This is the accepted manuscript made available via CHORUS. The article has been published as:

Orbital coupled dipolar fermions in an asymmetric optical ladder

Xiaopeng Li and W. Vincent Liu

Phys. Rev. A **87**, 063605 — Published 5 June 2013

DOI: [10.1103/PhysRevA.87.063605](https://doi.org/10.1103/PhysRevA.87.063605)

Orbital coupled dipolar fermions in an asymmetric optical ladder

Xiaopeng Li^{1,2} and W. Vincent Liu^{*1}

¹*Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania*

²*Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA*

We study a quantum ladder of interacting fermions with coupled s and p orbitals. Such a model describes dipolar molecules or atoms loaded into a double-well optical lattice, dipole moments being aligned by an external field. The two orbital components have distinct hoppings. The tunneling between them is equivalent to a partial Rashba spin-orbital coupling when the orbital space (s, p) is identified as spanned by pseudo-spin 1/2 states. A rich phase diagram, including incommensurate orbital density wave, pair density wave and other exotic superconducting phases, is proposed with bosonization analysis. In particular, superconductivity is found in the repulsive regime.

I. INTRODUCTION

Orbital degree of freedom [1] plays a fundamental role in understanding the unconventional properties in solid state materials [2]. Recent experiments in optical lattices have demonstrated that orbitals can also be used to construct quantum emulators of exotic models beyond natural crystals. Orbital lattices are attracting growing interests due to their unique and fascinating properties resulting from the spatial nature of the degenerate states. For example, the bosonic $p_x + ip_y$ superfluid [3–7] state has been prepared on a bipartite square lattice [8], and later the other complex superfluid with s and p orbitals correlated was observed on a hexagonal lattice [9].

Previous study on multicomponent cold gases mainly focused on hyperfine states of alkali atoms [10, 11]. In a cold gas of atoms with two approximately degenerate hyperfine states, the realized pseudo-spin SU(2) symmetry makes it possible to emulate Fermi Hubbard model in optical lattices [12–14]. To engineer spin-orbital couplings and the resulting topological phases, one has to induce Raman transitions between the hyperfine states to break the pseudo-spin symmetry [15–19]. In contrast, due to the spatial nature of the orbital degrees of freedom, the symmetry in orbital gases, such as that in $p_x + ip_y$ superfluid [8], can be controlled by simply changing the lattice geometry as shown in Ref. [8, 9, 20, 21], where unprecedented tunability of double-wells has been demonstrated. With a certain lattice geometry, a spin-orbital like coupling can naturally appear in an orbital gas with s and p -orbitals without Raman transitions [22]. Theoretical studies of orbital physics largely focusing on two or three dimensions suggest exotic orbital phases [3–6, 23–31] beyond the scope of spin physics.

In this article, we study a one dimensional orbital ladder with s and p orbitals coupled [22, 32]. We shall derive such an effective model for dipolar molecules or atoms [33–37] loaded in a double-well optical lattice. The tunneling rates (or effective mass) of each orbital component are highly tunable by changing the lattice

strength. The coupling between s and p orbitals mimics the spin-orbital couplings [22]. This orbital system suggests the possibility of exploring the equivalent of the exciting spin-orbital coupled physics in dipolar gases yet without requiring the use of synthetic gauge fields, and hence it provides an interesting, simple alternative route. A rich phase diagram, including incommensurate orbital density wave (ODW), pair density wave (PDW) [38–41], and other exotic superconducting phases, is found with bosonization analysis. The PDW phase realized here is a superconducting phase, that features an oscillating Cooper pair field with a period of π . The incommensurate ODW phase has an oscillating particle-hole pair, which tends to break the time-reversal symmetry. An exotic superconducting phase on the repulsive side is also discovered.

FIG. 1: (Color online) Schematic plot illustrating control of interactions with polar molecules or atoms loaded on s - and p -orbitals. Dipole moments aligned “head-to-head” (“head-to-tail”) in the left (right) figure provide repulsive (attractive) interaction.

II. MODEL

Consider a cold ensemble of polar molecules or atoms, e.g. $^{40}\text{K}^{87}\text{Rb}$ [33, 35, 36], $^{23}\text{Na}^{40}\text{K}$ [37], OH [42], or Dy [34], whose dipole moments are controlled by an external field as demonstrated in experiments. Long lived polar molecules have been realized in optical lattices [36]. Let the ensemble trapped by a ladder-like optical lattice of the type studied in [22]. As shown in the schematic

*email: w.vincent.liu@gmail.com

picture in [22], the lattice consists of two chains of potentials of unequal depth. We consider a single species of fermionic atoms/molecules occupying the s and p orbitals of the shallow and deep chains, respectively, with the low-lying s orbitals on the deep chain completely filled (FIG. 1). Alternatively, fermions can be directly loaded into the higher orbitals while keeping the lower s nearly empty by the techniques developed in recent experiments [8, 20]. Coherent meta-stable states in high orbitals with long life time up to several hundred milliseconds were demonstrated achievable [8]. To suppress chemical reactions of polar molecules, the latter approach is preferable. The single particle Hamiltonian of the sp -orbital ladder is then given as [22]

$$H_0 = \sum_j C_j^\dagger \begin{bmatrix} -t_s & -t_{sp} \\ t_{sp} & t_p \end{bmatrix} C_{j+1} + h.c. \quad (1)$$

where $C_j^\dagger = [a_s^\dagger(j), a_p^\dagger(j)]$, and $a_s^\dagger$ ($a_p^\dagger$) is the creation operator for the s -orbital (p -orbital). The lattice constant is set as the length unit in this paper. In the proposed optical lattice setup [22], the ratios t_s/t_p and t_{sp}/t_p are small (typically 0.1). We emphasize here that s -orbital is parity even and that p -orbital is parity odd. The relative signs of hoppings are dictated by the parity nature of the s - and p -orbitals [22]

The band structure is readily obtained by Fourier transform $C_j = \int \frac{dk}{2\pi} \tilde{C}(k) e^{ikj}$. The Hamiltonian in the momentum space reads as $H_0 = \int \frac{dk}{2\pi} \tilde{C}^\dagger(k) \tilde{\mathcal{H}}(k) \tilde{C}(k)$, with $\tilde{\mathcal{H}}(k) = h_0(k)\sigma_0 + \vec{h}(k) \cdot \vec{\sigma}$, where $h_0(k) = (t_p - t_s) \cos(k)$, $h_x(k) = 0$, $h_y(k) = 2t_{sp} \sin(k)$ and $h_z(k) = -(t_p + t_s) \cos(k)$. Here σ_0 is the identity matrix and $\sigma_{x,y,z}$ are Pauli matrices. The two bands are given by $E_\pm(k) = h_0(k) \pm \sqrt{h_y^2(k) + h_z^2(k)}$, which are shown in FIG. 2. The Hamiltonian is rewritten as $H_0 = \int \frac{dk}{2\pi} \sum_{\varphi=\pm} E_\varphi(k) \phi_\varphi^\dagger(k) \phi_\varphi(k)$. We define an angle variable θ by $\cos(\theta(k)) = h_z/|\vec{h}|$ and $\sin(\theta(k)) = h_y/|\vec{h}|$ to save writing. Here, we only consider lower than half filling, i.e., less than one particle per unit cell. The lower band is thus partially filled and the upper band is empty. Since we are interested in the low-energy physics, the spectrum E_- is linearized around the Fermi momenta $\pm k_{F\nu}$. Here, $\nu = A$ or B , and $\pm k_{FA}$ are inner Fermi points and $\pm k_{FB}$ are outer Fermi points (FIG. 2). The resulting Fermi velocities are $v_{F\nu} = |\frac{\partial E_-(k)}{\partial k}|_{k=k_{F\nu}}$. The operators capturing the low energy fluctuations are defined with right (Ψ) and left ($\bar{\Psi}$) moving modes $\Psi_A(k) = \phi_-(k_{FA} + k)$, $\bar{\Psi}_A(k) = \phi_-(-k_{FA} + k)$, $\Psi_B(k) = \phi_-(-k_{FB} + k)$ and $\bar{\Psi}_B(k) = \phi_-(k_{FB} + k)$. The field operators are introduced by $\psi_\nu(x) = \int \frac{dk}{2\pi} \Psi_\nu(k) e^{ikx}$ and $\bar{\psi}_\nu(x) = \int \frac{dk}{2\pi} \bar{\Psi}_\nu(k) e^{ikx}$. These field operators are related to lattice operators by

$$C(j) \rightarrow \lambda^A \psi_A(x) e^{ik_{FA}x} + \lambda^{A*} \bar{\psi}_A(x) e^{-ik_{FA}x} + \lambda^B \psi_B(x) e^{-ik_{FB}x} + \lambda^{B*} \bar{\psi}_B(x) e^{ik_{FB}x}, \quad (2)$$

where

$$\lambda^\nu = \begin{bmatrix} i \sin(\theta_\nu/2) \\ \cos(\theta_\nu/2) \end{bmatrix},$$

with $\theta_A = \theta(k_{FA})$ and $\theta_B = \theta(-k_{FB})$. The substitution in Eq. (2) and the energy linearization are valid for weakly interacting fermions at low temperature.

With polar molecules or atoms loaded on the sp -ladder, we include all momentum-independent interactions (momentum-dependent part is irrelevant in the Renormalization group flow [43]) allowed by symmetry. The Hamiltonian density of the interactions is given by

$$\mathcal{H}_{\text{int}} = \sum_{\nu\nu'} \frac{1}{2} g_4^{\nu\nu'} [J_\nu J_{\nu'} + \bar{J}_\nu \bar{J}_{\nu'}] + g_2^{\nu\nu'} J_\nu \bar{J}_{\nu'} + g_3 \{ \bar{\psi}_A^* \bar{\psi}_B \psi_A^* \psi_B + \bar{\psi}_B^* \bar{\psi}_A \psi_B^* \psi_A \}, \quad (3)$$

where $J_\nu =: \psi_\nu^* \psi_\nu$ and $\bar{J}_\nu =: \bar{\psi}_\nu^* \bar{\psi}_\nu$. For the symmetric case $t_s = t_p$, an Umklapp process

$$\mathcal{H}_{um} = g_u \{ \bar{\psi}_A^* \psi_A \psi_B^* \bar{\psi}_B + \bar{\psi}_B^* \psi_B \psi_A^* \bar{\psi}_A \} \quad (4)$$

becomes allowed for the reason that $k_{FA} + k_{FB} = \pi$. Since dipolar interactions between polar molecules or atoms decay as $1/r^3$, the leading interaction in the proposed double-well lattice setup [22] is

$$H_{\text{int}} = U \sum_j \left[a_s^\dagger(j) a_s(j) - \frac{1}{2} \right] \left[a_p^\dagger(j) a_p(j) - \frac{1}{2} \right].$$

The strength of U is tunable by changing the dipole moment, or by varying the distance between the shallow and deep wells (FIG. 1). By controlling this distance the leading interaction can be made significantly larger than sub-leading interactions (dipolar tails), which are neglected here.

In the weak interacting limit, the g -ology couplings are related to U by $g_4^{\nu\nu} = U$, $g_4^{AB} = g_4^{BA} = U \sin^2(\frac{\theta_A - \theta_B}{2})$, $g_2^{\nu\nu} = U \sin^2(\theta_\nu)$, $g_2^{AB} = g_2^{BA} = U$,

$$g_3 = U \sin(\theta_A) \sin(\theta_B),$$

and

$$g_u = U \cos(\theta_A) \cos(\theta_B),$$

at tree level [43]. Considering strong interactions or the finite ranged tail of dipolar interactions, the g -ology couplings will be renormalized due to neglected irrelevant couplings. By manipulating the direction of dipole moments with an external field, the interaction can be either repulsive or attractive (FIG. 1) [33, 35–37].

We follow the notation convention of Ref. [44], where the bosonization identity takes the form

$$\begin{aligned} \psi_\nu &= \frac{1}{\sqrt{2\pi}} \eta_\nu e^{-i\sqrt{\pi}(\varphi_\nu + \vartheta_\nu)} \\ \bar{\psi}_\nu &= \frac{1}{\sqrt{2\pi}} \bar{\eta}_\nu e^{i\sqrt{\pi}(\varphi_\nu - \vartheta_\nu)}, \end{aligned} \quad (5)$$

where η_ν is the Klein factor and ϑ_ν is the dual field of boson field φ_ν . The charge and orbital boson fields are further introduced here by $[\varphi_c, \varphi_o] = [\varphi_A, \varphi_B]T$, with the matrix T given by

$$T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}.$$

and their dual fields are $[\vartheta_c, \vartheta_o] = [\vartheta_A, \vartheta_B]T$. The Bosonized Hamiltonian density reads

$$\begin{aligned} \mathcal{H} &= \mathcal{H}_c + \mathcal{H}_o + \mathcal{H}_{\text{mix}}, \\ \mathcal{H}_c &= \frac{u_c}{2} \left[K_c \Pi_c^2 + \frac{1}{K_c} (\partial_x \varphi_c)^2 \right], \\ \mathcal{H}_o &= \frac{u_o}{2} \left[K_o \Pi_o^2 + \frac{1}{K_o} (\partial_x \varphi_o)^2 \right] \\ &\quad + \frac{1}{2\pi^2} \left[g_3 \cos(\sqrt{8\pi} \vartheta_o) + g_u \cos(\sqrt{8\pi} \varphi_o) \right], \\ \mathcal{H}_{\text{mix}} &= u_m \left[K_m \Pi_c \Pi_o + \frac{1}{K_m} (\partial_x \varphi_c) (\partial_x \varphi_o) \right], \end{aligned} \quad (6)$$

with

$$u_{\alpha=c/o} = \sqrt{(v_+ + \tilde{g}_4^{\alpha\alpha}/2\pi)^2 - (\tilde{g}_2^{\alpha\alpha}/2\pi)^2},$$

$$K_\alpha = \sqrt{\frac{2\pi v_+ + \tilde{g}_4^{\alpha\alpha} - \tilde{g}_2^{\alpha\alpha}}{2\pi v_+ + \tilde{g}_4^{\alpha\alpha} + \tilde{g}_2^{\alpha\alpha}}},$$

$$u_m = \sqrt{(v_- + \tilde{g}_4^{co}/2\pi)^2 - (\tilde{g}_2^{co}/2\pi)^2},$$

and

$$K_m = \sqrt{\frac{2\pi v_- + \tilde{g}_4^{co} - \tilde{g}_2^{co}}{2\pi v_- + \tilde{g}_4^{co} + \tilde{g}_2^{co}}},$$

where $v_+ = (v_{FA} + v_{FB})/2$, $v_- = (-v_{FA} + v_{FB})/2$ and the transformed coupling matrices $\tilde{g}_4$ and $\tilde{g}_2$ are given by $[\tilde{g}] = T^{-1}[g]T$. The mixing term $\mathcal{H}_{\text{mix}}$ vanishes for the symmetric case with $t_s = t_p$.

FIG. 2: (Color online) Sketch of the band structure of the sp -orbital ladder. Red dashed lines show the level of chemical potentials. (a), the symmetric case with $t_s = t_p$. (b), the asymmetric case with $t_s < t_p$. Here t_p is taken as the energy unit.

III. QUANTUM PHASES AND TRANSITIONS OF THE SYMMETRIC CASE

For the symmetric case with $t_s = t_p$ (FIG. 2), the Hamiltonian has an accidental Z_2 symmetry, $C_j \rightarrow (-1)^j \sigma_x C_j$ and Fermi momenta are related by $k_{FA} = \pi - k_{FB} \equiv k_F$. This Z_2 symmetry implies that $v_{FA} = v_{FB}$, $g_4^{AA} = g_4^{BB}$ and $g_2^{AA} = g_2^{BB}$. We find that the transformed coupling matrices $\tilde{g}_2$ and $\tilde{g}_4$ are diagonal and that the orbital-charge mixing term $\mathcal{H}_{\text{mix}}$ vanishes. In other words, the Z_2 symmetry guarantees orbital-charge separation. The charge part H_c is quadratic and the orbital part H_o is a sine-Gordon model [45].

With attraction, we have $K_o < 1$, $g_u > 0$, and the sine-Gordon term g_u is relevant (flows to $+\infty$) in the renormalization group (RG) flow [45]. This corresponds to an orbital gapped phase with $\cos(\sqrt{8\pi} \varphi_o)$ locked at -1 . In this phase, quantum fluctuations of φ_o become massive, and the divergent susceptibilities are the following: charge density wave (CDW) and PDW [38, 39, 41] given by the operators:

$$\begin{aligned} O_{\text{CDW}}(x) &= \psi_A^* \bar{\psi}_A e^{-2ik_{FA}x} - \psi_B^* \bar{\psi}_B e^{2ik_{FB}x} \\ &\propto e^{-2ik_F x} e^{i\sqrt{2\pi}\varphi_c} \sin(\sqrt{2\pi}\varphi_o) \end{aligned}$$

$$\begin{aligned} O_{\text{PDW}}(x) &= \psi_A \bar{\psi}_B e^{i(k_{FA} + k_{FB})x} + \psi_B \bar{\psi}_A e^{-i(k_{FA} + k_{FB})x} \\ &\propto (-1)^x e^{-i\sqrt{2\pi}\vartheta_c} \sin(\sqrt{2\pi}\varphi_o) \end{aligned}$$

Due to orbital-charge separation, the CDW and PDW correlation functions are readily given by

$$\langle O_{\text{CDW}}(x) O_{\text{CDW}}^\dagger(0) \rangle \propto e^{-2ik_F x} x^{-K_c}, \quad (7)$$

$$\langle O_{\text{PDW}}(x) O_{\text{PDW}}^\dagger(0) \rangle \propto (-1)^x x^{-1/K_c}. \quad (8)$$

Since $K_c > 1$ for attraction, the algebraic PDW order is dominant. In this phase, the superconducting pairing $\mathcal{O}_{\text{SC}} = a_s(j)a_p(j)$ oscillates in space with a period of π .

With repulsion, we have $K_o > 1$, and thus g_3 is relevant [45]. This gives an orbital gapped phase with $\cos(\sqrt{8\pi} \vartheta_o)$ locked at 1, because $g_3 < 0$. The fluctuations of ϑ_o are massive, and the divergent susceptibilities are ODW and superconducting SC^+ given by the operators:

$$\begin{aligned} O_{\text{ODW}}(x) &= e^{-i(k_{FA} - k_{FB})x} (\psi_A^* \bar{\psi}_B - \psi_B^* \bar{\psi}_A) \\ &\propto e^{-i(k_{FA} - k_{FB})x} e^{i\sqrt{2\pi}\varphi_c} \cos(\sqrt{2\pi}\vartheta_o) \\ O_{\text{SC}^+}(x) &= \psi_A \bar{\psi}_A + \psi_B \bar{\psi}_B \\ &\propto e^{-i\sqrt{2\pi}\vartheta_c} \cos(\sqrt{2\pi}\vartheta_o) \end{aligned}$$

Since $K_c < 1$ for repulsion, the dominant algebraic order here is ODW, for which the correlation function is given by

$$\langle O_{\text{ODW}}(x) O_{\text{ODW}}^\dagger(0) \rangle \propto e^{-i(k_{FA} - k_{FB})x} x^{-K_c}. \quad (9)$$

In the ODW phase, the particle-hole pairing in terms of lattice operators reads $\mathcal{O}_{\text{ODW}}(j) = C_j^\dagger \sigma_y C_j$. This

ODW order is incommensurate with an oscillation period $2\pi/(k_{FA} - k_{FB})$ in real space. If we go beyond the one-dimensional limit and consider small transverse tunnelings [22], a true long-range ODW order $\langle O_{ODW}(x) \rangle \propto e^{i(k_{FA} - k_{FB})x}$ is expected. Such an order breaks time-reversal symmetry.

The ODW and PDW phases predicted by Bosonization analysis are further verified in numerical simulations with matrix products state, in which open boundary condition is adopted. The superconducting correlation

$$C_{SC}(j' - j) = \langle a_p^\dagger(j) a_s^\dagger(j) a_s(j') a_p(j') \rangle$$

and the orbital density wave correlation

$$C_{ODW}(j' - j) = \langle C_j^\dagger \sigma_y C_j C_{j'}^\dagger \sigma_y C_{j'} \rangle$$

are calculated. In our calculation, the two points j and j' are 10 sites away from the boundaries to minimize the boundary effects. The convergence of these correlations is checked in numerical simulations. FIG. 3 shows the Fourier transform of these correlations, defined by $\mathcal{C}(k) = \sum_{j \neq 0} C(j) e^{-ikj}$, which approaches to its thermodynamic limit with increasing system size (FIG. 3). The sharp peaks of $\mathcal{C}_{SC}(k)$ at momenta $\pm\pi$ on the attractive side tell the quantum state has a PDW order shown in Eq. 8. On the repulsive side sharp dips of $\mathcal{C}_{ODW}(k)$ at finite momenta verify the incommensurate ODW order shown in Eq. 9. With numerical calculations, we also find the existence of PDW phase in the strongly attractive regime if $t_s \neq t_p$.

FIG. 3: (Color online) SC-PDW and CDW phases of the symmetric sp -orbital ladder with $t_s = t_p = 2t_{sp}$. $\mathcal{C}_{ODW}(k)$ and $\mathcal{C}_{SC}(k)$ show the Fourier transform of the orbital density wave and superconducting correlations, respectively. The numerical results are calculated for the system at two different sizes $L = 80$ and 100 at filling $\frac{1}{L} \sum_j \langle a_s^\dagger(j) a_s(j) + a_p^\dagger(j) a_p(j) \rangle = 0.7$. In the upper (lower) graph, the interaction $U = 3t_s$ ($U = -3t_s$).

IV. QUANTUM PHASES AND TRANSITIONS OF THE ASYMMETRIC CASE

For the asymmetric case— $t_s < t_p$ (FIG. 2), the Fermi velocity $v_{FB} > v_{FA}$ and the orbital-charge separation no longer holds. Thus, the orbital and charge degrees of freedom cannot be treated separately. The other difference with the symmetric case is that the Umklapp process g_u does not exist. Since the effects of g_4 couplings are just to renormalize the Fermi velocities [46–49]. For simplicity, we do not consider such effects and set $g_4^{\nu\nu'} = 0$ here. The one-loop RG equations are given by [49],

$$\begin{aligned} \frac{dg_2^{\nu\nu'}}{dl} &= \frac{g_3^2}{2\pi} \left[\frac{\delta_{\bar{\nu}\nu'}}{v_+} - \frac{\delta_{\nu\nu'}}{v_{F\bar{\nu}}} \right], \\ \frac{dg_3}{dl} &= \frac{g_3}{2\pi} \sum_{\nu} \left[\frac{g_2^{\nu\bar{\nu}}}{v_+} - \frac{g_2^{\nu\nu}}{v_{F\nu}} \right], \end{aligned} \quad (10)$$

where l is the flow parameter ($l \rightarrow \infty$) and $\bar{\nu} = A$ (B) for $\nu = B$ (A). The RG flow of the sine-Gordon term g_3 is obtained as

$$\begin{aligned} &\sqrt{|C|}/g_3(l) \\ &= F \left[-\text{sgn}(g_3 Y) \sqrt{\frac{2|C|D}{\pi v_+}} l + F^{-1} \left[\frac{\sqrt{|C|}}{g_3(0)} \right] \right], \end{aligned} \quad (11)$$

with

$$C = \frac{2v_{FA}v_{FB}v_+^2}{v_{FA}v_{FB} + v_+^2} \left[\frac{g_2^{AB}}{v_+} - \frac{g_2^{AA}}{2v_{FA}} - \frac{g_2^{BB}}{2v_{FB}} \right]^2 - g_3^2, \quad (12)$$

$$D = \frac{v_{FA}v_{FB} + v_+^2}{\pi v_{FA}v_{FB}v_+}, \quad (13)$$

and

$$Y = \frac{g_2^{AB}}{v_+} - \frac{g_2^{AA}}{2v_{FA}} - \frac{g_2^{BB}}{2v_{FB}}. \quad (14)$$

The function F is the hyperbolic function “sinh” (the trigonometric function “sin”) if $C > 0$ ($C < 0$). When $Y > 0$, g_3 always flows to ∞ and the system is in some gapped phase. When $Y < 0$, g_3 flows to ∞ only if $C < 0$. g_3 is irrelevant only if $C > 0$ and $Y < 0$. In the weak interacting regime, we have

$$Y/U = \frac{1}{v_+} - \frac{\sin^2(\theta_A)}{2v_{FA}} - \frac{\sin^2(\theta_B)}{2v_{FB}}. \quad (15)$$

We will consider the regime $Y/U > 0$ (this condition holds when t_{sp} is weak compared with $t_s + t_p$) in the following.

With repulsion ($U > 0$, $Y > 0$), g_3 is relevant and flows to $-\infty$ in RG flow. Then the dual orbital field ϑ_o is locked with $\cos(\sqrt{8\pi}\vartheta_o) = 1$ and its fluctuations ϑ_o are massive. The key effect of orbital-charge mixing

can be seen from its modification of the dynamics of the conjugate fields, given as

$$\Pi_{\vartheta_o} = \frac{K_o}{u_o} \partial_t \vartheta_o + \frac{K_o u_m}{K_m u_o} \partial_x \varphi_o, \quad (16)$$

$$\Pi_{\varphi_c} = \frac{1}{u_c K_c} \partial_t \varphi_c + \frac{K_m u_m}{K_c u_c} \partial_x \vartheta_o, \quad (17)$$

where Π_{ϖ} is the conjugate field of ϖ . The Lagrangian is constructed by

$$\mathcal{L}(x, t) = \Pi_{\vartheta_o} \partial_t \vartheta_o + \Pi_{\varphi_c} \partial_t \varphi_c - \mathcal{H}.$$

With massive fluctuations of ϑ_o integrated out, the Lagrangian of the charge field φ_c is given by

$$\mathcal{L}_c = \frac{1}{2\gamma} \left[\frac{1}{u} (\partial_t \varphi_c)^2 - u (\partial_x \varphi_c)^2 \right] + O((\partial \varphi_c)^4), \quad (18)$$

with the renormalized Luttinger parameter and sound velocity given by

$$\gamma = \frac{K_c}{\sqrt{1 - \frac{K_c K_o}{K_m^2} \frac{u_m^2}{u_c u_o}}}, \quad (19)$$

$$u = \sqrt{u_c^2 - u_m^2 \frac{u_c K_c K_o}{u_o K_m^2}}. \quad (20)$$

To zeroth order in the interaction U , the renormalized Luttinger parameter is

$$\gamma = \left[1 - \left(\frac{v_-}{v_+} \right)^2 \right]^{-1/2}. \quad (21)$$

Our result reproduces the perturbative result [49] when the orbital-charge mixing term is small. The diverging susceptibilities are ODW and SC^+ , and the corresponding correlation functions are given as

$$\langle O_{SC^+}(x) O_{SC^+}^\dagger(0) \rangle \propto x^{-1/\gamma}, \quad (22)$$

$$\langle O_{ODW}(x) O_{ODW}^\dagger(0) \rangle \propto e^{-i(k_{FA} - k_{FB})x} x^{-\gamma}. \quad (23)$$

With sufficiently weak repulsion $\gamma > 1$, the dominant order is SC^+ , of which the pairing in terms of lattice operators is $\mathcal{O}_{SC} = a_s(j) a_p(j)$. We emphasize here that this pairing does not oscillate in real space. Such a superconducting phase arising in the repulsive regime results from that charge mode φ_c is coupled with the orbital mode φ_o , which is strongly fluctuating with its conjugate field ϑ_o pinned. The sine-Gordon term g_3 causing this pinning effect is finite only when the coupling of sp -orbitals t_{sp} is finite, and g_3 is monotonically increasing when t_{sp} is increased. Thus the transition temperature of this repulsive superconducting phase can be increased by tuning t_{sp} , which makes this exotic superconducting phase potentially realizable in experiments. With stronger repulsion, the renormalized Luttinger parameter γ decreases. Eventually with repulsion larger than

some critical strength, we have $\gamma < 1$, and the repulsive superconducting phase gives way to the ODW phase.

With attractive interaction, the condition $Y/U > 0$ gives $Y < 0$. Thus g_3 is relevant and flows to $+\infty$ when $C < 0$. The sine-Gordon term $\cos(\sqrt{8\pi}\vartheta_o)$ is locked at -1 , and the dominant order is superconducting SC^- , given by

$$O_{SC^-} = \psi_A \bar{\psi}_A - \psi_B \bar{\psi}_B \propto e^{-i\sqrt{2\pi}\vartheta_c} \sin(\sqrt{2\pi}\vartheta_o). \quad (24)$$

In numerical simulations we find the SC^- phase competing with PDW in the strongly attractive regime. When g_3 is irrelevant ($C > 0$, $Y < 0$), the orbital ladder is in a two component Luttinger liquid phase exhibiting two gapless normal modes and each mode is a mixture of orbital and charge.

-symmetric $t_s = t_p$

-asymmetric $t_s < t_p$

FIG. 4: Schematic phase diagram of the sp -orbital ladder. In the asymmetric case, the condition that g_3 is relevant is taken (see the main text). The superconducting SC^+ phase appears in the repulsive regime due to quantum fluctuations.

Discussion of dipolar tails.— In Ref. [50], it is shown that the tail of dipolar interaction makes correlations decay as $1/r^3$ in the gapped phase, and that the tail does not change the critical properties at the critical point. The present study finds Luttinger liquid phases, which are critical. The power law correlations characterizing our predicted critical phases decay much slower than $1/r^3$. For example, with weak repulsive interaction, the scaling exponent for the SC^+ phase, $1/\gamma$, is found less than 1 from Eq. (21), well below the dipolar exponent 3. We thus conclude that the dipolar tail corrections should be negligible.

V. CONCLUSION

To conclude, we have studied quantum phases of a one-dimensional sp -coupled interacting Fermi gas, with both numeric and analytic methods. A PDW phase, featuring oscillating Cooper pair field, shows up naturally in the

attractive regime. An incommensurate ODW phase is found in the repulsive regime. A repulsive superconducting phase emergent from orbital-charge mixing is also discussed. In experiments, radio-frequency spectroscopy can be used to probe spectra functions [51, 52], which exhibit the signatures of pairings of the predicted phases. In orbital density wave phases, where there are diverging correlations $\langle C_j^\dagger \sigma_y C_j C_{j'}^\dagger \sigma_y C_{j'} \rangle$, the quench dynamics of occupation numbers of s and p orbitals is a probe of such orders [53].

Acknowledgement. We appreciate the very helpful discussions with A. Daley, E. Fradkin and Z. Nussinov, A. Paramekanti and E. Zhao. This work is supported by A. W. Mellon Fellowship (X.L.), NSF (PHY11-25915) (X.L.), AFOSR (FA9550-12-1-0079) (W.V.L.), ARO (W911NF-11-1-0230) and ARO-DARPA-OLE (W911NF-07-1-0464) (X.L. and W.V.L.), the National Basic Research Program of China (Grant No 2012CB922101), and Overseas Scholar Program of NSF of China (11128407) (W.V.L.).

-
- [1] M. A. Lewenstein and W. V. Liu, *Nature Physics* **7**, 101 (2011).
 - [2] Y. Tokura and N. Nagaosa, *Science* **288**, 462 (2000).
 - [3] A. Isacsson and S. M. Girvin, *Phys. Rev. A* **72**, 053604 (2005).
 - [4] W. V. Liu and C. Wu, *Phys. Rev. A* **74**, 013607 (2006).
 - [5] A. B. Kuklov, *Phys. Rev. Lett.* **97**, 110405 (2006).
 - [6] L.-K. Lim, C. M. Smith, and A. Hemmerich, *Phys. Rev. Lett.* **100**, 130402 (2008).
 - [7] X. Li, Z. Zhang, and W. V. Liu, *Phys. Rev. Lett.* **108**, 175302 (2012).
 - [8] G. Wirth, M. Ölschläger, and A. Hemmerich, *Nature Physics* **7**, 147 (2011).
 - [9] P. Soltan-Panahi, D.-S. Lühmann, J. Struck, P. Windpassinger, and K. Sengstock, *Nature Physics* **8**, 71 (2012).
 - [10] I. Bloch, J. Dalibard, and W. Zwerger, *Rev. Mod. Phys.* **80**, 885 (2008).
 - [11] M. Lewenstein, A. Sanpera, and V. Ahufinger, *Ultracold atoms in optical lattices: Simulating Quantum Many-body systems* (Oxford University Press, Oxford, 2012).
 - [12] T. Rom, T. Best, D. van Oosten, U. Schneider, S. Fölling, B. Paredes, and I. Bloch, *Nature* **444**, 733 (2006).
 - [13] A. Moreo and D. J. Scalapino, *Phys. Rev. Lett.* **98**, 216402 (2007).
 - [14] A. F. Ho, M. A. Cazalilla, and T. Giamarchi, *Phys. Rev. A* **79**, 033620 (2009).
 - [15] X.-J. Liu, M. F. Borunda, X. Liu, and J. Sinova, *Phys. Rev. Lett.* **102**, 046402 (2009).
 - [16] Y.-J. Lin, R. L. Compton, K. Jimenez-Garcia, J. V. Porto, and I. B. Spielman, *Nature* **462**, 628 (2009).
 - [17] P. Wang, Z.-Q. Yu, Z. Fu, J. Miao, L. Huang, S. Chai, H. Zhai, and J. Zhang, *Phys. Rev. Lett.* **109**, 095301 (2012).
 - [18] L. W. Cheuk, A. T. Sommer, Z. Hadzibabic, T. Yefsah, W. S. Bakr, and M. W. Zwierlein, *Phys. Rev. Lett.* **109**, 095302 (2012).
 - [19] J.-Y. Zhang, S.-C. Ji, Z. Chen, L. Zhang, Z.-D. Du, B. Yan, G.-S. Pan, B. Zhao, Y.-J. Deng, H. Zhai, et al., *Phys. Rev. Lett.* **109**, 115301 (2012).
 - [20] L. Tarruell, D. Greif, T. Uehlinger, G. Jotzu, and T. Esslinger, *Nature* **483**, 302 (2012).
 - [21] M. Ölschläger, G. Wirth, T. Kock, and A. Hemmerich, *Phys. Rev. Lett.* **108**, 075302 (2012).
 - [22] X. Li, E. Zhao, and W. V. Liu, *Nature Commun* **4**, 1523 (2013).
 - [23] E. Zhao and W. V. Liu, *Phys. Rev. Lett.* **100**, 160403 (2008).
 - [24] C. Wu, *Phys. Rev. Lett.* **100**, 200406 (2008).
 - [25] V. M. Stojanović *et al*, *Phys. Rev. Lett.* **101**, 125301 (2008).
 - [26] C. Wu, *Mod. Phys. Lett. B* **23**, 1 (2009).
 - [27] H.-H. Hung, W.-C. Lee, and C. Wu, *Phys. Rev. B* **83**, 144506 (2011).
 - [28] Z. Cai, Y. Wang, and C. Wu, *Phys. Rev. A* **83**, 063621 (2011).
 - [29] Q. Zhou, J. V. Porto, and S. Das Sarma, *Phys. Rev. B* **83**, 195106 (2011).
 - [30] K. Sun, W. V. Liu, A. Hemmerich, and S. Das Sarma, *Nature Phys.* pp. 67–70 (2012).
 - [31] H.-Y. Hui, R. Barnett, J. V. Porto, and S. Das Sarma, *Phys. Rev. A* **86**, 063636 (2012).
 - [32] Z. Zhang *et al*, *Phys. Rev. A* **82**, 033610 (2010).
 - [33] K.-K. Ni, S. Ospelkaus, M. H. G. de Miranda, A. Pe'er, B. Neyenhuis, J. J. Zirbel, S. Kotochigova, P. S. Julienne, D. S. Jin, and J. Ye, *Science* **322**, 231 (2008).
 - [34] M. Lu, S. H. Youn, and B. L. Lev, *Phys. Rev. Lett.* **104**, 063001 (2010).
 - [35] M. H. G. de Miranda, A. Chotia, B. Neyenhuis, D. Wang, G. Quemener, S. Ospelkaus, J. L. Bohn, J. Ye, and D. S. Jin, *Nat Phys* **7**, 502 (2011).
 - [36] A. Chotia, B. Neyenhuis, S. A. Moses, B. Yan, J. P. Covey, M. Foss-Feig, A. M. Rey, D. S. Jin, and J. Ye, *Phys. Rev. Lett.* **108**, 080405 (2012).
 - [37] C.-H. Wu, J. W. Park, P. Ahmadi, S. Will, and M. W. Zwierlein, *Phys. Rev. Lett.* **109**, 085301 (2012).
 - [38] E. Berg, E. Fradkin, and S. A. Kivelson, *Nat Phys* **5**, 830 (2009).
 - [39] A. Jaefari and E. Fradkin, *Phys. Rev. B* **85**, 035104 (2012).
 - [40] R. Wei and E. J. Mueller, *Phys. Rev. Lett.* **108**, 245301 (2012).
 - [41] N. J. Robinson, F. H. L. Essler, E. Jeckelmann, and A. M. Tsvelik, *Phys. Rev. B* **85**, 195103 (2012).
 - [42] B. K. Stuhl, M. T. Hummon, M. Yeo, G. Quemener, J. L. Bohn, and J. Ye, *Nature* **492**, 396 (2012).
 - [43] R. Shankar, *Rev. Mod. Phys.* **66**, 129 (1994).
 - [44] D. Sénéchal, *arXiv:cond-mat/9908262* (1999).
 - [45] T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, Oxford, 2003).
 - [46] M. Fabrizio, *Phys. Rev. B* **48**, 15838 (1993).
 - [47] L. Balents and M. P. A. Fisher, *Phys. Rev. B* **53**, 12133 (1996).
 - [48] H.-H. Lin, L. Balents, and M. P. A. Fisher, *Phys. Rev. B* **56**, 6569 (1997).
 - [49] U. Lederhann and K. Le Hur, *Phys. Rev. B* **61**, 2497 (2000).
 - [50] X.-L. Deng, D. Porras, and J. I. Cirac, *Phys. Rev. A* **72**,

- 063407 (2005).
- [51] J. T. Stewart, J. P. Gaebler, and D. S. Jin, *Nature* **454**, 744 (2008).
- [52] Q. Chen and K. Levin, *Phys. Rev. Lett.* **102**, 190402 (2009).
- [53] M. Killi, S. Trotzky, and A. Paramekanti, *Phys. Rev. A* **86**, 063632 (2012).