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Phys. Rev. A **91**, 013628 — Published 28 January 2015

DOI: 10.1103/PhysRevA.91.013628

# Fragmentation of a spin-1 mixture in a magnetic field

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We study the ground state quantum fragmentation in a mixture of a polar condensate and a ferromagnetic condensate when subject to an external magnetic field. We pay more attentions to the polar condensate, due to the fact that the fragmentation of polar condensate, which typically occurs only in a very weak magnetic field, can occur in the mixture at higher magnetic fields, where both atom numbers and the number fluctuations will keep in a macroscopic magnitude of order of $N$. The role of the ferromagnetic condensate is to provide a uniform and stable background which can delay the rapid shrink of the 0-component population and make it possible to capture the "super-fragmentation ". Our method has potential applications in measuring the inter-species spin-coupling interaction through adjusting the magnetic field.

PACS numbers: 03.75.Mn, 67.60.Bc, 67.85.Fg

## I. INTRODUCTION

Recent experimental breakthroughs in spinor Boson-Einstein Condensate, such as the sub-Poissonian spin correlations generated by atomic four-wave spin mixing [1], the atomic squeezed states realized in the spin-1 ultracold atomic ensembles [2], and the antiferromagnetic spatial ordering observed in a quenched one-dimensional spin-1 gas [3], are all in connection with the vacuum fluctuations and recall attentions to the finite particle number effect beyond the mean-field treatment. The vacuum fluctuations become a significant subject in more and more experimental facts, e.g., atomic quantum matter-wave optics, atomic spin squeezing and quantum information. As one of the active frontiers, the spin-1 ultracold atomic ensemble is often adopted. With the basic interaction form $V(\mathbf{r}) = (\alpha+\beta\mathbf{F}\cdot\mathbf{F})\delta(\mathbf{r})$, the properties of such a three-component spinor condensate [4] have been demonstrated experimentally [5] and two different phases reflecting fundamental properties of spin correlation are identified: the so-called polar and ferromagnetic states for $\beta > 0$ ($^{23}$Na) and $\beta < 0$ ($^{87}$Rb) atomic condensates respectively. The mixture of two spinor condensate with the ferromagnetic and polar atoms, respectively, show more attractive quantum effects [6–15]. With the help of sympathetic cooling, the BEC mixtures of Na and Rb have been realized and it is interesting to observe the interspecies interaction induced immiscibility between the two condensates [15].

The ground state of the condensate with $\beta > 0$ has been predicted to be either polar ($n_0 = N$) or antiferromagnetic ($n_1 = n_{-1} = N/2$) within the mean-field treatment, where the condensate is usually described by a coherent state. However, the many-body theory by Law, Pu and Bigelow [16] pointed out that the ground state of $\beta > 0$ atoms is a spin singlet with properties ($n_1 = n_0 = n_{-1} = N/3$) drastically different with the results predicted by the mean field theory. Soon, Ho and Yip [17] show that this spin singlet state is a fragmented condensate with anomalously large number fluctuations and thus has fragile stability. The remarkable nature of this super-fragmentation is that the single particle reduced density matrix gives three macroscopic eigenvalues ($N/3$) with large number fluctuations $\Delta n_{1,0,-1} \sim N$. Similar considerations were also addressed by Koashi and Ueda [18–20]. The signature of fragmentation is then refer to the anomalously large fluctuations of the populations in the Zeeman levels. This is a super-poissonian correlation character, and the large number fluctuations shrink rapidly as the experimentally adventitious perturbations exist, such as magnetic field or field gradient.

In this paper we will report the influence of external magnetic field on the spinor condensate with $\beta > 0$, but on the premise of doping many ferromagnetic atoms in it. The interspecies spin coupling interaction arises and we propose a valid procedure to observe and control the fragmented states. If the ferromagnetic atoms in the mixture are condensed, the ground state favors all atoms aligned along the same direction and provides a uniform and stable background which can delay the rapidly shrinking of the number fluctuations when the inter-species coupling interaction is adjusted. The back action from polar atoms on to the more stable ferromagnetic atoms is negligible. Doping ferromagnetic atoms into spin-1 polar condensate can effectively influence the vacuum fluctuations and will have potential applications in quantum information and quantum-enhanced magnetometry.

## II. HAMILTONIAN OF THE MIXTURE

We consider the mixture of two spinor condensates of $N_1$ ferromagnetic and $N_2$ polar atoms, respectively. The intra-condensate atomic spin-1 interaction takes the

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standard interaction form $V_k(\mathbf{r}) = (\alpha_k + \beta_k \mathbf{F}_k \cdot \mathbf{F}_k)\delta(\mathbf{r})$ with $k = 1, 2$. The inter-condensate interaction between the ferromagnetic and polar atoms is $V_{12}(\mathbf{r}) = \frac{1}{2}(\alpha + \beta \mathbf{F}_1 \cdot \mathbf{F}_2 + \gamma P_0)\delta(\mathbf{r})$, which is more complicated because collision can occur in the total spin $F_{\rm tot} = 1$ channel between different atoms [6, 7]. The parameters $\alpha, \beta$, and $\gamma$ are related to the $s$-wave scattering lengths in the three total spin channels and the reduced mass $\mu$ for atoms in different species, and $P_0$ projects an inter-species pair into spin singlet state. Within the single spatial-mode approximation (SMA) [16, 21, 23] for each of the two spinor condensates, the spin-dependent Hamiltonian for the mixture finally reads as

$$\hat{H} = c_1\beta_1 \hat{\mathbf{F}}_1^2 + c_2\beta_2 \hat{\mathbf{F}}_2^2 + c_{12}\beta \hat{\mathbf{F}}_1 \cdot \hat{\mathbf{F}}_2 + \frac{c_{12}\gamma}{3}\hat{\Theta}_{12}^\dagger \hat{\Theta}_{12}, \quad (1)$$

where $\hat{\mathbf{F}}_1 = \hat{a}_i^\dagger \mathbf{F}_{1ij} \hat{a}_j$ ($\hat{\mathbf{F}}_2 = \hat{b}_i^\dagger \mathbf{F}_{2ij} \hat{b}_j$) are defined in terms of the $3 \times 3$ spin-1 matrices with $i(j) = 1, 0, -1$, and $\hat{a}_i^\dagger (\hat{b}_i^\dagger)$ creates a ferromagnetic (polar) atom in the hyperfine state $i$. The operator

$$\hat{\Theta}_{12}^\dagger = \hat{a}_0^\dagger \hat{b}_0^\dagger - \hat{a}_1^\dagger \hat{b}_{-1}^\dagger - \hat{a}_{-1}^\dagger \hat{b}_1^\dagger, \quad (2)$$

creates a singlet pair with one atom each from the two species, similar to

$$\hat{\Theta}_2^\dagger = (\hat{b}_0^\dagger)^2 - 2\hat{b}_1^\dagger \hat{b}_{-1}^\dagger, \quad (3)$$

for intra-species spin-singlet pair [17, 18] when $\beta_2 > 0$. The interaction parameters are $c_1 = \frac{1}{2}\int d\mathbf{r}\, |\Psi(r)|^4$, $c_2 = \frac{1}{2}\int d\mathbf{r}\, |\Phi(r)|^4$ and $c_{12} = \int d\mathbf{r}\, |\Psi(r)|^2 |\Phi(r)|^2$, which can be tuned through the control of the frequency of the trapping potential [7].

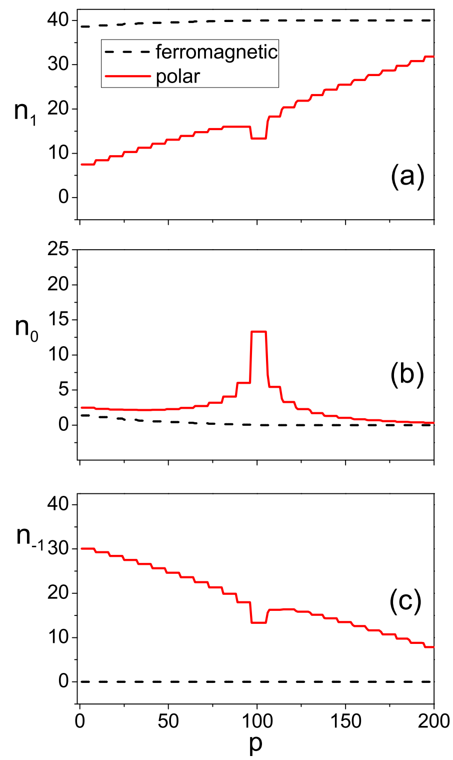


FIG. 1: (Color online) The dependence of atom numbers on $p$, at fixed values of $c_1\beta_1 = -1$, $c_2\beta_2 = 2$, and $c_{12}\beta = 2.5$. The total numbers of the two species are $N_1 = N_2 = 40$, and we consider the full-space with total magnetization $m$ a variable. Black dashed and red solid lines denote the number distributions in the ferromagnetic and polar condensate respectively. All interaction parameters are in the units of $|c_1\beta_1|$.

## III. FRAGMENTATION IN THE MAGNETIC FIELD

### A. Number distributions in a magnetic field

When the interspecies scattering parameters are calculated in the degenerate internal-state approximation (DIA) [24–27], the low-energy atomic interactions can be mostly attributed to the ground-state configurations of the two valence electrons, and the non-commutative term $\hat{\Theta}_{12}^\dagger \hat{\Theta}_{12}$ can be neglected [6, 7, 9]. The ground states are classified into four distinct phases: FF, $\mathrm{MM}_-$, $\mathrm{MM}_+$, and AA by three critical values of $c_{12}\beta = -\frac{(2N-1)c_2\beta_2}{N}$, 0, and $\frac{(2N-1)c_2\beta_2}{N+1}$ [9].

In this paper we discuss the atom number distribution and fluctuation in an external magnetic field. The spin-dependent Hamiltonian in the magnetic field reads,

$$\begin{aligned} \hat{H} &= c_1\beta_1 \hat{\mathbf{F}}_1^2 + c_2\beta_2 \hat{\mathbf{F}}_2^2 + c_{12}\beta \hat{\mathbf{F}}_1 \cdot \hat{\mathbf{F}}_2 \\ &\quad - c_1 p_1 \hat{F}_{1z} - c_2 p_2 \hat{F}_{2z}, \end{aligned} \quad (4)$$

where only the linear Zeeman terms are considered. As the SU(2) symmetry is broken in a spinor mixture, one can not eliminate the linear Zeeman effect through a spin rotation [28]. Meanwhile the quadratic Zeeman energy, typically 2 orders of magnitude weaker than the linear terms, is negligible in the calculation of number distributions. For the alkalis atoms such as $^{23}$Na and $^{87}$Rb in their subspace of the hyperfine spin F = 1, both the nuclear spins and the valence electron spins are the same for the two species, the linear Zeeman shifts are thus almost equal. In the following discussion, we will take $p = c_1 p_1 = c_1 p_2$ for simplicity.

We consider the direct product of the Fock states of the two species $|n_1, n_0, n_{-1}\rangle_1 \otimes |n_1, n_0, n_{-1}\rangle_2$, and do not restrict the model in the subspace with zero total magnetization [9, 10]. Instead, we consider the full space including all possible system magnetization $m = m_1 + m_2$. Using the full quantum approach of exact diagonalization, we can get the ground state of the system and study the response of the two species to the external magnetic field $p$ for $N_1 = N_2 = 40$. The three critical points for the phase boundaries are approximately $c_{12}\beta = -4, 0, 4$.

The field dependence of the population is shown in Fig. 1 for the $\mathrm{MM}_+$ phase at $c_{12}\beta = 2.5$, where polar atoms are partly polarized in the oppsite direction as the ferromagnetic atoms [9]. We notice that the ferromag-

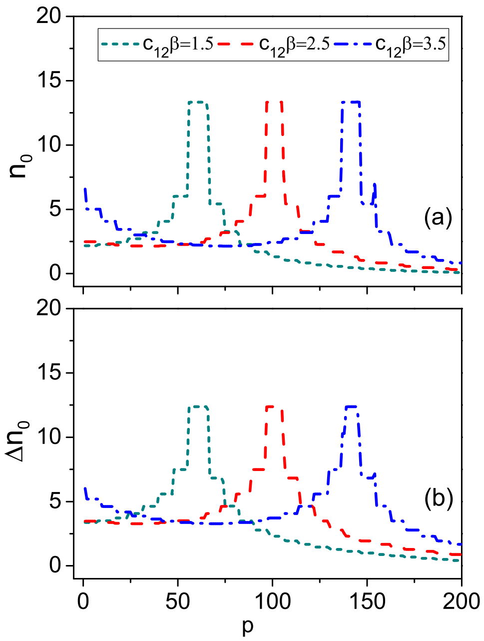


FIG. 2: (Color online) The dependence of atom numbers and fluctuations on $c_{12}\beta$ and $p$ at fixed values of $c_1\beta_1 = -1$ and $c_2\beta_2 = 2$. This column only shows the results of polar atoms with $n_0$ and $\Delta n_0$. When the extra magnetic field parameter $p$ (in the units of $|c_1\beta_1|$) increases, there are serval critical points associated with $c_{12}\beta$. All interaction parameters are in the units of $|c_1\beta_1|$.

netic atoms (black dashed lines) are very sensitive to the magnetic field, i.e. atoms quickly redistribute in the $n_1$ component and the magnetization of ferromagnetic condensate $m_1 = n_1 - n_{-1}$ saturates immediately. The ferromagnetic atoms actually form a stable condensate and provide a uniform magnetic background in the mixture. The polar atoms present a stepwise increase (decrease) in the atom number distribution $n_1(n_{-1})$ when the field increases. For small $p$ and positive $c_{12}\beta$, the system favors a negative magnetization ($m_2 = n_1 - n_{-1}$) of polar condensate, and $m_2$ will reverse and tend to saturate for large magnetic field. We notice that a special number distribution with $n_1 = n_0 = n_{-1} = \frac{N}{3}$ remarkably arises around the value of $p = 100$.

The situation becomes more simple if the parameter $c_{12}\beta$ is negative, that is, in the FF phase (or MM$_-$ phase), where polar atoms are fully (partly) polarized in the same direction as the ferromagnetic atoms. The enhanced ferromagnetic effect and the external magnetic field jointly suppress the atom number distribution $n_0$ and $n_{-1}$ of the polar condensate to zero, and at the same time saturate $n_1$ and the magnetization $m_2$ without magnetization reversal.

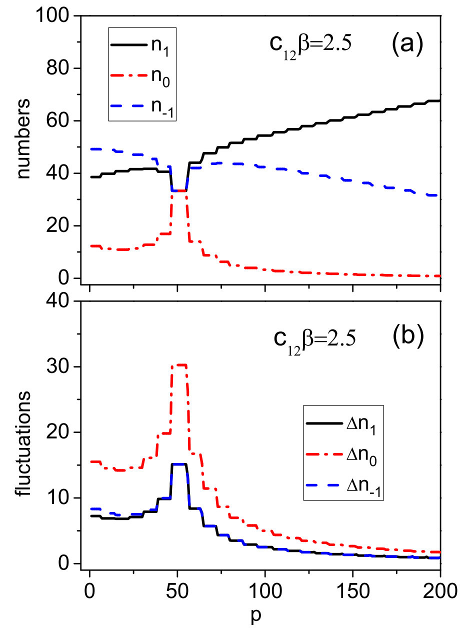


FIG. 3: (Color online) The dependence of atom number distributions and number fluctuations in the polar condensate on magnetic coefficient $p$ at fixed values of $c_1\beta_1 = -1$, $c_{12}\beta = 2.5$ and $c_2\beta_2 = 2$. The total numbers of the two species are $N_1 = 20$, $N_2 = 100$. Black solid, red dash-dot, and blue dashed lines denote atom numbers and the fluctuations on the $1, 0$, and $-1$ sub-levels respectively. All interaction parameters are in the units of $|c_1\beta_1|$.

### B. Retrieving the super-fragmented state

According to the spin space rotational invariant Hamiltonian [17–20],

$$\hat{H}_0 = c_2\beta_2\hat{\mathbf{F}}^2 = c_2\beta_2[\hat{N}_2^2 - \hat{\Theta}_2^\dagger\hat{\Theta}_2],$$

the super-fragmented state is named in [17] for the ground state of the pure spin-1 condensate with $c_2\beta_2 > 0$. This ground state is described by a many-body spin singlet with the form

$$|\phi_{\rm sup}\rangle \propto (\hat{\Theta}_2^\dagger)^{N_2/2}\,|0\rangle\,,$$

where $\hat{\Theta}_2^\dagger$ creates a singlet pair formed by two identical spin-1 bonsons. For spin-1 particles with three hyperfine spin state $|f, f_m\rangle = \hat{b}_m^\dagger\,|0\rangle$, the simplest spin singlet is formed by two spin-1 particles and described as

$$|F_{tot} = 0, F_m = 0\rangle = \sum C\,|f_1, f_{m1}\rangle\,|f_2, f_{m2}\rangle\,, \qquad (5)$$

under the condition $f_m = f_{m1} + f_{m2} = 0$, and with the remainder corresponding Clebsch-Gordon coefficient C, one can get

$$|F_{tot} = 0, F_m = 0\rangle = \frac{1}{\sqrt{3}}(\hat{b}_0^{\dagger 2} - 2\hat{b}_1^\dagger\hat{b}_{-1}^\dagger)\,|0\rangle\,. \qquad (6)$$

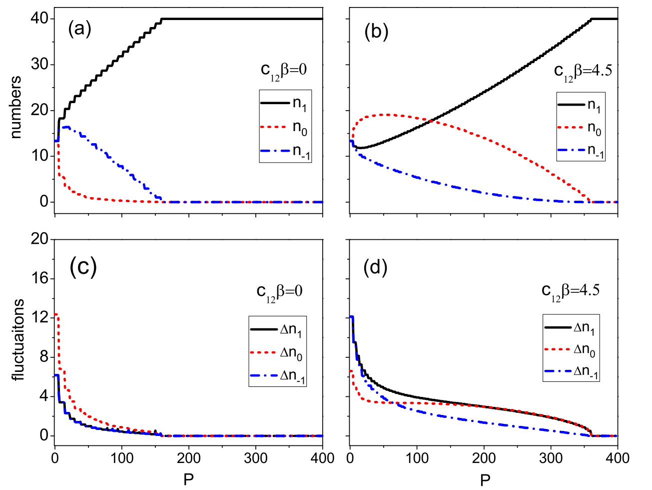


FIG. 4: (Color online) The dependence of atom number distributions $n_{1,0,-1}$ and the number fluctuations $\Delta n_{1,0,-1}$ of the polar condensate on both $c_{12}\beta$ and magnetic field $p$ at fixed values of $c_1\beta_1 = -1$ and $c_2\beta_2 = 2$. The total numbers of the two species are $N_1 = N_2 = 40$. Black solid, red dot, and blue dash-dot lines denote the numbers (fluctuations) on the 1, 0, and $-1$ sub-levels respectively. All interaction parameters are in the units of $|c_1\beta_1|$.

The many-body spin-singlet is constructed by applying $\hat{\Theta}_2^\dagger$ as many times as needed to get the desired number of particles [29]. The particle density matrix will be $(\hat{\rho})_{mn} = \left\langle \hat{b}_m^\dagger \hat{b}_n \right\rangle = \frac{N}{3}\delta_{mn}$, which fulfill the condition [30] that the ground state can contain several condensates. The number fluctuations, as the signature of fragmentation, can be calculated algebraically [17, 20] and satisfy $2\Delta n_1 = \Delta n_0 = 2\Delta n_{-1}$, with

$$\Delta n_0 = \frac{2\sqrt{N^2+3N}}{3\sqrt{5}}. \qquad (7)$$

Such a state with polar interaction was not likely realized in typical experiments due to its fragility towards any perturbation breaking spin rotational symmetry. For example, if subject to an external magnetic field, the ground state of the system [17–20] will be

$$|\phi_{\mathrm{mag}}\rangle \propto (\hat{b}_1^\dagger)^{m_2}(\hat{\Theta}_2^\dagger)^{(N_2-m_2)/2}\,|0\rangle\,, \qquad (8)$$

one can see a rapid shrink of the spin-0 component distribution $n_0$ and the fluctuations $\Delta n_{1,0,-1}$ when $m_2$ is increased. The super-fragmented state then reduces to a much more generic fragmented state: a two component number state with essentially zero fluctuations

$$|\phi_{\mathrm{num}}\rangle \propto (\hat{b}_1^\dagger)^{(N_2+m_2)/2}(\hat{b}_{-1}^\dagger)^{(N_2-m_2)/2}\,|0\rangle\,. \qquad (9)$$

For the spin-1 polar condensate doped with many ferromagnetic atoms, we can retrieve this super-fragmented state in the presence of an external field. For some special values of the magnetic field, both the spin-0 component population and number fluctuations would not shrink but revive to macroscopic orders of $N_2$. In Fig. 2, we illustrate the revival points for three inter-species coupling parameters $c_{12}\beta$ in the $\mathrm{MM}_+$ phase ($0 < c_{12}\beta < 4$). These revival points are found to move towards larger value of $p$ as $c_{12}\beta$ increases.

As learned from previous studies [23], the mean-field treatment is efficient for atomic interaction of the ferromagnetic type. The much more stable ferromagnetic condensate in the mixture can be formulated in the mean field treatment as a boson-enhanced effective magnetic field. This simplifies the Hamiltonian (4) as

$$\begin{aligned}\hat{H} &= c_1\beta_1\langle\hat{\mathbf{F}}_1^2\rangle + c_2\beta_2\hat{\mathbf{F}}_2^2 + c_{12}\beta\langle\hat{\mathbf{F}}_1\rangle\cdot\hat{\mathbf{F}}_2 \qquad (10)\\ &\quad -c_1p_1\langle\hat{F}_{1z}\rangle - c_2p_2\hat{F}_{2z}\\ &= c_2\beta_2\hat{\mathbf{F}}_2^2 + A\hat{F}_{2z} + C\end{aligned}$$

where $\langle\hat{\mathbf{F}}_1\rangle = \langle\hat{F}_{1z}\rangle = N_1$, $A = c_{12}\beta N_1 - c_2p_2$, $C = c_1\beta_1N_1(N_1+1) - c_1p_1N_1$. The criterion for the emergence of super-fragmented state is $p = c_{12}\beta N_1$, where the magnetic field ($p$), the optical trapping frequency ($c_{12}$), and the number of the doped ferromagnetic atoms ($N_1$) are all adjustable. When the magnetic field matches the condition that $c_{12}\beta N_1$ and $c_2p_2$ cancel each other, we may achieve the super-fragmented state in a magnetic field. The three critical points in Fig. 2 are found to agree with the numerical results exactly. The special value such as $p$=100 in the Fig.1 can be predicted exactly here with $p$=$c_{12}\beta N_1$=$2.5 \times 40 = 100$.

Next, we turn to the situation with population imbalance in the two species. Fig. 3 illustrates the location of the critical point when the inter-species coupling parameter $c_{12}\beta$ is fixed to be 2.5 and the atom numbers for the two species are $N_1 = 20$ and $N_2 = 100$. As the mean-field picture works well for the ferromagnetic atoms, we still get the crucial point $p = 2.5 \times N_1 = 50$ in Fig. 3. When equal population $n_1 = n_0 = n_{-1} = N/3$ occurs for the polar condensate, the number fluctuations also instantaneously reach to the macroscopic levels. Our numerical results for the fluctuation relation ($\Delta n_0 = 2\Delta n_{\pm 1}$) agree exactly with the algebraic results in [17] for pure polar condensate. With the emergence of equal population $N/3$ regarded as a sign of anti-ferromagnetic spin interaction, the inter-species spin coupling interaction $c_{12}\beta$ can be estimated by the location of the critical magnetic field.

### C. AA phase in a magnetic field

When the interaction parameter $c_{12}\beta > \frac{(2N-1)c_2\beta_2}{N+1}$, the system spontaneously breaks into a high symmetry state called AA phase. AA phase is another super-fragmented state which have been predicted in the absence of magnetic fields [9]. It is also a many-body spin singlet, which requires exactly the same atoms number of the two species ($N = N_1 = N_2$), and total spins from different species polarized in opposite directions. In the notation of the angular momentum representation

$$|F_1, F_2, F, m\rangle = \sum C^{F,m}_{F_1,m_1;F_2,m_2} |F_1, m_1\rangle |F_2, m_2\rangle\,, \tag{11}$$

AA phase is denoted as $|\phi_{AA}\rangle = |N, N, 0, 0\rangle$ with $F_1, F_2$ and $F$ the total spin quantum numbers of the ferromagnetic atoms, polar atoms, and the mixture and $m_1, m_2$ and $m$ the corresponding $z$-components. The intra-species angular momentum states involved in the AA phase, $|N, m_1\rangle$ and $|N, m_2\rangle$, should obey the constraint $m_1 + m_2 = 0$. The interesting feature of AA phase is the equal distribution of atoms in the six components $(N/3)$ and the large number fluctuations. To calculate the number distribution and the number fluctuation, one has to expanse the two species states $|N, m_1\rangle$ and $|N, m_2\rangle$ into the Fock states [9, 31], and the number fluctuations without magnetic field are calculated to be

$$\begin{aligned} \Delta n_0^{(1,2)} &= \frac{\sqrt{N^2+9N}}{3\sqrt{5}} \\ \Delta n_{\pm1}^{(1,2)} &= \frac{2\sqrt{N^2+3N/2}}{3\sqrt{5}}. \end{aligned} \tag{12}$$

However, unlike super-fragmented state, we can not give the perfect creating operator description of AA phase, due to the more complicated symmetry originated from the collision occurred in the total spin $F_{\rm tot} = 1$ channel.

In this section, we will numerically discuss the AA phase ($c_{12}\beta > 4$) subject to the external magnetic fields using the full quantum approach of exact diagonalization, and compare the results with the super-fragmented state in the pure condensate [17].

The features of these two typical fragmented ground states, which belong to two special phases characterized by typical values of the interaction parameter: $c_{12}\beta = 4.5$, and $c_{12}\beta = 0$, are illustrated in Fig. 4. First, we notice that the numerical results of the number distributions and fluctuations are exactly agree with the algebraic results on the special point $p$=0. The AA phase is as fragile as the pure polar singlet, with the number fluctuations drop rapidly (Fig. 4d), and the number distributions finally reduces to a generic number state $(b_1^\dagger)^{N_2}|0\rangle$. What interesting is that the responses of the $n_0$ component to the magnetic field are quite different. For a pure polar condensate (Fig. 4a), as $p$ increases, the 0-component distribution $n_0$ (red dashed line) shrink rapidly, which agree with the algebraic results in [17]. For the AA phase (Fig. 4b), we notice that $n_0$ does not shrink rapidly in the beginning, instead, it increases first and remains in a high value for a certain range of $p$. The applied external magnetic field can be used to characterize these two spin-singlets through tracing the atoms numbers of $n_0$ component of the polar atoms.

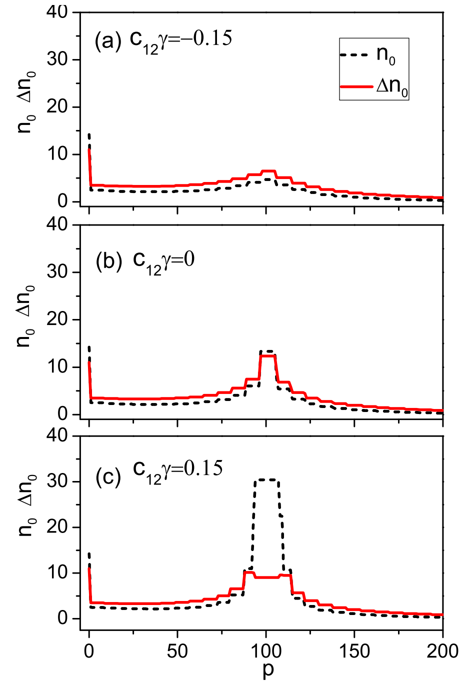


FIG. 5: (Color online) The dependence of atom number distribution $n_0$ and $\Delta n_0$ in the polar condensate on magnetic coefficient $p$ and $c_{12}\gamma$ at fixed values of $c_1\beta_1 = -1$, $c_{12}\beta = 2.5$ and $c_2\beta_2 = 2$. The total numbers of the two species are $N_1 = N_2 = 40$. Black dashed lines and red solid lines denote the value of $n_0$ and $\Delta n_0$ respectively. All interaction parameters are in the units of $|c_1\beta_1|$.

### D. The inter-species $P_0$ effect

If we refer to more general case beyond the DIA approximation, the $\gamma$ term of the Hamiltonian (1) should be considered. We notice that

$$\left[\hat{\mathbf{F}}^2_{1,2}, \hat{\Theta}^\dagger_{12}\hat{\Theta}_{12}\right] \neq 0, \left[\hat{\mathbf{F}}^2, \hat{\Theta}^\dagger_{12}\hat{\Theta}_{12}\right] = 0, \tag{13}$$

which means in general they do not belong to a set of commutative operators. However, we can numerically study the phase transition through the order parameter $\langle\hat{\Theta}^\dagger_{12}\hat{\Theta}_{12}\rangle$ [10]. To see more clearly the role played by the parameter $c_{12}\gamma$ on the fragmentation, we numerically diagonalize the Hamiltonian (1) with $N_1 = N_2 = 40$.

In Fig.5, we illustrate the influence of a small $c_{12}\gamma \neq 0$ to the population $n_0$ and $\Delta n_0$ of super-fragmented state which has been retrieved in MM$_+$ phase. We find that the crucial point is still located at $p = 2.5 \times N_1$, but a tiny $c_{12}\gamma = 0.15$ will elevate the $n_0$ component to a dominated value, meanwhile suppress the $n_1$ and $n_{-1}$ components to lower level. The high occupation on $n_0$ component is an evidence of the nematic order [2], and the signature of fragmentation disappears. For $c_{12}\gamma = -0.15$, on the contrary, the $n_0$ component is totally suppressed, with both $n_0$ and $\Delta n_0$ shrinking. Away from the critical point, the system is dominated by the magnetic field with the magnetization $m_2 = n_1 - n_{-1}$ increased linearly.

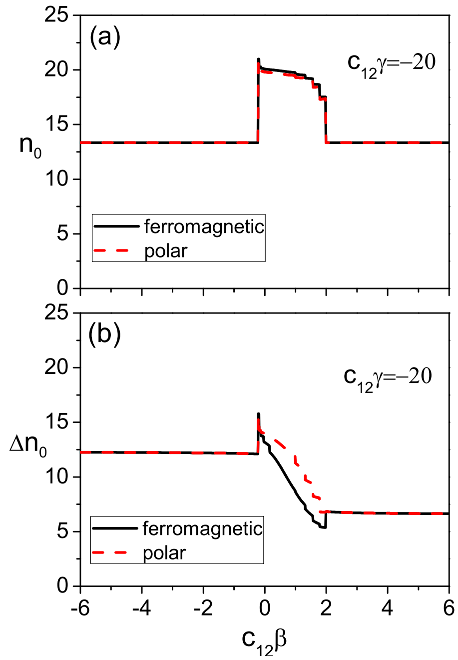


FIG. 6: (Color online) The dependence of atom numbers and fluctuations on $c_{12}\beta$, $c_{12}\gamma$ at fixed values of $p = 0$, $c_1\beta_1 = -1$ and $c_2\beta_2 = 2$. This graph only shows the results of $n_0$ and $\Delta n_0$, when the interaction parameter $c_{12}\gamma$ equals to $-20$. The total numbers of the two species are $N_1 = N_2 = 40$ and we restrict the problem in full-space without the external magnetic field. Black solid lines and red dash lines denote the ferromagnetic and polar condensate respectively. All interaction parameters are in the units of $|c_1\beta_1|$.

The negative $\gamma$ term encourages pairing two different types of atoms into singlets [10]. In Fig. 6, the influences of *negative* singlet pairing coefficient $c_{12}\gamma$ on the numbers and quantum fluctuations of the two species are illustrated. We notice that the typical $N/3$ number distributions arise both in the $c_{12}\beta < 0$ and $c_{12}\beta > 0$ regions when $c_{12}\gamma$ reaches to $-20$. The number fluctuation $\triangle n_0$ gives two steady values, which represent two typical fragmented ground state: the inter-species entangled fragmented state for $c_{12}\beta > 2$, and the pure species independent fragmented state for $c_{12}\beta < 0$. The fluctuations for these two states

$$\Delta n_0 = \frac{\sqrt{N^2+9N}}{3\sqrt{5}}, c_{12}\beta > 2,$$
$$\Delta n_0 = \frac{2\sqrt{N^2+2N}}{3\sqrt{5}}, c_{12}\beta < 0,$$

are found to match the numerical results in Fig. 6.

## IV. CONCLUSION

To conclude, we studied the ground state properties of a binary mixture of ferromagnetic and polar spinor condensates in a magnetic field. Using the full quantum approach of exact diagonalization, we can study the competition between magnetic linear Zeeman effect and the inter-species spin coupling interaction $c_{12}\beta$. The large vacuum fluctuation of number distributions on the three zeeman levels inside the polar condensate is worthy of investigation. We point out that the fragmentation properties of polar condensate can be adjusted through the magnetic field (p), trapping frequency ($c_{12}$), and the number of doped ferromagnetic atoms ($N_1$). The ferromagnetic condensate is involved to provide a uniform and stable background which can delay the rapidly shrinking of the large number fluctuations. We illustrated the influences of the magnetic parameter $p$, and identified two typical fragmented state with total spin $\langle \hat{\mathbf{F}}^2 \rangle = 0$. The positive inter-species spin coupling interaction ($c_{12}\beta > 0$) can effectively entangle the different species, while for $c_{12}\beta < 0$ the different species on their $F = 1$ manifold are essentially independent. We propose a possible mechanism to effectively measure the inter-species spin coupling interactions through applying a magnetic field, as well as discriminate the two types of many-body spin singlets. Our work highlights the significant promises for experimental work on sodium and rubidium atomic condensate mixtures and provide some useful information for the study of photo-association of heteronuclear molecules.

This work is supported by the NSF of China under Grant Nos. 11204204, 11347181, the NSF of Shanxi Province under Grant Nos. 2012021003-2, 2014021011-1 and the fund of Taiyuan University of Technology for young teachers. YZ is also supported by the National Basic Research Program of China (973 Program) under Grant No. 2011CB921601, Program for Changjiang Scholars and Innovative Research Team in University (PCSIRT)(No. IRT13076).

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