Quantum-discord amplification induced by a quantum phase transition via a cavity–Bose-Einstein-condensate system

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Phys. Rev. A **87**, 024101 — Published 19 February 2013
DOI: 10.1103/PhysRevA.87.024101

Quantum discord amplification induced by quantum phase transition via a cavity-Bose-Einstein-condensate system

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We propose a theoretical scheme to realize a sensitive amplification of quantum discord (QD) between two atomic qubits via a cavity-Bose-Einstein condensate (BEC) system which was used to realize the Dicke quantum phase transition (QPT) for the first time [Nature **464**, 1301 (2010)]. It is shown that influence of the cavity-BEC system upon the two qubits is equivalent to a phase decoherence environment. It is found that QPT in the cavity-BEC system is the physical mechanism of the sensitive QD amplification.

PACS numbers: 03.65.Ud, 03.65.Ta, 03.65.Yz

Quantum discord (QD) [1, 2] is considered to be a more general resource than quantum entanglement in quantum information processing [3–8]. As a physical quantity to characterize and quantify quantum correlation in a bipartite system, QD is different from quantum entanglement. For example, QD is nonzero in some separable states; QD can be increased by local operations [9–11] while quantum entanglement cannot. So compared with quantum entanglement, QD can give rise to novel unexpected phenomena. For instance, nonzero QD in some separable states is responsible for the quantum computational efficiency of deterministic quantum computation with one pure qubit [3, 4, 12] and also has been considered as a useful resource in quantum locking [5] and quantum state discrimination [6, 7]. On the other hand, any realistic quantum systems interact inevitably with their surrounding environments, which introduce quantum noise into the systems. It is an interesting topic to explore how to amplify QD by quantum noise induced by environments. Lo Franco and coauthors [13, 14] pointed out revival of quantum correlations without system-environment back-action and found alternate time regions of constant discord and decreasing and increasing discord in a non-dissipative environment. Authors of the present paper found that QD will be amplified for two non-interacting qubits immersed in a common phase decoherence environment [15]. Especially, when the two qubits are identical, the phase decoherence can induce a stable amplification of the initially-prepared QD for certain X-type states. In this paper, we propose a scheme to realize the controllable QD amplification of two atomic qubits by making use of an artificial phase decoherence environment consisting of a cavity-Bose-Einstein condensate (BEC) system.

The Dicke model [16, 17] describes a large number of two-level atoms interacting with a single cavity field mode. When atom-field coupling is increasing, the model predicts a quantum phase transition (QPT) [18] from the normal phase, where the atoms are in the ground state associated with vacuum field state, to the super-radiant phase, where both the atoms and field have collective excitations. Recently, a cavity-BEC system has been employed to realize the Dicke QPT experimentally and to explore symmetry breaking at the Dicke QPT [19]. Meanwhile, the QPT system usually displays ultrasensitivity in its dynamical evolution near the quantum critical point [20–24], which has been confirmed by an NMR experiment [25]. The purpose of this paper is to show that the QD of two initially correlated atomic qubits can be sensitively amplified via the cavity-BEC system near the critical point. We show that the cavity-BEC system can form an artificial phase decoherence environment for the two atomic qubits, and the QD of the two atomic qubits can be amplified by adjusting the QPT parameter of the cavity-BEC system.

The physical system under our consideration is shown in Fig. 1. A BEC with N identical two-level ^{87}Rb atoms is confined in an ultrahigh-finesse optical cavity. The atoms interact with a single cavity mode of frequency ω_c and a transverse pump field of frequency ω_p. We consider a situation where the frequencies ω_c and ω_p are detuned far from the atomic resonance frequency ω_R of each atom in the BEC so that the detunings far exceed the rate of atomic spontaneous emission, the atoms only scatter photons either along or transverse to the cavity axis. Before the pump field turns on, atoms in the BEC are supposed to be in the zero-momentum state $|p_x, p_z\rangle = |0, 0\rangle$. Once the pump field is turned on, some atoms are excited into momentum state $|p_x, p_z\rangle = |k, k\rangle \equiv \sum_{\upsilon_1,\upsilon_2=\pm1} |\upsilon_1 k, \upsilon_2 k\rangle$ due to the conservation of momentum, where k is the wave-vector, which is approximately equal to that of the cavity and pump fields. We take $\hbar = 1$ throughout the paper. Two momentum states $|0, 0\rangle$ and $|k, k\rangle$ are regarded as two-level states of the ^{87}Rb atom with energy separation $\omega_0 = k^2/m$ with m being the mass of ^{87}Rb atom. Define the collective operators $\hat{J}_z \equiv \sum_i |k, k\rangle_{ii}\langle k, k|$, $\hat{J}_+ = \hat{J}_-^\dagger \equiv \sum_i |k, k\rangle_{ii}\langle 0, 0|$ with the index i labeling the atoms, then cavity-BEC can be described by the Dicke model [19, 26]

$$\hat{H}_1 = \omega \hat{a}^\dagger \hat{a} + \omega_0 \hat{J}_z + \frac{\lambda}{\sqrt{N}} \left(\hat{a} + \hat{a}^\dagger\right)(\hat{J}_+ + \hat{J}_-), \qquad (1)$$

where $\hat{a}^\dagger(\hat{a})$ is the creation (annihilation) operator of the cavity field. The effective frequency $\omega = -\Delta_c + U_0 N/2$ includes the detuning $\Delta_c = \omega_p - \omega_c$ between the pump field and the cavity field and the frequency shift $U_0 N/2$ induced by the scattering of the BEC. Here $U_0 = g_0^2/\Delta$ is the frequency shift induced by a single atom with the maximal atom-cavity coupling strength

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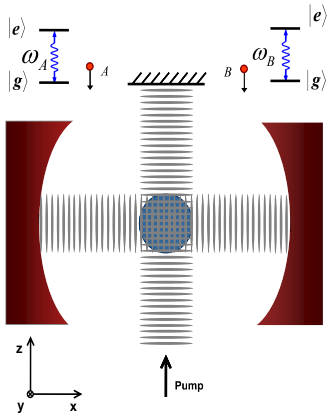

FIG. 1: (Color online) Schematic of the physical system under consideration: Two atomic qubits A and B with energy separation ω_A and ω_B are injected into the cavity in which an atomic BEC couples to a single-mode cavity field and a transverse pump field.

g_0 and detuning $\Delta = \omega_p - \omega_R$, and $\lambda = \sqrt{N} g_0 \Omega_p / 2\Delta$ is the coupling strength between the BEC and the cavity field with Ω_p denoting the maximal pump Rabi frequency which can be adjusted by the pump-field power.

We consider such a situation that the two atomic qubits pass through the cavity at the same time and interact with the single-mode cavity field. The Hamiltonian of the two atoms and the single-mode cavity field reads as

$$\hat{H}_2 = \omega \hat{a}^\dagger \hat{a} + \frac{\omega_A}{2}\hat{\sigma}_z^A + \frac{\omega_B}{2}\hat{\sigma}_z^B + \left(g_A \hat{a}^\dagger \hat{\sigma}_-^A + g_B \hat{a}^\dagger \hat{\sigma}_-^B + \mathtt{H.c.}\right), \tag{2}$$

where $\hat{\sigma}_z^{A(B)} = |e\rangle_{A(B)}\langle e| - |g\rangle_{A(B)}\langle g|$ is Pauli operator with $|e\rangle_{A(B)}$ and $|g\rangle_{A(B)}$ being the excited and ground states, $\sigma_+^{A(B)}(\sigma_-^{A(B)})$ the raising operator (lowering operator), $g_{A(B)}$ the coupling strength between the atomic qubit A(B) and the cavity field, and $\omega_{A(B)}$ the energy separation. Here we have made a rotating wave approximation. We consider the atom-cavity dispersive regime in which the atomic qubit is far off-resonant with the cavity field such that the detuning $\Delta_{A(B)} = \omega_{A(B)} - \omega$ is much larger than the corresponding coupling strength $g_{A(B)}$. In this regime, one can use the Fröhlich-Nakajima transformation [27, 28] to make the Hamiltonian in Eq. (2) become the following expression

$$\hat{H}_2^{'} = \frac{1}{2}\omega_A^{'}\hat{\sigma}_z^A + \frac{1}{2}\omega_B^{'}\hat{\sigma}_z^B + \left(\omega + \delta_A\hat{\sigma}_z^A + \delta_B\hat{\sigma}_z^B\right)\hat{a}^\dagger\hat{a}, \tag{3}$$

where $\omega_{A(B)}^{'} = \omega_{A(B)} + \delta_{A(B)}$ with $\delta_{A(B)} = g_{A(B)}^2/\Delta_{A(B)}$ being the frequency shift induced by the scattering between cavity field and atomic qubit A(B). The Hamiltonian of Eq. (3) corresponds to the so-called dispersive regime in cavity QED. Based on analogous atom-cavity dispersive interactions, there have been proposed both the generation of generalized binomial states of radiation and the realization of logical gates [29]. Then the effective Hamiltonian describing the two atomic qubits plus the cavity-BEC system is

$$\hat{H}_{eff} = \frac{1}{2}\omega_A^{'}\hat{\sigma}_z^A + \frac{1}{2}\omega_B^{'}\hat{\sigma}_z^B + \left(\delta_A\hat{\sigma}_z^A + \delta_B\hat{\sigma}_z^B\right)\hat{a}^\dagger\hat{a} + \hat{H}_1. \tag{4}$$

Now we consider dynamics of the two atomic qubits passing through the cavity-BEC system. We assume the two atomic qubits are initially prepared in a class of states with maximally mixed marginals ($\hat{\rho}^{A(B)} = \hat{I}^{A(B)}/2$) [30] described by the three-parameter X-type density matrix $\hat{\rho}_s(0) = 1/4(\hat{I}^{AB} + \sum_{i=1}^{3} c_i\hat{\sigma}_i^A \otimes \hat{\sigma}_i^B)$, where $\hat{I}^{AB}$ is the identity operator in the Hilbert space of the two atomic qubits, $i = 1, 2, 3$ mean x, y, z correspondingly, and c_i $(0 \leq |c_i| \leq 1)$ are real numbers satisfying the unit trace and positivity conditions of the density operator $\hat{\rho}_s(0)$. The cavity-BEC system is initially in the ground state $|G\rangle$ of the Hamiltonian $\hat{H}_1$. The dynamic evolution of the total system is controlled by the effective Hamiltonian in Eq. (4). The density operator of the total system at time t is written as $\rho_T(t) = U(\hat{\rho}_s(0) \otimes |G\rangle\langle G|)U^\dagger$ with $U = e^{-i\hat{H}_{eff}t}$. After tracing the degrees of freedom of the cavity-BEC system, we obtain the reduced density matrix of the two atomic qubits

$$\hat{\rho}_s(t) = \frac{1}{4}\begin{pmatrix} 1+c_3 & 0 & 0 & \mu(t)D_1(t) \\ 0 & 1-c_3 & \nu(t)D_2(t) & 0 \\ 0 & \nu^*(t)D_2^*(t) & 1-c_3 & 0 \\ \mu^*(t)D_1^*(t) & 0 & 0 & 1+c_3 \end{pmatrix}, \tag{5}$$

where we have introduced the following parameters

$$\begin{aligned} \mu(t) &= (c_1 - c_2)e^{-i(\omega_A^{'}+\omega_B^{'})t}, \quad \nu(t) = (c_1 + c_2)e^{-i(\omega_A^{'}-\omega_B^{'})t}, \\ D_1(t) &= \langle G|e^{i\hat{H}_{ee}t}e^{-i\hat{H}_{gg}t}|G\rangle, \quad D_2(t) = \langle G|e^{i\hat{H}_{eg}t}e^{-i\hat{H}_{ge}t}|G\rangle, \end{aligned} \tag{6}$$

with

$$\begin{aligned} \hat{H}_{ee} &= \delta_1\hat{a}^\dagger\hat{a} + \hat{H}_1, \hat{H}_{gg} = -\delta_1\hat{a}^\dagger\hat{a} + \hat{H}_1, \delta_1 = \delta_A + \delta_B, \\ \hat{H}_{eg} &= \delta_2\hat{a}^\dagger\hat{a} + \hat{H}_1, \hat{H}_{ge} = -\delta_2\hat{a}^\dagger\hat{a} + \hat{H}_1, \delta_2 = \delta_A - \delta_B. \end{aligned} \tag{7}$$

We consider the situation that the two atomic qubits pass through the cavity field region in a very short time such that $\delta_1 t \ll 1$ and $\delta_2 t \ll 1$. In fact, according to Ref. [19], the waist of the cavity field is 25 μm, the effective frequency shifts δ_A, δ_B are about 100 Hz. Above conditions are well satisfied if injected velocity of the atomic qubits meets $v > 10^{-3}$ m/s. By the short-time approximation, the factors $|D_1(t)|$ and $|D_2(t)|$ in Eq. (5) can be derived as

$$|D_1(t)| = \exp\left(-2\gamma\delta_1^2 t^2\right), |D_2(t)| = \exp\left(-2\gamma\delta_2^2 t^2\right), \tag{8}$$

where the decay factor $\gamma = \left\langle\left(\hat{a}^\dagger\hat{a}\right)^2\right\rangle - \left\langle\hat{a}^\dagger\hat{a}\right\rangle^2$ is the photon number fluctuation (PNF) of the cavity field in the ground state $|G\rangle$ [24].

From Eq. (5) we can see that the cavity-BEC system only affects off-diagonal elements of the density matrix for the two atomic qubits, hence it equivalently acts as a phase decoherence environment for the two atomic qubits. That is, the cavity-BEC system constitutes an artificial phase decoherence environment of the two qubits. The coupling constant λ in the Hamiltonians (1) and (7) is a QPT parameter of the cavity-BEC system. The QPT parameter is a controllable parameter of the artificial environment, and affects properties of the ground sate $|G\rangle$ of the Hamiltonian $\hat{H}_1$. It is worth noting that when the effective frequency shifts δ_A, δ_B are equal, i.e,

$\delta_2 = 0$, a decoherence-free subspace in the basis $\{|ge\rangle, |eg\rangle\}$ appears.

In order to obtain the detailed form of the PNF γ, in the following we find the ground state $|G\rangle$ according to the Ref. [17]. By the use of the Holstein-Primakoff transformation [31, 32] $\hat{J}_+ = \hat{c}^\dagger\sqrt{2j - \hat{c}^\dagger\hat{c}}$, $\hat{J}_- = \sqrt{2j - \hat{c}^\dagger\hat{c}}\hat{c}$, $\hat{J}_z = \hat{c}^\dagger\hat{c} - j$ with $j = N/2$, the Hamiltonian of the Eq. (1) is further reduced to

$$\hat{H}_1 = \omega\hat{a}^\dagger\hat{a} + \omega_0\hat{c}^\dagger\hat{c} + \lambda\left(\hat{a} + \hat{a}^\dagger\right)\left(\hat{c}^\dagger\sqrt{1 - \frac{\hat{c}^\dagger\hat{c}}{2j}} + \mathtt{H.c.}\right). \quad (9)$$

When the coupling strength λ is smaller than the critical coupling strength $\lambda_c = \sqrt{\omega\omega_0}/2$, the system is in the normal phase in which the BEC and the cavity field have low excitations. When the coupling strength is larger than the critical strength, the system is in the super-radiant phase in which both the BEC and the cavity field have collective excitations in the order of the atom number N.

In the normal phase at the thermodynamic limit $j \to \infty$, we can take $\sqrt{1 - \frac{\hat{c}^\dagger\hat{c}}{2j}} \approx 1$, Hamiltonian (9) then becomes

$$\hat{H}_1 = \omega\hat{a}^\dagger\hat{a} + \omega_0\hat{c}^\dagger\hat{c} + \lambda\left(\hat{a} + \hat{a}^\dagger\right)\left(\hat{c} + \hat{c}^\dagger\right), \quad (10)$$

which can be diagonalized as

$$\hat{H}_1 = \varepsilon_-\hat{d}_1^\dagger\hat{d}_1 + \varepsilon_+\hat{d}_2^\dagger\hat{d}_2, \quad (11)$$

where we have used the Bogoliubov transformation

$$\begin{aligned}\hat{a}^\dagger &= f_1\hat{d}_1^\dagger + f_2\hat{d}_1 + f_3\hat{d}_2^\dagger + f_4\hat{d}_2,\\ \hat{c}^\dagger &= h_1\hat{d}_1^\dagger + h_2\hat{d}_1 + h_3\hat{d}_2^\dagger + h_4\hat{d}_2. \end{aligned} \quad (12)$$

Two eigenfrequencies in Eq. (11) ε_- and ε_+ are given by

$$\varepsilon_\pm^2 = \frac{1}{2}\left[\omega^2 + \omega_0^2 \pm \sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 16\lambda^2\omega\omega_0}\right]. \quad (13)$$

In the normal phase, the coefficients of the Bogoliubov transformation about the cavity field in Eq. (12) are

$$f_{1,2} = \frac{1}{2}\frac{\cos\phi}{\sqrt{\varepsilon_-\omega}}\left(\omega \pm \varepsilon_-\right), \quad f_{3,4} = \frac{1}{2}\frac{\sin\phi}{\sqrt{\varepsilon_+\omega}}\left(\omega \pm \varepsilon_+\right), \quad (14)$$

where the mixing angle ϕ is given by $\tan 2\phi = \frac{4\lambda\sqrt{\omega\omega_0}}{\omega_0^2 - \omega^2}$. In the following discussion on the PNF we only need the coefficients f_i ($i = 1, 2, 3, 4$), so the coefficients h_i ($i = 1, 2, 3, 4$) are not given here.

In the super-radiant phase, in the Hamiltonian (9) we displace the bosonic modes $\hat{a}^\dagger \to \hat{a}^{\dagger'} + \sqrt{\alpha}$, $\hat{c}^\dagger \to \hat{c}^{\dagger'} - \sqrt{\beta}$ with $\sqrt{\alpha}$ and $\sqrt{\beta}$ describing the macroscopic mean fields in the order of $O(j)$. Neglecting terms with j in the denominator and taking $\sqrt{\alpha} = \frac{2\lambda}{\omega}\sqrt{\frac{j}{2}\left(1 - \xi^2\right)}$, $\sqrt{\beta} = \sqrt{j\left(1 - \xi\right)}$ with $\xi = \frac{\lambda_c^2}{\lambda^2}$, the Hamiltonian Eq. (9) is reduced to the following form

$$\begin{aligned}\hat{H}_1 &= \omega\hat{a}^{\dagger'}\hat{a}' + \tilde{\omega}_0\hat{c}^{\dagger'}\hat{c}' + \eta\left(\hat{c}^{\dagger'} + \hat{c}'\right)^2\\ &\quad + \tilde{\lambda}\left(\hat{a}' + \hat{a}^{\dagger'}\right)\left(\hat{c}^{\dagger'} + \hat{c}'\right), \end{aligned} \quad (15)$$

where the parameters $\tilde{\omega}_0$, $\tilde{\lambda}$ and η are given by

$$\begin{aligned}\tilde{\omega}_0 &= \frac{\omega_0}{2\xi}(1 + \xi), \quad \tilde{\lambda} = \lambda\xi\sqrt{\frac{2}{1 + \xi}},\\ \eta &= \frac{\omega_0\left(1 - \xi\right)\left(3 + \xi\right)}{8\xi\left(1 + \xi\right)}. \end{aligned} \quad (16)$$

The Hamiltonian in Eq. (15) can be also diagonalized as

$$\hat{H} = \varepsilon_-'\tilde{d}_1^{\dagger'}\tilde{d}_1' + \varepsilon_+'\tilde{d}_2^{\dagger'}\tilde{d}_2', \quad (17)$$

by the Bogoliubov transformation

$$\begin{aligned}\hat{a}^{\dagger'} &= f_1'\hat{d}_1^{\dagger'} + f_2'\tilde{d}_1' + f_3'\hat{d}_2^{\dagger'} + f_4'\tilde{d}_2',\\ \hat{c}^{\dagger'} &= h_1'\hat{d}_1^{\dagger'} + h_2'\tilde{d}_1' + h_3'\hat{d}_2^{\dagger'} + h_4'\tilde{d}_2', \end{aligned} \quad (18)$$

where the eigenfrequencies ε_-' and ε_+' read as

$$\varepsilon_\pm'^2 = \frac{1}{2}\left[\omega^2 + \frac{\omega_0^2}{\xi^2} \pm \sqrt{\left(\omega^2 - \frac{\omega_0^2}{\xi^2}\right)^2 + 4\omega^2\omega_0^2}\right]. \quad (19)$$

The coefficients of Bogoliubov transformation about the cavity field in the super-radiant phase are

$$f_{1,2}' = \frac{1}{2}\frac{\cos\phi'}{\sqrt{\varepsilon_-'\omega}}\left(\omega \pm \varepsilon_-'\right), \quad f_{3,4}' = \frac{1}{2}\frac{\sin\phi'}{\sqrt{\varepsilon_+'\omega}}\left(\omega \pm \varepsilon_+'\right). \quad (20)$$

where ϕ' is the mixing angle defined by $\tan 2\phi' = \frac{2\omega\omega_0\xi^2}{\omega_0^2 - \xi^2\omega^2}$. The coefficients h_i' ($i = 1, 2, 3, 4$) are also not given here since we do not need in the PNF calculation below.

We now investigate the PNF in the normal and super-radiant phase. In the normal (super-radiant) phase, the ground state is $|0, 0\rangle_{d_1, d_2}$ ($|0, 0\rangle_{d_1', d_2'}$). It is easy to get the PNF in the normal and super-radiant phase with the following form

$$\gamma = \begin{cases} 2f_1^2 f_2^2 + 2f_3^2 f_4^2 + \left(f_1 f_4 + f_2 f_3\right)^2, & \lambda < \lambda_c,\\ f_1'^2 f_2'^2 + 2f_3'^2 f_4'^2 + \left(f_1' f_4' + f_2' f_3'\right)^2 & \\ +\alpha\left[\left(f_1' + f_2'\right)^2 + \left(f_3' + f_4'\right)^2\right], & \lambda > \lambda_c. \end{cases} \quad (21)$$

Compared with the case of the normal phase, the displacement α due to collective excitation appears in the super-radiant phase. Figure 2 shows the PNF γ will experience drastic change near the critical coupling point $\lambda_c/\omega_0 = 10$. The closer the coupling strength λ near the critical coupling point, the larger the PNF γ. This inspires us to control the coherence decay rate of the two atomic qubits by adjusting the pump-field power to change the coupling strength in the region near the critical coupling.

In the following we consider the QD amplification of the two atomic qubits induced by the QPT of the cavity-BEC system. The QD [1] is defined as the difference between the total correlation and the classical correlation with the expression $\mathcal{D}\left(\hat{\rho}^{AB}\right) = \mathcal{I}\left(\hat{\rho}^A : \hat{\rho}^B\right) - C\left(\hat{\rho}^{AB}\right)$ with $\hat{\rho}^A$, $\hat{\rho}^B$ and

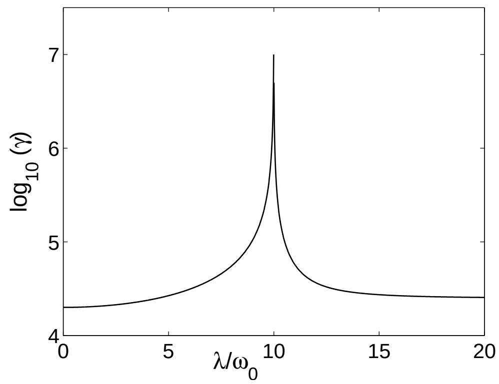

FIG. 2: (Color online) The logarithm to base 10 of the PNF γ changes with respect to the coupling strength λ. Related parameters $N = 10^5$, $\omega_0 = 0.05$MHz, $\omega = 20$MHz correspond to the experimental parameters in Ref. [19].

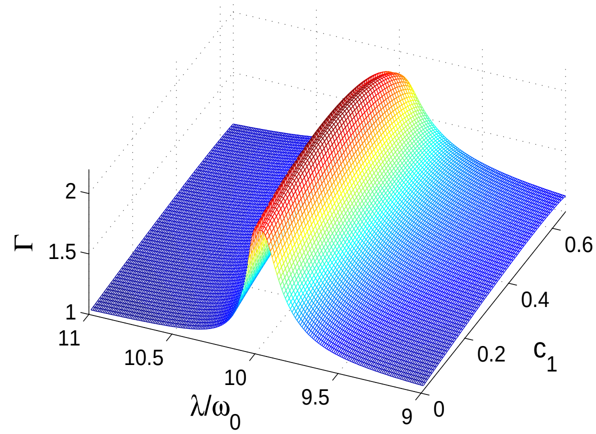

FIG. 3: (Color online) The QD amplification rate as a function of the coupling strength λ and the initial parameter c_1. Other parameters are set as $c_3 = c_1/2$, $c_2 = 0$, $\omega_0 = 0.05$ MHz, $\omega = 20$ MHz, $t_f = 1/\omega_0$, $\delta_1 = 0.001\omega_0$, $\delta_2 = 0$, and $N = 10^5$.

$\hat{\rho}^{AB}$ being the reduced density operators for subsystems A and B, and the total density operator, respectively. The total correlation in the state $\hat{\rho}^{AB}$ is measured by quantum mutual information $\mathcal{I}\left(\hat{\rho}^A : \hat{\rho}^B\right) = S\left(\hat{\rho}^A\right) + S\left(\hat{\rho}^B\right) - S\left(\hat{\rho}^{AB}\right)$ with $S\left(\hat{\rho}\right) = -\mathrm{Tr}(\hat{\rho}\log\hat{\rho})$ being the von Neumann entropy. The classical correlation between the two subsystems A and B is given by $C(\hat{\rho}^{AB}) = S\left(\hat{\rho}_A\right) - \min_{\{\hat{P}_k^B\}}\left[\sum_k p_k S\left(\hat{\rho}_k^A\right)\right]$ where $p_k = \mathrm{Tr}_{AB}[(\hat{I}^A \otimes \hat{P}_k^B)\hat{\rho}^{AB}(\hat{I}^A \otimes \hat{P}_k^B)]$ denotes the probability relating to the outcome k, and $\hat{I}^A$ denotes the identity operator for the subsystem A with $\{\hat{P}_k^B\}$ being a set of projects performed locally on the subsystem B.

The mutual information of the state given in Eq. (5) is derived as $\mathcal{I}\left(\hat{\rho}^A : \hat{\rho}^B\right) = 2 + \sum_{i=1}^4 \lambda_i \log\lambda_i$, where $\lambda_{1,2} = \frac{1}{4}(1 + c_3 \pm |\mu(t)D_1(t)|)$, $\lambda_{3,4} = \frac{1}{4}(1 - c_3 \pm |\nu(t)D_2(t)|)$ are four eigenvalues of $\hat{\rho}_s(t)$. And the classical correlation can be obtained as [15, 30] $C(\hat{\rho}_s(t)) = \sum\limits_{n=1}^{2} \frac{1+(-1)^n\chi}{2}\log_2\left[1 + (-1)^n\chi\right]$ with $\chi(t) = \max\left[|c_3|, (|\mu(t)D_1(t)| + |\nu(t)D_2(t)|)/2\right]$. Therefore, the QD can be written as

$$\mathcal{D}\left(\hat{\rho}_s(t)\right) = 2 + \sum_{i=1}^{4}\lambda_i \log_2\lambda_i - C(\hat{\rho}_s(t)). \qquad (22)$$

The QD can be amplified for some initial states such as the state parameters being set as $c_2 = 0$, $0 \leq c_1 = 2c_3 \leq 2/3$ when the qubits are in the phase decoherence environment [15]. For the present cavity-BEC environment, let the two atomic qubits enter the cavity at time $t = 0$ and leave the cavity at time t_f. Then we can define the QD amplification rate as $\Gamma = \mathcal{D}(t_f)/\mathcal{D}(0)$. In Figure 3 we have plotted the QD amplification rate Γ with respect to the coupling strength λ and the initial state parameter c_1 when $c_3 = c_1/2$, $c_2 = 0$, $\omega_0 = 0.05$ MHz, $\omega = 20$ MHz, $\lambda_c/\omega_0 = 10$, $t_f = 1/\omega_0$, $\delta_1 = 0.001\omega_0$, $\delta_2 = 0$, and $N = 10^5$. Figure 3 indicates that the initial QD can be amplified by the use of the cavity-BEC system through changing the QPT parameter λ. Specially, the QD amplification rate sensitively increases at the QPT point of the cavity-BEC system $\lambda = \lambda_c$. In this sense, the sensitive QD amplification can be understood as a quantum phenomenon induced by the QPT of the cavity-BEC system. It should be pointed out that one can control the QPT parameter λ by changing the Rabi frequency of the pump field Ω_p based on the relation $\lambda = \sqrt{N}g_0\Omega_p/2\Delta$.

In conclusion, we have proposed a scheme to realize the sensitive QD amplification of two atomic qubits via the cavity-BEC system by changing the QPT parameter of the the cavity-BEC system, and revealed the QPT mechanism of the sensitive QD amplification. We have indicated that the cavity-BEC system is equivalent to a phase decoherence environment for the two atomic qubits. Hence, it provides an artificial and controllable phase decoherence environment for quantum information processing. Essentially, the QD amplification is induced by the PNF of the cavity field. The PNF mainly depends on the fluctuation of the BEC density. At the point of the QPT, the density of the BEC in the cavity changes from a uniform distribution into a checker-board pattern. It should be mentioned that the present scheme should be within the reach of present-day techniques since the cavity-BEC system used in the scheme has been well established in recent experiments of observing the Dicke QPT [19]. The experimental realization of the scheme proposed in the present paper deserves further investigations.

Acknowledgments

This work was supported by the 973 Program under Grant No. 2013CB921804, the NSF under Grant No. 11075050, and the PCSIRTU under Grant No. IRT0964, and the HPNSF under Grant No. 11JJ7001.

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