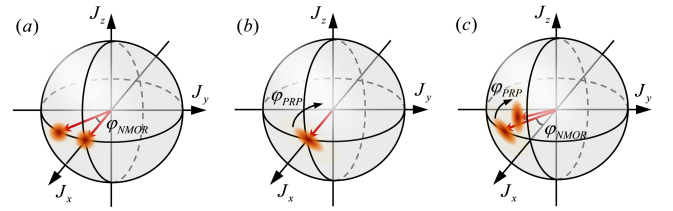


FIG. 1. (Color online) (a) Polarization rotation of a linearly polarized optical field under a magnetic field, depicted in a Poincaré sphere. The initial linear polarization is along  $J_x$ , and the balanced polarimeter detection measures the value of  $J_y$  [7]. Quantum uncertainty in the Stokes vector direction is represented by the noise ellipse (or circle, for a coherent state). (b) Output polarization of a linearly polarized light taking into account of off-resonant atom-light interaction without a magnetic field. The total Hamiltonian has a shearing effect on the uncertainty circle, turning it into an ellipse, resulting in a squeezed state of light. By adjusting the phase between  $x$  and  $y$  polarization component ( $\phi_{\text{PRP}}$ ), one can rotate the noise ellipse, making possible a sub-shot noise detection of  $J_y$ . (c) When a small magnetic field is applied, the noise in the magnetic field measurement (proportional to  $J_y$  fluctuations) is above the shot-noise level, since the long axis of the noise ellipse is almost along the equator of the Poincaré sphere. Rotation of the Stokes vector around the  $J_x$ -axis by  $\phi_{\text{PRP}}$ , as in (b), can decrease noise but at the price of a reduced  $J_y$  signal value.

larization squeezing [13, 14]. As shown in Fig. 1(b), the polarization fluctuation measurements can be reduced or increased compare to that of a coherent state by changing the quadrature angle  $\phi_{\text{PRP}}$ . Under realistic conditions, both of these interactions are present simultaneously. Our previous studies [15] have shown that the PSR squeezing persists even if a small non-zero longitudinal magnetic field is applied, and thus its effect on the noise NMOR measurements must be accounted for. Our results indicate that the PSR-induced modification of the quantum noise leads to increased polarization ro-

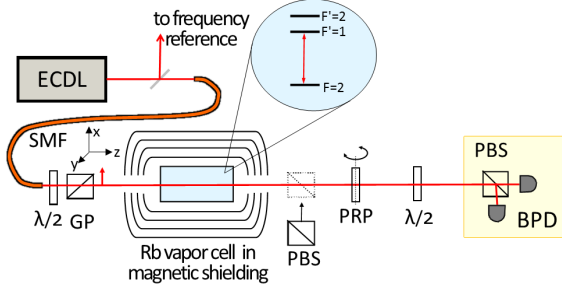


FIG. 2. (Color online) Schematics of the experimental setup (see text for abbreviations). Inset shows the detuning of the laser with respect to Rb atomic levels.

tation measurements noise, as the maximum axis of the noise ellipse is nearly aligned with direction of the polarization rotation, as illustrated in Fig. 1(c). Changing the quadrature phase  $\phi_{\text{PRP}}$  to detect the minimum noise quadrature simultaneously kills the useful signal, making the detected output practically independent on the  $\phi_{\text{NMOR}}$ .

The schematic of the experiment is shown in Fig. 2. We used a paraffin-coated cylindrical Pyrex cell (75 mm in length, 25 mm in diameter), containing isotopically enriched  $^{87}\text{Rb}$  vapor. The cell was mounted inside a four-layer magnetic shielding, and the number density of Rb atoms was controlled by adjusting the temperature of the pull-off tip of the cell. An external cavity diode laser (ECDL) was tuned to the  $5^2S_{1/2}F=2 \rightarrow 5^2P_{1/2}, F'=1$  transition of the  $^{87}\text{Rb}$  ( $\lambda \simeq 795$  nm). The laser output passed through a single-mode optical fiber (SMF) followed by a Glan-laser polarizer (GP) to prepare a high-quality linearly-polarized collimated beam with a diameter of approximately 2.3 mm. The input laser power in the cell was controlled by rotating a half wave plate before GP, with maximum available power 24 mW. The polarization rotation and the quantum noise of the output optical field were analyzed by rotating its polarization by  $45^\circ$  with a half-wave plate ( $\lambda/2$ ) with respect to the axes of a polarizing beam splitter (PBS), and then by sending the PBS outputs to a balanced photodetector (BPD). The strong linearly polarized field served as a local oscillator (LO) for the squeezing homodyne detection [14, 16]. The relative phase between the two polarizations (i.e. between the local oscillator and the analyzed vacuum field) was adjusted by horizontally tilting a birefringent phase-retarding plate (PRP)—a half-wave plate with optical axes aligned with the LO. This method provided extremely stable quadrature phase control; however, the lack of knowledge about PRP physical parameters made impossible to extract the exact numerical values of  $\phi_{\text{PRP}}$ . The shot noise level measurements were done with a re-

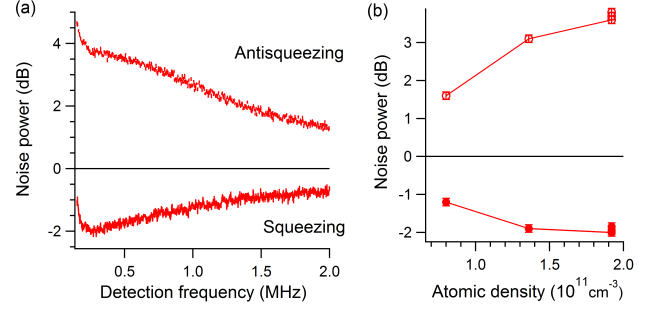


FIG. 3. (Color online) (a) Minimum and maximum measured quadrature noise spectra as a function of detection frequency. The optimal conditions to detect squeezing depend on the laser power, similar to the previous results [11]. (b) Atomic density dependence of the squeezed and antisqueezed quadratures. The incident laser power is  $P = 4$  mW. The noise power is normalized to the shot noise level (0 dB level), the spectrum analyzer frequency is set 200 kHz with the resolution and video bandwidths of 30 kHz and 30 Hz correspondingly. The error bars are one standard deviation.

movable polarizing beam splitter (PBS) placed after the Rb cell; when placed in the beam path it transmitted only the strong pump field (LO), and replaced the quantum field in the orthogonal polarization with coherent vacuum.

We first characterize the output polarization squeezed state without the magnetic field. Previous measurements, performed in uncoated vapor cells, found significant overall excess noise compared to a pure coherent state, resulting in large imbalance between minimum and maximum detected quadrature noise. Typical measured maximum quantum noise level was 10 – 20 dB above the standard quantum limit (SQL), while the corresponding squeezing reached only 2 – 3 dB below the SQL [10–12, 14]. Here, by using the vapor cell with paraffin wall coating to extend the atomic spin lifetime beyond 100 ms, we greatly reduced the amount of such excess noise and improved the purity of the output squeezed state *i.e.*, the difference between the squeezed and anti-squeezed quadratures with respect to the shot noise level is now much smaller, as shown in Fig. 3. The highest value of squeezing—approximately 2.0 dB below SQL—was obtained at the highest safe operational temperature ( $54^\circ\text{C}$ ) of our paraffin-coated cell. The corresponding antisqueezing quadrature noise was 3.5 – 4.0 dB above SQL. This improved purity in squeezing allows us to more accurately measure the orientation of the ellipse of the quantum optical noise and its effect on NMOR.

For the NMOR measurements, the longitudinal magnetic field was produced by a solenoid mounted inside the magnetic shielding. The polarization rotation angle was measured using a balanced polarimeter with zero phase between two polarizations (equivalent to no phase-retarding plate). We refer to the quantum noise, mea-

sured at these conditions, as *intensity* quadrature of the output light, which is also the  $J_y$  component on the Poincaré sphere. The *phase* quadrature corresponds to the  $J_z$  components on the Poincaré sphere, and was measured with the PRP phase changed by  $90^\circ$ . The *squeezed* and *antisqueezed* quadratures corresponded to the minimum and maximum quantum noise components, measured when the PRP phase was adjusted to put the short and the long axis of the photon noise ellipse (Fig. 1) along the  $J_y$  direction on the Poincaré sphere, respectively.

To observe the relationship between the NMOR signal and the measured quantum noise, we applied a modulated magnetic field in the Rb cell, and monitored the strength of the signal and noise for different tilt positions of the PRP (thus changing the phase  $\phi_{\text{PRP}}$ ). Figure 4 clearly shows that the maximum NMOR signal was observed for  $\phi_{\text{PRP}} = 0$ . At the same time, the measured noise background was above the shot-noise limit and nearly equal to the maximum antisqueezing quadrature noise. We could also experimentally set the position of the phase-retarding plate to  $\phi_{\text{PRP}} = 90^\circ$ , that corresponded to the phase quadrature measurements by nulling the rotation signal. For this case the measured noise was nearly at the shot noise level. Achieving the maximum noise suppression (corresponding to the squeezing quadrature measurement) required additional small  $\phi_{\text{PRP}}$  adjustment, and even though some NMOR signal was detectable, it was strongly suppressed, compared to the intensity quadrature measurements.

This analysis clearly shows that the squeezing and anti-squeezing noise quadratures did not perfectly match the intensity or phase quadratures, but that the squeezed noise ellipse was rotated by a small angle with respect to the intensity quadrature, as shown in Fig. 1(b), giving rise to the elevated noise level in the polarization rotation measurements. These observations are supported by the phenomenological PSR squeezing calculations [9], which predicted excess noise in the intensity quadrature, shot noise level at the phase quadrature, and non-zero squeezing angle for squeezed and antisqueezed quadratures.

We can gain some intuitive understanding of the squeezing process by employing a Poincaré sphere analysis of light polarization, shown in Fig. 1. We use a simplified level system to describe Rb atoms interacting with a monochromatic optical field, shown in the inset of the Fig. 2. In this system, the atom-light interaction part of the Hamiltonian has two parts, corresponding to resonant and off-resonant interactions with the Zeeman levels within the two hyperfine excited states [17]:

$$\hat{H}_{\text{int}} = A\hat{J}_z \cdot \hat{S}_z + \hat{H}_{\text{EIT}}. \quad (1)$$

The first term describes the off-resonant interaction of light with the atomic spin in the  $z$ -direction  $\hat{S}_z$ , mediated by the multiple upper excited states. Here, we neglect the residual absorption from that excited state.

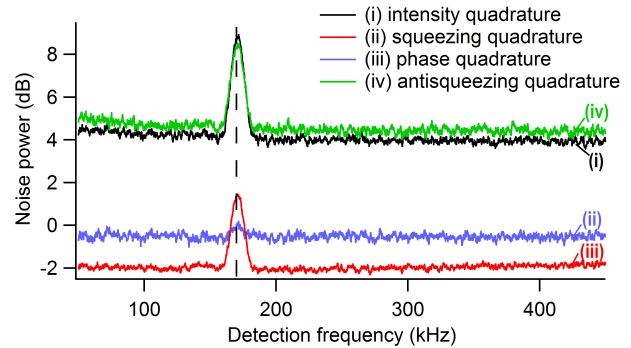


FIG. 4. (Color online) Nonlinear Faraday rotation and noise power measurements for various quadratures. Magnetic field is modulated at 170 kHz (vertical dashed line). Individual curves (i)-(iv) are recorded at fixed positions of the phase-retarding plate  $\phi_{\text{PRP}}$ . The incident laser power is  $P = 4$  mW, the spectrum analyzer settings are RBW=30 kHz, VBW=30 Hz. The noise power is normalized to the shot noise level, set to be 0 dB.

The second term describes the resonant EIT-like interaction, which is responsible for optically pumping the atoms into the non-interacting dark state [18, 19]. As a result, the atomic population distribution, and thus the mean atomic spin value, is determined by the differences in  $E_{\pm}$ , the circular components of the original optical field:  $S_z \propto J_z \propto E_+^2 - E_-^2$ , and thus the quantum fluctuations in  $J_z$  cause fluctuations in  $S_z$ . In this case, the off-resonant interaction term in Eq.(1) becomes proportional to  $J_z^2$ , which shears the originally symmetric ball of uncertainty into an ellipse with a long axis tilted towards the equator, as illustrated in Fig. 1(c). Interestingly, the  $J_z^2$  interaction is analogous to the one-axis twisting Hamiltonian  $S_z^2$  used to obtain spin-squeezed atomic ensemble [20, 21]. As discussed in [21], the shearing strength is proportional to atomic density, which in trend agrees with the density dependence of the squeezing and antisqueezing quadrature shown in Fig. 3(b).

The PRP tilting leads to a rotation of the noise ellipse around the  $J_x$  direction, and thus the noise ellipse has to be rotated almost by  $90^\circ$  to reach the minimum noise quadrature. Polarization rotation due to the applied magnetic field rotates the Stokes vector along the equator of the Poincaré sphere, and thus almost along the elongated axis of the noise ellipse, leading to excess detection noise described above. At the same time, any change in PRP phase results in the reduction of the detected rotation signal, as the Stokes vector rotates out of the equatorial plane, as shown in Fig. 1(c). This shows that the maximum response to the magnetic field is always accompanied by increased quantum noise. It may be possible still to take advantage of PSR squeezing, if the orientation of the PRP could be dynamically locked to the mean polarization direction, so that the noise ellipse can be rotated independently. Alternatively, one can use

the polarization squeezed light or twin-beam detection to decrease the detection noise floor [3, 4, 22]. However, realization of such methods in practice is rather technically involved.

The observation of additional intensity noise via the PSR interaction may have serious consequences for atomic magnetometers or other devices that are based on nonlinear polarization rotation. At low atomic density regime their performance is typically limited by the optical shot noise [23, 24], and is expected to improve with atomic density due to collective enhancement of coherent light-atom interactions. However, the detection intensity noise also grows with atomic density, because the shearing factor of the uncertainty ellipse for the Stokes vector is proportional to atomic density. For the wide range of laser powers and magnetic field parameters we explored, the intensity noise quadrature, corresponding to the maximum rotation signal, was always above the shot-noise level, as shown in Fig. 5(a). We verified that the signal-to-noise ratio cannot be further improved by adjustments of  $\phi_{\text{PRP}}$ . For example, when the detection scheme was adjusted to the minimum quantum noise (squeezed quadrature), the measured value of polarization rotation dropped by approximately 10 dB. The measured intensity quadrature noise increased linearly with atomic density (see inset in Fig. 5(b)).

We can now evaluate the dependence of the magnetic field measurement sensitivity as a function of atomic density and analyze the effect of the elevated intensity noise. We estimated the signal-to-noise of such a “magnetometer” by measuring the polarization rotation response to the modulated applied magnetic field and the corresponding noise floor (with the PRP removed) for the range of atomic densities. We also recorded the shot noise level, and estimated both realistic and shot-noise-limited values for the minimum detectable magnetic field, as shown in Fig. 5(b). It is clear that even though the measured magnetic field sensitivity of such a device continues to improve with the growth of the atomic density, realistically it improves at lower rate than estimated only by the shot noise limited sensitivity.

In conclusion, we have demonstrated a novel type of fundamental noise in atomic sensors. The noise distribution of a coherent state is modified and become anisotropic in phase-space, due to the combined effects of near- and off-resonant interactions between light and multi-level atoms. We show that this process adds significant excess noise to a nonlinear Faraday magnetometer, and the excess noise increases with the atomic density, making this effect more pronounced in the region where maximum sensitivity is expected. This noise mechanism is applicable to many systems where light is used to address atoms through simultaneous near- and off-resonant interactions. Our work should have broad impact to fundamental metrology and precision measurements.

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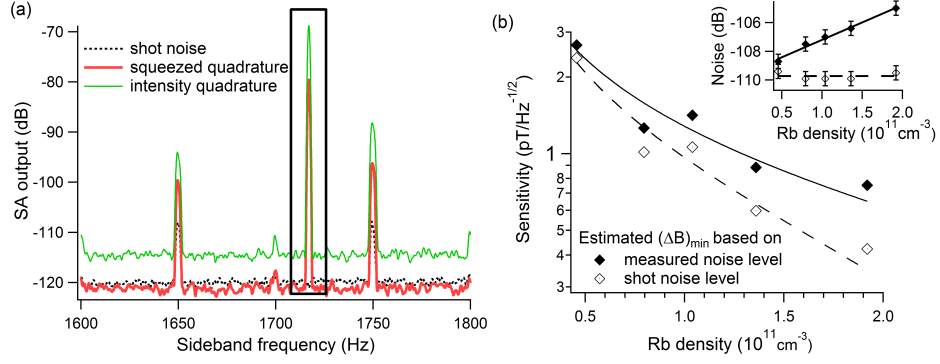


FIG. 5. (Color online) (a) Nonlinear Faraday rotation signal for a sinusoidally-modulated magnetic field ( $f_{mod} = 1,717$  Hz), measured in the intensity quadrature ( $\phi_{PRP} = 0$ ); in the squeezing quadrature ( $\phi_{PRP}$  is set to minimize the quantum noise); the measured shot noise level is also shown. The relevant signal is in the box, two neighboring peaks are due to technical noise. For these measurements the spectrum analyzer settings are RBW=VBW=1 Hz. (b) Measured quantum noise limited (solid diamonds) and estimated shot-noise limited (hollow diamonds) magnetic field sensitivity as functions of atomic density. *Inset*: Measured intensity quadrature noise and shot noise as a function of atomic density. The lines are to guide the eyes.

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