Processing quantum information in a hybrid topological qubit and superconducting flux qubit system

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Processing Quantum Information in Hybrid Topological Qubit and Superconducting Flux Qubit System

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A composite system of Majorana-hosted semiconductor nanowire and superconducting flux qubits, named top-flux-flux, is presented to process quantum information. We can electrically controll the coupling between the Majorana-based topological qubit and the readout flux qubit, supplying a convenient method to implement $\pi/8$ phase gate of the topological qubit. In addition, we design a scheme to transfer quantum information back and forth between the topological qubit and the flux qubit by employing Landau-Zener transition. With the demonstration of the entanglement of two topological qubits, it is very promising to do quantum information process with this hybrid system.

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I. INTRODUCTION

Topological quantum computation in which information is encoded into non-Abelian anyons is a promising approach to realize scalable quantum computer. Topological qubits hold the merit of resistance to some local fluctuations due to their non-local property. Recent progresses in the physical realization of non-Abelian anyons of Ising type-Majorana fermion (MF) have drawn much attention in this field. It has been shown theoretically that MF can exist as quasiparticles in many condensed matter systems, including $p_x + ip_y$ superconductor [1], topological insulator-superconductor heterostructures [2], and semiconductor-superconductor heterostructures [3–6]. Very recently, several groups have reported the observation of MF zero energy mode at the ends of semiconductor nanowire with the combination of spin-orbit coupling, proximity-induced superconductivity and applied magnetic field [7–9]. In addition, Alicea *et al.* have shown that in the nanowire networks MF, which obeys non-Abelian statistics, can be braided by simply adjusting gate voltages. This system may furnish as a platform for topological quantum computing.

For processing quantum information, two MF γ_1, γ_2 could be combined to form a Dirac fermion with creation and annihilation operator as:

$$f = \frac{\gamma_1 + i\gamma_2}{2}, \quad f^\dagger = \frac{\gamma_1 - i\gamma_2}{2} \tag{1}$$

The two states of the Dirac fermion, corresponding to $n = f^\dagger f = 0$ and 1, could function as a physical topological qubit. Because braiding any two MF can not change the parity of n, ususally the same parity states of two physical topological qubits (four MF) are used to code one logical topological qubit. For example, one typical choice is

$$|\psi\rangle = c_1|00\rangle + c_2|11\rangle \tag{2}$$

However, by braiding MF one can not generate a complete set of universal quantum logic gates required for quantum computation. It is well known that single qubit $\pi/8$ phase gate and non trivial two-qubit gate can not be realized solely by braiding the object MF without auxiliary qubit. In addition, it is usually very difficult to read out the state of MF, hindering the realization of the topological quantum computation. To remove these roadblocks, people have proposed many composite systems which consists of topological qubit and conventional qubits, including superconducting flux qubit [11–14], transmon [15], quantum dot [14, 16, 17]. Here we concentrate on the hybrid system of superconducting flux qubit and semiconductor nanowire which hosts MF.

It has been shown that with the help of flux qubit one can measure the state of topological qubit and implement $\pi/8$ phase gate using Aharonov-Casher effect [12]. One can also entangle two qubits (topological qubit or double dot qubit or combination of them) [11, 14] by jointly measuring them. However, processing quantum information with these hybrid quantum systems introduced a new problem: how to efficiently transfer information back and forth between topological qubits and other qubits. In this paper we give a possible solution to this problem. First, we propose a scheme to realize a controllable coupling between the flux qubit and topological qubit by setting appropriate gate voltage on the capacitor. With this in hand, we can improve the flexibility and fidelity of the $\pi/8$ phase gate of topological qubit. Then, we employ Landau-Zener transition to store the information of the flux qubit into the topological qubit. Finally, we present a scheme to retrieve the state of the topological qubit back to flux qubit. It is worth to emphasize that this is not a simple inverse of the storage process. Because one flux qubit is occupied to measure topological qubit, another flux qubit has to be used to receive the information. Borrowing the technique of tunable coupling between flux qubits [22], we have conceived a top-flux-flux composite system to realize the information retrieval scheme. Our proposals build a viable interface between topological and conventional solid-state qubits.

II. SYSTEM

To implement the $\pi/8$ phase gate for topological qubit and transfer information back and forth between different qubits, we have devised a hybrid system (see Fig. 1). The system consists of two flux qubits which coupled through the mediate loop, and a nanowire which is contact with one arm of the left flux qubit. In this section, we will illuminate the means by which we can purposely coupled and decouple these three elements of the composite system.

A. tunable coupling between topological and flux qubit and $\pi/8$ phase gate

The left flux qubit in Fig. 1 is made up of a superconducting loop interrupted by three Josephson junctions. The junctions have Josephson energy $E_j, \alpha E_j, E_j$, and charging energy $E_c, \alpha E_c, E_c$, respectively. In order to measure topological qubit and transfer information, we choose $\alpha > 1$ for utilizing Aharonov-Casher effect. The Hamiltonian of the flux qubit is

$$H = -\frac{1}{2}(\varepsilon\sigma_z + \Delta\sigma_x), \tag{3}$$

where σ_z and σ_x are Pauli matrices. $\varepsilon = 2I_p(\phi - \frac{\phi_0}{2})$ is the magnetic energy of two diabatic energy states $|L\rangle$ and $|R\rangle$, corresponding to the clockwise and counterclockwise persistent current respectively. The magnetic energy is adjustable by the external magnetic flux threading through the superconducting loop ϕ. ϕ_0 is the single flux quantum. I_p is the persistent current in the loop generated by ϕ. Δ is the tunneling splitting. At the energy level anti-crossing $\phi = \frac{\phi_0}{2}$ (so called optimal point), the two energy states is degenerate and the tunneling coupling mixes them, resulting a ground state $|g\rangle = (|L\rangle + |R\rangle)/\sqrt{2}$ and a excite state $|e\rangle = (|L\rangle - |R\rangle)/\sqrt{2}$ with an energy splitting Δ.

A nanowire is put on the bottom arm of the superconducting loop of the left flux qubit. Semiconductor nanowire under appropriate conditions can host some MF [5, 6]. The existence of MF in the nanowire has three prerequisites: spin-orbit coupling, magnetic field along the nanowire, proximity-induced superconductivity. When $\mu < \sqrt{Z_B^2 - \Delta_s^2}$, μ is the chemical potential of the nanowire, Z_B is the Zeeman splitting caused by the applied magnetic field, Δ_s is the induced superconducting gap, the nanowire is in topological superconductor phase; otherwise in untopological phase. For given Z_B, Δ_s, we can turn a segment of nanowire into topological phase by tuning μ piecewisely using the nearby electrodes. A pair of MF located at its two boundaries consist of a topological physical qubit with parity state 0 and 1. The even (or odd) subspace of two physical qubits (four MF) encode one logical qubit (see Eq. 2). Braiding the four MF pairwisely can realize some single-qubit operations for the logical qubit including $\sigma_x, \sigma_y, \sigma_z$ gate [18]. Moving MF is achieved by controlling the voltage of the electrodes to change the locations of topological-untopological phase boundaries. However, in one-dimensional nanowire it is impossible to braid MF because of spatial restriction. Fortunately, we can construct nanowire networks to exchange MF [10] (see Fig. 1).

If a pair of MF are created on the island between the most left and most right junction (the part inside the dashed rectangle in Fig. 1), the tunnel splitting is modulated by the total charge on island defined by two junctions and the gate capacitor

$$\Delta = \Delta_{max}|\cos(\pi q/2e)|, \tag{4}$$

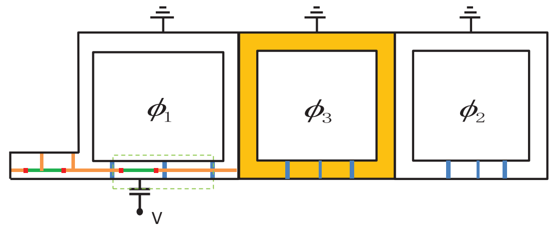

FIG. 1: (Color online) Sketch of top-flux-flux system. There are three main elements: nanowire networks which host two pairs of MF and two flux qubits. One pair of MF consist a topological physical qubit with parity state 0 and 1, and two physical qubits encode one topological logical qubit using the even subspace ($|00\rangle, |11\rangle$). When a pair of MF are loaded to the island of the left flux qubit (the part in the dashed rectangle), they could be coupled or decoupled to this flux qubit, depending on the gate voltage. The two flux qubits interact through the loop between them. We can also turn on and turn off the coupling of two flux qubits by applying a microwave pulse to the mediate loop with frequency equal to the difference of the two flux qubits. The microwave pulse modulates the magnetic flux of the mediate loop Φ_3.

where $q = en_p + q_{ext}$. $n_p = 0$ (1) denotes the eigenvalue of the even (odd) parity of the physical topological qubit formed from the two MF located on the island. q_{ext} is external bias charge which is proportional to the gate voltage applied through the gate capacitor. If we calibrate q_{ext} to 0 (modulo $2e$), the splitting will be maximized when the physical topological qubit is in even parity state. When the parity is odd, the splitting will be vanishing. Therefore, we can measure the qubit state (n_p) by probing the tunneling splitting of the flux qubit. The measured pair of MF and another pair of MF (located on the nanowire outside the dashed line) combine to a logical topological qubit in the subset ($|00\rangle, |11\rangle$). We can derive the state of the logical qubit by measuring one physical qubit.

However, the goal we are going to achieve using flux qubit is not only to measure the state of topological qubit, but also to assist topological quantum computation and coherently transfer information between different kinds of qubits. Hence, it is necessary to switch on and off the coupling efficiently and accurately. We find that this design is actually enable us to easily control the coupling between the topological qubit and flux qubit. From Equation 4, if we set $q_{ext} = e/2$ (modulo $2e$), both states of the topological qubit lead to the same tunneling splitting of the flux qubit. In this case, the eigenenergies of the flux qubit are not affected by the topological qubit, which indicates that the two qubits are effectively decoupled. Therefore, we can tune the gate voltage on the capacitor to turn on (or off) the coupling after initial voltage calibration.

The controllable coupling between qubits is indispensable in quantum computing. Especially, the $\pi/8$ phase gate in topological qubit system can not be obtained by braiding MF. With the help of a flux qubit which interacts with the topological qubit we can accomplish any singe qubit phase gate. Initially, the flux qubit is magnetically biased far away from $\phi = \phi_0/2$ and stay at its ground state. Before loading the MF to the island, the bias charge on the island is set to $e/2$ (modulo $2e$) by the gate voltage. Therefore, when the topological qubit is introduced to the system, it is uncoupled to the flux qubit. Then we switch on the coupling quickly. Depending on the state of the topological qubit, the ground state have eigenenergy

$$E_p = \begin{cases} -\frac{1}{2}\sqrt{\varepsilon^2+\Delta^2}, & if \quad n_p = 1 \\ -\frac{1}{2}\varepsilon, & if \quad n_p = 0 \end{cases} \tag{5}$$

Therefore, the two states of the topological qubit coupled to the flux qubit have a energy difference $\Delta E_p = \frac{1}{2}(\sqrt{\varepsilon^2+\Delta^2}-\varepsilon)$. We can realize any phase gate, including $\pi/8$ phase gate, by controlling the coupling time via tuning the gate voltage.

Previous works [12] also employed the energy difference ΔE_p of the ground state of the flux qubit resulting from the parities of different topological states to accumulate phase difference. After gate operation, they decouple the two qubits by biasing the external flux of flux qubit far away from the optimal point. However, that method can not turn off the coupling completely because even when we bias flux far from the optimal point to make $\varepsilon \gg \Delta$, the energy difference ΔE_p is non-vanishing up to the first order of Δ/ε. This brings extra error to the phase gate. Our scheme gets rid of this problem because we can turn off the coupling completely by electrical control.

B. coupling between two flux qubits

Another relevant part in our hybrid system is the second (right in Fig. 1) flux qubit. As discussed in section I, it is designed to transfer information from the topological qubit to the flux qubit. One may ask why not use SWAP gate to transfer information directly between different qubits. In fact, so far no transverse coupling ($\sigma_x\sigma_x$, $\sigma_y\sigma_y$) between topological qubits and other qubits has been worked out. Therefore, we can not realize SWAP operation by a single gate. Instead, we have to use quantum circuit shown later to transfer quantum state. In this process, one step is measuring the state of the original qubit. For transfering state from the flux qubit to the topological qubit, we need to measure the flux qubit, which is trivial. However, for the inverse process, we are going to measure the topological qubit. As shown before, one flux coupled with the topological qubit works as a detector. An extra flux qubit is required to receive the quantum state. In order to minimize the unwanted perturbation, we have to pay attention on the coupling of two flux qubits. Generally, two flux qubits can interact directly through geometric mutual inductance $MI_{p1}I_{p2}$ [21]. However, the generated interaction is not prone to be switched off, which makes individual qubit operation unrealistic. To realize a controllable coupling, we adopt the scheme proposed and demonstrated experimentally by Niskanen et al. [22, 23]. The left flux qubit (qubit 1) and the right flux qubit (qubit 2) are coupled through a third qubit between them. When both qubits are biased at the optimal point with splitting Δ_1, Δ_2 respectively, the interaction is actually off because the expectation value of the I_p is vanishing. Applying a microwave with frequency $\omega = |\Delta_1 - \Delta_2|$ to the coupler qubit turns on the interaction with form in rotating frame:

$$H_i = \Omega(\sigma_x^1\sigma_x^2 + \sigma_y^1\sigma_y^2) \tag{6}$$

Where Ω is the oscillation frequency between $|ge\rangle$ and $|eg\rangle$. Therefore, the two primary flux qubits can be effectively coupled and decoupled at will by applying microwaves to the coupler. It is worth to mention that other schemes of tunable coupling may be used for the flux qubit. We use this design because it can work at the optimal point which may result in a much longer coherence time.

III. TRANSFER INFORMATION FROM FLUX QUBIT TO TOPOLOGICAL QUBIT

It is well known that transferring information between two qubits can be realized by using CNOT gate combined with Hadamard gate and single-qubit measurement [19]. Following this method, we have designed a scheme for transferring information from qubit 1 to topological qubit (Fig 2(a)). Note that qubit 2 is not used in the transfer process, so we can decouple it from qubit 2 and leave aside it in this section. Initially, qubit 1 is at an unknown pure state $|\psi\rangle = a|g\rangle + b|e\rangle$. The topological qubit made of two topological physical qubit locates in the nanowire outside the island of qubit 1, and is prepared at $|00\rangle$. Then apply operations as following: Hadamard gate on the topological qubit, CNOT gate conditioned on the topological qubit, and measurement on the flux qubit. If the measurement result is $|g\rangle$, the transferring process is successfully complete; otherwise, an additional NOT operation should be applied to the topological qubit.

Let us turn to the question of how to implement each step. Actually, Hadamard gate on topological qubit can be achieved by braiding MF in one-dimension semiconductor nanowire network [10]. The measurement of flux qubit is in hand by now. However, the CNOT gate needs to be considered deliberately. We have worked out a method which employs the coupling between the two qubits and Landau-Zener transition.

Flux qubit is prepared in $|\psi\rangle$ at bias $\phi_i < \phi_0/2$. The condition $\varepsilon > \Delta$ is required for the Landau-Zener transition at the anti-crossing point well-defined. The topological qubit stays at a superposition state $|\varphi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ after a Hadamard operation, and one of the two physical topologcial qubit that encode the logical qubit is loaded to the island. The flux qubit and topological qubit is uncoupled at this moment. Then switch on the coupling by setting $q_{ext} = 0$ modulo $2e$, and sweep the flux bias through the anti-crossing point. If the topological qubit is $|00\rangle$ ($|11\rangle$), the tunneling splitting is Δ_{max} (0) . Based on this feature, we can choose a sufficient low sweep velocity to guarantee that the flux qubit evolutes adiabatically to the end without destroying its state if the topological qubit state is $|0\rangle$, and exchanges its ground state and excite state in the other case. Finally, turn off the coupling and measure the flux qubit. It is clear that the sweeping process is equivalent to a CNOT gate operation.

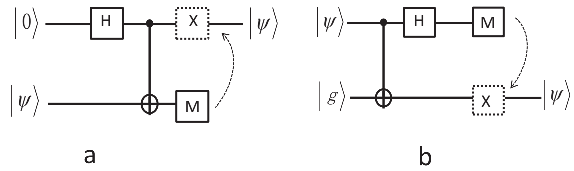

FIG. 2: Quantum circuit for quantum state transfer between topological qubit and flux qubit. (a)Transfer an unknown state $|\psi\rangle$ from the flux qubit to the topological qubit. (b)Transfer an unknown state $|\psi\rangle$ from the topological qubit to the flux qubit.

Noteworthily, here we do not use any microwave pulse.

Now, we estimate the minimum time to achieve the CNOT gate. In the Landau-Zener transition formulism, the transition (between the ground state and excite state) possibility is expressed as

$$\begin{aligned} P_1 &= 1, \qquad\qquad n_p = 1 \\ P_0 &= e^{-2\pi\Delta_{max}^2/4v}, \qquad n_p = 0 \end{aligned} \tag{7}$$

where $v = \Delta\varepsilon/\Delta t$. Assuming the final energy bias is $-\varepsilon$, we have $v = 2\varepsilon/\Delta t$. To make $P_0 = e^{-2\pi\Delta_{max}^2/4v} = e^{-2\pi\Delta_{max}^2\Delta t/8\varepsilon} \approx 0$, we get $2\pi\Delta_{max}^2\Delta t/8\varepsilon \gg 1$. If $\varepsilon = 2\Delta_{max} = 2\times 2\pi\ GHz$, Δt should much larger than 0.4 ns. We could choose $\Delta t = 10$ ns, which is much shorter than the coherence time of the flux qubit.

IV. TRANSFER INFORMATION FROM TOPOLOGICAL QUBIT TO FLUX QUBIT

In the previous section, we have addressed the question of how to "write" the data of the flux qubit to the topological qubit. Similarly, "read" the data of the topological qubit, i.e, transferring information from topological qubit to flux qubit, is also important for the hybrid system. In principle, it can be done by constructing a similar process like that in the last section. The initial state of the topological qubit and flux qubit is $|\psi\rangle = a|00\rangle + b|11\rangle$ and $|g\rangle$ respectively. Apply in order the operations: CNOT gate, Hadamard gate, and measurement on the topological qubit [Fig 2(b)]. At last, add a NOT (Identity) operation to the flux qubit if the readout result is $|11\rangle$ ($|00\rangle$). It seems straightforward to achieve in experiment, because we can use directly the relevant techniques illustrated in the previous section. However, attention must be paid to the differences between them:

1. After the CNOT operation, the topological qubit is subject to Hadamard operation and measurement. Meanwhile, the flux qubit needs to keep coherent. This require a longer coherence time of the flux qubit compared to that in the former section.

2. The object of the measurement is not the flux qubit but the topological qubit. Therefore one flux qubit is not enough here.

In order to make the information transferring feasible, we use both flux qubits. Qubit 1 functions as readout device of the topological qubit and information medium between topological qubit and qubit 2. Qubit 2 is a conventional three-junction flux qubit with $\alpha < 1$. Its eigenergies are insensitive to charge fluctuation due to the absence of the Aharonov-Casher effect. Hence, its coherence time could be longer than qubit 1, which makes it more suitable as an information receiver.

Now we explain our information transferring protocol in detail. Qubit 1 is prepared at ground state and biased at point A (Fig. 3), which is far away from the optimal point; qubit 2 is prepared at its excite state and biased at the optimal point. One may ask why qubit 2 is not prepared at the ground state as addressed at the beginning of this section. Actually, the purpose of preparing qubit 2 at the ground state and performing a CNOT operation on qubit 2 and the topological qubit is to produce the entangle state $(a|00g\rangle + b|11e\rangle)$ of them. We will demonstrate in the following that the entanglement state can also be generated with qubit 2 prepared at its excite state. Initial state of qubit 1 is approximately $|L\rangle$. Ever since one physcial topological qubit are loaded into the island of the qubit 1, turn on the coupling between topological qubit and qubit 1. Then, we sweep the bias of qubit 1 adiabatically to the anti-crossing point. As a result, the state of qubit 1 will remain at the ground state if the topological qubit is $|00\rangle$ because the splitting at the anti-crossing point is maximized. If the topological qubit is $|11\rangle$, qubit 1 will stay at its initial state $|L\rangle$ without mixing with $|R\rangle$. Then we add the microwave with frequency $\omega = |\Delta_1 - \Delta_2|$ to the coupler to switch on the interaction between qubit 1 and 2. Due to the resonance condition the flux qubits interact only if the topological qubit is $|00\rangle$. Choosing the microwave pulse with duration $1/2\Omega$, we have

$$(a|00g\rangle + b|11L\rangle)|e\rangle \rightarrow a|00eg\rangle + b|11Le\rangle \tag{8}$$

Now the topological qubit is entangled with two flux qubits. The next step is separating the qubit 1 from the entanglement. This is achieved by adiabatically sweeping the external flux bias of the qubit 1 across the anti-crossing point to point B (see Fig. 3) which is far away from the anti-crossing. At the end, the state $|L\rangle$ of qubit 1 is equal to $|e\rangle$. Therefore the final state of topological qubit and qubit 2 is $(a|00g\rangle + b|11e\rangle)$. The remaining operations are straightforward: braiding the MF to realize Hadamard gate, measure the topological qubit with qubit 1, and so on.

It is worth to note that adiabatic condition is needed in the bias sweeping process for qubit 1. The adiabatic condition can be characterized by Landau-Zener transition possibility, and is satisfied if the transition probability in the sweeping process is vanishing. The Landau-Zener transition probability is

$$P_{LZ} = e^{-2\pi\Delta_1^2/4v} \tag{9}$$

where Δ_1 is the energy splitting at the optimal point when topological qubit is at $|00\rangle$, $v = 2\varepsilon/\Delta t$, ε is the bias energy at the initial bias, Δt is the time of the sweeping. The adiabatic condition $P_{LZ} \approx 0$ is met when $e^{-2\pi\Delta_1^2/4v} \approx 0$. Assuming

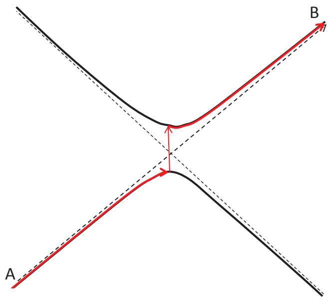

FIG. 3: (Color online) Energy diagram and the evolution of flux qubit 1 for quantum information transferring from flux qubit to topological qubit. The flux qubit 1 is prepared at ground state and biased at point A. Sweep the bias adiabatically to the anti-crossing point. Companied with different state of the topological qubit, flux 1 evolute along solid (red) line when topological qubit is $|0\rangle$, along the dashed line when topological qubit is $|1\rangle$. After shortly coupling with qubit 2 at the optimal point, bias is swept adiabatically to right until the point B, where qubit 1 stay at excite state for both states of the topological qubit.

$\varepsilon = 10\Delta_1 = 20\times 2\pi\, GHz$, the time scale of this process should be much longer than 1 ns. It is sufficient if we set the duration of the sweeping process $\Delta t = 20$ ns. Besides, the coupling time of qubit 1 and 2 scales as $1/2\Omega$ which is typically ~ 20 ns [23]. In all, the generation of the entanglement between the topological qubit and qubit 2 can be done within ~ 40 ns, which is much shorter than the coherence time(~ 2 μs) of qubit 2.

For clarity, we denote the state of the topological logical qubit with the complete form, such as $|\psi\rangle = a|00\rangle + b|11\rangle$. And indicate that in each process concerned in this paper only one topological physical qubit (including two MF) is needed to load to the island of one flux qubit.

V. DISCUSSION

People also suggested using transmon [20], a variant of superconducting charge qubit, to measure and communicate with topological qubit [15]. One can simply consider transmon as two superconductor islands connected by a dc SQUID. The advantage of using it is that the coupling with topological qubit can be switched on and off with exponential accuracy [15]. However, there are two drawbacks in their measurement scheme: 1. For measuring single topological qubit, all four MF consisting of the qubit should first be moved to one island of a transmon, then two of them are transfer to the other island. The process is much more complex than that of using flux qubit, and the situation is even worse when doing joint measurement. 2. The state calibration is very challenging. If the capacitances of the two islands are not symmetry, the period of the charge vs the frequency of the transmon is not $2e$ (see appendix of Ref [15] for detail). Therefore when the four MF are loaded into one island of the transmon, the two constituent state $|00\rangle$ and $|11\rangle$ correspond to different frequencies of the transmon, which makes the calibration of the charge on the islands impossible. Hence, the asymmetry of the the islands will reduce the practicability of transmon as a topological qubit measurer. On the contrary, flux qubits are not bothered by this kind of asymmetry [24]. From this point of view, flux qubit might be a better candidate for measuring topological qubit. That is why we choose it as a interface between topological and conventional qubit system.

At last, we would like to analyze some imperfections of our system. Firstly, before operation we need to initialize the topological qubit to a well-known state, say $|00\rangle$. However, this is more difficult than preparing of a conventional qubit, which usually can relax to its ground state by waiting for a sufficient long time at a very low temperature. Topological qubit is routinely degenerate unless the four MF consisting the qubit are very close to each other to generate energy splitting between the state $|00\rangle$ and $|11\rangle$. Whether we can use the relaxation method to initialize the topological qubit is still a open question. Secondly, there may be crosstalk between the gate voltage of the island and the voltage on the electrodes used to move MF. On one hand, in our scheme for switching on or off coupling between the topological qubit and the left-most flux qubit, the gate voltage needs to be tuned, which would change the chemical potential of the segment of the nanowire lay on the island. To ensure that the modification of chemical potential will not affect the existence of the two MF in the island, the topological phase part of the nanowire should meet the condition $\mu < \sqrt{Z_B^2 - \Delta_s^2}$ at both coupling stages. On the other hand, the voltage changes of the electrodes when loading MF to the island are supposed to affect the electrical charges in the island q_{ext}, which makes the former calibration of q_{ext} invalidate. How to overcome this problem is a big challenge to the community. We can use following steps to tackle it: calibrate of q_{ext} after a topological physical qubit with state 0 has loaded to the island; then move it out of the island; braid the four MF of the topological logical qubit; finally load the physical qubit again with the same voltage profile of the electrodes as that of the first loading. In this case, the calibration of q_{ext} would not be destroyed by moving MF.

Another imperfection of our system is incoherent tunneling of unpaired electrons through Josephson junction, a process named quasiparticle poisoning. The quasiparticle tunneling events were observed in the superconducting Josephson junction circuit even at temperature well below superconducting transition temperature [25]. The characteristic time scale for quasiparticle number fluctuations in Al which is often used to make superconducting qubit, is 2 ms below 160 mK [26]. This requires that the interaction between topological qubit and flux qubit lasting in a operation less than 2 ms. As indicated in Section 2 and 3, in the information transfer processes, the interaction times are much shorter than this value. For the $\pi/8$ phase gate, given $\epsilon = 40 \times 2\pi$ GHz, $\Delta = 1 \times 2\pi$ GHz, the during of this gate is just 20 ns which is also much shorter than 2 ms. It is worth to note that when switch off the interaction by setting $q_{ext} = e/2$ modulo $2e$, quasiparticle tunnelings would no longer affect the state of the topological qubit.

VI. CONCLUSION

We have constructed a viable interface between topological qubit system and conventional quantum system. In this top-flux-flux structure, we can electrically control the coupling between topological qubits and flux qubits, supplying a simple method to implement $\pi/8$ phase gate of a topological qubit. Combined with generating entanglement through joint measurement of two topological qubits and the braiding operations of MF, the hybrid system of semiconductor nanowire and flux qubit possesses a set of universal quantum logic gates for realizing universal quantum computation. Moreover, we propose schemes to transfer information back and forth between the flux qubit and the topological qubit via Landau-Zener transition, which are very important operations in processing quantum information with hybrid quantum systems. The feasibility of using our hybrid system to do quantum information process crucially depends on two preliminary experiments. One is demonstration of Aharonov-Casher effect in three-junction flux qubit, although this effect has already been observed in other systems. The other is confirmation the existence of MF in the one-dimensional nanowire, which is technically reachable currently.

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