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Peter Scherpelz, Dan Wulin, K. Levin, and A. K. Rajagopal

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Pseudogap Effects of Fermi Gases in the Presence of A Strong Effective Magnetic Field

Peter Scherpelz,[1] Dan Wulin,[1] K. Levin,[1] and A. K. Rajagopal[2,3,4]

[1] *James Franck Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637, USA*
[2] *Inspire Institute Inc., Alexandria, Virginia 22303, USA*
[3] *Harish-Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad, 211019, India*
[4] *Department of Materials Science & Engineering, Northwestern University, Evanston, IL 60208, USA*

We address the important question of how to characterize the normal state and the nature of non-condensed pairs in fermionic superfluids under the influence of a strong effective magnetic field. In ultracold gases, the magnetic field is implemented through rapid rotation or novel artificial field techniques. We consider the near unitary regime, where non-condensed pairs are likely to be present at temperatures above the condensation temperature T_c. We show (based on Gor'kov theory) that these pairs are associated with a precursor of a vortex configuration. Importantly, this non-uniform normal state appears to enable "Bose condensation" in a field which is otherwise problematic due to the effective one-dimensionality of Landau level dispersion.

The goal of the present paper is to study the nature and role of non-condensed pairs in fermionic superfluids under the influence of a strong effective magnetic field. This is a topic which has not been clearly addressed in the literature. We show here, as might be expected, that in anticipation of the superfluid vortex configuration, these non-condensed pairs exhibit some degree of inhomogeneity. More specifically, the non-condensed pairs occupy small distortions of the condensate vortex configuration. Our study, which focuses on the normal (or pseudogap) phase, above the transition $T_c(H)$, should lead to a reconsideration of previous work on rotating cold Fermi gases [1] where the upper critical rotation frequency was computed under the assumption that non-condensed pairs were not present at T_c, even in the unitary regime. Moreover, these inhomogeneous bosonic states may be relevant to "normal state vortex" models [2, 3] which are argued to be important in high temperature superconductivity.

This work bears on a central and puzzling aspect of superconductivity in high magnetic fields, even within the BCS framework. As has been emphasized in the literature, the degeneracy of Landau levels implies that the fluctuations around the BCS phase are effectively one-dimensional [4]; this is well known [5] to be problematic for stable superconductivity. By introducing non-condensed pairs associated with a finite and inhomogeneous pairing gap (or "pseudogap") at the onset of condensation, a strict "dimensional reduction"[4] is no longer present.

In this paper we show that the non-condensed pairs restore three-dimensional behavior to the normal state system, which is in turn necessary for the system to be able to condense. Thus, we demonstrate a connection between normal-state inhomogeneity and condensation, both of which arise from non-condensed pairs in this approach. Our approach is to be contrasted with previous attempts [6] to address the similar concerns raised by Schafroth [5] in Bose gases. Moreover, our ideas relate to earlier work on pair density waves [7], but are in contrast to previous studies on the interplay of a pseudogap and magnetic field [8] where density inhomogeneities were not contemplated.

Finally, an additional motivation for this paper is to stimulate experimental searches for predicted precursors of the below-T_c vortex configurations. We note the community excitement over proposals to establish strong effective magnetic fields through "artificial" means [9], [10, 11]. This makes our work on the interplay of rapid rotation and superfluidity particularly topical.

Landau-Ginzburg (LG) theory provides physical insight into non-condensed pairs. In zero magnetic field they are associated with finite center of mass momentum $\boldsymbol{q}$, while in non-zero field, the natural counterpart should be associated with slightly distorted configurations of the superconducting state. In both cases these represent gapless excitations (as $\boldsymbol{q} \to 0$, or as their configuration approaches that having lowest energy). These non-condensed pairs are distinct from previously considered Landau-Ginzburg vortex lattice fluctuations which only address the condensate [12, 13].

Rewriting the Gor'kov Equations We next show how a BCS-type theory allows characterization of both condensed and non-condensed pairs in a rather parallel fashion. The real-space Gor'kov coupled equations for the gap $\Delta(\boldsymbol{r})$ and the fermionic Green's function $G(\boldsymbol{r}, \boldsymbol{r}'; i\omega)$ are:

$$G(\boldsymbol{r}, \boldsymbol{r}'; i\omega) = G^0(\boldsymbol{r}, \boldsymbol{r}'; i\omega) - \int d\boldsymbol{r}'' d\boldsymbol{r}''' G^0(\boldsymbol{r}, \boldsymbol{r}''; i\omega) \times \Delta(\boldsymbol{r}'') G^0(\boldsymbol{r}''', \boldsymbol{r}''; -i\omega) \Delta^\dagger(\boldsymbol{r}''') G(\boldsymbol{r}''', \boldsymbol{r}'; i\omega) \quad (1)$$

$$\Delta^\dagger(\boldsymbol{r}) = \frac{g}{\beta} \sum_{i\omega} \int d\boldsymbol{r}' G(\boldsymbol{r}', \boldsymbol{r}; i\omega) G^0(\boldsymbol{r}', \boldsymbol{r}; -i\omega) \Delta^\dagger(\boldsymbol{r}') \quad (2)$$

Throughout this paper $i\omega$ ($i\Omega$) will be used to denote discrete fermionic (bosonic) Matsubara frequencies, with the traditional subscripts omitted for clarity. Introducing a Landau level basis for the fermions indexed by $m = (N, p, k_z)$ where N is the Landau level, p the degenerate Landau level index, and k_z the momentum parallel to the magnetic field, we write the bare Green's function $G^0(\boldsymbol{r}, \boldsymbol{r'}; i\omega) = \sum_n \psi_n(\boldsymbol{r})\psi_n^\dagger(\boldsymbol{r'})/(i\omega - \xi_n)$ where ξ_n is the single-particle energy; the dressed Green's function is $G(\boldsymbol{r}, \boldsymbol{r'}; i\omega) = \sum_{mm'} G_{mm'}(i\omega)\psi_m(\boldsymbol{r})\psi_{m'}^\dagger(\boldsymbol{r'})$ [14]. The self energy, given by $\Sigma(\boldsymbol{r}, \boldsymbol{r'}; i\omega) = -\Delta(\boldsymbol{r})\Delta^\dagger(\boldsymbol{r'})G_0(\boldsymbol{r'}, \boldsymbol{r}; -i\omega)$, is rewritten as $\Sigma_{mm'}(i\omega) = -\sum_n G_n^0(-i\omega)\Delta_{mn}\Delta_{m'n}^\dagger$, and the number equation necessary for a self-consistent solution is $N = \frac{2}{\beta}\sum_{m,i\omega} G_{mm}(i\omega)$. Defining $\Delta_{mn} \equiv \int d\boldsymbol{r}\Delta(\boldsymbol{r})\psi_m^\dagger(\boldsymbol{r})\psi_n^\dagger(\boldsymbol{r})$, and integrating over position variables yields

$$G_{mm'}(i\omega) = G_m^0(i\omega)\delta_{mm'} - \sum_{ln} G_m^0(i\omega)\Delta_{ml}G_l^0(-i\omega)\Delta_{ln}^\dagger G_{nm'}(i\omega). \quad (3)$$

$$1 = \frac{g}{\beta}\sum_{i\omega}\sum_{mm'n}\frac{\Delta_{m'n}\Delta_{mn}^\dagger}{\int d\boldsymbol{r}\,|\Delta(\boldsymbol{r})|^2} G_{mm'}(i\omega)G_n^0(-i\omega). \quad (4)$$

Eq. (4) is equivalent to that found elsewhere in Refs. [15, 16]. We stress that the nonlinear gap equation of Eq. (4) applies to all $T \le T_c$, and that at this point we have solely rewritten the Gor'kov equations in a Landau level basis, with no further approximation.

Characterizing Non-condensed Pairs Except in the weak-coupling limit, where all pairing is only in the condensate, pairs may form in kinetically excited states [17]. They appear above the superfluid transition temperature T_c, corresponding to a pseudogap phase, and should also persist below the transition. Associated with the z-direction, parallel to the field, excited pairs may have non-zero total momentum $q_z = k_{z_1} + k_{z_2}$. The perpendicular co-ordinates, however, are characterized by Landau level indices. Both condensed and non-condensed pairs may lie in the same Landau level. We parameterize different real-space gap configurations $\Delta(\boldsymbol{r})$ by ζ and denote the condensate configuration as ζ_0. In general non-condensed pairs will occupy configurations for which ζ is close to ζ_0. To transform to state-space, we can associate with each distortion a normalized real-space gap configuration $\Delta^0(\boldsymbol{r}, \zeta)$, where $\int d\boldsymbol{r}\,\left|\Delta^0(\boldsymbol{r}, \zeta)\right|^2 = 1$, from which $\Delta_{mn}^0(\zeta) \equiv \int d\boldsymbol{r}\Delta^0(\boldsymbol{r}, \zeta)\psi_m^\dagger(\boldsymbol{r})\psi_n^\dagger(\boldsymbol{r})$ can also be calculated.

We now rewrite the gap equation, Eq. (4), by defining a pair susceptibility $\chi(\zeta, q_z; i\Omega) \equiv$

$$\frac{1}{\beta}\sum_{i\omega,m,m',n}\Delta_{mn}^0(\zeta)\Delta_{nm'}^{0\dagger}(\zeta)G_{mm'}(i\omega)G_n^0(i\Omega - i\omega) \quad (5)$$

so that Eq. (4) assumes a simple form

$$1 + g\chi(\zeta_0, 0; 0) = 0. \quad (6)$$

Equation (6) of Gor'kov theory suggests that there is a specifically defined t-matrix (or a summation of particle-particle ladder diagrams), which is related to the pair susceptibility in Eq. (5) and which diverges at and below T_c. More precisely [18] one can write this t-matrix as

$$t^{\mathrm{pg}}(\zeta, q_z; i\Omega) \equiv \frac{g}{1 + g\chi(\zeta, q_z; i\Omega)}. \quad (7)$$

Feedback effects from this t-matrix will lead to a pseudogap self energy contribution [18] (in parallel with what is found in Gor'kov theory for the condensate self energy and associated condensate t-matrix $t^{\mathrm{sc}}(\zeta, q_z; i\Omega) \equiv -\delta(\zeta - \zeta_0)\delta(q_z)\delta(i\Omega)\int d\boldsymbol{r}\,|\Delta^{\mathrm{sc}}(\boldsymbol{r})|^2$, which leads to $\Sigma_{mm'}(i\omega)$ above). Here $\Sigma_{mm'}^{\mathrm{pg}}(i\omega) =$

$$\frac{1}{\beta}\sum_{\zeta,q_z,i\Omega,n}\Delta_{mn}^0(\zeta)\Delta_{nm'}^{0\dagger}(\zeta)t^{\mathrm{pg}}(\zeta, q_z; i\Omega)G_n^0(i\Omega - i\omega). \quad (8)$$

It is this self energy which enters the dressed Green's function as well as the pair susceptibility; in this way it yields a self-consistently determined t-matrix. Of particular interest here is the nature of the inhomogeneous pairing gap in the unusual (pseudogapped) normal phase which is contained in the self energy.

Structure of the Vortex Lattice We emphasize that other than starting with the Gor'kov equations in real space which circumscribe the form of a t-matrix, the only approximations made here are to associate this t-matrix with a self energy and Green's function for the low-energy noncondensed pairs [17]. We next address the robust *qualitative* phenomena that result from this system of equations.

As $t^{\mathrm{pg}}(\zeta_0, 0; 0)$ diverges at and below T_c, we expect that even somewhat above T_c, $t^{\mathrm{pg}}(\zeta, q_z; i\Omega)$ will be strongly peaked for $(\zeta, q_z; i\Omega) \approx (\zeta_0, 0; 0)$. Because t^{pg} enters directly into the self-energy, the result is that fermions will tend to pair in states close to ζ_0, as those contributions to the self-energy will dominate. If we were to anticipate the real-space gap structure associated with these non-condensed pairs, we can expect it to reflect a sum of the $|\Delta(\boldsymbol{r}, \zeta)|^2$ functions, weighted by the $|t^{\mathrm{pg}}|$. The latter parameter, as defined in Eq. (7), roughly represents the number of non-condensed pairs. The resulting real-space structure is presented in Fig. 1. More concretely, any pinning or symmetry breaking, such as an impurity or trap center, will necessarily reveal this real-space gap structure which reflects the state-space peak present in t^{pg} and Σ^{pg}.

We may also arrive at the same qualitative figure from a different point of view. If the lowest Landau level of the non-condensed pairs were truly degenerate, condensation would not occur due to the one-dimensionality of the system. Condensation requires a broadening of the Landau levels, which can be interpreted as causing t^{pg} (which represents a statistical occupation of pair states) in Eq. (7) to form a narrow peak around the preferred superconducting vortex configuration. This in turn necessitates the above-T_c real-space structure. We note the nonlinearity in Eq. (5) allows us to incorporate the inhomogeneity of the pairing gap (recall G depends on Δ). This Gor'kov-based approach should be contrasted with

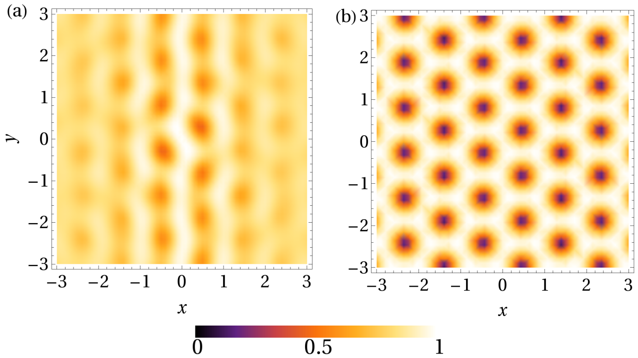

FIG. 1. (Color online) **(a)**: A density plot of the total squared energy gap $|\Delta^{\rm pg}|^2$ (normalized to its largest value) in real space, corresponding to the same parameters as in Fig. 3c. This calculation is done through a discrete sampling of 1,360 points in ζ-space, and in both plots the axes are given in units of $x_0 = \sqrt{\pi} l_H$. **(b)**: A density plot of the condensate energy gap $|\Delta(\boldsymbol{r}, \zeta_0)|^2$ only, which corresponds to the total energy gap at zero temperature, again normalized to its largest value.

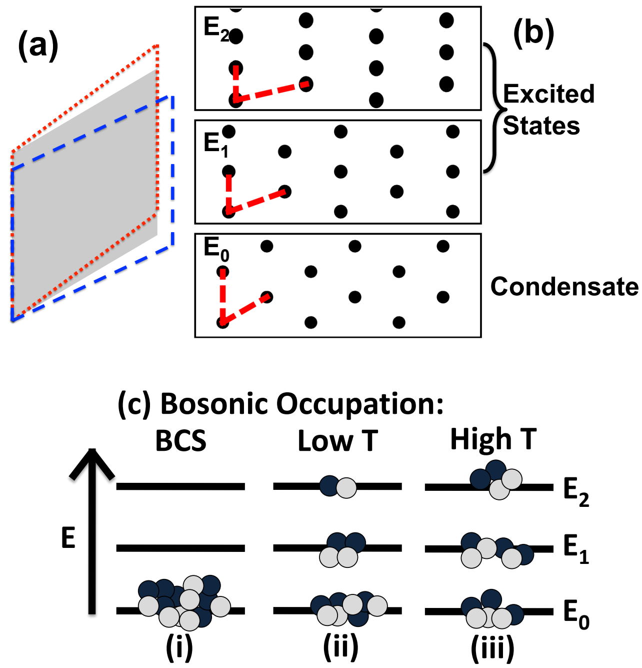

FIG. 2. (Color online) A diagram of the different distortions of the Abrikosov lattice, the resulting splitting in energy levels, and the different occupation statistics that result. **(a)**: The shaded gray unit cell is the optimal lattice configuration, whereas the red dotted unit cell corresponds to an excitation of b_y/a while the blue dashed unit cell corresponds to an excitation of b_x/a. **(b)**: Real-space diagrams of three different values for b_x/a, showing zeroes of $\Delta^0(\boldsymbol{r})$ for each configuration (black circles) and lattice vectors (dashed red lines). E_0 is the optimal configuration, while E_1 and E_2 are progressively higher in energy. **(c)**: The pairs of dark blue (spin up) and light gray (spin down) fermions are now able to occupy a continuum of energy levels corresponding to different lattice configurations. For the three displayed configurations, **(i)**: The BCS approach results in occupation of only the optimal configuration (also the case in this system at $T = 0$). In contrast, in this system for $T \neq 0$, higher energy levels can also be occupied by pairs. **(ii)**: An example of occupation statistics at a low temperature – most pairs are in E_0. **(iii)**: At a higher temperature, more pairs are in excited states.

approaches [19, 20] which consider two bare Green's functions at the instability onset temperature.

Calculation We turn now to more precise calculations and introduce some simplifications which help quantitatively establish and characterize a vortex-like structure above T_c. First, we restrict our consideration to intra-Landau level pairing [15, 16, 18]. We also presume that the lowest-energy state ζ_0 is a triangular Abrikosov lattice, as in the free system. Mathematically, we use the Landau gauge $\boldsymbol{A} = (0, H\hat{\boldsymbol{x}}, 0)$ and an Abrikosov lattice with unit vectors $\boldsymbol{a} = (0, a, 0)$ and $\boldsymbol{b} = (b_x, b_y, 0)$, where $ab_x = \pi l_H^2$ with $l_H = \sqrt{\hbar c/eH}$ the magnetic Hall length.

We emphasize that these non-essential assumptions allow us to arrive at a tractable scheme and were shown to be a good approximation in the high-field regime [18, 21]. Non-condensed pairs will occupy a two-dimensional continuum of other Abrikosov lattice configurations. The two mean-field distortions available to excited pairs are associated with changing b_x/a, and b_y/a (see Fig. 2) [22]. Both of these are higher in energy relative to the optimal Abrikosov lattice. We then follow Ref. [22] in setting $\zeta = b_y/a + ib_x/a$, for which the optimal triangular lattice condensate configuration is $\zeta_0 = 1/2 + i\sqrt{3}/2$.

Because of the mixing through the self-energy of an infinite number of real-space gap configurations, the calculation of the pair susceptibility χ is not analytically tractable. To make progress without the distraction of heavy numerics, we approximate the pair susceptibility as $\chi(\zeta, q_z; i\Omega) \approx$

$$\frac{1}{\beta} \sum_{i\omega} \sum_{m,m'} \phi^2_{mm'}(\zeta) \tilde{G}_{mm'}(\zeta; i\omega) G^0_N(q_z - k_z; i\Omega - i\omega) \quad (9)$$

where we have, in effect, decomposed $G_{mm'}$ into separate contributions $\tilde{G}_{mm'}$, each associated with a distinct lattice structure. In Eq. (9), G^0 is written in terms of the Landau level N and z-momentum of $m = (N, p, k_z)$ (with $N_m = N_{m'}$ and $k_{z_m} = k_{z_{m'}}$) and $\phi^2_{mm'}(\zeta) = \sum_n \Delta^0_{mn} \Delta^{0\dagger}_{nm'}$.

In a diagonal pairing scheme [16], we find that $\tilde{G}_{mm}(\zeta, i\omega) =$

$$(i\omega + \xi_m) / \left[(i\omega)^2 - \xi_m^2 - |\Delta^{\rm pg}|^2 \phi^2_{mm}(\zeta) \right]. \quad (10)$$

To establish the internal consistency of these results we have used the fact that the small chemical potential of the pairs implies that $t^{\rm pg}$ is strongly peaked around $(q_z, i\Omega) = (0, 0)$ near and below T_c.[23] This leads to the Green's function $\tilde{G}$ in Eq. (10) which has a familiar BCS form with the gap function of the non-condensed pairs given by $|\Delta^{\rm pg}|^2 \equiv -\frac{1}{\beta} \sum_{\zeta, q_z, i\Omega} t^{pg}(\zeta, q_z; i\Omega)$.

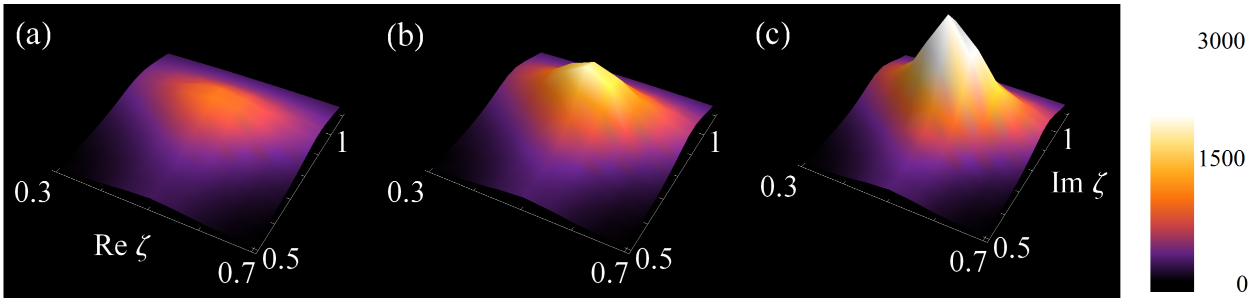

FIG. 3. (Color online) A three-dimensional plot of $|t^{\rm pg}(\zeta, 0; 0)|$ against (dimensionless) lattice configurations ζ. All values are normalized to a constant fermionic chemical potential above the lowest Landau level ($\mu - \hbar\omega_c/2 = 1$), and each plot has different $(\Delta^{\rm pg}, T)$: **(a)** $(5.75, 1.1118)$; **(b)** $(5.90, 1.0480)$; **(c)** $(6.00, 1.00)$. After choosing $\Delta^{\rm pg}$ and T for **(c)**, g is chosen such that in **(c)**, $|g^{-1}| - \chi(0, 0; 0) = 3 \times 10^{-4}$, which puts **(c)** just slightly above T_c. The other two plots share the same interaction g, but have $\Delta^{\rm pg}$ and T chosen to represent a similar system further from T_c. This is done by choosing a progressively smaller $\Delta^{\rm pg}$ and solving for T such that the minimum $|t^{\rm pg}|$ is the same in all three plots. The fermions lie in the lowest Landau level for simplicity.

In effect, the main approximation we have made is to consider each vortex configuration as an "independent system," sharing self-consistently the same magnitude of the energy gap $|\Delta^{\rm pg}|^2$, but possessing a distinct real-space gap parameter $\Delta^0(\boldsymbol{r}, \zeta)$ which in turn provides a unique form factor $\phi^2_{mm'}(\zeta)$ in the self-energy. This is similar in approach to considering an LG energy functional for the different forms of the gap parameter.

In our calculations we use the magnetic translation group (MTG) basis [16, 24, 25], although we have also explored an alternative orbit-center based pairing basis [15, 18]. MTG pairing relates to the Abrikosov lattice and is associated with a Bloch-like index $\boldsymbol{k} = (k_x, k_y)$. The unit cell for the MTG has unit vectors $2\boldsymbol{a}$ and $\boldsymbol{b}$, which means in turn that the state basis is dependent on ζ. Pairing for the ζ appropriate to the basis occurs with a single partner between opposite $\boldsymbol{k}$ [16], i.e. $\Psi^{\rm pair}_{N,\boldsymbol{k},q_z}(\boldsymbol{r}) = \psi^{\rm fermion}_{N,\boldsymbol{k},k_z,\uparrow}(\boldsymbol{r}) \psi^{\rm fermion}_{N,-\boldsymbol{k},-k_z+q_z,\downarrow}(\boldsymbol{r})$. $\Delta^0_{mn}(\zeta)$ for these pairs is computed elsewhere [16, 18].

Results In Fig. 3 we illustrate how superfluidity with pre-formed pairs takes place in the presence of a high effective magnetic field. We plot the t-matrix vs. lattice configuration ζ for three different sets of effective temperatures, demonstrating that as condensation is approached from higher temperatures, the occupation of lattice states near the ideal triangular Abrikosov lattice ($\zeta = \zeta_0$) begins to peak. Precisely at $T = T_c$, a delta function results at $\zeta = \zeta_0$. Nevertheless at the transition there is still considerable weight associated with other lattice configurations reflecting the fact that the pseudogap $|\Delta^{\rm pg}|^2$ remains finite. The condensate contribution corresponding to a perfect triangular lattice necessarily has small weight near T_c.

Of particular interest is the real space reflection of these distorted Abrikosov lattice contributions. To illustrate this precursor vortex configuration we evaluate

$$|\Delta^{\rm pg}(\boldsymbol{r})|^2 = \frac{1}{\beta} \sum_{\zeta, \boldsymbol{q}_z, i\Omega} t^{\rm pg}(\zeta, q_z; i\Omega) \left|\Delta^0(\boldsymbol{r}, \zeta)\right|^2, \qquad (11)$$

which is a weighted average of the gap (squared). This is compared with the counterpart for a fully condensed system in Fig. 1. By addressing the square of the gap, we emphasize that there is no phase information in the normal state pseudogap. It should be noted that the point $\boldsymbol{r} = 0$ is chosen as a point of "symmetry breaking" for the translational symmetry available in the selection of each $\Delta^0(\boldsymbol{r}, \zeta)$.

The state-space occupation in Fig. 3 is also responsible for the ability of the system to condense. Above T_c, the dispersion is no longer one-dimensional due to this breaking of the Landau level degeneracy for pairs, and the resulting inhomogeneous occupation of the available pairing states. Such inhogomgeneity changes the system from the system considered in earlier approaches [5] which cannot condense.

Implications for Bosonic Systems Our results have addressed fermionic gases near unitarity. Within Gor'kov-based theories, extended to incorporate BCS-BEC crossover, the endpoint is a fermionic BEC which becomes noninteracting in the strict BEC limit [17, 26]. The effect described here depends on interfermion interactions. Thus, a fermionic BEC of this type does not yield a suitable starting point for a treatment of interacting bosons.

However, after the submission of this paper, we became aware of more recent studies addressing non-condensed bosons below T_c in rotating bosonic systems with a vortex lattice [27]. There it is claimed that "Adding quantum fluctuations to the mean field theory leads to excitations of neighboring k-states which correspond to vortex lattices that are offset with respect to the macroscopically occupied one. We can say that quantum fluctuations smear the original mean field lattice." While the new work analyzes non-condensed bosons resulting from quantum fluctuations near $T = 0$, the qualitative picture of fluctuations (now thermal) of the vortices would remain [28]. More generally, there do not appear to be any theories that address interacting bosonic systems in the high temperature, normal regime [28]. Addressing this high temperature Bose gas system will require a very different formalism that goes beyond the Gross-Pitaevskii equation (to handle a large occupation of non-condensed bosons) and maintain a fourth-order, $\boldsymbol{r}$-dependent bosonic interaction, such as in Ref. [29].

Conclusions In this paper we have addressed an important and in principle testable prediction in the cold Fermi gases: the presence of precursor vortex configurations in the normal (pseudogap) phase as illustrated in Fig. 1. We stress that our results are natural consequences of Gor'kov theory which constrains the form of the t-matrix which we then associate with non-condensed pairs. From this the qualitative phenomena we observe then necessarily follow. The experimental implementation of this work in cold gases will depend on reaching a regime in which the Landau level spacing is much larger than the gap, although we expect the qualitative aspects regarding pair density inhomogeneities to apply to lower fields. While rapid rotation may be able to reach this regime, current proposals for artificial fields [9–11] also show promise.

We have also discussed the fact that normal-state inhomogeneity of the gap satisfies a necessary condition for condensation in a magnetic field, namely avoiding a one-dimensional normal state dispersion. Our work presents a formalism in which this inhomogeneity arises naturally from non-condensed pairs. There may be even stricter requirements to ensure that the condensate is stable, and future work may focus on a more extensive description of condensate fluctuations, and their impact on condensed and non-condensed pairs.

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