# Excitations of a trapped two-component Bose-Einstein condensate

Christopher Ticknor

Phys. Rev. A **88**, 013623 — Published 15 July 2013

DOI: 10.1103/PhysRevA.88.013623

# Excitations of a trapped two component Bose Einstein Condensate

Christopher Ticknor
*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA and*
*Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA*

We present analysis of the excitation spectrum for a 2 component quasi2D Bose Einstein Condensate. We study how excitations change character across the miscible to immiscible phase transition. We find that the bulk excitations are typical of a single-component BEC with the addition of interface bending excitations. We study how these excitations change as a function of the interaction strength.

PACS numbers: 03.75Hh,67.85-d

## I. INTRODUCTION

Ultracold atoms provide an excellent forum to study complex quantum mechanical behavior. An example is the superfluid to Mott insulator transition [1], where experimental efforts have imaged this as a quantum phase transition at the single atom level [2]. A quantum phase transition is a fundamental change in the ground state as a parameter is altered, in the superfluid-Mott insulator example the parameter is lattice depth. Near this quantum phase transition the temperature dependence of superfluidity and quantum criticality has been studied [3, 4].

Another system in ultracold atoms, which exhibits a quantum phase transition, is the 2 component Bose Einstein Condensate (BEC), where by tuning the interactions between the components the gas can change from a miscible to an immiscible phase, as has been experimentally demonstrated in Rb [5]. Theoretical studies showed that if $g_{11}g_{22} - g_{12}^2$ is greater (less) than zero, the gas is miscible (immiscible) [6, 7]. Here $g_{ij}$ is the coupling strength in the mean field treatment between the $i^{th}$ and $j^{th}$ component. Interestingly, 2 component BECs have been used to create vortices [8] and study non-equilibrium dynamics [9–11].

In this work, we look at the character of the excitations across the quantum phase transition in the 2 component BEC system. For a miscible system, the modes are collective in nature, and the condensates move either in-phase or out-of-phase with each other. For the immiscible system, the excitations are either collective or interface excitations. This work offers a new perspective into the nature the miscible-immiscible transition with characterization of the excitations.

There has been previous work on excitation spectra for trapped 2 component BEC [12]. That work studied symmetry breaking as a function of particle number and found that a mode goes to zero when the system becomes immiscible, but they did not study the nature of the quasiparticles. Ref. [13] studied the ground state and characterized the mode that goes soft (energy goes to zero), here we extend the analysis to many low-lying modes of the system.

## II. METHODS

To obtain the excitation spectrum, we solve the Bogoliubov de Gennes equations [6, 7] for a trapped gas with contact interactions. First, we must solve the Gross Pitaevskii equation for the 2 condensates ($\phi_i$):

$$H_{GP}\phi_i = \left( H_0 + \sum_j g_{ij}|\phi_j|^2 \right) \phi_i = \mu_i \phi_i. \tag{1}$$

$H_0$ is the kinetic energy and trapping potential and $\mu_i$ is the chemical potential for the $i^{th}$ component. We normalize $\phi_i$ so that $\int dV|\phi_i|^2$=1. Now we consider the ground state and its excitations to be of the form: $e^{-i\mu_i t/\hbar}\{\phi_i + \lambda(u_i e^{-i\omega t} + v_i^* e^{i\omega t})\}$. Substituting this into the time dependent version of Eq. (1) and collecting powers of $e^{\pm i\omega t}$ and linear terms in $\lambda$, we find the excitations are given by:

$$\begin{pmatrix} H_{GP}-\mu_i & 0 \\ 0 & H_{GP}-\mu_i \end{pmatrix} \begin{pmatrix} u_i^\alpha \\ v_i^\alpha \end{pmatrix} + \sum_j \begin{pmatrix} g_{ij}\phi_i\phi_j & g_{ij}\phi_i\phi_j \\ g_{ij}\phi_i\phi_j & g_{ij}\phi_i\phi_j \end{pmatrix} \begin{pmatrix} u_j^\alpha \\ v_j^\alpha \end{pmatrix} = \omega_\alpha \begin{pmatrix} u_i^\alpha \\ -v_i^\alpha \end{pmatrix}. \tag{2}$$

We have assumed $\phi_i$ is real, and that the different components have equal number and mass. The second term contains both the exchange and anomalous term, which couples $u_i^\alpha$ to $u_j^\alpha$ and $u_i^\alpha$ to $v_j^\alpha$, respectively. These terms would be non-local if the interaction was finite range. These excitations are normalized in the standard way: $\int dV|u^\alpha|^2$-$|v^\alpha|^2 = 1$ [14].

To perform this work, we focus on the quasi2D case where high resolution experimental imaging is possible [2, 15]. For clarity, we will focus on two examples: one is miscible and the other immiscible, both far away from the transition so the character of the excitations is clear. We consider $g_{11} = g$ =100 and $g_{22}$=1.01g; for the miscible example, we pick $g_{12}$=0.5g and an immiscible example, we pick $g_{12}$=2g, where $g = N\sqrt{8\pi}\hbar^2 a_s/ml_z$ is the strength of the contact interaction, $a_s$ is the 3D s-wave the scattering length ($a_s \ll l_z$), $l_z = \sqrt{\hbar/m\omega_z}$ is the axial harmonic oscillator length and $\omega_z$ is the trapping frequency in the tightly confined direction. We only consider $N = N_1 = N_2$ and equal masses. We rescale the

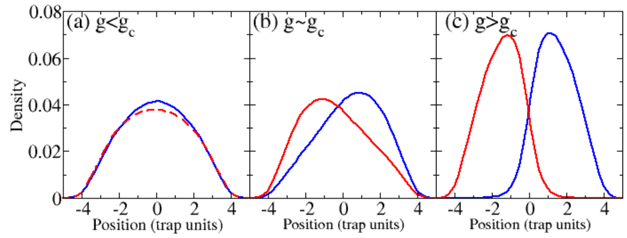


FIG. 1: (color online) The evolution of the ground state from miscible (a) to immiscible (c). In (a) we show $g_{12}$ only slightly less than $g_{12}^c$, and in (b) we show the quantum gas just after quantum phase transition when the ground state breaks rotational symmetry and $g_{12}$ only slightly greater than $g_{12}^c$. For (c) we have shown the standard example of an immiscible gas with $g_{12}/g = 2$. The BEC labeled 1 (2) is shown as blue (red).

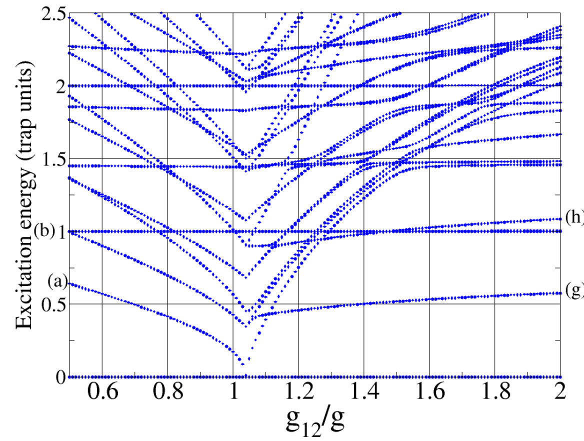


FIG. 2: (color online) Excitation spectrum for a transition from miscible to immiscible. The critical $g_{12}^c$ is at 1.04g for trapped example with $g$=100. Note the presence of strong avoided crossings and broken degeneracies after rotational symmetry has been broken, $g_{12} > g_{12}^c$. For the miscible system as $g$ is increased, the out-of-phase collective excitations dramatically lower in energy. Some of the excitations shown in Fig. 3 (4) are labeled on the left (right) of this figure.

equations into trap units, so the energy scale is $\hbar\omega_\rho$ and the length scale is $l_\rho = \sqrt{\hbar/m\omega_\rho}$ where $\omega_\rho$ is the trapping frequency in the x-y plane. We can loosely relate this to experiments, for g=100 if we pick $N$=1000, $\omega_z/\omega_\rho$=100, and $a_s$=100 $a_0$, then this example corresponds to radial trapping frequencies, of $2\pi\times38$ Hz for K and $2\pi\times11$ Hz for Cs. It is worth mentioning that the chemical potentials for each component are about equal ($\mu = \mu_1 \sim \mu_2$). More importantly, for the immiscible system, $\mu$ is $8.4\hbar\omega_\rho$ and this gives a healing length of $\xi \sim 0.35 l_\rho$ (for the miscible system $\mu \sim 7.4\hbar\omega_\rho$). Further details of how we solve these equations (1,2) appear in Ref. [16].

## III. RESULTS

In Fig. 1 we see the ground state changes character as we vary $g_{12}$. The component with the smaller $g_{ii}$ is more dense in the middle of the trap. We define $g_{12}^c$ as the value of $g_{12}$ when the ground state changes character to a broken symmetry state which begins the immiscible regime. When $g_{12} < g_{12}^c$, the ground state of the system has azimuthal symmetry and the 2 BECs overlap, (a). However as $g_{12}$ is increased to $g_{12}^c$, the ground state suddenly changes, and the azimuthal symmetry is broken, see (b). As $g_{12}$ is further increased, the 2 BECs separate further and decrease their overlap as it becomes energetically costly, (c). A similar evolution of the ground state as a function of $g_{12}$ was reported in Ref. [17].

It is challenging to find the ground state for all $g_{12}$. To do so, we use the conjugate gradient method. We have found the best initial guess is one with a slightly broken symmetry and poor overlap with the final group state. We seed the noise so that the interface would be along the y axis. If the overlap between the initial guess and the ground state is too large then it is easy to get stuck in a local energy minimum. When the conjugate gradient method fails to find the true ground state, the solutions to the Bogoliubov de Gennes equations have complex eigenvalues. When we vary $g_{12}$, the previous solution is thrown out. In this way we reliably find the excitation spectrum of the 2 component BEC with only real eigenvalues.

In Fig. 2 we show Bogoliubov excitation energies ($\omega_\alpha$) as a function of $g_{12}$. This shows the transition from miscible to immiscible at $g_{12}^c/g \sim 1.04$, where a mode goes soft. Homogeneous theory predicts this transition at $g_{12}^c = \sqrt{g_{11}g_{22}}$ [6, 7]. The discrepancy is explained by the trap and finite size of the gas [20]. In fact, if we were to increase g (keep $g_{ij}/g$ fixed); $g_{12}^c$ decreases toward 1. For example if g=400 ($\mu \sim 16\hbar\omega_\rho$) then $g_{12}^c \sim 1.01g$. A recent study explored how the trapping and the kinetic energy contributions impact the criteria for immiscibliity [18]. Our findings are consistent with their results.

To further understand Fig. 2, we start with $g_{12} < g_{12}^c$ where the systems is miscible. The quasi-particles are readily classified based on their azimuthal symmetry and the relative motion of the 2 condensates. As $g_{12}$ increases towards $g_{12}^c$, many modes decrease in energy. Then at $g_{12}^c$ two modes go soft or their energies go to zero. For $g_{12} > g_{12}^c$, many degeneracies are broken, and there are many avoided crossing as $g_{12}$ is further increased. This is where the excitations mix and change character. For the miscible side of spectrum, there are energy crossings, but they are protected symmetry and do not couple. To further understand this transition, we look at the mode which goes soft at $g_{12}^c$.

In Fig. 3 we show density perturbations associated with 2 low-energy excitations for the miscible system (far left of Fig. 2 with $g = 100$). Density perturbations from Bogoliubov de Gennes theory (T=0) are given by $\phi_i(u_i^\alpha + v_i^\alpha)$ for the mode $\alpha$. In Fig. 3 we show both (a)

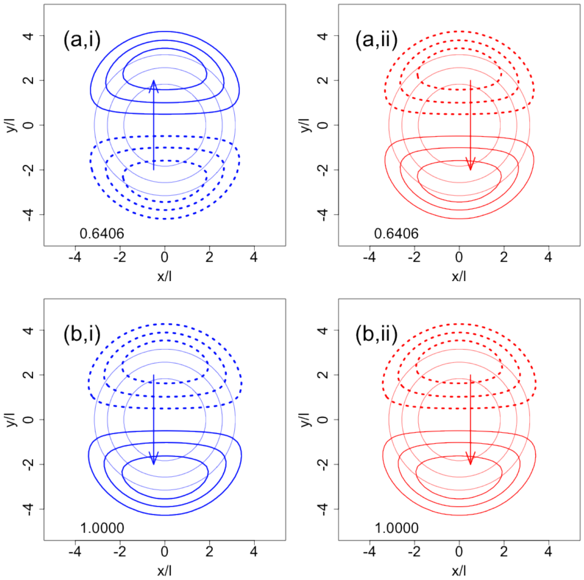


FIG. 3: (color online) We show the density perturbations as a function of x and y. Theses perturbations are co-spatial, but for clarity they have been separated. The density moves out of the dashed regions and into the solid regions. The condensate densities are shown as faint contours. In (a) we show the out-of-phase slosh mode where (i) moves up and (ii) moves down. This mode has no center-of-mass motion. In (b) we show the in-phase slosh mode where (i) and (ii) move together. This is a Kohn mode with center-of-mass motion. For this example $g_{11} = g = g_{22}/1.01 = 2g_{12}$ with $g = 100$. Energy is shown for each quasi-particle in trap units.

the out-of-phase and (b) the in-phase slosh modes. The condensates for this example are co-spatial and nearly identical. They look similar to those in Fig. 1 (a). In this figure, we separated the two components for clarity. The density moves from the regions define by the dashed lines to the regimes defined by the solid lines. The color of the perturbations matches the color for the associated condensate. We have drawn arrows to illustrate the motion of the density perturbations. The contours are shown for 0.25, 0.5 and 0.75 of the maximum value of the perturbation, and condensate density is shown in the background. The energy of the mode is reported on the figure in trap units.

In Fig. 3 (a) we show a slosh mode, but the motions of the 2 BECs are out-of-phase with each other. The solid lines coincide with the dashed lines for the other condensate's motion. For example, the blue condensate, (i), sloshes from $y < 0$ to $y > 0$ while the red condensate, (ii), sloshes from $y > 0$ to $y < 0$. There is no center-of-mass motion in this case. In contrast, (b) shows a mode with center-of-mass motion which is a Kohn mode of energy $1\hbar\omega_\rho$. The motion of each condensate coincides with the other; both the blue and red condensate slosh from $y > 0$ to $y < 0$ in phase with each other. It is important to note that the out-of-phase slosh mode has the lowest energy, and goes soft at the quantum phase transition.

For $g_{12} < g_{12}^c$, modes with $|m| > 0$, where $m$ is the azimuthal quantum number, have a degenerate twin. In our case with real quasi-particle modes, degenerate modes are related by a rotation of $\pi/2m$. For example in 3 (a) and (b) there are degenerate twins are just rotated by $\pi/2$.

Referring back to the energy spectra in Fig. 2, we see that as $g_{12}$ increases towards $g_{12}^c$ the energy of the mode in Fig 3 (a) decreases (while (b) stays at $1\hbar\omega_\rho$). More generally, all out-of-phase modes significantly lower in energy as $g$ is increased to $g_{12}^c$. In fact, the out-of-phase modes with $m = 1$, $m = 2$, $m = 3$, and $m = 0$ are all below $1\hbar\omega_\rho$ at $g_{12}^c$. Then at $g_{12}^c$ the energy of the out-of-phase slosh goes to zero and one mode stays zero for $g_{12} > g_{12}^c$. The other mode (rotated by $\pi/2$) shoots up in energy as $g_{12}$ is further increased beyond $g_{12}^c$.

The ground state spontaneously breaks rotational symmetry - which the Hamiltonian has - and this leads to an extra zero energy Goldstone mode in the excitations spectrum [19]. There are already two Goldstone modes associated with broken phase symmetry of each condensate, and they are: $u_i^{(1)} = v_i^{(1)} = \phi_i$ and $u_i^{(2)} = -v_i^{(2)} = \phi_i$. If one looks more closely at the third mode with $\omega = 0$, one finds it is a rotation of the interface ($u_i^{(3)} = v_i^{(3)} \neq \phi_i$). This extra zero energy mode has already been observed in the 2 component BECs [13]. Goldstone modes have been discussed in more detail for spinor BECs [20, 21], and Ref. [22] found similar behavior for a Goldstone mode in an attractive condensate with a Bogoliubov de Gennes treatment.

As we have said for the miscible system, the classification of the modes is simple: we use azimuthal symmetry and relative motion of the two condensates. But for the immiscible system, there is no rotational symmetry, so the characterization of the excitations must be different. To study this in more detail, we look at several quasi-particles.

In Fig. 4 we show the density perturbations associated with quasi-particle excitations for the immiscible system ($g_{11} = g$, $g_{22} = 1.01g$, $g_{12} = 2g$, and g=100). There are two types of quasi-particle modes: first, bulk excitations which look like those from a standard condensate, and second, interface excitations where the excitations are localized to the interface between the 2 condensates. Since there is no azimuthal symmetry, to classify the bulk modes we need to access how their motion is oriented relative to the interface. In Fig. 4, to depict the motion of the density perturbations, we show arrows in a few examples. The density moves from the dashed regions to the solid regions. We also show the energy of the excitation in trap units. The contours are shown for 0.25, 0.5 and 0.75 of the maximum value of the perturbation and condensate density shown in the background.

The collective modes look like those in a standard BEC, however the two BECs now act collectively to retain the excitation character. First, we look at the slosh modes of the systems. They are shown in Fig. 4 (a,b). In (a) the slosh mode with the center-of-mass displace-

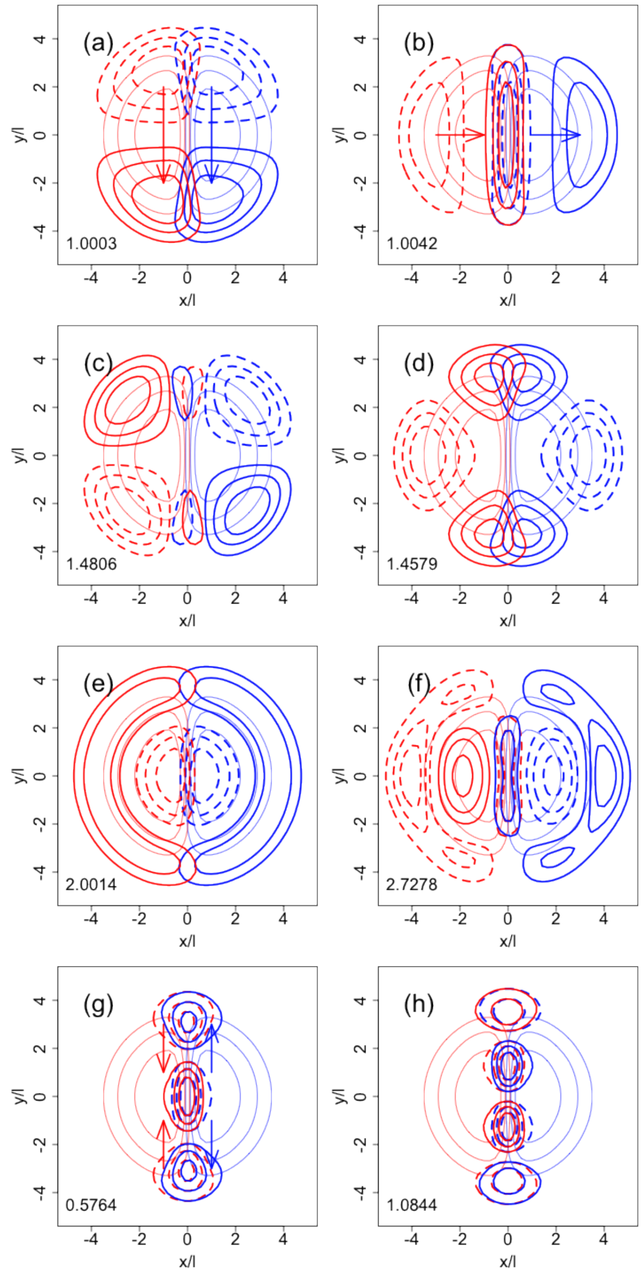


FIG. 4: (color online) The quasi-particle density perturbations for the immiscible system, $g_{12} = 2$. The x and y axes are in trap units. (a,b) are slosh modes, (c,d) are quadrupole modes (e,f) are breathing modes, and (g,h) are interface modes. The energy is reported in trap units and for this example $g_{11} = g_{22}/1.01 = g_{12}/2 = 100$.

ment parallel to the interface (Kohn mode) is shown and in (b) a slosh mode and center-of-mass displacement is perpendicular to interface (also a Kohn mode) is shown. The arrows show that in (a) both the blue and red condensate sloshes from $y > 0$ to $y < 0$. For example (b) both the blue and red condensate sloshes from $x < 0$ to $x > 0$.

Next we show the quadrupole modes in Fig. 4 (c,d). (c) shows a quadrupole mode with a nodal line along interface, and (d) shows a quadrupole mode where the density increases at interface. These excitations are typical of a single component BEC where the excitations are related by a $\pi/2m$ rotation. But in this case, the 2 BECs collude to make the excitation. Additionally, these two excitations are very similar in energy.

We show two breathing modes in Fig. 4 (e,f). They can be classified as in-phase and out-of-phase motion of the 2 condensates. In (e) we show an in-phase breathing mode, where both BECs inhale at once, or they both move into or out of the center of the trap in unison. In (f) we show an out-of-phase breathing mode, where one condensate inhales and the other exhales. The energies of these modes are notably different: 2 and 2.73 $\hbar\omega_\rho$.

There is another class of excitation in the immiscible 2 component BEC: interface excitations. Two examples are shown in Fig. 4 (g, h). These excitations are localized along the interface, and in general they are out-of-phase excitations, i.e. the density of one moves to where the other is leaving. Note these are low energy excitations, in fact (g) is the lowest energy excitations, for the system at 0.58 $\hbar\omega_\rho$ and (h) is only slightly higher than the two Kohn modes at 1.08 $\hbar\omega_\rho$. If $g$ is increased the mode in (h) will decrease below $1\hbar\omega_\rho$. So if the chemical potential is increased, then the interface modes become lower in energy. To illustrate this, we look at how the excitation energies change as a function of $g$ while keeping the ratio of the interactions fixed ($g_{12}/2 = g_{11} \sim g_{22}$). This is shown in Fig 5 (a). For reference, on the far right where $g$=600 and $\mu \sim 20\hbar\omega_\rho$, we have marked the interface modes with red $\times$'s, there are 16 interface modes with energy under $3.5\hbar\omega_\rho$. In contrast, on the far left where $g = 100$, we have marked the interface modes with red $+$'s and there are only 7 modes under $3.5\hbar\omega_\rho$. As one moves up in excitation energy, each new excitation simply adds another bend to the interface. One more important point of Fig. 5 (a), is that as $g$ increases the energies of interface modes decrease.

Fig. 5 (b) and (c) show two examples of the higher energy interface modes. First, Fig. 5 (b) is a mode with 6 bends in the interface. An interesting point about (c) is that it is a hybrid mode, it also has some density perturbations near the edge of the gas, away from the interface. The mode shown in (c) is in a region where several modes are crossing and their character is changing. If we further increase $g$, the interface mode lowers in energy and loses it collective nature and looks more like mode in (b). Related excitations have been studied in non-equilibrium simulations of 2 component BECs where Rayleigh-Taylor instabilities have been predicted [23–25]. In these studies, the value of $g_{11}$ is changed and this drives one BEC into the other, the interface then becomes unstable and a Rayleigh-Taylor instability forms.

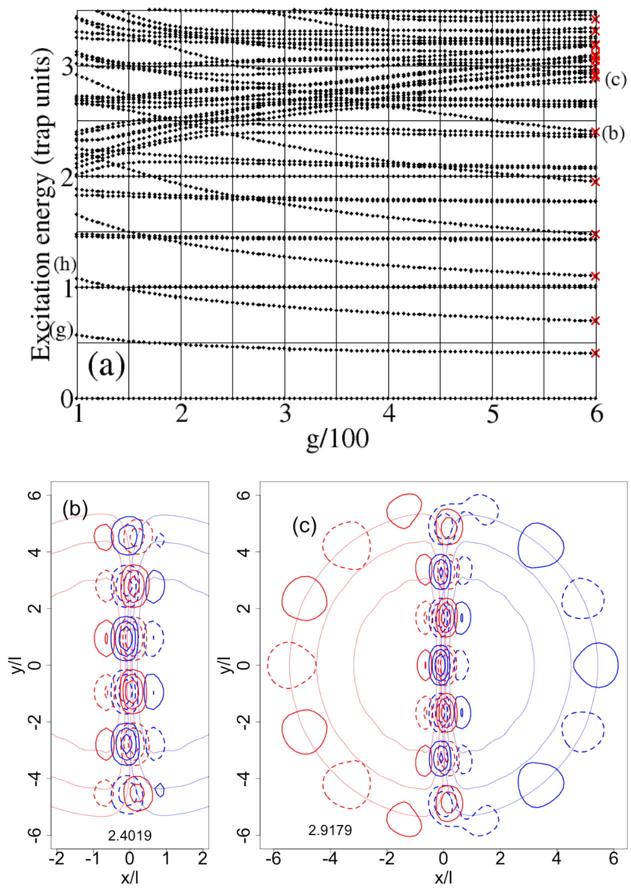


FIG. 5: (color online) The excitation spectrum as a function of g for the immiscible example ($g_{12}/2 = g_{11} = g_{22}/1.01$). The quasiparticles from Fig. 4 are on the far left side of spectrum. The interface excitations are marked by $\times$ or $+$ on either side of the figure. The energies of the quasiparticle shown in Fig. 4 (g,h) are labeled. In (b) and (c) the density perturbations for the interface excitations when $\mu \sim 20\hbar\omega_\rho$ or $g$=600, they are labeled on the right side of (a). Their energy is reported in trap units.

## IV. CONCLUSIONS

In conclusion, we have characterized the excitations of a 2 component BEC within the Bogoliubov de Gennes framework. We found that as $g_{12}$ is increased from a miscible regime to an immiscible regime, Fig. 2, generally, all of the out-of-phase excitations lower in energy. The energy of the out-of-phase slosh mode goes to zero, Fig. 3 (a). This mode becomes new Goldstone modes when the rotational symmetry is spontaneously broken in the immiscible system. We looked at the excitations of the immiscible systems when $g = g_{12}/2 = g_{11} = g_{22}/1.01$ in Fig. 4. We found that there are bulk modes which look similar to the excitations of a single trapped BEC. There are also excitations localized at the boundary between the condensates. One of these interface modes is lowest energy mode when the BECs are immiscible, and there are many other low energy interface modes, see the red +'s or x's in Fig. 5 (a). Furthermore, if one goes to a more strongly interacting BEC regime (while still immiscible), the interface modes lower in energy.

Future work will seek to understand the relationship between Bogoliubov excitations across the miscible-immiscible transition and critical phenomena. The effect of temperature on this system will be studied within the Hartree Fock Bogoliubov framework, where Bogoliubov excitations are thermally occupied. Additionally, we will look at how this collective excitation changes with nonlocal dipolar interactions [16, 26].

### Acknowledgments

The author gratefully acknowledges support through a LDRD ECR grant, LANL which is operated by LANS, LLC for the NNSA of the U.S. DOE under Contract No. DE-AC52-06NA25396. This research was supported in part by the National Science Foundation under Grant No. NSF PHY11-25915. The author also gratefully acknowledges conversations with E. Timmermans, L. A. Collins, and R. M. Wilson.

[1] M. Greiner *et al.*, Nature **415**, 39 (2002).
[2] W. S. Bakr *et al.* Nature **462**, 74 (2009).
[3] S. Trotzky *et al.*, Nat. Phys. **6**, 998 (2010).
[4] X. Zhang *et al.*, Science. **335**, 1070 (2012).
[5] S. B. Papp, J. M. Pino, and C. E. Wieman, Phys. Rev. Lett. **101**, 040402 (2008).
[6] H. Pu and N. Bigelow, Phys. Rev. Lett. **80**, 1130 (1998); *ibid.* **80**, 1134 (1998).
[7] E. Timmermans, Phys. Rev. Lett. **81**, 5718 (1998).
[8] M. R. Matthews *et al.*, Phys. Rev. Lett. **83**, 2498 (1999).
[9] K. M. Mertes *et al.*, Phys. Rev. Lett. **99**, 190402 (2007).
[10] R. P. Anderson, C. Ticknor, A. I. Sidorov, and B. V. Hall Phys. Rev. A **80**, 023603 (2009).
[11] E. Nicklas *et al.*, Phys. Rev. Lett. **107**, 193001 (2011).
[12] D. Gordon and C. M. Savage, Phys. Rev. A **58** 1440 (1998).
[13] J. G. Kim and E. K Lee, Phys. Rev. E **65**, 066201 (2002).
[14] A. L. Fetter and J. D. Waleck, *Quantum Theory of Many-Particle Systems*, (Dover, New York 2003).
[15] C.-L. Hung, X. Zhang, N. Gemelke, and C. Chin, Nature

**470**, 236 (2011).
[16] C. Ticknor, Phys. Rev. A **85**, 033629 (2012); Phys. Rev. A **86**, 053602 (2012).
[17] R. Navarro, R. Carretero-Gonzlez, and P. G. Kevrekidis, Phys. Rev. A **80**, 023613 (2009).
[18] L. Wen *et al.* Phys. Rev. A **85**, 043602 (2012).
[19] For a review see: T. Brauner, Symmetry **2**, 609 (2010).
[20] T.-L. Ho and V. B. Shenoy, Phys. Rev. Lett. **77**, 3276 (1996).
[21] S. Uchino, M. Kobayashi, and M. Udea, Phys. Rev. A **81**, 063623 (2010); S. Uchino, M. Kobayashi, M. Nitta, and M. Ueda Phys. Rev. Lett. **105**, 230406 (2010).
[22] R. Kanamoto, H. Saito, and M. Ueda, Phys. Rev. Lett. 94, 090404 (2005).
[23] K. Sasaki, N. Suzuki, D. Akamatsu, and H. Saito, Phys. Rev. A **80**, 063611 (2009); N. Suzuki, H. Takeuchi, K. Kasamatsu, M. Tsubota, and H. Saito, Phys. Rev. A **82**, 063604 (2010); H. Takeuchi, N. Suzuki, K. Kasamatsu, H. Saito, and M. Tsubota, Phys. Rev. B **81**, 094517 (2010); T. Kadokura, T. Aioi, K. Sasaki, T. Kishimoto, and H. Saito, Phys. Rev. A **85**, 013602 (2012).
[24] S. Gautam and D. Angom, Phys. Rev. A **81**, 053616 (2010).
[25] A. Bezett, V. Bychkov, E. Lundh, D. Kobyakov, and M. Marklund, Phys. Rev. A **82**, 043608 (2010); D. Kobyakov, V. Bychkov, E. Lundh, A. Bezett, and M. Marklund, Phys. Rev. A **86**, 023614 (2012).
[26] R. M. Wilson, C. Ticknor, J. L. Bohn, and Eddy Timmermans, Phys. Rev. A **86**, 033606 (2012).