Generation of squeezed states in a movable mirror via dissipative optomechanical coupling

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Generation of squeezed state in a movable mirror via the dissipative optomechanical coupling

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We investigate the generation of squeezed state in a movable mirror in the dissipative optomechanics in which the oscillating mirror modulates both the resonance frequency and the linewidth of the cavity mode. Via feeding a broadband squeezed vacuum light accompanying a coherent driving laser field into the cavity, the master equation for the cavity-mirror system is derived by following the general reservoir theory. When the mirror is weakly coupled to the cavity mode, we find that the driven cavity field can effectively perform as a squeezed vacuum reservoir for the movable mirror via utilizing the completely destructive interference of quantum noise. The efficient transfer of squeezing from the light to the movable mirror occurs, which is irrespective of the ratio between the cavity damping rate and the mechanical frequency. Moreover, when the mirror is moderately coupled to the cavity mode, the photonic excitation can preclude the completely destructive interference of quantum noise. As a consequence, the mirror deviates from the ideal squeezed state.

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I. INTRODUCTION

Rapid progress on optomechanics towards sensing and control of the zero-point motion of mechanical oscillators has been made via the engineering of high-quality micromechanical oscillators coupled to high-finesse cavity modes [1–4], because exploration of quantum behavior in these mechanical systems will spark new insights into quantum information processing (QIP) [5–7], measurement science [8–11], and fundamental tests of physical laws [12], etc. Recently, some experimental investigations for observing quantum mechanical effects in the mechanical systems have been demonstrated [13, 14]. Indeed, these technical developments also offer the possibility to observe nonclassical state of the mechanical oscillator [15, 16]. Specifically, achieving squeezed states in mechanical oscillators, in which the variance of one quadrature of motion is below the zero-point motion, is an important goal because of their applications in ultrahigh precision measurements such as the detection of gravitational waves [17–19]. By now, different schemes have been proposed for the generation of quantum squeezing of movable mirrors [20–25]. For example, Huang *et al.* [23] proposed a potential scheme to generate squeezing by putting an optical parameter amplifier inside a cavity, Seok *et al.* [24] presented a theoretical analysis of the motional squeezing of a cantilever magnetically coupled to a classical tuning fork via microscopic magnetic dipoles, and Jähne *et al.* [25] investigated the creation of squeezed states of movable mirror transferred from a squeezed light driving the cavity via the dispersive coupling under the assumption of the resolved-sideband limit.

However, from a practical perspective, it is preferable to deviate from the resolved-sideband limit, since it allows one to use small drive detunings compared with the cavity decay rate and achieve much larger effective cavity-mechanical oscillator couplings. Recently, the dissipative cavity-mirror systems have been investigated in both microwave and optical domains [26, 27], in which the driven cavity can effectively act like a zero-temperature bath via a destructive interference of quantum noise in the non-sideband-resolved regime, and hence the ground-state cooling and low-power quantum-limited position transduction are both possible. The enhanced cooling rate and elimination of optically-induced heating will be benefit for squeezing transfer from the squeezed light driving the cavity to the movable mirror, as mechanical squeezing is fairly vulnerable to thermal and optically-induced heating scattering mechanisms. Thus, in this paper we will present that in the dissipative optomechanics the performance of the squeezing transfer can be improved under the condition of the perfect elimination of heating processes arising from the interference of quantum noise, and finally a better mechanical squeezed state can be achieved.

In this paper, we propose a scheme that is capable of generating mechanical squeezing via engineering reservoir in an optomechanical setup having a strong dissipative coupling. The setup consists of an effective Fabry-Pérot interferometer (FPI) with one movable ideal end mirror. The equivalent FPI is achieved from a Michelson-Sagnac interferometer (MSI) with a movable membrane, explicitly shown in Refs. [27–29]. When we feed a much weaker broadband squeezed vacuum light accompanying a coherent cooling laser field into the cavity, the cavity field couples to the movable mirror via both the tunable dispersive and dissipative interactions. Then, distinct from the common Heisenberg-Langevin approach adopted in Refs. [23, 25–27], we follow the general reservoir theory based on the density operator in which the reservoir variables are adiabatically eliminated in the interaction picture. When the movable mirror is weakly coupled to cavity mode, the master equation for the movable mirror can be derived by adiabatically eliminating the cavity field. It shows that under the conditions of laser cooling to the ground motional state as discussed in Refs. [26, 27], i.e. elimination of the heating scattering process due to the completely destructive in-

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terference of quantum noise, the driven cavity can effectively perform as a squeezed vacuum reservoir for the movable mirror. The efficient transfer of squeezing from the squeezed light to the movable mirror occurs, which is irrespective of the ratio between the cavity damping rate and the mechanical frequency. Moreover, when the mirror is moderately coupled to the cavity mode, we solve the full motional equations for cavity-mirror system with a purely dissipative optomechanical coupling. We find that the photonic excitation can preclude the completely destructive interference of quantum noise and induce extra thermal phonon excitation in the mirror, which results in the deviation from the ideal squeezed state. However, the movable mirror is still effectively squeezed around its ground mechanical state in this dissipative optomechanics beyond the weak-coupling regime, which is numerically proved.

The paper is structured as follows. In Sec. II we introduce the FPI and derive the motion equation for the mirror-cavity system via adiabatically eliminating the reservoir variables. In Sec. III we analyze the cooling and squeezing of the movable mirror in the weak-coupling regime and results beyond the weak-coupling regime are presented in Sec. IV. In the last the conclusion is drawn in Sec. V.

II. DISSIPATIVE OPTOMECHANICAL SYSTEM DRIVEN WITH A SQUEEZED RESERVOIR

A. Description of the model

We consider an optomechanical system consisted of an effective Fabry-Pérot interferometer (FPI), sketched in Fig. 1, which can be achieved from the Michelson-Sagnac interferometer (MSI) with a movable membrane [27–29]. The movable mirror $\mathcal{M}$ oscillates along the x-axis with the frequency ω_m and couples to a cavity mode with the resonant frequency ω_a via the dispersive and dissipative couplings, which corresponds to the shifts of the cavity's resonant frequency and damping rate respectively due to the mechanical motion. The full Hamiltonian is a sum of Hamiltonians for the free cavity H_{c}, free movable mirror H_{m}, free reservoir field H_{R}, cavity-mirror interaction H_{int} and cavity-reservoir interaction $H_{\mathrm{c\text{-}R}}$ ($\hbar = 1$):

$$
\begin{aligned}
H &= H_{\mathrm{c}} + H_{\mathrm{m}} + H_{\mathrm{R}} + H_{\mathrm{c\text{-}R}} + H_{\mathrm{int}},\\
H_{\mathrm{c}} &= \omega_a a^\dagger a, \quad H_{\mathrm{m}} = \omega_m b^\dagger b, \quad H_{\mathrm{R}} = \int d\omega \omega a_\omega^\dagger a_\omega,\\
H_{\mathrm{int}} &= g_0\left[\alpha a^\dagger a + i\beta\sqrt{\frac{L}{2\pi c}}\int d\omega (a_\omega^\dagger a - a^\dagger a_\omega)\right](b + b^\dagger),\\
H_{\mathrm{c\text{-}R}} &= i\sqrt{\frac{\kappa_c}{\pi}}\int d\omega (a_\omega^\dagger a - a^\dagger a_\omega). \qquad (1)
\end{aligned}
$$

The operators a and b are the annihilation operators of cavity and phonon modes. The operator a_ω describes the continuous modes of optical reservoir coupled to the cavity mode and κ_c is the damping rate of the cavity field without the motion of the mirror. The parameters α (dispersive) and β (dissipative) respectively represent the cavity frequency's (ω_a) and damping rate's (κ_c) linear dependence on the small displacement x with $x = x_0(b^\dagger + b)/\sqrt{2}$, where x_0 is the zero-point motion amplitude of the movable mirror. The effective length of the interferometer is L. This optomechanical setup can realize the strong dissipative coupling, even in the order of cavity linewidth in the absence of dispersive coupling $\alpha = 0$ [27].

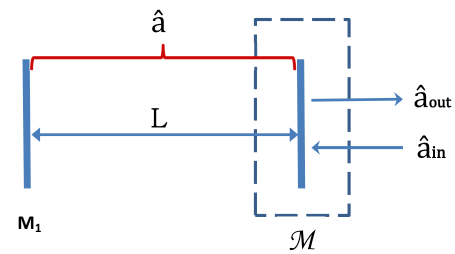

FIG. 1. (Color online) Sketch of the effective Fabry-Pérot interferometer (FPI) coupled to the cavity mode via the dispersive and dissipative couplings. The cavity is driven by a squeezed vacuum field accompanying a coherent driving laser.

The dispersively and dissipatively coupled optomechanical system has been investigated to cool the mechanical oscillator to its ground state in microwave and optical domains in the Heisenberg-Langevin approach [26, 27, 30]. In this paper, we present the dynamics of the movable mirror based on the density operator in which the reservoir variables can be adiabatically eliminated by using the reduced density operator for the system. The optical reservoir has two contributions on the cavity field: the c-number part $\langle a_\omega \rangle = \sqrt{2\pi}\bar{a}_{\mathrm{in}} e^{-i\omega_R t}$ corresponding to the coherent cooling laser of frequency ω_R and random noise part δa_ω describing the broadband squeezed vacuum reservoir with central frequency ω_s. The noise operator δa_ω has zero mean value and second moments are [31]

$$
\begin{aligned}
\langle \delta a_\omega^\dagger \delta a_{\omega'} \rangle &= N\delta(\omega - \omega'),\\
\langle \delta a_\omega \delta a_{\omega'}^\dagger \rangle &= (N+1)\delta(\omega - \omega'),\\
\langle \delta a_\omega \delta a_{\omega'} \rangle &= M\delta(\omega + \omega' - 2\omega_s),\\
\langle \delta a_\omega^\dagger \delta a_{\omega'}^\dagger \rangle &= M^*\delta(\omega + \omega' - 2\omega_s), \qquad (2)
\end{aligned}
$$

where $N = \sinh^2(r)$, $M = \sinh(r)\cosh(r)e^{i\varphi}$ with r the squeezing parameter of the squeezed vacuum light and φ the phase of the squeezed vacuum light. Evidently, N is the mean photon number and M is the two-photon correlation of the squeezed reservoir.

B. Adiabatically elimination of the squeezed reservoir

We assume that the bandwith of the squeezed reservoir is not only larger than typical spontaneous dissipation rates of the cavity field but also large compared to detunings and the effective coupling strength between the cavity and mirror. The Markovian master equation for the cavity-mirror system can be obtained via adiabatically eliminating the squeezed vacuum reservoir variables [32]. Following the general reservoir theory in textbook [33], the system-reservoir interaction is given by $\mathscr{V}(t) = H_{\text{c-R}}(t) + H_{\text{int}}(t)$ in the interaction picture. By tracing over the reservoir coordinates under Born-Markovian approximations, the reduced density operator ρ_s for the cavity-mirror system is given by the equation

$$\begin{aligned}\dot{\rho}_s =& -i\text{Tr}_R[\mathscr{V}(t), \rho_s(t)\otimes\rho_R(t_i)] \\ &- \text{Tr}_R\int_{t_i}^{t}[\mathscr{V}(t), [\mathscr{V}(t'), \rho_s(t)\otimes\rho_R(t_i)]]dt'. \end{aligned} \tag{3}$$

By substituting the c-number component and the two-time correlation functions in Eq. (2) into the Eq. (3), the motion equation for the density operator ρ_s can now be obtained as

$$\dot{\rho}_s = -i[H_0, \rho_s] + \mathcal{L}_1\rho_s + \mathcal{L}_2\rho_s. \tag{4}$$

The Hamiltonian H_0 consists of the free Hamiltonians of cavity and phonon modes, which is given by

$$H_0 = -\Delta a^\dagger a + \omega_m b^\dagger b \tag{5}$$

with $\Delta = \omega_R - \omega_a$ the detuning of the cavity resonant frequency from the coherent driving light frequency. The Liouvillian operators $\mathcal{L}_2$ and $\mathcal{L}_1$ contain the dissipations of the cavity and phonon modes and interactions between them, which are expressed as

$$\begin{aligned}\mathcal{L}_1\rho_s =& -i[g_0\alpha a^\dagger a(b+b^\dagger) + i\sqrt{2}(\bar{a}_{in}^* C - \bar{a}_{in}C^\dagger), \rho_s], \\ \mathcal{L}_2\rho_s =& M^* e^{i2\Delta_s t}(C^2\rho_s + \rho_s C^2 - 2C\rho_s C) \\ &+ Me^{-i2\Delta_s t}({C^\dagger}^2\rho_s + \rho_s {C^\dagger}^2 - 2C^\dagger\rho_s C^\dagger) \\ &+ N(2C^\dagger\rho_s C - CC^\dagger\rho_s - \rho_s CC^\dagger) \\ &+ (N+1)(2C\rho_s C^\dagger - C^\dagger C\rho_s - \rho_s C^\dagger C), \end{aligned} \tag{6}$$

with the composite operator $C = [\sqrt{\kappa_c} + g_0\beta\sqrt{\frac{L}{2c}}(b+b^\dagger)]a$ and $\Delta_s = \omega_s - \omega_R$ the detuning between the central frequency of squeezing vacuum reservoir and the frequency of coherent driving light.

The master equation in Eq. (4) is difficult to be exactly solved because of the existence of the nonlinear terms. However, outside the strong-coupling regime as discussed in single-photon optomechanics [34], it is valid to proceed the linearization on the full quantum dynamics by assuming that each operator in the system can be written as the sum of its mean value and a small fluctuation [35]:

$$a = \bar{a} + d, \quad b = \bar{b} + f, \tag{7}$$

where the classical components $\bar{a} = \langle a\rangle$, $\bar{b} = \langle b\rangle$. Moreover, our considerations are explicitly focused outside the strong-coupling limit, i.e. the single-photon optomechanical coupling strengths $(g_0\alpha, g_0\beta\sqrt{\kappa_c L/2c}) \ll (\omega_m, \kappa_c)$. Thus to the lowest order of the strengths $g_0\alpha$ and $g_0\beta\sqrt{\kappa_c L/2c}$, the mean phonon operator $\bar{b} \approx 0$ and the mean cavity operator $\bar{a}$ obeys the equation

$$\frac{d}{dt}\bar{a} = (i\Delta - \kappa_c)\bar{a} - \sqrt{2\kappa_c}\bar{a}_{in}. \tag{8}$$

The steady-state solution for $\bar{a}$ is obtained as

$$\bar{a} = \frac{\sqrt{2\kappa_c}}{i\Delta - \kappa_c}\bar{a}_{in}. \tag{9}$$

In this shifted representation of Eq. (7), the evolution of the cavity-mirror system is governed by the contributions respectively caused by the motions of the uncoupled cavity and phonon modes and the interaction between them, which reads

$$\frac{d}{dt}\rho_s = \mathcal{L}^{\text{d}}\rho_s + \mathcal{L}^{\text{f}}\rho_s + \mathcal{L}^{\text{d-f}}\rho_s. \tag{10}$$

The uncoupled cavity contribution obeys the equation

$$\begin{aligned}\mathcal{L}^{\text{d}}\rho_s =& i[\Delta d^\dagger d, \rho_s] + \kappa_c M^* e^{i2\Delta_s t}(d^2\rho_s + \rho_s d^2 - 2d\rho_s d) \\ &+ \kappa_c Me^{-i2\Delta_s t}({d^\dagger}^2\rho_s + \rho_s {d^\dagger}^2 - 2d^\dagger\rho_s d^\dagger) \\ &+ \kappa_c N(2d^\dagger\rho_s d - dd^\dagger\rho_s - \rho_s dd^\dagger) \\ &+ \kappa_c(N+1)(2d\rho_s d^\dagger - d^\dagger d\rho_s - \rho_s d^\dagger d), \end{aligned} \tag{11}$$

which parallels the evolution of a cavity field coupled to an outside squeezed vacuum reservoir. The uncoupled mirror follows the equation

$$\begin{aligned}\mathcal{L}^{\text{f}}\rho_s =& -i[\omega_m f^\dagger f, \rho_s] + g_0^2\beta^2\frac{L}{2c}\big[(2N+1)|\bar{a}|^2 - M^* e^{i2\Delta_s t}\bar{a}^2 - Me^{-i2\Delta_s t}\bar{a}^{*2}\big] \\ &\times\big[2(f+f^\dagger)\rho_s(f+f^\dagger) - (f+f^\dagger)^2\rho_s - \rho_s(f+f^\dagger)^2\big]. \end{aligned} \tag{12}$$

The interaction between the cavity field and movable mirror is described by the Liouvillian operator

$$\begin{aligned}\mathcal{L}^{\text{d-f}}\rho_s =& -ig_0\Big[\Big(\alpha(\bar{a}^* d + \bar{a}d^\dagger) + i\beta\sqrt{\frac{L}{c}}(\bar{a}_{in}^* d - \bar{a}_{in}d^\dagger)\Big)(f+f^\dagger), \rho_s\Big] \\ &+ 2\Big\{g_{\text{eff}}M^* e^{i2\Delta_s t}\big[d(f+f^\dagger)\rho_s + \rho_s d(f+f^\dagger) - (f+f^\dagger)\rho_s d - d\rho_s(f+f^\dagger)\big] + h.c.\Big\} \\ &+ 2\Big\{g_{\text{eff}}N\big[d^\dagger\rho_s(f+f^\dagger) - \rho_s d^\dagger(f+f^\dagger)\big] + g_{\text{eff}}(N+1)\big[(f+f^\dagger)\rho_s d^\dagger - d^\dagger(f+f^\dagger)\rho_s\big] + h.c.\Big\}, \end{aligned} \tag{13}$$

with

$$g_{\text{eff}} = g_0\beta\sqrt{\frac{\kappa_c L}{2c}}\bar{a} \tag{14}$$

the effective dissipative coupling strength between the cavity field and movable mirror. Similarly, the effective dispersive coupling strength is characterized by $g_0\alpha\bar{a}$.

III. COOLING AND SQUEEZING THE MOVABLE MIRROR IN THE WEAKLY COUPLING REGIME

A. Adiabatic elimination of the cavity field

We first consider the weakly optomechanical coupling regime, in which the cavity field weakly couples to the movable mirror such that the effective strengths $g_0\alpha\bar{a}$ and g_{eff} are much smaller than cavity damping rate κ_c, the cavity variable arrives at the steady state much faster than the mirror variable and can be adiabatically eliminated. Thus, the motional equation for the reduced density operator of the movable mirror can be also calculated paralleling the method for derivation of the cavity-mirror system in the last subsection by tracing over the cavity variable. Applying the second-order perturbation method with respect to the effective coupling strengths $g_0\alpha\bar{a}$ and g_{eff}, the reduced density operator for the movable mirror ρ_f now becomes

$$\frac{d}{dt}\rho_f = \text{Tr}_{\text{d}}\int_{t_0}^{t}\mathcal{L}^{\text{d-f}}(t)\mathcal{L}^{\text{d-f}}(t')\rho_d(t_0)\otimes\rho_f(t)dt', \tag{15}$$

where $\rho_d(t_0)$ is the steady-state density operator of cavity field, governed by the Liouvillian operator in Eq. (11). With the definition of the detuning $\delta = \Delta_s - \omega_m$ and assumption of $\delta \ll (\Delta_s, \omega_m)$ to accommodate for cavity-induced energy shift, after some calculations the resulting motion equation for the mirror is described by the master equation with the rotating-wave approximation

$$\begin{aligned}\frac{d}{dt}\rho_f =& -i[H_f,\rho_f] + |\Theta(\omega_m)|\big[N(f^\dagger\rho_f f - ff^\dagger\rho_f + h.c.)\\ &+ (N+1)(f\rho_f f^\dagger - f^\dagger f\rho_f + h.c.)\big]\\ &+ \big[\Theta(\omega_m)M^* e^{i2\delta t}(2f\rho_f f - f^2\rho_f - \rho_f f^2) + h.c.\big]\\ &+ |\Theta(-\omega_m)|\big[(N+1)(f^\dagger\rho_f f - ff^\dagger\rho_f + h.c.)\\ &+ N(f\rho_f f^\dagger - f^\dagger f\rho_f + h.c.)\big],\end{aligned} \tag{16}$$

with

$$\Theta(\omega_m) = \frac{g_{\text{eff}}^2}{\kappa_c}\frac{(2\Delta + \omega_m + \frac{\alpha}{\beta}\sqrt{\frac{2\kappa_c c}{L}})^2}{[i(\Delta+\omega_m)+\kappa_c]^2}. \tag{17}$$

The Hamiltonian H_f represents the optically-induced energy shift of the oscillator frequency and is given by

$$\begin{aligned}H_f =& \frac{|g_{\text{eff}}|^2}{\kappa_c^2}\Big\{\Big[(\frac{\alpha}{\beta}\sqrt{\frac{2\kappa_c c}{L}} + \Delta)^2 + \kappa_c^2\Big]\Big[\theta_1(-\omega_m) + \theta_1(\omega_m)\Big]\\ &- 2\kappa_c^2\Big[\theta_2(\omega_m) + \theta_2(-\omega_m)\Big]\Big\}f^\dagger f,\end{aligned} \tag{18}$$

with $\theta_1(\omega_m) = (\Delta+\omega_m)/[(\Delta+\omega_m)^2+\kappa_c^2]$, $\theta_2(\omega_m) = (2\Delta+\omega_m+\frac{\alpha}{\beta}\sqrt{\frac{2\kappa_c c}{L}})/[(\Delta+\omega_m)^2+\kappa_c^2]$. In general, when $\Theta(-\omega_m) = 0$, i.e. the detuning fulfills the relation

$$\Delta = \omega_m/2 - \frac{\alpha}{\beta}\sqrt{\frac{2\kappa_c c}{L}}/2, \tag{19}$$

which is just the optimal detuning for ground-state cooling of mechanical oscillator appeared in the dissipative optomechanics [26], and simultaneously the detuning δ fulfills the relation

$$\delta = \frac{|g_{\text{eff}}|^2}{\kappa_c^2}\frac{2\Delta(\Delta^2-\omega_m^2+\kappa_c^2) - 4\kappa_c^2\omega_m}{(\Delta+\omega_m)^2+\kappa_c^2} \tag{20}$$

to accommodate for the "optical spring effect" [25, 36] described in Eq. (18), the efficient transfer of squeezing can occur. The movable mirror is described by the master equation

$$\begin{aligned}\frac{d}{dt}\rho_f =& \frac{\gamma_{\text{opt}}}{2}|M|e^{i\varphi'}(2f\rho_f f - f^2\rho_f - \rho_f f^2)\\ &+ \frac{\gamma_{\text{opt}}}{2}|M|e^{-i\varphi'}(2f^\dagger\rho_f f^\dagger - f^{\dagger 2}\rho_f - \rho_f f^{\dagger 2})\\ &+ \frac{\gamma_{\text{opt}}}{2}N(2f^\dagger\rho_f f - ff^\dagger\rho_f - \rho_f ff^\dagger)\\ &+ \frac{\gamma_{\text{opt}}}{2}(N+1)(2f\rho_f f^\dagger - f^\dagger f\rho_f - \rho_f f^\dagger f),\end{aligned} \tag{21}$$

where optically-induced damping rate is

$$\gamma_{\text{opt}} = 2\frac{|g_{\text{eff}}|^2}{\kappa_c}\frac{4\omega_m^2}{(\Delta+\omega_m)^2+\kappa_c^2} \tag{22}$$

and $\varphi' = \arg\left\{\frac{4\omega_m^2\bar{a}^2}{[i(\Delta+\omega_m)+\kappa_c]^2}\right\} - \varphi$ is a new squeezing phase factor. It is obvious that the cavity field behaves like the squeezed vacuum reservoir for the movable mirror with the required frequencies of optical reservoir

$$\begin{aligned}\omega_R &= \omega_a + \omega_m/2 - \frac{\alpha}{\beta}\sqrt{\frac{2\kappa_c c}{L}}/2,\\ \omega_s &\approx \omega_R + \omega_m\end{aligned} \tag{23}$$

due to the negligibility of δ compared with ω_R, ω_m. Indeed, we can choose appropriate initial phase of the input squeezed vacuum light φ or coherent driving light $\bar{a}_{in}$ to make $\varphi' = 0$ for simplicity.

Now considering the experimental realizable parameters in Refs. [27] and [29], mechanical oscillator's effective mass is $m = 100$ng, frequency is $\omega_m = 2\pi\times 103$kHz, intrinsic damping rate is $\gamma_m = 2\pi\times 0.025$Hz, cavity's damping rate is $\kappa_c = 2\pi\times 196$kHz and the tunable dispersive optomechanical coupling for which $\alpha = 0$ can be also achieved. When the input power is 10mW, the corresponding effective dissipative cavity-mirror coupling strength in this experimentally realized FPI system reaches $2|g_{\text{eff}}| \approx 0.07\kappa_c$, which is well within the weak-coupling regime to validate the adiabatically eliminating approach for the cavity field.

B. Cooling of the movable mirror

The squeezed-state mechanical mirror has many applications under the conditions of ground-state cooling [37], therefore cooling down the mechanical oscillator is a vital step toward the practical implementation. In absence of optomechanical coupling the movable mirror is still coupled to a mechanical bath. The mirror is damped at the intrinsic rate γ_m which leads to a mean phonon number in thermal equilibrium n_{th}. In presence of the mechanical bath and optomechanical coupling, the total damping rate γ_{tot} becomes a sum of intrinsic damping rate γ_m and optically-induced damping rate γ_{opt}

$$\gamma_{\text{tot}} = \gamma_m + \gamma_{\text{opt}}, \tag{24}$$

and the steady-state mean phonon number becomes

$$n_{\text{st}} = (\gamma_m n_{\text{th}} + \gamma_{\text{opt}} N)/(\gamma_m + \gamma_{\text{opt}}). \tag{25}$$

In fact, for the particular case of no injection of squeezed vacuum noise into the cavity, i.e. $M = N = 0$, the final occupation number is $n_{\text{st}} = \gamma_m n_{\text{th}}/(\gamma_m + \gamma_{\text{opt}})$. In general, for high-Q mechanical oscillators and efficient laser cooling, it is feasible to take the relation $\gamma_m n_{\text{th}} \ll \gamma_{\text{opt}}$. For example, with the parameters shown in last section, the optically-induced damping rate for the movable mirror becomes $\gamma_{\text{opt}} = 2\pi \times 320$Hz, which is 4 orders of magnitude higher than the intrinsic damping rate γ_m. Thus it is possible to achieve ground-state cooling, which is also independent of the ratio κ_c/ω_m. These results coincide with those in Refs. [26] and [27], which are obtained with the use of the Heisenberg-Langevin approach. The cooling scheme can be physically explained as follows: via utilizing the completely destructive interference of quantum noise, the driven cavity effectively acts as a zero-temperature bath irrespective of the ratio κ_c/ω_m, leading the movable mirror to cool down to the ground state.

We can neglect the contribution of the phononic heat bath under the conditions of the small thermal heating rate compared with the optically-induced cooling rate. Then via feeding the squeezed vacuum noise into the cavity, the steady-state mean phonon number is $n_{\text{st}} = N = \sinh^2(r)$ calculated from Eq. (25), which coincides with the average input photon number of the squeezed reservoir. For example, for the squeezing parameter $r = 1$, the phonon number is $n_{\text{st}} = 1.38$. The movable mirror is still close to the ground state. In the following, we will show that it also offers the possibility to realize the efficient squeezing of the movable mirror transferred from the light field in the dissipative optomechanical system, which is outside the resolved-sideband limit arising from the destructive interference of quantum noise.

C. Squeezing of the movable mirror

In order to study the squeezing of the movable mirror, we need to evaluate the variances of the generalized quadrature operators

$$X = (f + f^\dagger)/\sqrt{2}, \quad Y = i(f^\dagger - f)/\sqrt{2}. \tag{26}$$

From the master equation in Eq. (21), after some calculations we obtain position and momentum fluctuations in a simple form by neglecting the thermal noise

$$\langle X^2 \rangle = N + \frac{1}{2} - |M| = \frac{1}{2}e^{-2r},$$
$$\langle Y^2 \rangle = N + \frac{1}{2} + |M| = \frac{1}{2}e^{2r}. \tag{27}$$

Obviously, the position squeezing of the movable mirror occurs and the mirror is in an ideal squeezed state. The squeezing factor of the movable mirror is r, equal to that of the input squeezed noise. It means that the squeezing is perfectly transferred from the light reservoir to the movable mirror in this dissipative optomechanical system. On the physical ground, the squeezing of the movable mirror is vulnerable to the heating processes, including the thermal bath and optically-induced heating. Thus the success in the elimination of optically-induced heating scattering and enhancement of cooling rate arising from the destructive interference of quantum noise guarantees the ideal squeezing transfer to the movable mirror from the squeezed light.

For the mechanism of transfer of squeezing from light to a membrane based on the resolved-sideband cooling scheme [25], in which is the cavity field and mirror is purely dispersively-coupled, ideal squeezed state is only possible under the conditions of the suppressed heating scattering well within the resolved-sideband limit. The squeezing for the mirror starts to degrade outside the resolved-sideband regime because the optically-induced heating process becomes to take into account, which influences the squeezing transfer. In contrast, in this dissipative optomechanics, the movable mirror is in the ideal squeezed state independent upon the ratio of κ_c/ω_m due to the perfect elimination of the optical-induced heating via utilizing destructive interference of quantum noise. The cavity field mimics an ideal squeezed vacuum environment for the movable mirror without requiring the cavity to be in so-called good cavity limit. Moreover, the cooling rate is not restricted by the low cavity decay rate, making the squeezed state be robust against the thermal noise. Therefore, the perfect squeezing of the movable mirror close to its ground state can be achieved in the non-resolved-sideband regime. These analytical results for the steady-state mean phonon number and squeezing will be numerically validated in the following.

IV. COOLING AND SQUEEZING THE MIRROR BEYOND THE WEAK-COUPLING REGIME

We have presented the perfect squeezing transfer from the squeezed vacuum light to the movable mirror as a result of interference in the weak-coupling limit in last section. To address whether the destructive noise interference effect persists beyond the simplest weak-coupling regime, we will focus on the purely dissipative optomechanics $\alpha = 0$ for simplicity, which is also achievable [27]. Since a broadband squeezed vacuum is assumed, the Markovian master equation for the cavity-mirror system obtained via adiabatically eliminating the squeezed vacuum reservoir variables in Eq. (4) is still valid [32]. To proceed, the classical components $\bar{a}$ and $\bar{b}$ are unchanged and now we need the full solutions for Eqs. (10)–(13).

We turn to calculate a close set of motional equations for second moments $\{\langle d^2\rangle, \langle d^{\dagger 2}\rangle, \langle d^\dagger d\rangle, \langle d(f+f^\dagger)\rangle, \langle d(f-f^\dagger)\rangle, \langle d^\dagger(f+f^\dagger)\rangle, \langle d^\dagger(f-f^\dagger), \langle (f+f^\dagger)^2\rangle, \langle (f-f^\dagger)(f+f^\dagger)\rangle, \langle (f-f^\dagger)^2\rangle\}$, from which we will obtain the steady-state mean phonon number and squeezing for the movable mirror. These motional equations are presented in the Appendix, and in there we obtain the explicit expression of steady-state mechanical occupation number in Eq. (A.7). For the moderately strong input driving fields and under the conditions of $\Delta = \omega_m/2$, we expand the result up to first order in the $|g_{\text{eff}}|^2/\kappa_c^2$, which becomes

$$n_{\text{st}} = N + (1+2N)|g_{\text{eff}}|^2/\kappa_c^2. \qquad (28)$$

The term proportional to $|g_{\text{eff}}|^2/\kappa_c^2$ corresponds to optically-induced heating for the movable mirror, which is resulted from the non elimination of photonic excitation as compared with the weak-coupling regime. It indicates that photonic excitation precludes the complete destructive interference of the quantum noise appeared in the weak-coupling regime and induces the extra thermal phonon excitation. When we replace the input squeezed reservoir by the vacuum field, i.e, $N=0$, the result coincides with the expression in Ref. [27] by neglecting the small intrinsic damping rate. Also, the optimal $\langle f^2\rangle$ is related to well-chosen Δ_s to accommodate for cavity-induced energy shift for movable mirror, and we can numerically find the appropriate detuning Δ_s around ω_m to obtain the optimum squeezing state for the mirror.

We numerically calculate the steady-state mean phonon number and the squeezing for the position operator with some effective coupling values between cavity mode and movable mirror characterized by $G_{\text{eff}} = 2|g_{\text{eff}}|$, and the numerical results are demonstrated in Fig. 2. The minimum phonon number and the optimal squeezing are achieved at $\Delta = \omega_m/2$, which coincides with result in Eq. (19). In special, in the weakly coupling regime, for example, $G_{\text{eff}}/\kappa_c = 0.1$ indicated by the red solid curve, the numerical results $n_{\text{st}} = 1.395$ and $\langle X^2\rangle = 0.08$ around $\Delta = \omega_m/2$, agree with the corresponding analytical results which are 1.38 and 0.068 obtained in Eqs. (25) and (27). Further, including the higher-order correction in Eq. (28), steady-state mean phonon number 1.39 is better agreement with the numerical result.

In addition, for the moderate coupling strength, the incomplete destructive quantum interference hinders the optimal cooling for the movable mirror because of the existence of higher-order optical-induced heating in $|g_{\text{eff}}|^2/\kappa_c^2$. Simultaneously, $|\langle f^2\rangle|$ can not be larger than M. The resulting relation

$$\sqrt{n_{\text{st}}(n_{\text{st}}+1)} > |\langle f^2\rangle| \qquad (29)$$

is fulfilled, which means that the mirror deviates from the ideal squeezed state with the increased coupling strength. However, the squeezed state for the movable mirror can still occur beyond the weak-coupling regime numerically indicated in Fig. 2, in which the curves demonstrate the ability of

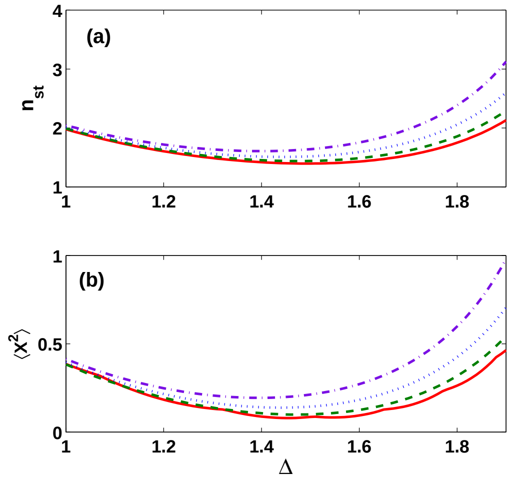

FIG. 2. (Color online) The steady-state mean phonon number n_{st} in (a) and the position squeezing $\langle X^2\rangle$ in (b) as functions of the detuning Δ with the different effective coupling strengths between the cavity field and movable mirror, with the parameters (in units of κ_c) $\omega_m = 3\kappa_c$, $\Delta_s \approx \omega_m$, $r=1$, and $\alpha = 0$. The effective coupling strengths are taken as G_{eff}/κ_c=0.1 (red solid line), 0.2 (green dashed line), 0.3 (blue dotted line), 0.4 (purple dash-dotted line) respectively and the optimal squeezing is obtained via carefully tuning Δ_s around ω_m because of the cavity-induced energy shift.

dissipative optomechanical system in producing the squeezing for the position operator around its motional ground state. When $\alpha \neq 0$, the general form of motion equations presented in Eq. (A.2) is unchanged from the Liouvillian operator in Eq. (13). One can discuss the squeezing property of the mirror following the same procedure and the main results should not be changed.

V. CONCLUSION

In conclusion, we present an optomechanical system consisted of an effective FPI with one movable ideal end mirror, which is capable of generating mechanical squeezing via engineering reservoir. Via feeding a broadband squeezed vacuum light accompanying a coherent driving laser field into the cavity, the cavity field is coupled to the movable mirror through both the tunable dispersive and dissipative interactions. The motion equation for the cavity-mirror system is derived by following the general reservoir theory in which the reservoir variables are adiabatically eliminated. When the mirror is weakly coupled to the cavity mode, the driven cavity can effectively perform as a squeezed vacuum reservoir via utilizing the complete destructive interference of quantum noise. Thus, the perfect transfer of squeezing from the light to the movable mirror occurs, which is irrespective of the ratio between the cavity damping rate and the mechanical frequency. When the mirror is coupled to the cavity field beyond the weak-coupling regime, the photonic excitation can preclude the complete destructive interference of quantum noise, leading to the mirror deviation from the ideal squeezed state. However, in the dissipative optomechanics the squeezed state of the mirror can still be produced for the moderate coupling strength.

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Appendix

We derive the motional equations for the second-order moments of the cavity and the mirror variables from the Eqs. (10)–(13), which are written into a vector form as

$$\vec{X} = \Big(\langle d^2\rangle, \langle d^{\dagger 2}\rangle, \langle d^\dagger d\rangle, \langle df_+\rangle, \langle df_-\rangle, \langle d^\dagger f_+\rangle, \langle d^\dagger f_-\rangle, \langle f_+^2\rangle, \langle f_- f_+\rangle, \langle f_-^2\rangle\Big)^T, \tag{A.1}$$

where $f_+ = f + f^\dagger$, $f_- = f - f^\dagger$ and T denotes the transpose of the vector. The second moments obey the equation

$$\frac{d}{dt}\vec{X} = \underline{A}\vec{X} + B^{(+)}e^{i2\Delta_s t} + B^{(-)}e^{-i2\Delta_s t} + B^{(0)}. \tag{A.2}$$

The coefficient matrix $\underline{A}$ is

$$\underline{A} = \begin{pmatrix} 2(i\Delta-\kappa_c) & 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2(i\Delta+\kappa_c) & 0 & 0 & 0 & \xi^* & 0 & 0 & 0 & 0 \\ 0 & 0 & -2\kappa_c & \xi^*/2 & 0 & \xi/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i\Delta-\kappa_c & -i\omega_m & 0 & 0 & \xi/2 & 0 & 0 \\ \chi^* & 0 & -\chi & -i\omega_m & i\Delta-\kappa_c & 0 & 0 & 0 & \xi/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -(i\Delta+\kappa_c) & -i\omega_m & \xi^*/2 & 0 & 0 \\ 0 & -\chi & \chi^* & 0 & 0 & -i\omega_m & -(i\Delta+\kappa_c) & 0 & \xi^*/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i2\omega_m & 0 \\ 0 & 0 & 0 & \chi^* & 0 & -\chi & 0 & -i\omega_m & 0 & -i\omega_m \\ 0 & 0 & 0 & 0 & 2\chi^* & 0 & -2\chi & 0 & -i2\omega_m & 0 \end{pmatrix}, \tag{A.3}$$

with $\chi = 2g_0\beta\sqrt{\frac{L}{c}}\bar{a}_{in}, \zeta = 4g_{\text{eff}}, \xi = -\chi - \zeta$, and the non-homogeneous terms are

$$\begin{aligned} B^{(+)} =& \big(0,\ 2\kappa_c M^*,\ 0,\ 0,\ 0,\ 0,\ -\zeta M^*,\ 0,\ 0,\ \frac{\zeta^2 M^*}{2\kappa_c}\big)^T, \\ B^{(-)} =& \big(2\kappa_c M,\ 0,\ 0,\ 0,\ \zeta^* M,\ 0,\ 0,\ 0,\ 0,\ \frac{\zeta^{*2} M}{2\kappa_c}\big)^T, \\ B^{(0)} =& \big(0,\ 0,\ 2\kappa_c N,\ 0,\ -\zeta N,\ 0,\ \chi^* + \zeta^*(N+1),\ i2\omega_m, \\ & 0,\ i2\omega_m - (2N+1)\frac{|\zeta|^2}{2\kappa_c}\big)^T. \end{aligned} \tag{A.4}$$

We expand the time-dependent $\vec{X}$ into a sum of the slowly varying components composed of $\vec{X}^{(0)}, \vec{X}^{(+)}, \vec{X}^{(-)}$ with the harmonic oscillating frequencies 0, $2\Delta_s$, $-2\Delta_s$,

$$\vec{X} = \vec{X}^{(0)} + \vec{X}^{(+)}e^{i2\Delta_s t} + \vec{X}^{(-)}e^{-i2\Delta_s t}. \tag{A.5}$$

Thus the steady-state solutions for the Eq. (A.2) are given as

$$\begin{aligned} \vec{X}^{(0)} &= -\underline{A}^{-1}B^{(0)}, \\ \vec{X}^{(+)} &= (i2\Delta_s - \underline{A})^{-1}B^{(+)}, \\ \vec{X}^{(-)} &= -(i2\Delta_s + \underline{A})^{-1}B^{(-)}, \end{aligned} \tag{A.6}$$

from which we obtain the mean phonon number $\langle f^\dagger f\rangle$ and $\langle f^2\rangle$. The explicit expression of steady-state mean phonon number $\langle f^\dagger f\rangle$ takes the form

$$\begin{aligned} \langle f^\dagger f\rangle =& N + \frac{1+2N}{4} \\ & \times \Big\{\frac{\omega_m(\Delta^2-\kappa_c^2)(\omega_m-2\Delta) - \Delta^2(\Delta^2+\kappa_c^2)}{\omega_m\Delta(\Delta^2-\kappa_c^2)} \\ & + \frac{(\Delta^2+\kappa_c^2)^2}{\Delta^2-\kappa_c^2}\Big[\frac{-\Delta(\Delta^2+\kappa_c^2)}{\chi^2\Delta(\Delta^2-3\kappa_c^2)+\omega_m(\Delta^2+\kappa_c^2)^2} \\ & + \frac{\chi^2\Delta/2 - (\Delta^2+\kappa_c^2)\omega_m}{\Delta(\Delta^2+\kappa_c^2)(2\Delta^2-2\kappa_c^2-\omega_m^2)+\chi^2\Delta^2\omega_m}\Big]\Big\}. \end{aligned} \tag{A.7}$$

Under the conditions of the optimal detuning $\Delta = \omega_m/2$, we expand $\langle f^\dagger f\rangle$ up to the order in $|g_{\text{eff}}|^2/\kappa_c^2$ and obtain

$$\langle f^\dagger f\rangle = N + (1+2N)|g_{\text{eff}}|^2/\kappa_c^2. \tag{A.8}$$

Moreover, the optimal $\langle f^2\rangle$ is related to the well-chosen detuning Δ_s to accommodate for the cavity-induced energy shift of oscillating frequency, and here we can numerically find out the Δ_s around ω_m to obtain the optimum squeezing, which is shown in Fig. 2.

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