Adiabatic evolution of light in an array of parallel curved optical waveguides

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Adiabatic evolution of light in parallel curved optical waveguide array

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Adiabatic evolution of light in parallel curved optical waveguide array is investigated theoretically. This problem is shown to bear a close connection with the process of coherent population transfer in a "bow-tie" model in quantum physics. Under certain conditions on the geometry of the waveguides and the optical properties of the system complete light transfer between the outer waveguides is achieved. A special attention is paid to the case of three waveguides, which is analysed using the solutions of the well-known "bow-tie" model. The analytic solution is used to design recipes for creating arbitrary superpositions of light intensity between the waveguides, with possible applications in achromatic optical multiple-beam splitters. For more than three waveguides complete light transfer between the outer waveguides and beam splitting is demonstrated numerically.

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I. INTRODUCTION

The analogies between wave optics and quantum mechanics were made since the dawn of quantum mechanics in the pioneering works of de Broglie [1] and Schrödinger [2]. The wavefunction itself is named by the analogy with wave optics. In the past decade analogies have been going in the opposite direction: some of the very well-known techniques from coherent quantum control of atoms and molecules found analogues in the realm of optical physics. Examples include Rabi oscillations [3], Landau-Zener tunnelling [4–6] and stimulated Raman adiabatic passage (STIRAP) [7–10]. The number of quantum-optical analogies appearing in the literature is still growing rapidly, as described recently in a comprehensive review with a special focus at the use of waveguide structures [11].

In this paper we propose an experiment for light transfer between the two outer waveguides of an waveguide array by using the ideas of adiabatic population transfer in a multistate quantum system with crossing energies. This technique promises to be both efficient and robust against variations of the parameters, such as the transitory curvature of the WGs and the couplings between them; therefore, the technique is expected to be achromatic. In addition to the complete light transfer between the two outer WGs in the array the technique allows to create arbitrary superpositions of light intensity between the WGs; therefore, the scheme can serve as an achromatic optical multiple beam splitter. In contrast to the previous achromatic adiabatic multiple beam splitter, which uses an analogue of STIRAP and which is unidirectional [12, 13], here the splitting device works in forward and backward directions of light propagation equally well.

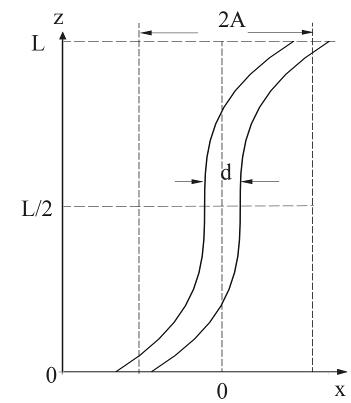

FIG. 1: Schematic diagram of a directional coupler for the observation of LZ dynamics. The coupler is made of two equal optical WGs with a cubically bent axis separated by a distance d.

II. PARALLEL CURVED WG ARRAY

An optical realization of Landau-Zener-Stückelberg-Majorana (LZSM) tunneling [14] in WG couplers with a cubically bent axis was proposed by Longhi [5] (see Fig. 1). The propagation of the amplitudes $a_1(z)$ and $a_2(z)$ of light waves trapped in the two WGs in the scalar and paraxial assumptions for the electromagnetic field is described by a system of two coupled differential equations written in a matrix form as

$$i\frac{d}{dz}\begin{bmatrix} a_1(z) \\ a_2(z) \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2kz & \beta_{12} \\ \beta_{12} & -2kz \end{bmatrix}\begin{bmatrix} a_1(z) \\ a_2(z) \end{bmatrix}, \quad (1)$$

with

$$k \simeq \frac{48\pi d A n_s}{\lambda L^3}, \quad \beta_{12} = \beta_2 - \beta_1.$$

Here d is the distance between the two WGs of the coupler, $2A$ is the maximum lateral shift of the WG axis

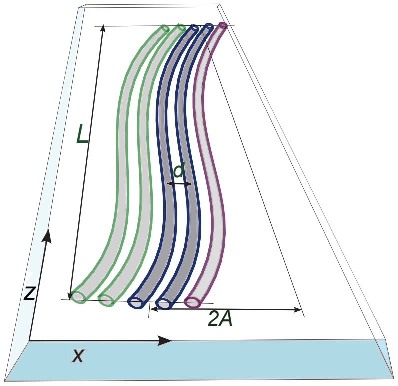

FIG. 2: (Colour online) Schematic diagram of an array made of N parallel curved identical WGs separated by a distance d.

from the input ($z = -L/2$) and output ($z = L/2$) WG planes, n_s is the refractive index of the substrate, λ is the wavelength, L is the coupler length and β_j ($j = 1, 2$) are the propagation constants. The absolute squares of the amplitudes $a_1(z)$ and $a_2(z)$ are the dimensionless light intensities in the WGs, normalized to the total input light intensity: $I_1(z) = |a_1(z)|^2$ and $I_2(z) = |a_2(z)|^2$. Obviously, $I_1(z) + I_2(z) = 1$ in the lossless case. This realization was experimentally demonstrated by Dreisow *et al.* [6] and the results were in good agreement with the theoretical LZSM model for a linear crossing of energy levels with a constant coupling of finite duration.

Here we generalize this model to an array of N parallel curved WGs. We consider propagation of a monochromatic wave with a wavelength $\lambda = 2\pi/k$ in a WG array of length L made of N identical single-mode WGs separated by a distance d in the transverse x direction. The propagation axis of the array is assumed to be weakly curved along the paraxial propagation direction z, as seen in Fig. 2. Then the equation of light evolution reads

$$i\frac{d}{dz}\mathbf{a}(z) = \mathbf{H}_N\mathbf{a}(z), \qquad (2)$$

with $\mathbf{a}(z) = [a_1(z), a_2(z), \dots, a_N(z)]^T$, where $a_j(t)$ is the amplitude of the wave trapped in j-th WG, and $I_j(z) = |a_j(z)|^2$ is the corresponding light intensity. The "Hamiltonian" in the scalar and paraxial electric-field approximations, and with the assumption of nearest-neighbor tight binding, has the three-diagonal form

$$\mathbf{H}_N = \frac{1}{2}\begin{bmatrix} \Delta_1 & \beta_{12} & & \cdots & 0 \\ \beta_{21} & \Delta_2 & & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & & \Delta_{N-1} & \beta_{N-1,N} \\ 0 & \cdots & & \beta_{N,N-1} & \Delta_N \end{bmatrix}, \qquad (3)$$

with $\Delta_j = (N + 1 - 2j)kz$. Hamiltonians of this type are well known and well studied in quantum optics, where they describe coherently driven chainwise connected quantum system of discrete energy states [17, 18]. We use this analogy below to describe several possible achromatic devices for light transfer and multiple beam splitting in WG arrays.

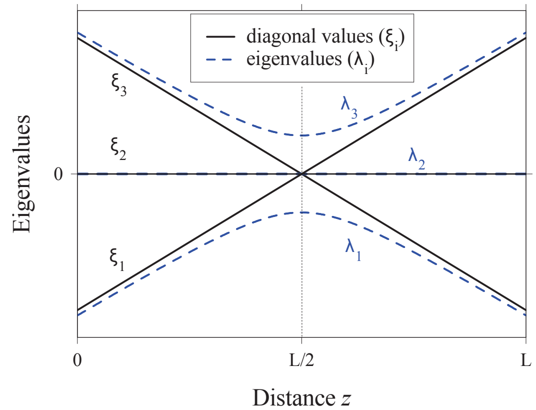

FIG. 3: (Colour online) "Bow-tie" crossing: diagonal values (ξ_i) and eigenvalues (λ_i) vs z for the "Hamiltonian" $\mathbf{H}_3$ of a three-WG system.

TABLE I: Light intensities I_n ($n = 1, 2, 3$) after the three-WG coupler driven by the "Hamiltonian" of Eq. (4) in the limit $L^2 \gg 1/k$ for different initial conditions (I_1^0, I_2^0, I_3^0). Here $p = \exp(-\pi\beta_{12}^2/4k)$ and $q = \exp(-\pi\beta_{23}^2/4k)$.

(I_1^0, I_2^0, I_3^0)	I_1	I_2	I_3
(1,0,0)	p^2	$(1-p)(p+q)$	$(1-p)(1-q)$
(0,1,0)	$(1-p)(p+q)$	$(1-p-q)^2$	$(1-q)(p+q)$
(0,0,1)	$(1-p)(1-q)$	$(1-q)(p+q)$	q^2

III. THREE-WAVEGUIDE ARRAY

We begin with an array of three coupled optical WGs. The evolution of light propagating in this array is described by Eq. (2) where the "Hamiltonian" reads

$$\mathbf{H}_3 = \frac{1}{2}\begin{bmatrix} 2kz & \beta_{12} & 0 \\ \beta_{12} & 0 & \beta_{23} \\ 0 & \beta_{23} & -2kz \end{bmatrix}. \qquad (4)$$

The diagonal values ξ_i and the eigenvalues λ_i of $\mathbf{H}_3$ are shown in Fig. 3. We assume, without loss of generality, that $k > 0$. This "Hamiltonian" is exactly the same as the one for a three-level quantum system (with the substitution $z \to t$), with a "bow-tie" energy diagram. This problem has been solved analytically by Carroll and Hioe [15] in the case when the couplings are constant and the WG length is large, $L^2 \gg 1/k$. The Carroll-Hioe solution is summarized in Table I; note that p and q can take any real value between 0 and 1. This analytic solution allows us to readily derive the conditions for complete light transfer between the outer WGs and for beam splitting at arbitrary ratios.

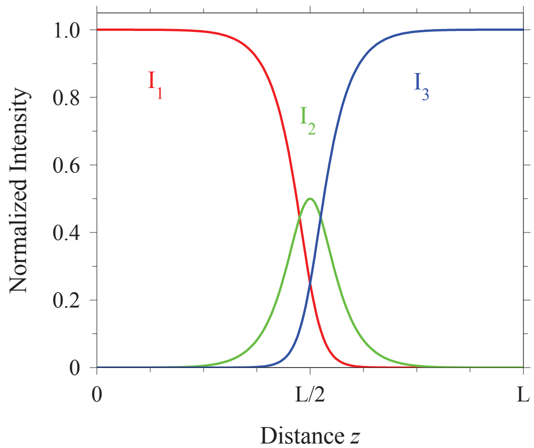

FIG. 4: (Colour online) Adiabatic light transfer $1 \to 3$ between WGs 1 and 3 in a three-WG array. We assume Gaussian-shaped couplings, $\beta_{12}(z) = \beta_{23}(z) = \beta_0 \exp[-(z - L/2)^2/\zeta^2]$, with $\beta_0 = 50/\zeta$, $k = 250/\zeta^2$, $L = 4\zeta$. Here ζ is used as the unit of length and $1/\zeta$ as the unit of frequency.

A. Complete light transfer $1 \to 3$

An important special case is the complete light transfer $1 \to 3$. The transition probability $1 \to 3$, according to Table I is

$$\begin{aligned} P_{1\to3} &= (1-p)(1-q) \\ &= [1-\exp(-\pi\beta_{12}^2/4k)][1-\exp(-\pi\beta_{23}^2/4k)]. \end{aligned} \quad (5)$$

We conclude that complete light transfer from WG 1 to WG 3, $I_3 \to 1$, takes place when $p \to 0$ and $q \to 0$. These conditions, which require

$$\beta_{12}^2 \gg k, \qquad \beta_{23}^2 \gg k, \quad (6)$$

imply adiabatic evolution [16]. From here we conclude that the light transfer in this three-WG array is highly achromatic (i.e., independent of k) in the frequency range in which the adiabatic condition (6) is fulfilled.

It is important to note that, as Table I shows, the transition probabilities $P_{1\to3}$ and $P_{3\to1}$ are equal. The implication is that this WG device produces two-way light transfer, i.e., complete light transfer occurs in both directions. This important feature, which derives from the level crossing nature of the device, makes it distinctly different from STIRAP-based devices [7–10], which are uni-directional.

We note that these findings are not limited to the Carroll-Hioe model, which assumes constant couplings and constant parameter k. It is only necessary that the adiabatic evolution conditions are satisfied, which in the general case require that at each crossing the square of the coupling between the corresponding WGs is far greater than the derivative of the difference between the diagonal elements of the "Hamiltonian".

An example of adiabatic light transfer $1 \to 3$ between WGs 1 and 3 is shown in Fig. 4. The light flows smoothly from WG1 to WG3, while some of it resides temporarily in the middle WG2.

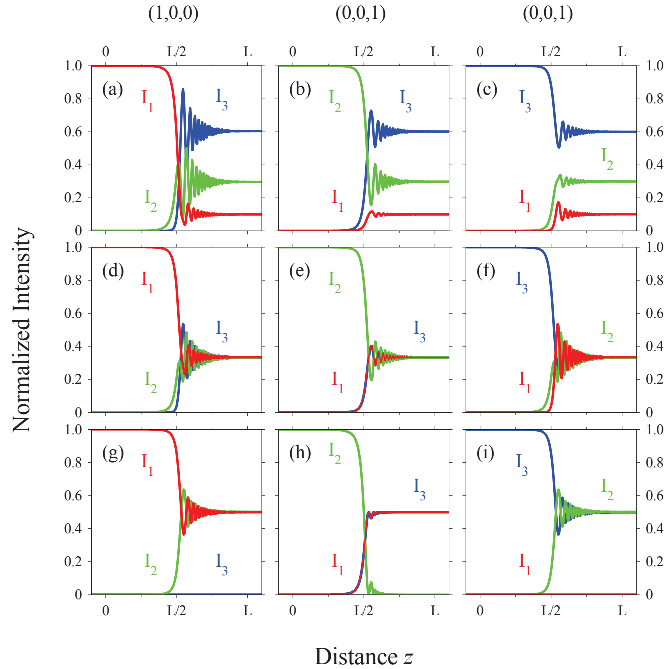

FIG. 5: (Colour online) Operation of a three-WG variable beam splitter for different initial conditions (I_1^0, I_2^0, I_3^0): $(1,0,0)$ (left column), $(0,1,0)$ (middle column), $(0,0,1)$ (right column). The top row demonstrates beam splitter with intensity ratio $\frac{1}{10} : \frac{3}{10} : \frac{6}{10}$, the middle row with ratio $\frac{1}{3} : \frac{1}{3} : \frac{1}{3}$, and the bottom row with ratio $\frac{1}{2} : 0 : \frac{1}{2}$. The parameter k is taken to be $k = 10/\zeta^2$ in all frames and $L = 4\zeta$. The couplings are assumed to have Gaussian space dependence, $\exp[-(z - L/2)^2/\zeta^2]$, with maximum values at $L/2$ as listed in the table below.

(a) $\beta_{12} = 4.786/\zeta$ $\beta_{23} = 6.463/\zeta$	(b) $\beta_{12} = 1.153/\zeta$ $\beta_{23} = 3.124/\zeta$	(c) $\beta_{12} = 3.415/\zeta$ $\beta_{23} = 2.254/\zeta$
(d) $\beta_{12} = 3.306/\zeta$ $\beta_{23} = 5.561/\zeta$	(e) $\beta_{12} = 2.173/\zeta$ $\beta_{23} = 2.173/\zeta$	(f) $\beta_{12} = 5.561/\zeta$ $\beta_{23} = 3.306/\zeta$
(g) $\beta_{12} = 2.626/\zeta$ $\beta_{23} = 0$	(h) $\beta_{12} = 3.713/\zeta$ $\beta_{23} = 3.713/\zeta$	(i) $\beta_{12} = 0$ $\beta_{23} = 2.626/\zeta$

B. Beam splitting

A potentially very important application of the three-WG array is to be used as an optical beam splitter. The Carroll-Hioe solution in Table I allows us to readily find the necessary WG parameters for beam splitting with variable light intensity ratios. According to the Carroll-Hioe solution, light propagating in WG 1 will be split in three equal parts by the WG array, $(1,0,0) \to (1/3, 1/3, 1/3)$, if the couplings satisfy the relations

$$p = \frac{1}{\sqrt{3}}, \qquad q = \frac{3-\sqrt{3}}{6}. \quad (7)$$

These values of p and q can be produced by choosing the values of the WG parameters β_{12}, β_{23} and k appropriately. Similar beam splitting is achieved for light arriving in WG 3, with the exchange of the values of p and q.

If the light arrives in WG 2 it will be split in three equal parts by the WG array, $(0,1,0) \to (1/3,1/3,1/3)$, if the couplings obey

$$p = q = \frac{3-\sqrt{3}}{6} \quad \text{or} \quad p = q = \frac{3+\sqrt{3}}{6}. \tag{8}$$

It is also possible to split light arriving in WG 2 into two equal parts, $(0,1,0) \to (1/2,0,1/2)$; this requires

$$p = q = \frac{1}{2}. \tag{9}$$

Examples of operation of three-WG variable beam splitters is illustrated in Fig. 5. We have determined the values of the couplings β_{12} and β_{23} from the Carroll-Hioe model, although we have used Gaussian-shaped couplings in the simulations. These values are seen to produce the desired beam splitting ratios very accurately.

IV. MULTIPLE-WG ARRAY ($N > 3$)

For an array of arbitrarily many WGs N, described by the "Hamiltonian" (3), an exact analytic solution is not known yet. Nevertheless, it follows from general arguments that complete light transfer $1 \to N$ is always possible in the adiabatic limit. The reason is that two of the adiabatic states — with the lowest and the largest eigenenergies of $\mathbf{H}_N$ — reduce asymptotically to states 1 and N in the beginning and the end. For $k > 0$, the lowest eigenenergy $\lambda_1(z)$ and the highest eigenenergy $\lambda_N(z)$ have the asymptotics

$$|1\rangle \leftarrow \ |\lambda_1(z)\rangle \ \to |N\rangle, \tag{10a}$$
$$|N\rangle \leftarrow \ |\lambda_N(z)\rangle \ \to |1\rangle. \tag{10b}$$

For $k < 0$ similar relations apply, with states $|\lambda_1(z)\rangle$ and $|\lambda_N(z)\rangle$ exchanging their places.

In order to estimate the conditions for adiabatic propagation of light, we consider an array of four WGs and, for the sake of simplicity, we assume that all couplings are equal: $\beta_{12} = \beta_{23} = \beta_{34} \equiv \beta$. The "Hamiltonian" now reads

$$\mathbf{H}_4 = \frac{1}{2}\begin{bmatrix} 3kz & \beta & 0 & 0 \\ \beta & kz & \beta & 0 \\ 0 & \beta & -kz & \beta \\ 0 & 0 & \beta & -3kz \end{bmatrix}. \tag{11}$$

The diagonal values ξ_i and the eigenvalues λ_i of this matrix are shown in Fig. 6. If light comes in WG1, and if the propagation is nearly adiabatic then the light will follow predominantly the lowest eigenvalue $\lambda_1(z)$, which associates with the WG 1 (and ξ_1) initially and WG4 (respectively ξ_4) in the end. The adiabatic condition reads

$$[\lambda_2(0) - \lambda_1(0)]^2 \gg 3k, \tag{12}$$

and for the case of equal couplings it is

$$\beta^2 \gg 12k. \tag{13}$$

Examples of complete adiabatic light transfer between

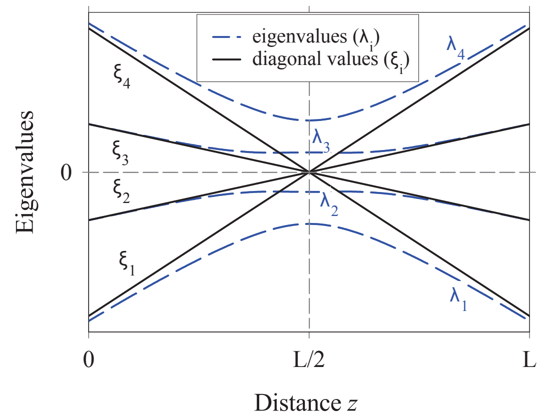

FIG. 6: (Colour online) The diagonal values (ξ_i) and the eigenvalues (λ_i) of the "Hamiltonian" (11) vs z for a four-WG array.

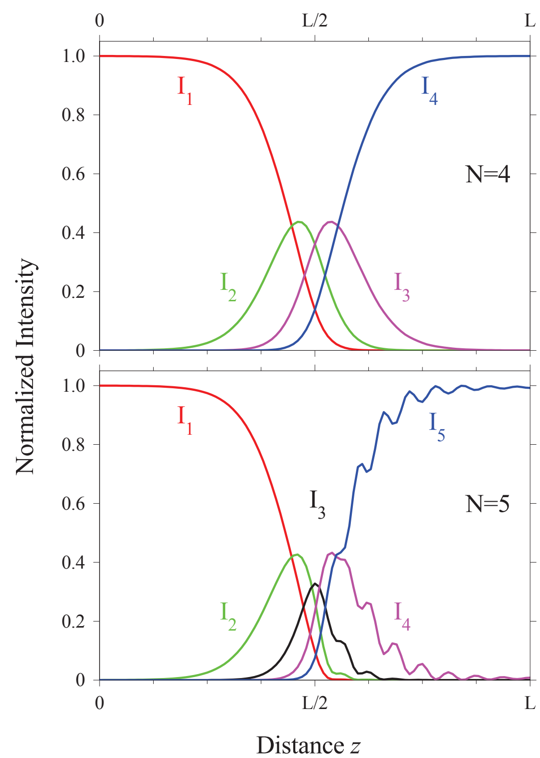

FIG. 7: (Colour online) Adiabatic light transfer $1 \to N$ between WGs 1 and N in an array of N=4 WGs (top) and $N = 5$ WGs (bottom). All couplings $\beta_{j,j+1}$ are equal to β_0 sech(z/ζ) with $\beta_0 = 10/\zeta$, $k = 10/\zeta^2$ and $L = 10\zeta$.

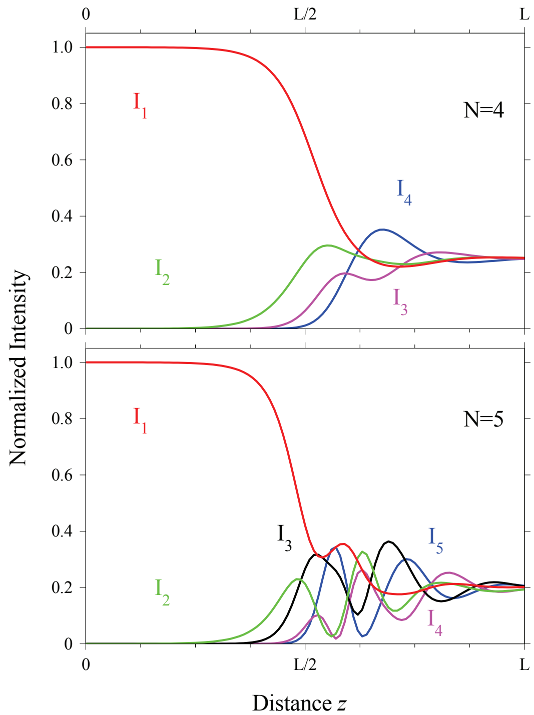

FIG. 8: (Colour online) Operation of a beam splitter with intensity ratio $\frac{1}{N}$: $\frac{1}{N}$: $\cdots$: $\frac{1}{N}$ for initial conditions $(I_1^0, I_2^0, ...I_N^0)$: $(1, 0, 0, ...0)$ for 4 (top frame) and 5 WGs (bottom frame). All couplings $\beta_{j,j+1}$ have sech spatial dependence $\mathrm{sech}(z/\zeta)$ but different maximum magnitudes (listed below) at $L/2$, and $L = 10\zeta$. The WG parameters: (top) $N = 4$, $\beta_{12} = 0.6/\zeta$, $\beta_{23} = 1.02/\zeta$, $\beta_{34} = 1.21/\zeta$, $k = 1.5/\zeta^2$; (bottom) $N = 5$, $\beta_{12} = 1.012/\zeta$, $\beta_{23} = 2.993/\zeta$, $\beta_{34} = 1.777/\zeta$, $\beta_{45} = 4.100/\zeta$, $k = 2.625/\zeta^2$.

the outermost WGs are shown in Fig. 7 for arrays of 4 (top) and 5 (bottom) WGs.

Because an analytic solution to the bow-tie chain model is not known the performance of the multiple-WG device as a beam splitter can be investigated only numerically. We have found that a proper choice of the couplings can make the device act as a multiple beam splitter. We demonstrate splitting of the light intensity in equal parts for $N = 4$ and 5 WGs in Fig. 8.

V. CONCLUSIONS

We have introduced a method to create multiple optical beam splitting and complete light transfer in an array of multiple WGs by using ideas from the well studied dynamics of multistate quantum systems in the presence of "bow-tie" level crossings. This light transfer devices use adiabatic passage of light and hence they are expected to be robust against variations of the light wavelength, the WGs couplings and the WGs geometry. The operation of the devices as multiple beam splitters requires careful tuning of the coupling between the WGs. For an array of three WGs, we have used the exact Carroll-Hioe "bow-tie" model to analytically determine the parameters needed to construct a variable three-beam splitter. In WG arrays with more than three WGs multiple beam splitting is demonstrated numerically for four and five WGs.

Acknowledgments

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[1] L. de Broglie, Ondes et quanta, Compt. Rend. **177**, 507 (1923).

[2] E. Schrödinger, Naturwissenchaften **23**, 807 (1935).

[3] S. Longhi, Phys. Rev. A **71**, 065801 (2005).

[4] R. Khomeriki and S. Ruffo, Phys. Rev. Lett. **94**, 113904 (2005).

[5] S. Longhi, J. Opt. B, Quantum Semiclassical Opt. **7**, L9 (2005).

[6] F. Dreisow, A. Szameit, M. Heinrich, S. Nolte, A. Tünnermann, M. Ornigotti and S. Longhi, Phys. Rev. A **79**, 055802 (2009).

[7] E. Paspalakis, Opt. Commun. **258**, 31 (2006).

[8] S. Longhi, Phys. Rev. E **73**, 026607 (2006).

[9] S. Longhi, G. DellaValle, M. Ornigotti, and P. Laporta, Phys. Rev. B **76**, 201101(R) (2007).

[10] Y. Lahini, F. Pozzi, M. Sorel, R. Morandotti, D.N. Christodoulides, and Y. Silberberg, Phys. Rev. Lett. **101**, 193901 (2008).

[11] S. Longhi, Laser & Photon. Rev. **3**, 243 (2009).

[12] C. Ciret, V. Coda, A. A. Rangelov, D. N. Neshev, and G. Montemezzani, Opt. Lett. **30**, 634 (2005).

[13] A. A. Rangelov and N. V. Vitanov, Phys. Rev. A **85**, 055803 (2012).

[14] L. D. Landau, Physik Z. Sowjetunion **2**, 46 (1932); C. Zener, Proc. R. Soc. Lond. Ser. A **137**, 696 (1932); E. C. G. Stückelberg, Helv. Phys. Acta **5**, 369 (1932); E. Majorana, Nuovo Cimento **9**, 43 (1932).

[15] C. E. Carroll and T. F. Hioe, J. Phys. A **19**, 1151 (1986); V. N. Ostrovsky and H. Nakamura, J. Phys. A **30**, 6939 (1997).

[16] N. V. Vitanov, T. Halfmann, B. W. Shore, and K. Bergmann, Annu. Rev. Phys. Chem. **52**, 763 (2001).

[17] S. Guèrin, Phys. Rev. A **56**, 1458 (1997).

[18] S. Thomas, S. Guèrin, and H. R. Jauslin, Phys. Rev. A **71**, 013402 (2005).