High-efficiency cascaded wavelength conversion based on adiabatic evolution

Junxiong Wei, Changshui Chen, He Jiang, Wei Li, and Tian Han

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Wei junxiong, Chen changshui*, Jiang he, Li wei, Han tian

Laboratory of Nanophotonic Functional Materials and Devices & MOE Key Laboratory of Laser Life Science and Institute of Laser Life Science, South China Normal University, Guangzhou, 510631, Guangdong, China

*Corresponding Author

E-mail: ***cschen@aiofm.ac.cn***

Fax: 0086+20+85211768

ABSTRACT

We demonstrate that the frequency conversion in the cascaded processes can be mapped uniquely into an associated three wave mixing processes in adiabatic evolution. After solving the coupling wave equations of three wave mixing processes, we immediately draw the solutions of the corresponding cascaded coupled equations. Furthermore, we introduce a rather simple model, which displays the main features of the STIRAP situation but allows for analytic evaluation of all quantities. It performs two simultaneous three wave mixing processes efficiently and without significant generation of an intermediate frequency. At last, we consider in detail the effects of the phase mismatch to the Optical STIRAP efficiency. It shows that the analytic bounds are seen to describe the transfer region very accurately.

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1. INTRODUCTION

Mid-IR lasers in the 3-5μm wavelength region have many applications, such as military countermeasures, remote monitors of the special environment, spectrum, and so on [1,2]. Combined with Quasi Phase Matching (QPM) and based on the Optical Parametric Oscillators (OPO)[3] or Difference Frequency Generation(DFG)[4], nonlinear optical frequency conversion can effectively produce the infrared light source. However, recent advances in quasi-phase-matching (QPM) technology which is based on periodically poled nonlinear crystals have motivated great interest in the physics and applications of multistep optical parametric processes [5-7]. By using an analogy to stimulated Raman adiabatic passage (STIRAP) in atomic physics [8] Gil Porat proposed [9,10] a scheme in which the input frequency (ω_1) is directly converted into an output frequency (ω_4) without significant generation of the intermediate frequency (ω_3). A unique feature of STIRAP is that the intermediate frequency ω_3 will never generate. The reason is that throughout the adiabatic evolution of the multistep parametric processes remains trapped in a dark vector $C_0(z)$, which is only a superposition of frequency ω_1 and ω_4 and does not involve the intermediate frequency ω_3. If the modulations order are counterintuitive, then the dark vector is initially associated with frequency ω_1 and finally with frequency ω_3, thus providing an adiabatic route from ω_1 to ω_3. Because the existence of the dark vector $C_0(z)$ is vital for STIRAP, and utilization of the adiabatic elimination procedure can also directly generate frequency (ω_4) , maintaining the perfect phase-matching is usually considered crucial for STIRAP. This is indeed correct when the pump and Stokes couplings possess approximately equal peak values, which is favorable for STIRAP and which is also the assumption stated in the references.[10].

In this paper, we introduce a new model, which displays the main features of the STIRAP situation but allows analytic evaluation of all quantities. It is based on that the cascaded conversions processes can be mapped uniquely into an associated three wave mixing processes. We formulate a general result and introduce the analytic approximations from the adiabatic theory of three wave mixing processes. This could provide us with the solution to a nontrivial case of the STIRAP process in the adiabatic frequency conversion situation. What's more, the sensitivity of the cascaded wavelength conversions process by STIRAP technique to phase mismatch has been

analyzed, the method is suggested in ref.[11], which STIRAP technique as a function of the two-photon detuning. By analyzing the emerged adiabatic basis crossings and the positions of the nonadiabatic couplings, we are able to derive the accurate bounds of the high conversion region, and estimates of the width of the phase mismatching. Finally, a technologically feasible method for carrying out such a process was proposed and numerically demonstrated to be effective.

2. DYNAMICAL EQUATIONS

We consider two simultaneous three wave mixing processes (STWM), as is shown in figure 1. The two simultaneous Difference Frequency Generation (DFG) processes are simultaneously realized in a single super lattice.

Under plane-wave approximation and considering the QPM condition, the coupling equations for the dimensionless field amplitudes ψ_j which describe the cascaded interactions are given by equations (2-1)

$$\frac{d}{dz}\psi_{\mathrm{p}} = \frac{-i\omega_p d_{eff}}{n_p c} f_1 \psi_s \psi_i e^{i\Delta k_1 z} \quad \text{(2-1a)} \qquad \frac{d}{dz}\psi_s = \frac{-i\omega_s d_{eff}}{n_s c}(f_1 \psi_p \psi_i^* e^{-i\Delta k_1 z} + f_2 \psi_i \psi_{mid-IR} e^{i\Delta k_2 z}) \quad \text{(2-1b)}$$

$$\frac{d}{dz}\psi_i = \frac{-i\omega_i d_{eff}}{n_i c}(f_1 \psi_p \psi_s^* e^{-i\Delta k_1 z} + f_2 \psi_s \psi_{mid-IR}^* e^{-i\Delta k_2 z}) \qquad \text{(2-1c)}$$

$$\frac{d}{dz}\psi_{\text{mid-IR}} = \frac{-i\omega_{mid-IR} d_{eff}}{n_{mid-IR} c} f_2 \psi_s \psi_i^* e^{-i\Delta k_2 z} \qquad \text{(2-1d)}$$

where z is the position along the propagation axis, λp, λs, λi, $\lambda_{\text{mid-IR}}$ represent the wavelengths of the pump, signal, idler, and mid-IR respectively. Difference frequency $\omega_i = \omega_p - \omega_S$ and $\omega_{\text{mid-IR}}=\omega_S-\omega_i$, the electric field at the frequency ω is $E = \sqrt{P_0^{\omega} / 2\varepsilon_0 c n}\psi(z)\exp(i\omega t - ikz) + c.c$,

P_0^{ω} is input power of the frequency ω field. c is the speed of light in vacuum, f_1 and f_2 is magnitudes of the Fourier coefficients, d_{eff} is second order nonlinear coefficient, the phase mismatches is Δk , n_j is the refractive index.

$$\Delta k_1 = n_s \omega_s / c + n_i \omega_i / c - n_p \omega_p / c + 2\pi / \Lambda_1 , \Delta k_2 = n_i \omega_i / c + n_{mid-IR} \omega_{mid-IR} / c - n_s \omega_s / c + 2\pi / \Lambda_2$$

Undepleted pump approximation: "undepleted pump approximation" is the incident signal field E_s is much stronger than the other fields and therefore its amplitude is nearly constant

(undepleted) during the evolution. So the coupled equations that govern the evolution of the two STWM processes can be written in three linear equations[12]:

$$i\frac{d}{dz}\varphi(z)=M(z)\varphi(z) \qquad M(z)=\begin{bmatrix}\Delta k_1 & \Omega_p^* & 0\\ \Omega_p & 0 & \Omega_s^*\\ 0 & \Omega_s & -\Delta k_2\end{bmatrix} \tag{2-2}$$

$$\varphi(z)=[\varphi_1,\varphi_2,\varphi_3]^T \quad \Omega_P=\psi_s\sqrt{g_1g_{31}} \quad \Omega_s=\psi_s\sqrt{g_4g_{32}}$$

$$\varphi_1(z)=\psi_s(z)\psi_p(z)e^{-i\Delta k_1 z}\sqrt{g_{31}g_4/2}\,,\ \varphi_2(z)=\psi_s(z)\psi_i(z)\sqrt{g_1g_4/2}$$

$$\varphi_3(z)=\psi_s(z)\psi_{mid-IR}(z)e^{-i\Delta k_2 z}\sqrt{g_1g_{32}/2}$$

Here $g_1=\frac{-i\omega_p d_{eff}}{n_p c}f_1$, $g_{31}=\frac{-i\omega_i d_{eff}}{n_i c}f_1$, $g_{32}=\frac{-i\omega_i d_{eff}}{n_i c}f_2$, $g_4=\frac{-i\omega_{mid-IR} d_{eff}}{n_{mid-IR} c}f_2$,

equations (2-2) have the same form as the dynamics of quantum mechanical three-level Λ system. The off-diagonal elements Ω_p and Ω_s provide couplings between the optical fields, which corresponds to the pump and Stokes Rabi frequencies of three-level Λ system, both of them are proportional to the signal field E_s and the magnitudes of the Fourier coefficients.

It is convenient to single out the phase mismatch terms of matrix M(Z) by dividing it into two parts:

$$\mathrm{M(z)=M_0(z)+M_\Delta(z)}\ ,\quad M_0(z)=\begin{bmatrix}0 & \Omega_p^* & 0\\ \Omega_p & 0 & \Omega_s^*\\ 0 & \Omega_s & 0\end{bmatrix}\ ,\quad M_\Delta(z)=\begin{bmatrix}\Delta k_1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\Delta k_2\end{bmatrix} \tag{2-3}$$

where the matrix $M_0(z)$ corresponds to the usual STIRAP Hamiltonian $H_0(t)$, and $M_\Delta(z)$ accounts for the two-photon detuning δ, respectively. We suppose that at z=0 the optical field is, $\varphi_1(z=0)=1$,$\varphi_2(z=0)=0$, $\varphi_3(z=0)=0$, and we are interested in the conversion at z=L(crystal length), $P_n(z=L)=|\varphi_n(z=L)|^2$ (n=1, 2, and 3).

We will first consider the case of perfect phase-matching $\Delta k_1=\Delta k_2=0$. The eigenvalues of matrix M(Z) are:

$$\varepsilon_0(z)=0\,,\ \varepsilon_+(z)=\sqrt{\Omega_p^2+\Omega_s^2}\,,\ \varepsilon_-(z)=-\sqrt{\Omega_p^2+\Omega_s^2} \tag{2-4}$$

The corresponding eigenvectors of M(Z) that form the adiabatic basis are:

$$\lambda_0=[-\Omega_s^*,0,\Omega_p]^T\ \ \lambda_+=[\Omega_p^*,-\sqrt{\Omega_p^2+\Omega_s^2},\Omega_s]^T,\ \ \lambda_-=[\Omega_p^*,\sqrt{\Omega_p^2+\Omega_s^2},\Omega_s]^T \tag{2-5}$$

Next, let us define the mixing angle θ: $\tan\theta(z)=\Omega_p/\Omega_s$, then the adiabatic basis are:

$$C_+(Z)=\frac{\sin\theta}{\sqrt{2}}\varphi_1(Z)-\frac{1}{\sqrt{2}}\varphi_2(Z)+\frac{\cos\theta}{\sqrt{2}}\varphi_3(Z) \tag{2-6a}$$

$$C_0(Z)=\cos\theta\varphi_1(Z)-\sin\theta\varphi_3(Z) \tag{2-6b}$$

$$C_-(Z)=\frac{\sin\theta}{\sqrt{2}}\varphi_1(Z)+\frac{1}{\sqrt{2}}\varphi_2(Z)+\frac{\cos\theta}{\sqrt{2}}\varphi_3(Z) \tag{2-6c}$$

STIRAP technique is based on the zero-eigenvalue of dark vector $C_0(Z)$, which is a coherent superposition of the initial optical fields $\varphi_1(z)$ to final optical fields $\varphi_3(z)$ only. When the coupling order is counterintuitive, it means that optical fields $\psi_3(z)$ and $\psi_4(z)$ are first coupled, and the coupling between $\psi_1(z)$ and $\psi_3(z)$ is introduced at a later point .

$$\lim_{z\to 0}\frac{\Omega_p(z)}{\Omega_s(z)}=0,\ \lim_{z\to L}\frac{\Omega_s(z)}{\Omega_p(z)}=\infty \tag{2-7}$$

During the conversion, the mixing angle θ(z) rotates from θ(z=0)=0 to θ(z=L)=π/2. When the system can be forced to stay in the dark vector at all conversion processes, all of the optical power will be transferred from $\varphi_1(z)$ to $\varphi_3(z)$ without ever going through $\varphi_2(z)$. In order to ensuring adiabatic evolution, it is required that the coupling between each pair of adiabatic vectors is negligible compared with the difference between the energies of these states, so the adiabatic condition is:

$$\left|\left\langle\frac{\mathrm{d}}{dz}C_0(Z)\middle|C_\pm(Z)\right\rangle\right|\ll|\varepsilon_0-\varepsilon_\pm| \tag{2-8}$$

The adiabaticity condition simplifies and becomes: $|d\theta/dz|\ll\sqrt{\Omega_p^2+\Omega_s^2}$, it requires the changes of the coupling coefficients to be very gradual[10].

When beyond the adiabatic limit or perfect phase-matching condition can't be satisfied, the analysis is more difficult. Next we will develop a new method that the cascaded conversions processes can be mapped uniquely into an associated three wave mixing processes. After solving the coupling wave equations of three wave mixing processes, we immediately draw the solutions of the corresponding cascaded coupled equations.

3. Optical Bloch Equations For Difference Frequency Generation

3.1. Theoretical Model

Comparing between the undepleted pump approximation of DFG process and the dynamics of a two-level atomic system, it have been found that the equations possess the same forms [13,14]. What's more, the coupling equations for complex-valued amplitudes can be recast as three couple equations for real-valued variables, the resulting is Optical Bloch equation.

Under undepleted pump approximation, the DFG coupled equations can be simplified as[14]:

$$i\frac{d}{dt}\begin{bmatrix} A_1(z) \\ A_3(z) \end{bmatrix}=\frac{1}{2}\begin{bmatrix} -\Delta k(z) & 2|q| \\ 2|q| & \Delta k(z) \end{bmatrix}\begin{bmatrix} A_1(z) \\ A_3(z) \end{bmatrix} \tag{3.1-1}$$

By using rotation transformation, we could obtain the dressed fields (adiabatic amplitudes):

$$i\frac{d}{dz}\begin{bmatrix} \tilde{A}_1(z) \\ \tilde{A}_3(z) \end{bmatrix}=\frac{1}{2}\begin{bmatrix} -\Omega(z) & -i2\dot{\theta}(z) \\ i2\dot{\theta}(z) & \Omega(z) \end{bmatrix}\begin{bmatrix} \tilde{A}_1(z) \\ \tilde{A}_3(z) \end{bmatrix} \tag{3.1-2}$$

The adiabaticity condition for (3.1-2) is $|\dot{\theta}| \ll \Omega$, where $q(z)=2\pi\omega_1\omega_3 d_{eff}A_2\Big/\sqrt{k_1k_3}\,c^2$ $\tan\theta(z)=2|q|\big/\Delta k(z)$ $\Omega(z)=0.5*\sqrt{\Delta k^2(z)+4|q|^2}$, overdot means a space derivative. The two complex-valued amplitudes can be recast as three coupled equations for real-valued variables[15-17]. We now define new unit vector $B(z)=\left[u(z),v(z),w(z)\right]^T$, and the components of this vector are: $u(z)=2\,\mathrm{Re}(A_1A_3^*)$, $v(z)=2\,\mathrm{Im}(A_1A_3^*)$, $w(z)=|A_1|^2-|A_3|^2$.

The final result is Optical Bloch equation:

$$i\frac{d}{dz}B(z)=\begin{bmatrix} 0 & -\Omega_1 & 0 \\ \Omega_1 & 0 & -\Omega_2 \\ 0 & \Omega_2 & 0 \end{bmatrix}B(z) \tag{3.1-3}$$

Here, $\Omega_1=\Delta k(z)$, $\Omega_2=2|q|$. After replacing the amplitudes v(z) and w(z) by the amplitudes $\tilde{v}(z)=iv(z)$ and $\tilde{u}(z)=-u(z)$, and exchanging the places of w(z) and u(z), the Bloch equation takes the form:

$$i\frac{d}{dz}\begin{bmatrix} w \\ \tilde{v} \\ \tilde{u} \end{bmatrix}=\begin{bmatrix} 0 & \Omega_2 & 0 \\ \Omega_2 & 0 & \Omega_1 \\ 0 & \Omega_1 & 0 \end{bmatrix}\begin{bmatrix} w \\ \tilde{v} \\ \tilde{u} \end{bmatrix} \tag{3.1-4}$$

Eq. (3-4) is exactly of the form Eq. (2-2), when we set $M_\Delta(z)=0$, $\Omega_1=\Omega s$, $\Omega_2=\Omega p$.

The optical field amplitudes φ(z) of the two STWM processes are related to the DFG processes amplitudes A(z) by:

$$\varphi_1(z)=|A_1|^2-|A_3|^2,\ \varphi_2(z)=2i\,\mathrm{Im}\,A_1A_3^*,\ \varphi_3(z)=-2\,\mathrm{Re}\,A_1A_3^* \qquad (3.1\text{-}5)$$

and to the adiabatic amplitudes $\tilde{A}(z)$ by:

$$\varphi_1(z)=(\left|\tilde{A}_1(z)\right|^2-\left|\tilde{A}_3(z)\right|^2)\cos\theta(z)+2\,\mathrm{Re}\left[\tilde{A}_1(z)\,\tilde{A}_3^*(z)\right]\sin\theta(z) \qquad (3.1\text{-}6a)$$

$$\varphi_2(z)=-2\mathrm{iIm}\left[\tilde{A}_1(z)\,\tilde{A}_3^*(z)\right] \qquad (3.1\text{-}6b)$$

$$\varphi_3(z)=2\,\mathrm{Re}\left[\tilde{A}_1(z)\,\tilde{A}_3^*(z)\right]\cos\theta(z)-(\left|\tilde{A}_1(z)\right|^2-\left|\tilde{A}_3(z)\right|^2)\sin\theta(z) \qquad (3.1\text{-}6c)$$

These relations hold for any coupling order. When we have solved the adiabatic DFG functions (3.1-2), we immediately drew the solutions to the corresponding two STWM coupled equations (2-2). Such a one-to-one relation between the solutions holds only for restricted sets of initial conditions.

Because the optical field starts from pump field A_1 to signal field A_3 at the end, the initial conditions is normalized intensities $|A_1(z=0)|^2=1$, $|A_3(z=0)|^2=0$. Now we define the two nonlinear coupling as follows :

$$\Omega_1(z)=\Omega_0(z)\cos(\pi z/2L),\ \Omega_2(z)=\Omega_0(z)\sin(\pi z/2L) \qquad (3.1\text{-}7)$$

It can be easily seen that for all propagation process the amplitude Ω stays constant(Ω=Ω_0), it behavior is shown in Fig. 2A**.**When the adiabatic condition $|\dot{\theta}|\ll\Omega$ maintains, the adiabatic solutions for counterintuitive coupling are exactly given by:

$$\varphi_1(z)=\left[1-\theta'^2(1-\cos(z\eta))/\eta^2\right]\cos\theta+\left[\theta'\sin z\eta/\eta\right]\sin\theta \qquad (3.1\text{-}8a)$$

$$\varphi_2(z)=\Omega_0\theta'\sin^2\left(z\eta/2\right)/\eta^2 \qquad (3.1\text{-}8b)$$

$$\varphi_3(z)=\left[\theta'\sin z\eta/\eta\right]\cos\theta+\left[\theta'^2(1-\cos(z\eta))/\eta^2-1\right]\sin\theta \qquad (3.1\text{-}8c)$$

Where $\eta=\sqrt{(\Omega_0^2/4)+\theta'^2}$. It is more difficult to achieve analytic evolution for two STWM equations(2-2). A useful feature of this model is that it allows a simple analytic solution not only for the final wave energy but also for their spatial evolution.

3.2. Numerical Simulation Under Ideal Conditions

As an example, we considered the generation of mid-IR at 4.3 μm by using the CW pump beam at 1.06μm. The LN crystal length is 50 mm, and wavelengths of the signal and the idler beams are set to be 1.7μm and 2.8 μm respectively. We set the temperature to be 25^0C in our model. Using the LN dispersion relation and neglecting the thermal expansion, the other structural parameters can be calculated from the sellmeier equation. By numerical integration of Eqs.(2-1), Figure 2b shows the normalized intensities of the interacting waves along the nonlinear crystal.

By numerical simulation, we conclude that the final intensities $\left|\varphi_n(z=L)\right|^2$ (n=1, 2, and 3) can only depend on the combination $L\Omega_0$. Within the adiabatic limit, $L\Omega_0 \to \infty$, the energies on the optical field $\varphi_1(z)$ and $\varphi_3(z)$ are exchanged in an expected adiabatic manner, the results are shown in Fig. 2b. The figure is close to adiabatic, where less than 0.6% of the energy has been transformed into the intermediate wave, and the total energy has been transferred to the final wave. In addition, when the pump intensity is high enough, the intermediate frequency ω_3 would almost never generate. At last, we also shows the transfer efficiency as a function of the interaction length L and the interaction amplitude Ω_0, for the special case when Ω_P and Ω_S have equal maximum value, simulation results are displayed in fig. 3

We can see that the energy on the intermediate wave φ2 oscillates, when $L\Omega_0$ is not very large; In the ideal case, $L\Omega_0 \to \infty$, the energy on the intermediate wave tends to zero, and the energy on the final wave tends to unity. Numerical modeling results which exhibiting a clear 'STIRAP signature'' are shown in Fig. 2. Fig.3.clearly shows the variation tendency of the intermediate optical filed intensity and final conversion efficiency following the change of adiabatic parameter, which could be helpful for us to utilize the STIRAP technology to design the system at the last step.

It is worth pointing out that the effects of the two coupling order are crucial for the perfect phase-matching $\Delta k_1=\Delta k_2=0$ case. By exploring the application of the stimulated Raman adiabatic passage (STIRAP) in the realm of frequency conversion, we must consider the realistic situation; the parameters which control the efficiency of the processes shift will cause deviation from the

condition of perfect phase match, so we must also take into account the effects of the phase mismatch $\Delta k1=\Delta k2\neq 0$.

4. Effects of phase mismatch to the Optical STIRAP

When the condition of perfect phase match can't satisfied, the coupled equations in the adiabatic basis (2-6) reads[11]:

$$i\,dC(z)/dz = M^{ad}(z)C(z) \tag{4-1}$$

Where $$M^{ad}(z) = M_0{}^{ad}(z) + M_\Delta{}^{ad}(z) \tag{4-2}$$

$$M_0^{ad}(z) = \begin{bmatrix} \Omega/2 & i\dot{\theta}/\sqrt{2} & 0 \\ -i\dot{\theta}/\sqrt{2} & 0 & -i\dot{\theta}/\sqrt{2} \\ 0 & i\dot{\theta}/\sqrt{2} & -\Omega/2 \end{bmatrix} \tag{4-3a}$$

$$M_\Delta^{ad}(z) = \frac{\Delta}{2}\begin{bmatrix} \cos 2\theta/2 & -\sin 2\theta/\sqrt{2} & \cos 2\theta/2 \\ -\sin 2\theta/\sqrt{2} & -\cos 2\theta & -\sin 2\theta/\sqrt{2} \\ \cos 2\theta/2 & -\sin 2\theta/\sqrt{2} & \cos 2\theta/2 \end{bmatrix} \tag{4-3b}$$

As is shown in Eqs. (4-3), the phase mismatching induces nonadiabatic couplings between the dark basis C_0(Z) and the other two adiabatic basis, $C_+(z)$ and $C_-(z)$. The coupling expression is (4-4), respectively.

$$\rho_{+0}(z) = \left[2i\dot{\theta}(z) - \Delta\sin 2\theta(z)\right]/\sqrt{2} \tag{4-4a}$$

$$\rho_{-0}(z) = \rho_{+0}(z) \tag{4-4b}$$

Where $\rho_{+0}(z)$ is the coupling of basis $C_0(z)$ and $C_+(z)$, and $\rho_{-0}(z)$ is the coupling of basis $C_0(z)$ and C-(z). These couplings will reduce the STIRAP conversion efficiency. In addition, the phase mismatching will lead to the energies of all adiabatic basis shifts[11]:

$$\varepsilon_{+0}(z) = \left[\Omega(z) + 3\Delta\cos 2\theta(z)/2\right]/2 \tag{4-5a}$$

$$\varepsilon_{-0}(z) = \left[3\Delta\cos 2\theta(z)/2 - \Omega(z)\right]/2 \tag{4-5b}$$

Where $\varepsilon_{+0} = \varepsilon_+ - \varepsilon_0, \varepsilon_{-0} = \varepsilon_- - \varepsilon_0$, is the energies shifts between the dark basis $C_0(z)$ and the other two adiabatic basis $C_+(z)$ and $C_-(z)$. For $\varepsilon_{+0}=0$ (or $\varepsilon_{-0}=0$), it represents the energy is crosses.

As illustrated in Fig. 4(a) where the energy splitting are plotted versus the mixing angle. Obviously, no matter $\Delta k<0$ or $\Delta k>0$, the energy shifts $\varepsilon_{-0}(z)$ and $\varepsilon_{+0}(z)$ is always crosses zero. Hence, the energies of basis $C_0(z)$ and $C_-(z)$ or $C_+(z)$ always crosses.

As far as STIRAP is concerned, the main loss occurs at the crossings of the adiabatic basis $C_0(z)$ with $C_-(z)$ and $C_+(z)$. When the crossing is close to the maxima of the nonadiabatic couplings (assume maxima is $\rho(Z_{max})$), there is a significant nonadiabatic interaction at this crossing and hence a significant energy loss from the dark basis $C_0(z)$[11,18-20]. So the crossing of the energies with relation to the maxima of the nonadiabatic couplings, determines two bounds of phase-matching Δ. As is shown in Fig. 4(b), the shifts in the $\varepsilon_{-0}(z)$ are larger than $\varepsilon_{+0}(z)$. It implicates that the crossing in $\varepsilon_{-0}(z)$ is shifted farther from Z_{max} than $\varepsilon_{+0}(z)$, so the main energy losses from the dark basis $C_0(z)$ take place through the crossing in $\varepsilon_{+0}(z)$.

For $\Delta>0$, the place of crossing in the $\varepsilon_{+0}(z)$ occurs after Zmax(z_{+0}>Zmax), and z_{+0} approaches Zmax as Δ increases; consequently, the transfer efficiency approaches zero. In order to simplify the results, we choose the place of Pump couplings Ω_p maxima (Zpmax) as a reference point, for the crossing place, then

$$\cos 2\theta(z_{p\max}) = -1 \qquad \Omega(z_{p\max}) \approx \Omega_p(z_{p\max}) \tag{4-6}$$

From Eq.(4-5a) we could obtain $\Delta = 2\Omega(z)/3$. For $\Delta<0$, the place of crossing z_{+0} in the $\varepsilon_{+0}(z)$ occurs before Z_{max}(z_{+0}< Z_{max}), and z_{+0} approaches Z_{max} as $|\Delta|$ increases; We choose Zsmax which is the place of maxima of the Stokes couplings Ω_s as a reference point for the crossing place, then:

$$\cos 2\theta(z_{s\max}) = 1, \quad \Omega(z_{s\max}) \approx \Omega_s(z_{s\max}) \tag{4-7}$$

By setting the $\varepsilon_{+0} = 0$ from Eq.(4-5a),we could obtain the $\Delta = -2\Omega(z)/3$. So the bounds of phase mismatching Δ in Optical STIRAP are:

$$-2\Omega_P(z)/3 \le \Delta \le 2\Omega_S(z)/3 \tag{4-8}$$

Hence, the phase mismatching width is proportional to $\Omega(z)$. For the (3-7) triangle model, $\Omega(z)$ $=\Omega_0(z)$. We show in Fig. 5 the Optical STIRAP evolution for different number of phase mismatching Δ. In all cases the energy is converted from frequency ($\omega 1$) to frequency ($\omega 4$) in the end in a stepwise manner. The transient generation of the intermediate frequency (ω_3) is enhanced and the transfer efficiency is damped as Δk increases: from $\Delta kL=0$ for without significant

generation of the intermediate frequency (ω3) to ΔkL=1.5 with oscillates for output wave. So the above conclusions are confirmed completely, and the analytic bounds are seen to describe the conversion region very accurately. We note that although the example in Fig. 4 uses trigonometric model, the analysis method is equally suitable for nonlinear coupling coefficients of Gaussian Modulation or any other shape.

5. Numerical Simulation Of Optical STIRAP In A Nonperiodic Optical Superlattice

In the end of the section we shortly discuss Nonperiodic Optical Superlattice for generation of Optical STIRAP in cascaded parametric oscillator. To achieve the purposes of both processes were phase matched and the coupling coefficients were modulated as desired, ref.[10] propose using phase-reversal quasi-phase matching technique. For example, we can construct a product of two binary functions as follows:

$$G(z)=sign[\sin(\frac{2\pi}{\Lambda_1}z+\frac{\pi}{\Lambda_1}l_2)-\sin(\frac{\pi}{\Lambda_1}l_2)]\times sign[\sin(\frac{2\pi}{\Lambda_2}z+\frac{\pi}{\Lambda_2}l_2^{'})-\sin(\frac{\pi}{\Lambda_2}l_2^{'})] \qquad (5\text{-}1)$$

Where $\Lambda_1=l_1+l_2$, $\Lambda_2=l_1^{'}+l_2^{'}$ is the period, $l_1(l'_1)$ is length of positive domains, $l_2(l'_2)$ is length of inverted domains. Using simple Fourier analysis yields:

$$G_{QPM}(z)\approx\frac{1}{\pi}(2D_2-1)\sin(2\pi D_1)\exp(\pm i\Delta k_1)+\frac{1}{\pi}(2D_1-1)\sin(2\pi D_2)\exp(\pm i\Delta k_2) \quad (5\text{-}2)$$

$D_i=l_1/\Lambda_i$ is the duty cycle. This is the modulation of the second-order nonlinear coefficient $\chi^{(2)}$. If we choose $\Lambda_1=\Delta k_1$ and $\Lambda_2=\Delta k_2$, and only kept phase-matched terms, then

$$f_1=(2D_2-1)\sin(2\pi D_1)\,,\ f_2=(2D_1-1)\sin(2\pi D_2)$$

D_1 and D_2 determine the magnitude of the effective coupling coefficient for each process. Varying the two duty cycles along the crystal achieves the required modulation. The simulation results are presented in the Fig. 6a; good correspondence is obtained with the ideal case results. We also explored the conversion evolution for different number of phase mismatching Δ. Simulation results are depicted in Figure 6b**.** Obviously, for $\Delta kL\geqslant 1.5$, the transient generation of the intermediate frequency (ω3) is significant, and the output wave shows strong oscillates. In order to using PRQPM for phase-matching and coefficient modulation, here we not take into consideration technological restrictions. Actually, the adiabaticity condition requires the changes of the

coefficients to be very gradual, which means domains length as small as possible; Otherwise, it will generate intermediate frequency (ω3) obviously. It has been discussed in detail in Ref.[10].

It is worth pointing out that the effects of the two coupling order are crucial for the perfect phase-matching $\Delta k1=\Delta k2=0$ case. But it is not required for large phase mismatch. By analogy to the adiabatic elimination procedure, it can also directly generate frequency (φ4) from frequency (φ1) assuming that both of DFG processes exhibits a very large phase mismatch but their sum is rather small. This approximation has been discussed in Ref.[9]. But due to inherently large phase-mismatches, high conversion efficiency was difficult to achieve.

5. CONCLUSION

In this paper, we have shown that the cascaded wavelength conversions process in the undepleted pump approximation can be mathematically formulated and geometrically visualized in complete analogy with the framework of atomic STIRAP. We have developed an approximate analytical trigonometric model to describe the two STWM processes, which is based on the adiabatic DFG process, wherein the pump amplitude is assumed constant along the nonlinear crystal. It holds the advantage that it does allow a full solution. The trigonometric model, which is analytically solvable enables us to show explicitly that, as the adiabaticity increases, the conversion efficiency for the counterintuitive coupling sequence approaches unity, while it oscillates for the nonadiabatic transition. By analyzing the achieved adiabatic evolution, we were able to derive surprisingly simple and accurate bounds of the high-conversion efficiency region, and simple estimates of the adiabatic parameters magnitude.

Moreover, we also take into account the effects of the phase mismatch to the Optic STIRAP technique. By analyzing the emerged adiabatic basis crossings and the positions of the nonadiabatic couplings, we were able to derive simple and accurate bounds of the high conversion region, and estimates of the width of the phase mismatching. We point out that other approachs, (e.g, based on coherence length maps [21]for the involved nonlinear processes) is also useful to analysis the effects of the phase mismatch, but the analysis processes are too complicated to be use here. Finally, a technologically feasible method for carrying out such a process was proposed and numerically demonstrated to be effective.

The results in this paper also can be of potential significance in applications of STIRAP where the two coupling, pump and Stokes, are of different physical nature, as in the DFG process wherein the Stokes coupling is replaced by phase mismatching ΔK. Suitably extended, we here draw upon a mathematical equivalence of the STIRAP STWM processes with adiabatic DFG process, to suggest a potentially useful technique, DFG-STIRAP, it may allow complete transfer of energy from one wavelength to another in single three wave mixing process.

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FIGURE CAPTIONS

Fig.1. (color online)Schematic illustration of cascaded difference frequency generation process for efficient mid-IR generation.

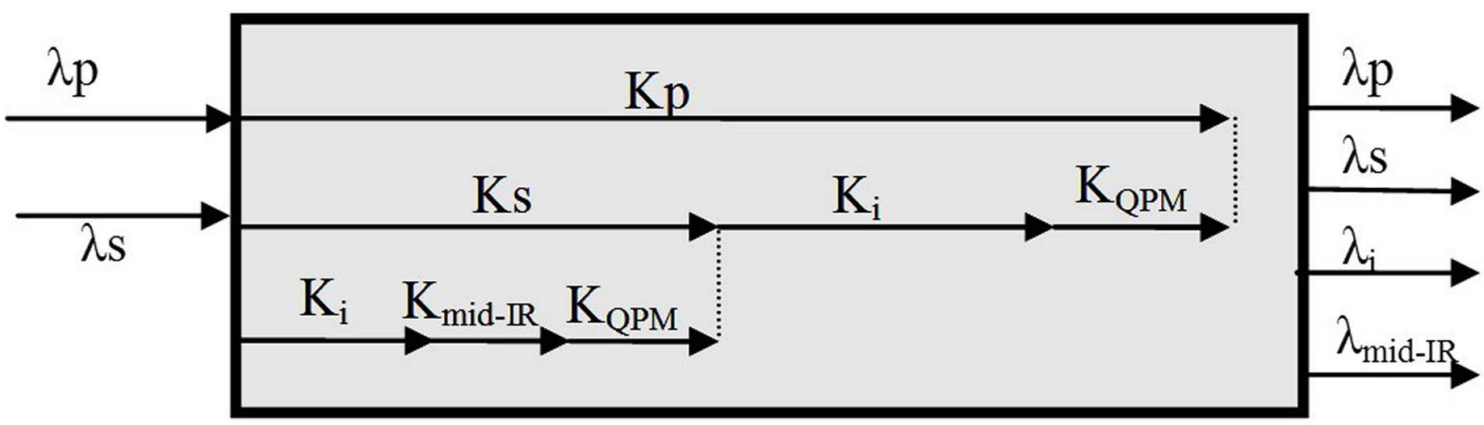
Fig.2. (color online)Numerical simulation of the intensities of the interacting waves along the nonlinear medium in above model. The field intensities are calculated from Eqs. (1) for f1=5sin(πz/2L), f2=5cos (πz/2L). Part (a) is Normalized coupling coefficients of the two nonlinear processes. Part (b) is "counterintuitive" order resulting, and the inset shows the intermediate wave intensity.

Fig.3. (color online)Numerical simulation of the intensities of the interacting waves along the nonlinear medium. Part(a) is the final intensity for intermediate wave plotted as a function of the L and Ω0. Part (b) is the final conversion efficiency for final wave plotted as a function of the L and Ω_0.

Fig.4. (color online)Spatial evolution of the energy splitting ε_{+0} (ε_{-0}) and the nonadiabatic coupling ρ for trigonometric model (3.1-7).Part(a) is the spatial evolution of energy splitting with difference sign of Δk. Part (b) is the evolution of the energy splitting ε_{+0} (ε_{-0}) and the nonadiabatic coupling ρ for difference value of $|\Delta k|$

Fig.5. (color online)Numerical simulation of the normalized intensities of the interacting waves along the nonlinear medium with different number of phase mismatching Δ, with amplitude Ω0 =40, under ideal conditions.

Fig.6. (color online)Numerical simulation of the intensities of the interacting waves along the nonlinear medium with phase-reversal quasi-phase matching technique. The field intensities are calculated from Eqs. (2-1). Part (a) is phase matching results. Part(b) is the final conversion efficiency for Δk have difference values.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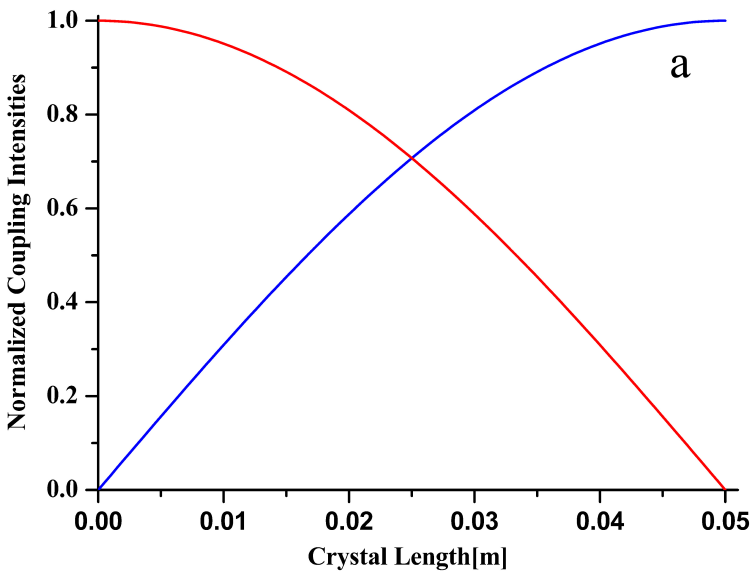
λs
Kp
Ks
Ki
KQPM
Ki
Kmid-IR
KQPM
λp
λs
λi
λmid-IR


a
Normalized Coupling Intensities
1.0
0.8
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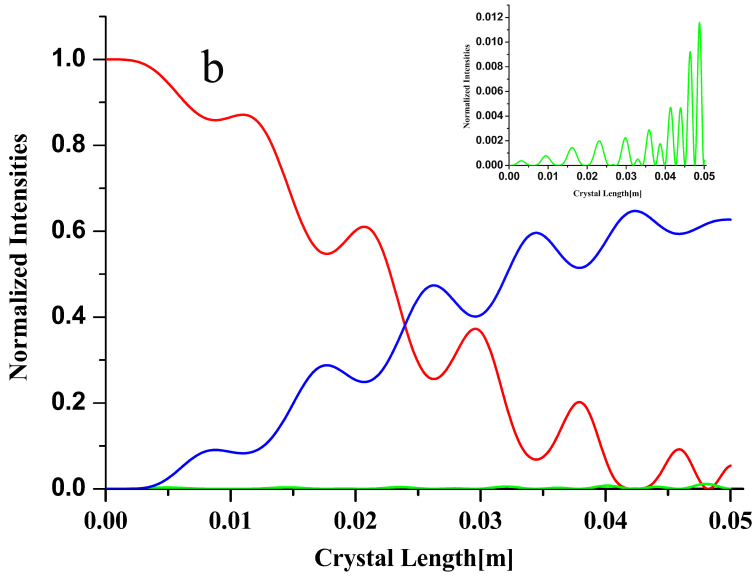
0.4
0.2
0.0
0.00
0.01
0.02
0.03
0.04
0.05
Crystal Length[m]


b
Normalized Intensities
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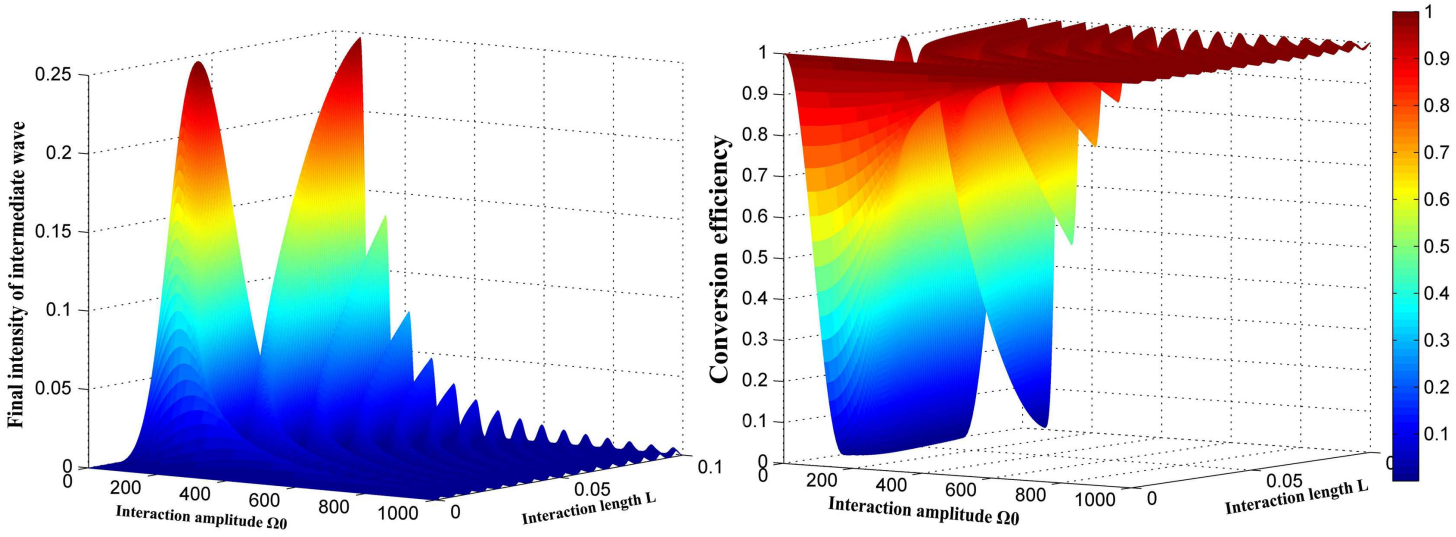
0.6
0.4
0.2
0.0
0.00
0.01
0.02
0.03
0.04
0.05
Crystal Length[m]
Normalized Intensities
0.012
0.010
0.008
0.006
0.004
0.002
0.000
0.00
0.01
0.02
0.03
0.04
0.05
Crystal Length[m]


Final intensity of intermediate wave
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0.05
0
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200
400
600
800
1000
Interaction amplitude Ω0
0
0.05
0.1
Interaction length L
Conversion efficiency
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
200
400
600
800
1000
Interaction amplitude Ω0
0
0.05
Interaction length L
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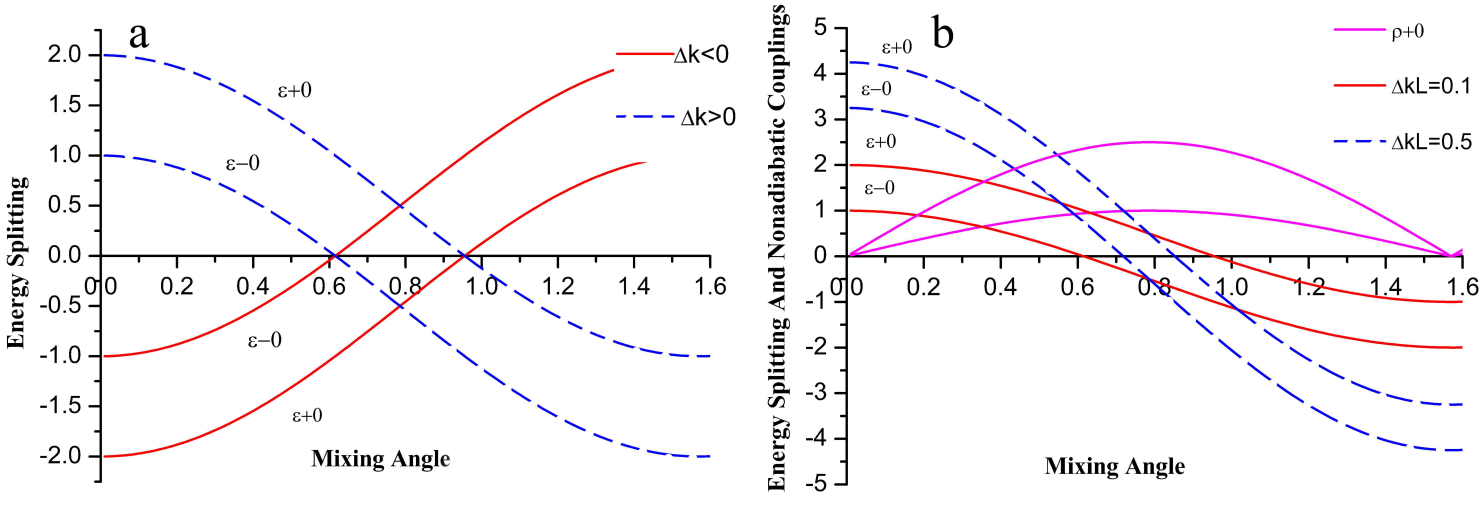
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1


a
Energy Splitting
Mixing Angle
ε+0
ε−0
ε−0
ε+0
Δk<0
Δk>0
b
Energy Splitting And Nonadiabatic Couplings
Mixing Angle
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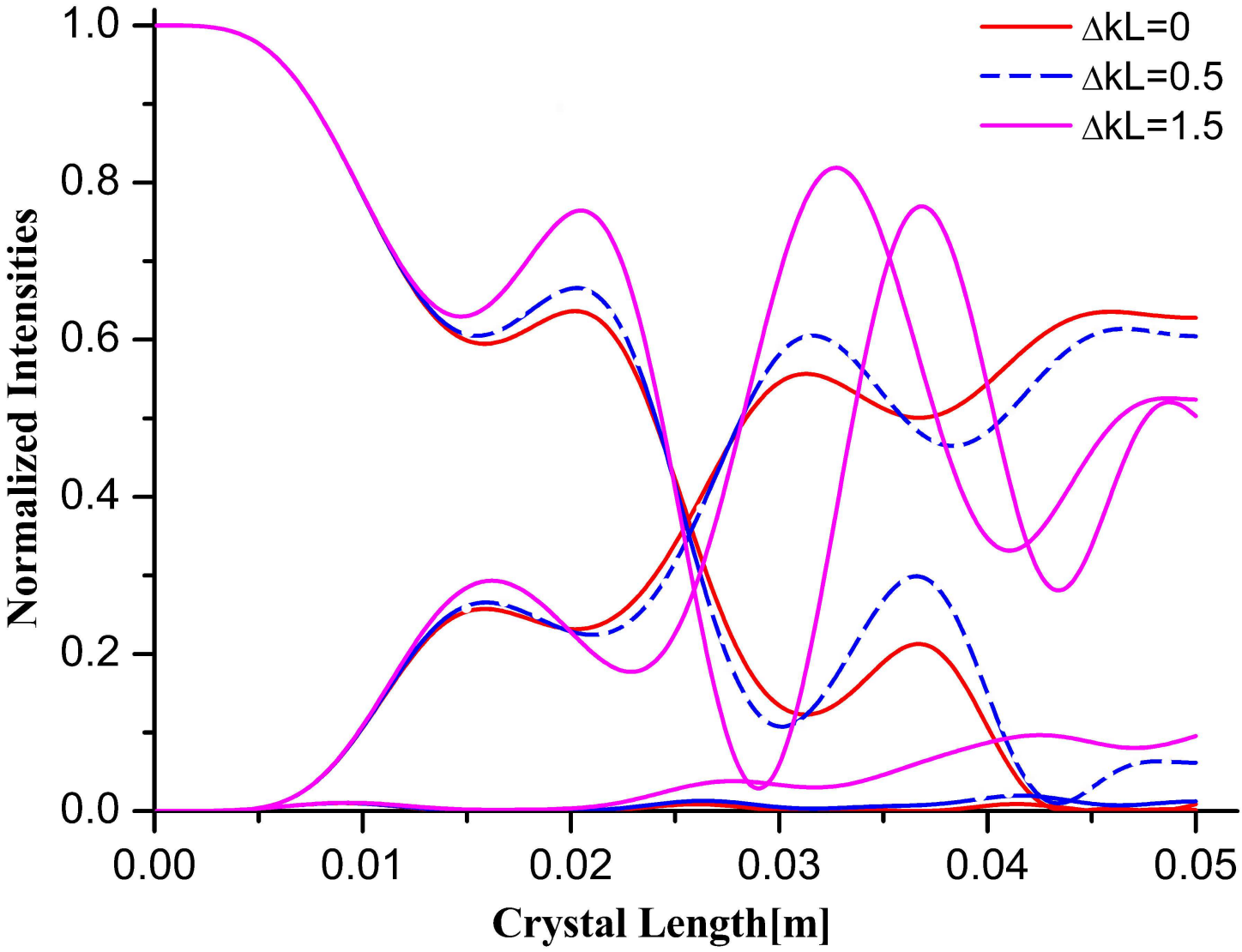
ε−0
ε+0
ε−0
ρ+0
ΔkL=0.1
ΔkL=0.5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ΔkL=0.5
ΔkL=1.5
Normalized Intensities
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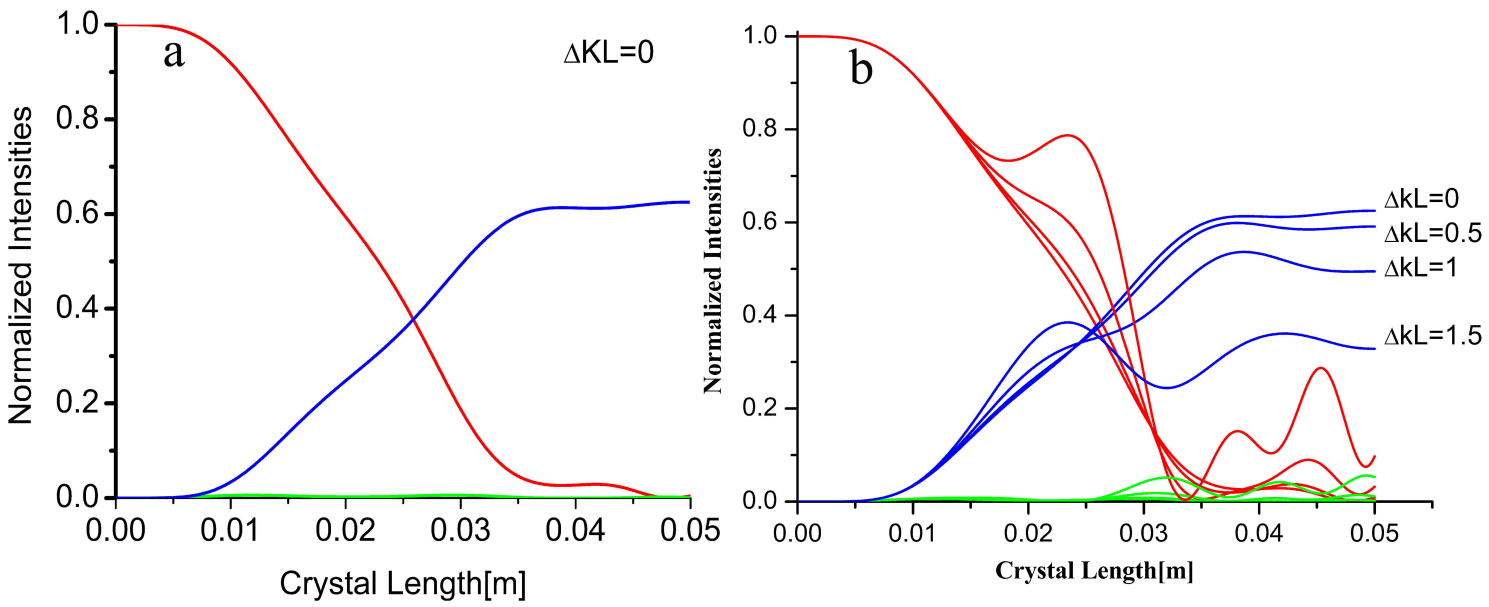
0.8
0.6
0.4
0.2
0.0
0.00
0.01
0.02
0.03
0.04
0.05
Crystal Length[m]


a
ΔKL=0
Normalized Intensities
Crystal Length[m]
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0.6
0.4
0.2
0.0
0.00
0.01
0.02
0.03
0.04
0.05
b
ΔkL=0
ΔkL=0.5
ΔkL=1
ΔkL=1.5
Normalized Intensities
Crystal Length[m]