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Theoretical Analysis of Dipole Induced Electromagnetic Transparency

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We present a detailed, realistic analysis of the implementation of a proposal for dipole induced electromagnetic transparency (DIET) [R. Puthumpally-Joseph, M. Sukharev, O. Atabek and E. Charron, Phys. Rev. Lett. 113, 163603 (2014)] using an ensemble of cold atoms at high density. Using both direct numerical simulations and simple analytical models we show how, in a realistic N-level quantum system, narrow transparency windows can appear at large densities. The existence of such windows is attributed to quantum interference effects in overlapping resonances. Our analysis is applied to the D1 transition of Rb atoms and we show that, at high densities Rb can behave as a simple three-level emitter exhibiting all the properties of DIET. Some interesting effects such as slow light are also presented and their limits in the context of DIET are discussed.

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I. INTRODUCTION

The research of phenomena related to light-matter interaction has been experiencing significant growth for the past decade [1] in the domain of nano-optics. From the fundamental point of view, one of the most intriguing question lies in the dynamics and in the optical properties of many-body systems. For instance, new collective modes were predicted and measured in materials composed of interacting quantum emitters (atoms, molecules, etc.) [2]. Experiments in that regard are noticeably advanced compared to theory, as it has long been realized that treating multi-body systems is quite challenging. Even systems comprised of just a few interacting quantum dipoles are difficult to investigate [3, 4]. semiclassical descriptions of light-matter interaction, in which a classical electric field interacts with a quantum system, enormously simplify modeling leading to results that both support experiments and predict new phenomena [2, 5, 6].

Collective effects [7] arise from the fact that the electric field experienced by an arbitrary quantum emitter is the sum of both the applied field and the radiation from all other atoms in the system under consideration. When it is strong enough, the radiation from the dipoles set them coupled. This is the source of cooperativity. One of the important fundamental cooperative effects, the collective Lamb shift which is due to the exchange of virtual photons between the particles in a dense sample of atoms was recently observed [8]. Dependent of the cooperativeness in homogeneous and inhomogeneous systems were studied both experimentally [9, 10] and theoretically [4, 11, 12]. At present, research on those cooperative effects concerns both cold [13–17] and thermal [8, 18, 19] samples of atoms since they offer the observation of different interesting quantum phenomena. Because of the ability of light to carry effectively quantum information, understanding the optical response and the light scattering in dense samples is also interesting for many applications such as slow and stopped light [20, 21], quantum information [22, 23], trapped light [24] and optical memories [25].

In the linear regime, the propagation of light through any medium can be investigated by extracting its electric susceptibility [26, 27]. It was shown recently [6] that the electric susceptibility of systems comprised of three level quantum emitters at high densities exhibit a Fano-type profile that results in a new phenomenon of induced transparency called Dipole Induced Electromagnetic Transparency (DIET). DIET is due to the collective response of the system towards the external electromagnetic (EM) field via dipole-dipole interactions. Following this work, in this paper we present a realistic theoretical analysis of the implementation of a proposal for the experimental observation of DIET in dense rubidium atom samples, a system of timely interest [8, 10]. We also derive a generalized analytical formula for the susceptibility of a dense N-level quantum system, and we obtain a Fabry-Perot expression, from which we can predict very accurately the transmission and reflection spectra of dense N-level quantum systems. Our goal is to describe the physics related to the key interference mechanism at work in DIET. It is also to show that this effect is observable in a realistic physical system, and to bring some simple theoretical tools that can be used to predict the optical response of a dense N-level quantum system.

The paper is organized as follows. The next section discusses in detail our theoretical description for N-level systems and the analytic model used to analyze the results. Section III provides results and a comprehensive discussion of how linear optical properties of a system comprised of interacting two-level emitters depends on various material parameters such as the particle density. In Section IV, we extend our discussion to systems having more than one excited states. First we consider interacting three-level emitters. The results provided demonstrate dipole induced electromagnetic transparency (DIET). This phenomenon is explained using an analytical model and is extended to the multilevel D1 transitions of 85Rb. Section V discusses possible appli-
ations of our results and finally, we conclude our work
in Section VI.

II. THEORETICAL MODELS

Our goal is to describe the linear dynamics of quantum
emitters in a self-consistent manner taking into account
collective effects. The most comprehensive and thus com-
plete description would be to implement a fully quantum
model based on the Jaynes-Cummings Hamiltonian [28].
On the other hand, the quasi-classical theory of light-
matter interaction recently applied to hybrid materials
has proven to lead to both qualitative and quantitative
agreement with experiments [29]. Keeping also in mind
that the majority of experiments in nano-optics, such as
white light spectroscopy, for instance, are performed
under conditions corresponding to high occupation num-
bers of photons in a given EM mode, the quasi-classical
description is well justified.

The idea is to separate the description of EM waves
from the dynamics of quantum emitters considering the
spatio-temporal evolution of EM radiation using classical
Maxwell’s equations while applying the full machinery of
quantum mechanics to describe the response of quantum
emitters to EM excitation. This results in a system of
coupled Maxwell-Bloch equations [30]. If one wants to
observe collective effects, it is imperative to solve the cor-
responding equations of motion self-consistently without
any decoupling [31].

A. Theoretical Model and Numerical Simulations

The dynamics of the EM field components, $\vec{E}$ and $\vec{H}$,
is simulated using Maxwell’s equations

\[ \frac{\partial \vec{H}}{\partial t} = -\nabla \times \vec{E}, \]
\[ \frac{\varepsilon_0}{\mu_0} \frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{H} - \frac{\partial \vec{P}(\vec{r}, t)}{\partial t}, \]

where $\varepsilon_0$ and $\mu_0$ are the permittivity and the permea-
bility of free space, respectively, and $\vec{P}(\vec{r}, t)$ is the macro-
scopic polarization of the system at position $\vec{r}$ and time $t$.
The dynamics of the latter is considered both quantum
mechanically and classically. In both cases we as-
sume that the emitters are continuously distributed in
space and we neglect static emitter-emitter interactions.
Under such conditions one can express the macroscopic
polarization as

\[ \vec{P}(\vec{r}, t) = n_0 \langle \vec{\mu} \rangle, \]

where $n_0$ is the number density of emitters. The average
dipole moment, $\langle \vec{\mu} \rangle$, is either determined directly from
classical equations of motion or quantum mechanically
by evaluating

\[ \langle \vec{\mu} \rangle = \text{Tr} \left[ \hat{\rho}(\vec{r}, t) \vec{\mu} \right], \]

where $\hat{\rho}(\vec{r}, t)$ denotes the density matrix of the system.
Each quantum emitter has $(N - 1)$ excited states repre-
sented by $|j\rangle$, with $j \geq 1$ and $N \geq 2$, coupled to the
ground state $|0\rangle$ via the time dependent EM interaction
$V_{\text{int}}(\vec{r}, t)$. The density matrix $\hat{\rho}(\vec{r}, t)$ satisfies the following
Liouville-von Neumann equation

\[ i\hbar \frac{\partial \hat{\rho}}{\partial t} = [\hat{H}, \hat{\rho}] - i\hbar \hat{\Gamma} \hat{\rho}, \]

where $\hat{H} = \hat{H}_0 + V_{\text{int}}(\vec{r}, t)$ is the total Hamiltonian and $\hat{\Gamma}$
is the superoperator describing relaxation and dephasing
processes taken in the Lindblad form under Markov ap-
proximation [32]. Off-diagonal elements of $\hat{\Gamma}$ include the
pure dephasing rate $\gamma^*$ and the diagonal elements include
the radiationless decay rate $\Gamma$ of the excited states.

The field free Hamiltonian $\hat{H}_0$ is given by

\[ \hat{H}_0 = \sum_{j=0}^{N-1} \hbar \omega_j |j\rangle\langle j|. \]

We define the Bohr frequencies as $\omega_{0j} = \omega_j - \omega_0$ and
the interaction of a single emitter with the EM fields is
written as

\[ \hat{V}_{\text{int}}(\vec{r}, t) = \sum_{j=1}^{N-1} \hbar \Omega_j(\vec{r}, t) (|j\rangle \langle j| + |j\rangle \langle 0|), \]

where $\{\Omega_j(\vec{r}, t)\}$ are the instantaneous Rabi frequencies
associated with the interaction between the quantum sys-
tem and the local EM fields. In the present work we as-
sume that the excited states are not directly coupled to
each other but only to the ground state $|0\rangle$. The corre-
responding system of coupled differential equations reads

\[ \dot{\rho}_{0j} = \sum_{j' \geq 1} i\Omega_{jj'}(\vec{r}, t)(\rho_{0j'} - \rho_{j'0}) + \Gamma \rho_{jj'}, \]
\[ \dot{\rho}_{jj} = i\Omega_{jj'}(\vec{r}, t)(\rho_{0j} - \rho_{0j'}) - \Gamma \rho_{jj'}, \]
\[ \dot{\rho}_{jj} = i\Omega_j(\vec{r}, t)(\rho_{0j} - \rho_{0j'}) + \left[ i\omega_{0j} - \Gamma \right] \rho_{0j}, \]

where

\[ \gamma = \frac{2\gamma^* + \Gamma}{2}. \]

Eqs. (1) and (4), coupled through the evolution of the quantum
polarization (2), form the basis of the model. As this system of equations
is propagated in space and time on a grid, one should note that each grid point is
effectively a point-wise dipole with the amplitude of an
individual emitter times the number density. This essen-
tially means that dipole-dipole interactions within a
single grid point are neglected. This approximation al-
though valid at low densities has to be corrected for high
densities. Treating the dipoles exactly to include those
contributions is extremely difficult and it is almost im-
possible to solve the system of equations by using cur-
cent computational facilities [4]. An alternative way to
\[ E_{\text{local}} = \vec{E} + \frac{\vec{P}}{3\varepsilon_0}. \] (9)

This local electric field enters the Liouville-von Neumann equation (7) through the Rabi frequencies \( \Omega_j(\vec{r}, t) = \mu_{0j} E_{\text{local}}(\vec{r}, t)/\hbar \), where \( \mu_{0j} \) denotes the transition dipole moment between states \( |0\rangle \) and \( |j\rangle \).

Using the proposed model we simulate the linear optical response of a vapor comprised of interacting quantum emitters as schematically depicted in Fig. 1. To simulate a first order elastic scattering/absorption we implement the short pulse method \([30]\). A weak ultra-short incident pulse polarized along \( \hat{z} \) is launched at normal incidence, and the electric field is measured on the output side, 2\( \mu m \) far from the source. We verified that the results are independent of those distances as long as the pulse acts normal to the layer.

### B. Semiclassical Approximation

In the linear regime, \( \rho_{00} \gg \{\rho_{j}\} \), for \( j \geq 1 \), and the homogeneous system of \( N \)-level quantum emitters can be described by a set of \( (N-1) \) coupled harmonic dipole oscillators. The macroscopic polarization associated with the transition between \( |0\rangle \) and \( |j\rangle \), \( P_j \), follows a classical equation of motion

\[ \dot{P}_j(t) + \gamma_{cl} \dot{P}_j + \omega_0^2 P_j = \left( \frac{n_0 q_j^2}{m_j} \right) E_{\text{local}}(t), \] (13)

where \( \gamma_{cl} = 2\gamma \) and \( m_j \) is the effective mass of the oscillating charge \( q_j \) driven by the electric field \( E_{\text{local}}(t) \). Taking into account Eq. (9) and subsequently applying a Fourier transform, we obtain

\[ W_j(\omega) \dot{\tilde{P}}_j(\omega) = \varepsilon_0 \omega_0^2 \tilde{P}_j \tilde{E}_x(\omega) + \frac{\omega_P^2}{3} \sum_k \tilde{P}_k(\omega), \] (14)

where

\[ W_j(\omega) = \omega_0^2 - \omega^2 + i\gamma_{cl}\omega \] (15)

and where \( \omega_P \) is the plasma frequency representing the collective oscillations of the dipoles in the system

\[ \omega_P^2 = \frac{n_0 q_0^2}{\varepsilon_0 m_J}. \] (16)

With the assumption that the maximum amplitude of dipole oscillations in the absence of a driving field is given by the quantum harmonic oscillation length, we have

\[ \omega_P^2 = \frac{2\omega_0 n_0 \mu_J^2}{3\hbar \varepsilon_0}. \] (17)
In weak electric fields where \( \hat{P}_j(\omega) = \chi_j \varepsilon_0 \hat{E}_x(\omega) \), Eq. (14) gives the susceptibility \( \chi_j(\omega) \) associated with the \( j \)th dipole. Summing over all dipole oscillators in the material, we obtain the total susceptibility of the system

\[
X(\omega) = \sum_j \chi_j(\omega) = \frac{\sum_j \omega_{j}^2 W_j(\omega)}{1 - \frac{1}{3} \sum_j \omega_{j}^2 / W_j(\omega)}.
\] (18)

III. INTERACTING TWO-LEVEL EMITTERS

First we consider a slab of interacting simple two-level emitters characterized by the transition energy \( \hbar \omega_{01} = 2 \) eV, the transition dipole \( \mu_1 = 2 \) D, and the total decoherence rate \( \gamma = 10.5 \) THz. We calculate the response of the system to an extremely short pulse of width \( \tau = 0.18 \) fs using coupled Maxwell-Liouville-von Neumann equations in different collective interaction regimes [36]. The results are analyzed using the semiclassical approximation described in Section II B.

A. Semiclassical approximation and the Spectra

The electric susceptibility of interacting two-level emitters in the semiclassical model can be written as

\[
X(\omega) = \frac{\omega_{01}^2}{\omega_{01}^2 - \omega^2 + i\gamma \omega},
\] (19)

where \( \omega_{01} = \sqrt{\omega_{01}^2 - \frac{1}{4} \mu_1^2} \) is the shifted resonant frequency of the system. The shift in transition frequency, \( \Delta_1 \), which is due to the Lorentz-Lorenz correction included in the formulation of the local electric field via Eq. (9) can be estimated from the semiclassical model. The first order term gives the well-known Lorentz-Lorenz shift (LL shift) [12, 26, 37].

\[
\Delta_1 = n_0 \mu_1^2 / 9 \hbar \varepsilon_0,
\] (20)

The case of strong dipole-dipole interactions is characterized by \( \Delta_1 \gg \gamma \), while \( \Delta_1 \ll \gamma \) corresponds to the weak interaction regime.

Typical reflection spectra are shown in Fig. 2 as a function of the reduced detuning, \( \delta = (\omega - \omega_{01}) / \gamma \), for different values of \( \Delta_1 / \gamma \) for a slab of thickness \( \ell = 600 \) nm. In the weak interaction regime \( (\Delta_1 / \gamma = 10^{-3}) \), the system reflects a very small amount of the total energy (of the order of \( 10^{-5} \)) around the transition frequency. Such a response is similar to that of a single emitter since it is characterized by a typical Lorentzian profile. As the value of \( \Delta_1 / \gamma \) increases, and especially for \( \Delta_1 > \gamma \), the interaction between the dipoles becomes dominant and the system starts to respond to the applied field collectively. For \( \Delta_1 / \gamma \geq 10 \), the reflection spectrum broadens resulting in a window of frequencies within which more than 80% of incident energy is reflected. This reflection window was explained in 2000 by R. J. Glauber and S. Prasad in Ref. [11, 12] by considering the exponentially decaying modes of coherent excitation of the medium that depend strongly on the frequency.

For a slab of thickness \( \ell \) larger than the dipole wavelength \( \lambda_{01} = 2 \pi c / \omega_{01} \), the reflection at the vacuum/slab interface (shown as interface 1 in Fig. 1) dominates and the contribution from the possible multiple reflections inside the slab can be neglected. For such a slab, the width of the reflection window can be estimated from the reflection probability at this interface.

\[
\mathcal{R}(\omega) = \left| \frac{1 - n_r(\omega)}{1 + n_r(\omega)} \right|^2,
\] (21)
Transmission and Extinction (panel (a)) show the data for $\Delta_1/\gamma = 10^{-3}$ and the dotted green lines (right vertical scale) show the data for $\Delta_1/\gamma = 12$. All other parameters are the same as in the Figure 2.

where $n_r(\omega) = \text{Re} [\sqrt{1 + X(\omega)}]$ is the real part of the effective refractive index of the system. In the case of total reflection, $R(\omega) = 1$ and the real part of the electric susceptibility is $-1$. Equating the real part of Eq. (19) to $-1$ leads to the range of frequencies within which the reflection probability reaches its maximum: $\omega_{01} - \Delta_1 \leq \omega \leq \omega_{01} + 2\Delta_1$ and hence the frequency window with the maximum reflection shows a width of $3\Delta_1$ which agrees with the quantum simulation [6]. For relatively thin slab of width $\ell \leq \lambda_{01}$, the width of the reflection window is noticeably below $3\Delta_1$. This is due to the fact that in this analysis we have neglected contributions from multiple reflections that occur inside the slab. For a thin system, contributions from the two different vacuum-medium interfaces are negligible and obviously affect the reflection spectra.

Fig. 3 shows the extinction (panel (a)) and transmission (panel (b)) spectra at $\Delta_1/\gamma = 10^{-3}$ (blue solid line) and $\Delta_1/\gamma = 12$ (green dashed line). For $\Delta_1/\gamma = 10^{-3}$, almost all the incident energy is transmitted. Clearly in the weak interaction regime, the local polarization barely modifies the local electric field experienced by a given emitter. Hence the emitters respond to the field as if they were independent. As the mutual interaction between the dipoles increases, the system becomes opaque and reflects most of the incident energy [12]. In complement with reflection and transmission, the remaining energy is absorbed by the system and decays due to radiationless transitions described by $\Gamma$ in Eq. (4). In particular at $\Delta_1/\gamma = 12$, the system loses its transparency, as seen in Fig. 3(b), and behaves almost like a mirror around a broad range of frequencies near the transition frequency.

It should be emphasized that the reflection from strongly interacting dipoles resembles Bragg mirrors (also referred to in the literature as distributed Bragg reflectors) [38]. Bragg mirrors are composed of alternating dielectric layers with different refractive indices. The high reflectivity is achieved by multiple reflection from layers of the mirror. This leads to a controllable constructive interference. Spatially varied refractive index profiles determine the fringes in the spectrum. In contrast to Bragg mirrors our system is completely homogeneous. Since we perform simulations in a linear regime, the refractive index of the system is a constant throughout the layer. This rules out the possibility of having multilayer interferences. All the effects observed in the spectra are due to the collective response of the system. The width and the position of the reflection window are characterized by the physical properties of the system such as the transition dipole, transition frequency, material density. The qualitative explanation for the modifications in the spectra when the system enters the strong interaction regime relies on the fact that the polarization of the system is enhanced due to the strong dipole-dipole interactions. Prior to the detailed quantitative analysis of the spectral features, we want to estimate the accuracy of the semiclassical model and compare it with results obtained via exact numerical simulations.

Fig. 4 shows the relative errors of the electric susceptibilities for interacting two-level emitters obtained using the semiclassical model with respect to the numerical integration of the Maxwell-Bloch equations, Eq. (19) in the weak ($\Delta_1/\gamma = 10^{-3}$) and strong ($\Delta_1/\gamma = 12$) interaction regimes. The solid blue line shows the relative error for weakly interacting dipoles and the red dashed line, is the same for strongly interacting dipoles. Both relative errors are peaked around the corresponding transition frequencies with a maximum amplitude of about 0.08% and we conclude that the analytical model is accurate enough to analyze the numerical results.

B. Fabry-Pérot Modes

The semiclassical model explains the spectral features such as the shift in the resonance frequency and the width of the reflection window for a system of interacting two-level emitters in the strong interaction regime. The features that are not yet interpreted by the model are sidebands (see Fig. 2 for $\Delta_1/\gamma = 10$). Those however can be interpreted as simple Fabry-Pérot modes [8, 39]. The semiclassical model does not yet contain any information about the geometry of our system, assuming homogeneous distribution of the emitters. With two or more interfaces present, one can expect to observe interferences between reflected and transmitted fields due to the possible multiple reflections inside the slab. In this case, the system can be seen as a Fabry-Pérot etalon consisting of two parallel partially reflecting interfaces shown as the interfaces 1 and 2 in Fig. 1. EM fields at frequencies outside the transmission window are partially reflected and transmitted, resulting in non-zero optical path dif-
ferences and hence giving rise to an interference pattern as seen in Fig. 5 for instance. Introducing the decay of the EM fields inside the slab and taking into account multiple reflections with proper boundary conditions [39], we find the reflection \( r(\omega) \) and transmission \( t(\omega) \) coefficients for the system as

\[
\begin{align*}
    r(\omega) &= \frac{-2\sin(n(\omega)k\ell)[n(\omega)^2 - 1]e^{-ik\omega\ell}}{[n(\omega) + 1]^2 - [n(\omega) - 1]^2\sin(2n(\omega)k\ell)}, \\
    t(\omega) &= \frac{4n(\omega)e^{-\kappa\ell}}{[n(\omega) + 1]^2 - [n(\omega) - 1]^2e^{-2n(\omega)k\ell}},
\end{align*}
\]

where \( n(\omega) = \sqrt{1 + X(\omega)} \) is the refractive index, \( k = 2\pi/\lambda \) is the propagation constant of the applied field in the vacuum and \( \kappa \) is the imaginary part of the propagation constant inside the medium. The associated reflection and transmission spectra predicted by the semiclassical model are given by

\[
\begin{align*}
    R(\omega) &= |r(\omega)|^2, \\
    T(\omega) &= |t(\omega)|^2.
\end{align*}
\]

The expressions (23) together with (22) can accurately reproduce the positions of the maxima and minima of the sidebands in the calculated spectra. Fig. 5 shows the reflection spectrum calculated by integrating Maxwell-Bloch equations (blue dashed line) and the same obtained from the equation (23 a) shown by the solid red line. The two curves are in perfect agreement.

IV. MULTILEVEL SYSTEMS

Let us consider a ground state coupled to \((N-1)\) excited states. As in the previous case, the infinite slab of emitters is exposed to a transverse electric field. In the strong interaction regime, the Lorentz-Lorenz correction results in redshifts for all transitions. We thus expect to observe \((N-1)\) reflection windows of width \(3\Delta_j\) each. The most interesting case is when the transitions are significantly overlapping, leading to interference effects. For three level systems, in the presence of such overlapping resonances, we observed DIET which is due to the interference between two indistinguishable excitation pathways [6].

The calculated reflection spectrum for a three level system with overlapping transitions is shown in Fig. 6 as a blue solid line. It is compared with the reflection spectrum (red dotted line) obtained from a slab of two types of uncoupled two-level emitters having the same physical parameters which is calculated as the product of the reflection probabilities of each dipole. For the uncoupled dipoles the reflection spectrum behaves completely differently compared to the case when they are coupled. For uncoupled dipole systems, the total reflection is peaked where the two independent reflection windows corresponding to each dipole are contributing. This region corresponds to a clear overlap of two resonances.

The strong interaction between the dipoles in the system broadens the transition that results in the overlapping of two closely spaced allowed transitions and hence sets up a competition between two possible excitation pathways. We obtain a narrow window where the excitation probability becomes zero. The presence of a hole in otherwise flat reflection spectra is a clear signa-
ture of the destructive interference between the radiation emitted by coupled dipoles and hence a signature of DIET [6]. Except for the observed hole in the reflection spectrum, all features discussed in the previous section are clearly present in the three-level system spectra. At the frequency where there is no reflection, we observe a strong transmission if damping processes in the system are not too efficient. The observed transparency, similar to electromagnetically induced transparency (EIT) [40, 41], may lead to many potential applications such as slow light discussed later in this paper. The position of the transparency window can be controlled to a certain extend by changing the material parameters [6].

The same type of interference effect can happen if the system has a series of closely spaced energy levels. One of the potential actual systems for studying DIET are Rb atoms. Rb is used experimentally both as a thermal gas [8] and as a cold atomic gas [10]. Dipole-dipole interaction depends on the square of the transition dipoles and Rb atoms are characterized by large $S$ to $P$ transition dipoles [42]. We consider the D1 transition, i.e., $5^2S_{1/2} \rightarrow 5^2P_{1/2}$ of $^{85}$Rb. Both $5^2S_{1/2}$ and $5^2P_{1/2}$ are split into two sub-levels due to the hyperfine interaction [43] and hence there are four dipole-allowed transitions. Thus the D1 transitions of $^{85}$Rb isotope can be considered as a superposition of two three-level systems having same excited states but two different ground states. Since the energy splitting between the excited states is smaller than the splitting of the ground states, it gives a possibility to observe DIET between two excited states that are coupled to a specific ground state as well as between two excited states coupled to two different ground states for different atomic densities.

The susceptibility of a system consisting of two ground states can be written as

$$X(\omega) = \frac{\sum_j \omega^2_{P_j}/W_j + \sum_{j'} \omega^2_{P'_{j'}/W_{j'}}}{1 - \frac{1}{4} \left( \sum_j \omega^2_{P_j}/W_j + \sum_{j'} \omega^2_{P'_{j'}/W_{j'}} \right)}$$ \quad (24)

where the index $j$ corresponds to the transitions from the sub-level $F = 2$ and $j'$ is for the transitions from $F = 3$. It is important to note that Eq. (24) can also be used to analyze systems composed of different types of atoms or molecules. The LL shifts for D1 transitions of Rb can be written as

$$\Delta_{FF'} = \Delta_0 S_{FF'}^2,$$ \quad (25)

where $\Delta_0 = n_0 \mu^2 / 9 \hbar \varepsilon_0$ is the LL shift defined in the absence of hyperfine splitting and $S_{FF'}$ is a measure of relative strength of the transitions [42].

The reflection and susceptibility of a slab of thickness $\ell = 600$ nm comprised of $^{85}$Rb for $\Delta_0/\gamma = 3.8 \times 10^{-3}$ are shown in Fig. 7. The reduced detuning $\delta$ is defined with respect to 377.107 THz, the D1 transition frequency in the absence of hyperfine splitting [42]. The double peaks in the negative detuning region are due to the transitions $F = 3 \rightarrow F' = 2$ and $F = 3 \rightarrow \ldots.\ldots$
FIG. 8. (Color online) Susceptibility (panel (a), log scale), reflection (panel (b), linear scale) and transmission (panel (c), linear scale) as functions of the reduced detuning of $^{85}$Rb with $\Delta_0/\gamma = 21$. Vertical red dashed lines show the positions of destructive interference between the different transition dipoles.

$F' = 3$ and those in the positive detuning region are due to the transitions $F = 2 \rightarrow F' = 2$ and $F = 2 \rightarrow F' = 3$. The separation of the double peaks is due to the hyperfine splitting of the ground state. Since the system is in the weak interaction regime, the transitions are independent and they are not overlapping. It explains the reflection spectrum in Fig. 7(b) with a series of Lorentzian profiles corresponding to different transitions.

Since the dipoles are relatively large for D1 transitions, relatively small increments in the gas pressure will couple the different transitions and hence lead to the DIET regime. The calculated reflection and transmission spectra and the susceptibility for such a system with $\Delta_0/\gamma = 21$ are given in Fig. 8. Panel (a) shows the susceptibility, panel (b) presents the reflection spectrum, and panel (c) shows the transmission.

Overlapping of the transitions to $F' = 2$ and $F' = 3$ from the ground states result in two Fano profiles in the susceptibility. Two sharp minima in the reflection spectrum are the signatures of destructive interferences between the transition dipoles. Concurrently we see two narrow peaks in the transmission spectrum. These frequencies at which DIET takes place are shown as red dashed lines in the Figure.

Further increase of the dipole-dipole interaction mixes the $F' = 2$ and $F' = 3$ excited states into a single excitation state which is coupled to two ground states ($F = 2$ and $F = 3$ of $^{85}S_{1/2}$). It leads to DIET due to the overlapping of transitions to the excited state from two different ground states. Fig. 9 shows such an effect in strong interacting samples of $^{85}$Rb with $\Delta_0/\gamma = 200$. Panel (a) in Fig. 9 is the modulus of the susceptibility, panel (b) and (c) are the reflection spectra for slabs of thickness $\ell = 600$ nm and $\ell = 2.5$ $\mu$m, respectively, and panels (d) and (e) are the corresponding transmissions.

The susceptibility shows one dominant and two faint Fano type profiles. The latter clearly shown in the inset in panel (a) are due to the splitting of the excited states. The dominant Fano profile corresponds to the overlapping of two coupled dipoles $F = 3 \rightarrow F' = 2, 3$ and $F = 2 \rightarrow F' = 2, 3$. It leads to DIET which is well resolved in the reflection (Fig. 9(b) and (c)) and transmission (Fig. 9(d) and (e)) spectra for slabs of thickness 600 nm and 2.5 $\mu$m. This can be understood if we assume
that the D1 transitions are a mixture of two three-level systems having the same excited states. In such a picture, the dominant DIET in the strong interaction regime can be interpreted as an interference effect between the transitions to a single excited state from two different ground states. The two small Fano profiles in the susceptibility are due to the splitting of the excited states. They also result in DIET which explains two small transmission peaks on the left and right of the dominant DIET signal from the 600 nm slab. These two transmission peaks are not resolved for the 2.5 µm slab due to damping effects. The system, therefore, acts almost as if it was comprised of three-level emitters [6]. However, the splitting of the excited state leaves spectral holes in the reflection spectra (Fig. 9(c)).

V. POTENTIAL APPLICATIONS

The destructive interference of two overlapping transitions can be used to modify the spectral envelope of an incident pulse, i.e., as a pulse shaper. If we consider a slab of interacting multi-level quantum emitters with a reflection window wider than the FWHM, \(\delta \omega\) of the incident pulse, the collective response of the quantum emitters induce a selective reflection and transmission of the incident pulse. This is illustrated in Fig. 10 for the D1 transitions of \(^{85}\text{Rb}\) at \(\Delta_0/\gamma = 200\), with a pulse of FWHM, \(\delta \omega = 4\pi\) GHz. Panel (a) in Fig. 10 compares the normalized reflected pulse envelope (red line) with the incident pulse shape (blue dashed line) of such a pulse with the central wavelength 794.98 nm for a 600 nm thick slab of \(^{85}\text{Rb}\). Panel (b) is the same as panel (a) but for a 2.5 µm thick slab. The normalized transmitted pulse envelopes (red line) are shown in panel (c) for 600 nm and in panel (d) for 2.5 µm. A zoom around the central wavelength is given in the inset of panel (d) to see the shape of the pulse transmitted through the 2.5 µm slab.

From the Figs. 10(a) and (b) it is clear that the reflection from the 600 nm slab is not as efficient as the reflection from the 2.5 µm slab since this slab transmits a large part of the incident pulse (Fig. 10(c)). It is due to the mismatching between the slab thickness \(\ell = 600\) nm and the transition wavelength \(\lambda_0 \simeq 795\) nm that creates long tails of Fabry-Pérot modes near the reflection window which modify the transmission probability (see Fig. 9(b) and (d)). But for the 2.5 µm slab, the condition \(\ell > \lambda_0\) is satisfied leading to the wide reflection window. This minimizes the transmission of the pulse of width \(\delta \omega < 3\Delta_0\) through the slab except at the positions of DIET.

If the system is comprised of emitters with many optically active excited states that can be populated by the incident laser pulse (such as molecules with ro-vibrational
levels for instance [31]), each transition interacts with the others and interferes constructively or destructively at different frequencies leading to reflected pulses with more than a single frequency amplified or removed from the spectrum. In particular, spectra of the the transmitted pulse in Fig. 10(c) and (d) show that DIET can vary dramatically depending on the system parameters.

In systems comprised of interacting multilevel emitters we observe a minimum in the reflection window with a highly nonlinear dispersion (see Fig. 8 and 9). It opens several intriguing applications, including slow light [20, 44]. Fig. 11 shows the slowing down of the transmitted pulse in a layer of $^{85}$Rb with $\Delta_0/\gamma = 200$. Panel (a) shows the refractive index. The blue solid line is the imaginary part and the red dashed line is the real part of the refractive index. Panel (b) shows the group index $n_g(\omega) = \text{Re}[n(\omega) + \omega(dn/d\omega)]$ and the group velocity $v_g(\omega) = c/n_g(\omega)$ is shown in panel (c).

The system has a very large group index $n_g$ within the window of frequency where DIET occurs (see the Fig. 11(d) and the inset of Fig. 10(b)). It peaks at $1.55 \times 10^7$ similar to a recent experiment on slow light [44] and the group velocity (panel (c)) drops below 20 m/s. But in the system we consider this lowest velocity may not be observed since the imaginary part of the refractive index (see panel (a) in Fig. 11) is large that set the system as a lossy medium of light at this frequency. It also explains the quick drop in the transmission pulse shown in the inset of Fig. 10(d). The transmitted pulse peaked around the reduced detuning $\delta = 13$ dies out at $\delta = 20$. Within this short window of the reduced detuning the group index $n_g$ is of the order of $10^6$ and the velocity is decreased down to 40 m/s. Within this window, the imaginary part of the refractive index is small enough to allow the experimental observation of slowed down transmitted pulse.

VI. CONCLUSION

In this manuscript, we study collective effects in systems of interacting multilevel quantum emitters and their dependence on different physical parameters such as density. Such effects are studied both numerically using Maxwell-Bloch equations and analytically using a semiclassical model.

When damping is sufficiently low, as in atomic or molecular vapors at low temperatures, for instance, narrow transmission windows are observed. Such narrow windows in an otherwise completely opaque material are due to quantum interferences between different dipoles corresponding to different induced transitions. A clear evidence of such an interaction is the presence of Fano profiles in the susceptibility of the system. It is shown that this effect is amplified when the dipole-dipole coupling increases. We call the transparency of the system induced by the coupling between different kinds of dipoles Dipole Induced Electromagnetic Transparency (DIET). We note that DIET is similar to the well-known electromagnetic induced transparency (EIT) [40, 41] the former however is fundamentally different from EIT. EIT requires a strong laser pulse that couples two different quantum states, while DIET is inherently internal, i.e. requires overlapping resonances induced by strong dipole-dipole couplings.

The presence of additional levels such as ro-vibrational levels in molecules for instance or multiple optically active electronic states in case of atoms can change the response of the system due to the presence of additional energetically allowed transition dipoles which leave their signatures as spectral holes in the reflection spectrum and as narrow transmission peaks in the transmission spectrum. Such windows are induced via coherent cancellation of the dipoles through a destructive quantum interference effect. In addition, we have shown that this effect can be controlled by changing the number density of quantum emitters. This destructive interference of two or more transition dipoles can be used for shaping laser pulses or for slow light. It is however not as efficient as EIT in terms of slow light for instance, since high atomic or molecular densities are usually associated with increased losses.

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