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Frozen Gaussian quantum discord: A proposal for realization in photonic crystal cavity array system

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Protecting quantum correlation from decoherence is one of the crucial issues in quantum information processing. It was commonly recognized that any initial quantum correlation of a composite system diminishes asymptotically or abruptly to zero under local Markovian decoherence. Here we show that, contrary to this recognition, a noticeable Gaussian quantum discord of a continuous-variable bipartite system can be frozen in the steady state in the non-Markovian dynamics if each of the subsystems forms a localized mode with its local reservoir. The condition for this quantum discord frozen can be reached by engineering appropriately the structure of the reservoirs. The possible realization of our results in coupled cavity array system formed by photonic crystal is proposed.

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I. INTRODUCTION

Quantum correlation plays an essential role in quantum information science. In the early days of quantum information, quantum correlation is characterized by entanglement, which is viewed as the main resource for quantum information processing [1]. It engenders the dramatic speedup of quantum computer over its classical counterpart. Recently, it was found that entanglement is not the only reason to cause such speedup and the similar speedup can also be achieved in the so-called deterministic one-qubit quantum computation by use of the zero-entanglement states [2, 3]. It has been attributed to another measure of quantum correlation [4], i.e. quantum discord (QD) [5, 6]. These results indicate that entanglement cannot exhaust quantum correlation and QD characterizes the quantumness of correlations more generally than entanglement.

The study of quantum correlations under decoherence attracts much attention in recent years, as this study is expected to supply some insight to beat the detrimental effects caused by decoherence on quantum correlation. It is found that QD [7–10] exhibits some peculiar features which are absent for entanglement. Firstly, QD of a two-qubit system under individual decoherence decays to zero in asymptotical manner [11–16], which is much different from the sudden death behavior of entanglement in the same setting [17, 18]. The experimental [19] and theoretical [20, 21] works also confirm the similar results for the Gaussian QD of continuous-variable systems. Secondly, QD can be developed transiently from certain initially classical state under a single local Markovian dissipation channel both for discrete- [22, 23] and continuous- [24] variable systems. This is unattainable with entangle-

ment. Thirdly, QD under decoherence shows a sudden change from the “classical decoherence” regime to the “quantum decoherence” regime [25, 26]. In the former regime, the classical correlation decays while QD is frozen to its initial value; in the latter one, QD starts to decay while classical correlation is frozen. This interesting phenomenon has been observed in optical [16] and NMR [27] systems.

All these features indicate that QD is more robust than entanglement against decoherence. As a result, QD could be more preferred resource in quantum information processing. However, one finds that QD discussed above decays exclusively to zero in the long-time limit. To overcome the detrimental effects of decoherence on quantum information processing, it is of course desirable to preserve the initial quantum correlation in the long-time limit.

In this work, we propose a scheme to stabilize QD by engineering appropriately the reservoirs to introduce the non-Markovian effect, an issue actively studied recently [28–32]. By studying the correlation dynamics of a continuous-variable bipartite system, we show that a finite Gaussian QD can be frozen in the steady state. The essential physics is the formation of a localized mode in the subsystems and the non-Markovian effect. An experimental accessible scheme is proposed to observe the frozen QD by using a coupled cavity array system realized especially in photonic crystal system [33–35]. The result and its possibly experimental realization could be significant in quantum information processing.

II. MODEL AND DYNAMICS

Consider two noninteracting harmonic oscillators coupled to two independent reservoirs. The Hamiltonian of each local subsystem is ($\hbar = 1$)

$$\hat{H}^k = \omega_k \hat{a}_k^\dagger \hat{a}_k + \sum_l \omega_{kl} \hat{b}_{kl}^\dagger \hat{b}_{kl} + \sum_l (g_{kl} \hat{a}_k^\dagger \hat{b}_{kl} + \text{h.c.}), \quad (1)$$

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where \hat{a}_k and \hat{b}_{kl} (\hat{a}_k^\dagger and \hat{b}_{kl}^\dagger) are, respectively, the annihilation (creation) operators of the k -th harmonic oscillator with frequency ω_k and its corresponding reservoir. The coupling strength between them is given by g_{kl} . The system is highly pertinent to quantum-optical setting where the system oscillators can describe the quantized optical fields in cavity [36] or in circuit [37] QED, mechanical oscillators in opto-mechanics [38], and atomic ensemble under large- N limit [39]. Currently, most quantum optical experiments are performed at low temperatures and under vacuum condition. Thus, we assume the reservoirs to be at zero temperature in this work.

The exact decoherence dynamics of the system can be derived by Feynman and Vernon's influence-functional theory [40, 41]. The reduced density matrix of the system expressed in the coherent-state representation is given by

$$\rho(\bar{\alpha}_f, \alpha'_f; t) = \int d\mu(\alpha_i) d\mu(\alpha'_i) \mathcal{J}(\bar{\alpha}_f, \alpha'_f; t | \bar{\alpha}_i, \alpha'_i; 0) \times \rho(\bar{\alpha}_i, \alpha'_i; 0). \quad (2)$$

The coherent-state representation is defined as $|\alpha\rangle = \prod_{k=1}^2 \exp(\alpha_k a_k^\dagger) |0_k\rangle$, which are the eigenstates of annihilation operators and obey the resolution of identity, $\int d\mu(\alpha) |\alpha\rangle \langle \alpha| = 1$ with the integration measures defined as $d\mu(\alpha) = \prod_k e^{-\bar{\alpha}_k \alpha_k} \frac{d\bar{\alpha}_k d\alpha_k}{2\pi i}$. Here $\bar{\alpha}$ denotes the complex conjugate of α . The propagating function $\mathcal{J}(\bar{\alpha}_f, \alpha'_f; t | \bar{\alpha}_i, \alpha'_i; 0)$ is expressed as the path integral governed by an effective action consisting of the free actions of the forward and backward propagators of the system and the influence functional obtained from the integration of reservoir degrees of freedom. After evaluating the path integral, we get

$$\mathcal{J}(\bar{\alpha}_f, \alpha'_f; t | \bar{\alpha}_i, \alpha'_i; 0) = \exp \left\{ \sum_{k=1,2} [u_k(t) \bar{\alpha}_{kf} \alpha_{ki} + \bar{u}_k(t) \alpha'_{ki} \alpha'_{kf} + [1 - |u_k(t)|^2] \alpha'_{ki} \alpha_{ki}] \right\}, \quad (3)$$

where $u_k(t)$ satisfies

$$\dot{u}_k(t) + i\omega_k u_k(t) + \int_0^t f_k(t-\tau) u_k(\tau) d\tau = 0 \quad (4)$$

with $u_k(0) = 1$ and $f_k(x) \equiv \int J_k(\omega) e^{-i\omega x} d\omega$ under the continuous limit of the environmental modes. Combining with Eq. (3), the time-dependent state can be obtained from any initial state by evaluating the integration in Eq. (2). The exact decoherence dynamics, determined by Eq. (4), essentially depends on the so-called spectral density $J_k(\omega) \equiv \sum_l |g_{kl}|^2 \delta(\omega - \omega_k)$, which characterizes the coupling strength of the different environmental modes to the system with respect to their frequencies. In the continuum limit it takes the form $J_k(\omega) = \eta_k \omega \left(\frac{\omega}{\omega_c}\right)^{n-1} e^{-\frac{\omega}{\omega_c}}$, where ω_c is a cutoff frequency, and η_k is a dimensionless coupling constant. The environment is classified as Ohmic if $n = 1$, sub-Ohmic if $0 < n < 1$, and super-Ohmic for $n > 1$ [42]. Different spectral densities manifest different non-Markovian decoherence dynamics.

To compare with the conventional Born-Markovian approximate description to such system, a master equation can be derived by taking the time derivative to Eq. (2)

$$\dot{\rho}(t) = \sum_{k=1,2} \{-i\Omega_k(t) [\hat{a}_k^\dagger \hat{a}_k, \rho(t)] + \Gamma_k(t) [2\hat{a}_k \rho(t) \hat{a}_k^\dagger - \hat{a}_k^\dagger \hat{a}_k \rho(t) - \rho(t) \hat{a}_k^\dagger \hat{a}_k]\}, \quad (5)$$

where $\Gamma_k(t) + i\Omega_k(t) \equiv -\dot{u}_k(t)/u_k(t)$. It can be seen that Eq. (5) keeps the Lindblad form but with time-dependent shifted frequency $\Omega_k(t)$ and decay rate $\Gamma_k(t)$. All the backactions induced by the non-Markovian effect have been incorporated into these time-dependent coefficients self-consistently.

III. DYNAMICAL FROZEN OF GAUSSIAN QD

Consider explicitly the initial state of the system as two-mode squeezed state $|\psi(0)\rangle = \exp[r(\hat{a}_1 \hat{a}_2 - \hat{a}_1^\dagger \hat{a}_2^\dagger)] |00\rangle$ with r being the squeezing parameter. The time evolution of such state under Eq. (2) keeps the Gaussianity. The Gaussian state can be fully characterised by the covariance matrix $\sigma_{12} = \begin{pmatrix} \alpha_1 & \gamma \\ \gamma^T & \alpha_2 \end{pmatrix}$, where α_k are the 2×2 covariance matrices for the k -th subsystems, and γ is the matrix containing the correlations between (x_1, p_1) and (x_2, p_2) with $\hat{x}_k = \frac{\hat{a}_k + \hat{a}_k^\dagger}{\sqrt{2}}$ and $\hat{p}_k = \frac{\hat{a}_k - \hat{a}_k^\dagger}{\sqrt{2}i}$. σ_{12} can be easily estimated experimentally from the homodyne measurements to the amplitude quadratures \hat{x}_k and \hat{p}_k . The QD for Gaussian state can be calculated as follows. The total correlation for a bipartite system is given by the mutual information $\mathcal{I}(\rho) = S(\rho_1) + S(\rho_2) - S(\rho)$, where S is the von Neumann entropy and $\rho_{1(2)}$ is the reduced density matrix of the 1 (2) subsystem. Another measure of mutual information that only quantifies the amount of classical correlations extractable by a Gaussian measurement is $\mathcal{C}_1(\rho) = S(\rho_1) - \inf_{\sigma_M} S(\rho_1 | \sigma_M)$ where σ_M is the covariance matrix of the measurement on mode 2. As it only captures the classical correlations, the difference, $\mathcal{D}_1 = \mathcal{I}(\rho) - \mathcal{C}_1(\rho)$, is a measure of Gaussian quantum correlation that is coined Gaussian QD. An explicit expression for this QD has been found [10]:

$$\mathcal{D}(\sigma_{12}) = \mathfrak{f}(\sqrt{I_2}) - \mathfrak{f}(\nu_-) - \mathfrak{f}(\nu_+) + \mathfrak{f}(\sqrt{m}) \quad (6)$$

with $\mathfrak{f}(x) = \left(\frac{x+1}{2}\right) \log \frac{x+1}{2} - \left(\frac{x-1}{2}\right) \log \frac{x-1}{2}$ and

$$m = \begin{cases} \frac{2I_3^2 + (I_2 - 1)(I_4 - I_1) + 2|I_3| \sqrt{I_3^2 + (I_2 - 1)(I_4 - I_1)}}{(I_2 - 1)^2}, \text{ a) } \\ \frac{I_1 I_2 - I_3^2 + I_4 - \sqrt{I_3^4 + (I_4 - I_1 I_2)^2 - 2C^2(I_4 + I_1 I_2)}}{2I_2}, \text{ b) } \end{cases} \quad (7)$$

where a) applies if $(I_4 - I_1 I_2)^2 \leq I_3^2 (I_2 + 1)(I_1 + I_4)$ and b) applies otherwise. Here $I_k = \det \alpha_k$, $I_3 = \det \gamma$, $I_4 = \det \sigma_{12}$ are the symplectic invariants and $\nu_\pm^2 = \frac{1}{2}(\delta \pm \sqrt{\delta^2 - 4I_4})$ with $\delta = I_1 + I_2 + 2I_3$ are the symplectic eigenvalues. The explicit form of the time evolution

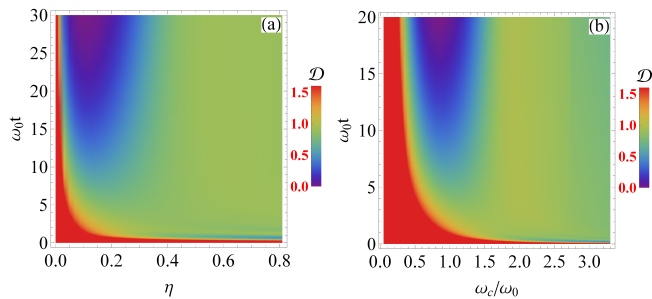


FIG. 1: (Color online) The densityplot of Gaussian QD vs t for super-Ohmic spectral density in different η (a) and ω_c (b). $r = 1.0$ and $\omega_c/\omega_0 = 1.0$ in (a), $\eta = 0.08$ in (b) have been used.

of the two-mode squeezed state and its corresponding covariance matrix are given in appendix A. With the obtained covariance matrix (A7), the Gaussian QD can be evaluated straightforwardly.

Choosing the super-Ohmic spectral density, explicitly $n = 3$, as an example, we plot in Fig. 1 the evolution of Gaussian QD for the initial two-mode squeezed state. It has been shown that the super-Ohmic spectral density can describe the phonon bath in one or three dimensions, depending on the symmetry properties of the strain field [43] and a charged particle coupled to its own electromagnetic field [44]. Compared with the Ohmic and sub-Ohmic spectral densities, the super-Ohmic one is higher-frequency dominate, which will cause a strong modification to the short-time decoherence dynamics of the system. We can see from Fig. 1(a) that the Gaussian QD decays to zero and a larger η induces a faster decay, which are qualitatively consistent with the results under Markovian approximation, only when the coupling is vanishingly weak. With the increase of η , it is remarkable to find that the decay of the Gaussian QD tends to slow down even to be totally stabilized. This is dramatically contrary to one's expectation that a stronger coupling between the system and the reservoir always induces a more severe decoherence to the system. The similar Gaussian QD frozen can also be achieved with the increase of the cutoff frequency in Fig. 1(b).

We argue that the formation of a localized mode between each of the harmonic oscillators and its local reservoir plays essential role in this QD frozen. To verify this, we perform a Fourier transform to Eq. (4) and obtain

$$y(E) \equiv \omega_0 - \int_0^\infty \frac{J(\omega)}{\omega - E} d\omega = E. \quad (8)$$

One can see that $y(E)$ is a monotonically decreasing function in the region $E \in (-\infty, 0)$. It means that Eq. (8) may have one and only one negative root if the system parameters fulfill $y(0) < 0$. On the other hand, no further discrete root exists in the region $(0, +\infty)$ because that would make the integration in $y(E)$ divergent. After the inverse Fourier transform, the obtained $u_k(t)$ contributed from this discrete negative root will have a vanishing de-

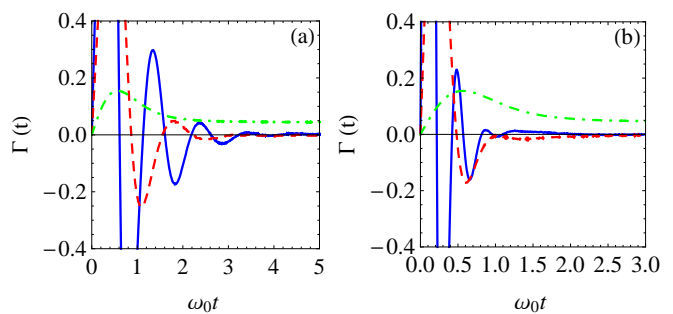


FIG. 2: (Color online) The decay rate when $\omega_c/\omega_0 = 1.0$ and $\eta = 0.08$ (dot-dashed green), 0.5 (dashed red), and 1.0 (solid blue) in (a) and when $\eta = 0.08$ and $\omega_c/\omega_0 = 1.0$ (dot-dashed green), 2.0 (dashed red), and 3.0 (solid blue) in (b). The localized mode is formed when $\eta > 0.5$ for (a) and $\omega_c > 1.84\omega_0$ for (b).

coherence $\Gamma_k(t)$. This vanishing decay rate causes the decoherence inhibited to the system. It means that the discrete negative root for Eq. (8) actually corresponds to a stationary state to Eq. (4), which preserves the quantum coherence in its superposed components during time evolution. We call this stationary state the localized mode of the whole system [28]. For our super-Ohmic spectral density, we can readily show that the localized mode is formed when $\omega_0 - 2\eta\frac{\omega_0^3}{\omega_0^2} < 0$ is fulfilled. This criterion gives a basic judgement on the condition under which the Gaussian QD frozen is present.

To verify dynamical consequence of the formed localized mode, we plot in Fig. 2 the decay rate in the case Fig. 1(a,b). We can see that if the localized mode is absent, the decay rate keeps to be positive and tends to a positive value, which, as expected, will induces monotonic decoherence to the system, as shown in Fig. 1(a) when $\eta < 0.5$ and in Fig. 1(b) when $\omega_c < 1.84\omega_0$. On the contrary, if the localized mode is present, the decay rate is transiently negative, which manifests that the lost information/energy of the system returns back from the reservoir. Another character different from the case when the localized mode is absent is that the decay rate tends to zero asymptotically. This vanishing decay rate causes the decoherence of the system to cease in the long-time limit. This give an explanation why a strong coupling can induce a suppressed decoherence in Fig. 1. Such anomalous decoherence also manifests as the deviation from the exponential decay of $|u(t)|^2$ under Born-Markovian approximation, as shown in appendix B.

From above analysis, we can conclude that the Gaussian QD frozen is present due to an interplay between the formed localized mode and the non-Markovian effect. The localized mode provides an ability to freeze the Gaussian QD, while the non-Markovian effect provides a dynamical way to freeze the Gaussian QD. The mechanism of the stable Gaussian QD frozen in our system is linked to the non-Markovian memory effect of the harmonic oscillator with its local reservoir when the lo-

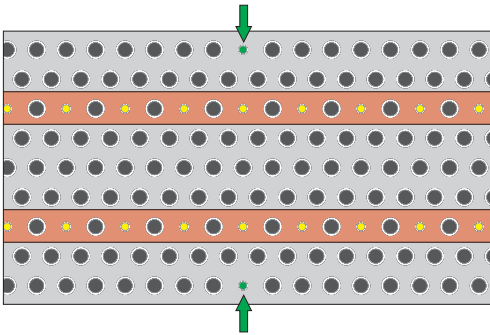


FIG. 3: (Color online) Two initially correlated cavity fields propagating in two cavity arrays formed in photonic crystal.

calized mode is formed. It is much different to the case of two harmonic oscillators coupled to a common reservoir [45, 46], where a stable QD is established due to an indirect interaction between the two harmonic oscillators induced effectively by the common reservoir.

It is noted that our result is also of benefit to the analysis of entanglement under the same decoherence setting. Since the decoherence is suppressed when the localized mode is formed, we also could expect a finite entanglement preservation in the steady state. In this case, the Gaussian QD shows no qualitative difference from the entanglement. However, in the parameter regime where the localized mode is absent, it can be confirmed that the entanglement always decays to zero more rapidly than the Gaussian QD does. This is consistent in the previous result that the QD is more robust than the entanglement to local decoherence [11–16, 19–21].

IV. PHYSICAL REALIZATION

With the basic criterion at hands, we can see that the Gaussian QD frozen we elaborated is a generic phenomenon in open quantum systems irrespective of the form of the spectral density. A best candidates to observe our prediction is the system of two arrays of coupled cavities, which can now be realized experimentally in micro-disc cavities coupled by one tapered optical fiber [47], in photonic crystal system [33–35], and synthesized in optical waveguide array system [48, 49]. In Fig. 3 we depict the schematic illustration to this scheme realized in photonic crystal system. Here two initially correlated quantized optical fields are fed into the two system cavities. With some probability the optical fields in the two system cavities will hop respectively to the two spatially separated coupled cavity arrays. Each of the local system is governed by $\hat{H}^{(1)} = \omega_0 \hat{a}^\dagger \hat{a} + \omega_C \sum_{j=0}^{N-1} \hat{b}_j^\dagger \hat{b}_j + (g \hat{a}^\dagger \hat{b}_0 + \xi \sum_{j=0}^{N-2} \hat{b}_j^\dagger \hat{b}_{j+1} + \text{h.c.})$. A Fourier transform $\hat{b}_j = \sum_k \hat{b}_k e^{ikjx_0}$ recasts $\hat{H}^{(1)}$ into

$$\hat{H}^{(1)} = \omega_0 \hat{a}^\dagger \hat{a} + \sum_k \epsilon_k \hat{b}_k^\dagger \hat{b}_k + \frac{g}{\sqrt{N}} \sum_k (\hat{a}^\dagger \hat{b}_k + \text{h.c.}) \quad (9)$$

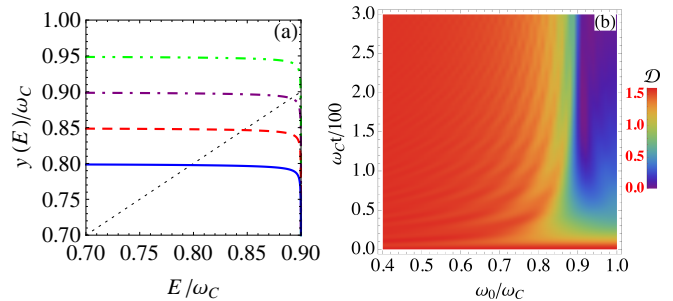


FIG. 4: (Color online) (a): The formation of a localized mode manifested by intersection point of the dotted line with the lines when $\omega_0 = 0.95\omega_C$ (dot-dot-dashed green), $0.9\omega_C$ (dot-dashed purple), $0.85\omega_C$ (dashed red), and $0.8\omega_C$ (solid blue). (b): The densityplot of Gaussian QD vs t in different ω_0 . $\xi = 0.05\omega_C$, $g = 0.02\omega_C$, and $N = 200$ have been used.

with $\epsilon_k = \omega_C + 2\xi \cos kx_0$ and x_0 being the spatial separation between the two neighbor cavities of the cavity arrays. One can notice that the dispersion relation of the field in such structured reservoirs shows finite band width, which can induce a strong non-Markovian even in the weak and intermediate coupling regimes.

In Fig. 4(a), we plot the possible formation of the localized mode manifested by the intersection points between the dotted line and each line in different parameter regimes. It can be seen that if there is no intersection point, which means the localized mode is absent, then the Gaussian QD, as shown in Fig. 4(b), decays to zero. Whenever the localized mode is formed, certain finite Gaussian QD can be frozen in the steady state. As an interesting observation, we can find that the frozen Gaussian QD in this case is even as large as its initial value. It means that the detrimental effect from decoherence is almost eliminated. Another interesting observation is that the frozen QD can be obtained even there is no strong coupling between the system and the reservoirs. This reduces greatly the experimental difficulty in the practice.

V. CONCLUSIONS

We have revealed a mechanism under which the decoherence of QD can be avoided and a finite QD can be frozen in the steady state. The underlying physics is the interplay between the formed localized mode and the non-Markovian effect. We have also proposed an experimentally accessible scheme to observe our prediction in coupled cavity array system realized in photonic crystal platform [33–35]. Our result suggests the controllability of decoherence by reservoir engineering [50, 51]. Our finding provides a significant progress in the practical continuous-variable quantum information processing.

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Appendix A: The covariance matrix

The initial state can be represented in the coherent-state representation as

$$\rho(\bar{\alpha}_i, \alpha'_i; 0) = \frac{\exp[-\tanh r(\bar{\alpha}_{1i}\bar{\alpha}_{2i} + \alpha'_{1i}\alpha'_{2i})]}{\cosh^2 r}. \quad (\text{A1})$$

Substituting Eq. (A1) into Eq. (2), we can obtain the evolved state as

$$\rho(\bar{\alpha}_f, \alpha'_f; t) = a \exp\left[\sum_{k \neq k'} \left(\frac{b}{2} \bar{\alpha}_{kf} \bar{\alpha}'_{k'f} + c \bar{\alpha}_{kf} \alpha'_{k'f} + \frac{b^*}{2} \alpha'_{k'f} \alpha'_{k'f}\right)\right], \quad (\text{A2})$$

where

$$a = \frac{1}{\cosh^2 |r| [1 - \tanh^2 |r| (1 - |u(t)|^2)^2]}, \quad (\text{A3})$$

$$b = \frac{-\tanh |r| |u(t)|^2}{1 - \tanh^2 |r| (1 - |u(t)|^2)^2}, \quad (\text{A4})$$

$$c = \frac{\tanh^2 |r| (1 - |u(t)|^2) |u(t)|^2}{1 - \tanh^2 |r| (1 - |u(t)|^2)^2}. \quad (\text{A5})$$

For the continuous-variable (Gaussian-type) bipartite state, its density matrix is characterized by the covariance matrix defined as the second moments of the quadrature vector $\hat{X} = (\hat{x}_1, \hat{p}_1, \hat{x}_2, \hat{p}_2)$,

$$\sigma_{ij} = \langle \Delta \hat{X}_i \Delta \hat{X}_j + \Delta \hat{X}_j \Delta \hat{X}_i \rangle, \quad (\text{A6})$$

where $\Delta \hat{X}_i = \hat{X}_i - \langle \hat{X}_i \rangle$, and $\hat{x}_i = \frac{\hat{a}_i + \hat{a}_i^\dagger}{\sqrt{2}}$, $\hat{p}_i = \frac{\hat{a}_i - \hat{a}_i^\dagger}{i\sqrt{2}}$. From the time-dependent state (A2), the covariance matrix for the harmonic oscillators can be calculated straightforwardly,

$$\sigma = 2 \begin{pmatrix} \frac{y(1+d)}{2(1-d)^2} & 0 & \frac{a\text{Re}[b]}{x} & \frac{a\text{Im}[b]}{x} \\ 0 & \frac{y(1+d)}{2(1-d)^2} & \frac{a\text{Im}[b]}{x} & -\frac{a\text{Re}[b]}{x} \\ \frac{a\text{Re}[b]}{x} & \frac{a\text{Im}[b]}{x} & \frac{y(1+d)}{2(1-d)^2} & 0 \\ \frac{a\text{Im}[b]}{x} & -\frac{a\text{Re}[b]}{x} & 0 & \frac{y(1+d)}{2(1-d)^2} \end{pmatrix}, \quad (\text{A7})$$

where $x = [(1-c)^2 - |b|^2]^2$, $y = \frac{a}{1-c}$, and $d = c + \frac{|b|^2}{1-c}$.

Appendix B: Anomalous decoherence

Accompanying with the formation of the localized mode of the whole system, the dynamics of the reduced

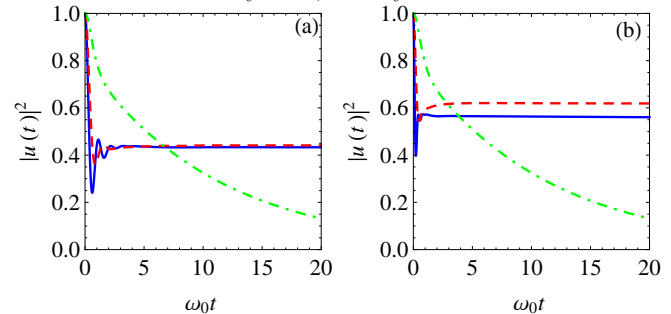


FIG. 5: (Color online) The corresponding $|u(t)|^2$ of Fig. 2(a) and (b). The localized mode is formed when $\eta > 0.5$ for (a) and $\omega_c > 1.84\omega_0$ for (b).

system is inhibited. This can be verified by the time-dependent behaviors of $u(t)$. In Fig. 5, we plot the evolution of $|u(t)|^2$ corresponding to the parameter regimes used in Fig. 2(a,b), respectively. We can see that with the formation of the localized mode above the critical point $\eta = 0.5$ for Fig. 5(a) and $\omega_c = 1.84\omega_0$ for Fig. 5(b), the time-dependent behavior of $|u(t)|^2$ shows qualitatively changes. If the localized mode is absent, $|u(t)|^2$ decays to zero monotonically, which is consistent with the results under Born-Markovian approximation. On the other hand, if the localized mode is present, $|u(t)|^2$ tends to a finite value after transient oscillation. It indicates the ceasing of the decoherence in the long-time limit, which is also consistent with the vanishing decay rate in Fig. 2. It deviates qualitatively from the results under Born-Markovian approximation. This shows clearly that the non-Markovian effect can not only induce transient oscillation, but also induce dramatic change on the steady state behavior to the open quantum system. Equipped with this anomalous decoherence, it is not hard to understand the Gaussian QD frozen revealed in our work.

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