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Noise Properties of Coherent Perfect Absorbers and Critically-coupled Resonators

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The performance of a coherent perfect absorber (time-reversed laser) is limited by quantum and thermal noise. At zero temperature, the quantum shot noise dominates the signal for frequencies close to the resonance frequency, and both vanish exactly at the resonance frequency. We compute the sensitivity of the absorbing cavity as a background-free detector, limited by finite signal or detector bandwidth.

In recent work, the authors and collaborators have proposed [1] and demonstrated [2] the phenomenon of coherent perfect absorption, or "time-reversed lasing". Applying the time-reversal operation to the classical electromagnetic equations yields the following statement: if a cavity containing a gain medium reaches the lasing threshold at frequency ω_0 for a certain amplifying refractive index $n(\vec{r}) = n_1(\vec{r}) - in_2(\vec{r})$, then a cavity containing a dissipative refractive index $n^*(\vec{r})$ will perfectly absorb an input mode at ω_0 corresponding to the time-reverse of the lasing mode. We refer to such a lossy cavity as a coherent perfect absorber (CPA). Assuming the input signal is perfectly monochromatic, the above "CPA theorem" is rigorously true within classical electromagnetic theory, where the effects of quantum and thermal noise are neglected. The CPA is a generalization to arbitrary geometries and arbitrary numbers of input channels of the well-known concept of a critically-coupled resonator (CCR) [3], an optical device which can be used for switching, modulation, enhanced photodetection and sensing [4, 5], and which may be regarded as the single channel limit of a CPA. Because a CPA is associated with a vanishing output signal in the classical zero-temperature limit, a CPA (or CCR) can function as a background-free detector or interferometer, similar to a Mach-Zender interferometer (MZ). The fundamental limits to its effectiveness in this role are determined by quantum and thermal noise, which are the subject of the present paper.

Quantum fluctuations, such as spontaneous emission, break the symmetry between emission and absorption. Hence, noise processes in a CPA (or CCR) differ from those in a laser or amplifier. Within semiclassical theory, the laser has zero linewidth; by including quantum fluctuations, one obtains the Schawlow-Townes (ST) linewidth [6], which decreases inversely with the output power well above threshold. As is well-known, the ST linewidth arises from the dephasing of the above-threshold laser field due to quantum noise (usually characterized as "one noise photon per relaxation time per mode"). Because a CPA does not contain an inverted medium like a laser, the spontaneous emission in the absence of the input field vanishes at low temperature, and there is no direct analog of the ST linewidth in a CPA. At zero temperature, the frequency characteristics of the output are determined entirely by the input, and the CPA absorption resonance has approximately twice the passive cavity linewidth, as expected for critical coupling. However, this linewidth is further modified by an analog of the Petermann factor [7, 8]; this effect has not, to our knowledge, been recognized in absorbing systems.

In principle, even when the CPA is at zero temperature, there may be some output noise arising from spontaneous emission (fluorescence) from atoms in the absorbing medium which are excited by the coherent input [9]. However, in the solid state systems which are used for CCRs and CPAs, there are typically many non-radiative degrees of freedom into which the absorbed energy can decay. For instance, in a semiconductor material such as silicon (used in Ref. [2] as an experimental demonstration of a CPA), most of the absorbed energy is eventually converted to heat and drawn out to a thermal bath. Throughout this paper, we will assume that the energy absorbed by the CPA flows into non-radiative degrees of freedom, instead of producing spontaneous emission at the input frequency. Hence, we are studying a *lower bound* on the CPA output noise, but one that is relevant to practical implementations.

Under these assumptions, the only remaining source of quantum noise at zero temperature is the partition or shot noise of the photons. At the perfect absorption resonance, there is no partitioning of the input photons and hence both the average output and its variance vanish for a truly monochromatic input on resonance. In practice, however, the unavoidable linewidth of the input field, $\Delta_{\rm in}$, combines with quantum shot noise to generate a finite noise floor even at the resonance frequency. The noise dominates the signal within an interval $\delta\omega_{\rm x} \propto \sqrt{\Delta_{\rm in}/P_{\rm in}}$ around the resonance frequency, where $P_{\rm in}$ is the power of the input signal. This behavior is similar to a MZ, with the absorption into an external reservoir playing the role of an unobserved MZ output port. However, in contrast to the MZ, the resonant frequency response of the cavity can give rise to para-

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metrically better signal-to-noise ratio in the vicinity of the background-free point [10]. For $T \neq 0$, the CPA also emits thermal radiation; at the absorption resonance, the thermal emissivity takes the black-body value for a one-port system (CCR), but the emission can be significantly reduced in a multi-port CPA.

We can analyze the noise properties of the CPA using the standard input-output framework of quantum optics [11, 12]. Input and output photon operators are connected by the classical electromagnetic scattering matrix $S_{ij}(\omega)$, where $j = 1, 2, \ldots, N$ denote the scattering channels. The S-matrix for a CPA is sub-unitary due to the presence of an absorbing reservoir; it describes a "gray-body" which elastically scatters some fraction of incident photons and absorbs the rest. The scattering channels denote different spatial states in the asymptotic region which suffice to represent an arbitrary incoming or outgoing field at ω, e.g. incoming and outgoing angular momentum channels in a two-dimensional scattering geometry. The input photon operators are denoted by a_j, and the output photon operators by b_j. The special case of a single scattering channel ($N = 1$) corresponds to a standard CCR. We assume that the CPA is coupled to an ideal external reservoir, so that the absorption of light produces negligible heating and hence negligible change in the output noise. The photon operators are related by an input/output relation [11–13]

$$b_i(\omega) = \sum_j S_{ij}(\omega)\, a_j(\omega) + \sum_\nu U_{i\nu}(\omega)\, c_\nu(\omega), \qquad (1)$$

where the c_ν's are ladder operators for reservoir quanta. Here we have assumed a coupling to the reservoir which adds the minimum amount of quantum noise [14]. The requirement that a, b, and c obey canonical commutation relations, e.g. $[a_i(\omega), a_j^\dagger(\omega')] = \delta_{ij}\, \delta(\omega - \omega')$, yields the fluctuation-dissipation relation [12]

$$SS^\dagger + UU^\dagger = \mathbf{1}, \qquad (2)$$

where $\mathbf{1}$ is the $N \times N$ identity matrix. Equation (2) generalizes the unitarity relation of the lossless system, and implies that the eigenvalues of the S-matrix generically have magnitude smaller than unity.

We are interested in the shot noise in the output field, for a coherent input at some frequency ω. To work with equal-frequency correlators, it is convenient to rescale the continuum operators $\{a_i, b_i, c_\nu\}$ to discrete operators $\{\hat{a}_i, \hat{b}_i, \hat{c}_\nu\}$, which are normalized so that the equal-frequency commutator is unity, e.g. $[\hat{a}_i(\omega), \hat{a}_j^\dagger(\omega)] = \delta_{ij}$. Here and in the following, we omit the ω dependence from the notation. Next, the input photon operator can be rewritten using a displacement transformation [14, 16]

$$\hat{a}_i = \alpha_i + \hat{a}'_i, \qquad (3)$$

where α_i is a coherent state amplitude, and $\hat{a}'_i$ is an operator accounting for fluctuations around the coherent state, which likewise obeys the canonical commutation relation $[\hat{a}'_i, \hat{a}'^\dagger_j] = \delta_{ij}$. Hence,

$$\hat{b}_i = \sum_j S_{ij}\,(\hat{\alpha}_j + \hat{a}'_j) + \sum_\nu U_{i\nu}\, \hat{c}_\nu. \qquad (4)$$

The operator $\mathcal{N}_i \equiv \hat{b}_i^\dagger \hat{b}_i$ gives the output photon flux per unit frequency (at frequency ω) in channel i. (Correspondingly, $\mathcal{I}_i = \hbar\omega_0 \mathcal{N}_i$ describes the spectral density.) Using (4), we can calculate the expectation value and correlation function for $\mathcal{N}_i$, using standard Gaussian n-point operator correlators [16]. We take $\langle \hat{a}'_i \rangle = \langle \hat{c}_\nu \rangle = 0$, and $\langle \hat{a}'^\dagger_i \hat{a}'_j \rangle = 0$ (zero net fluctuation around the specified coherent input amplitude), and $\langle \hat{c}_\mu^\dagger \hat{c}_\nu \rangle = \delta_{\mu\nu}\, f(T)$ where $f(T) = [\exp(\hbar\omega/k_B T) - 1]^{-1}$ and T is the temperature of the reservoir. The result for $\langle \mathcal{N}_i \rangle$ is

$$\langle \mathcal{N}_i \rangle = |(S\alpha)_i|^2 + \left[1 - (SS^\dagger)_{ii}\right]\, f(T). \qquad (5)$$

Thus the total output is

$$\langle \mathcal{N} \rangle = \sum_i \langle \mathcal{N}_i \rangle = |S\alpha|^2 + f(T)\, \mathrm{Tr}(\mathbf{1} - SS^\dagger), \qquad (6)$$

and the noise is

$$\begin{aligned} \langle \delta\mathcal{N}^2 \rangle &\equiv \sum_{ij} \left[\langle \mathcal{N}_i \mathcal{N}_j \rangle - \langle \mathcal{N}_i \rangle \langle \mathcal{N}_j \rangle \right] \\ &= |S\alpha|^2 \\ &\quad + f(T) \left\{ 2|S\alpha|^2 - 2|S^\dagger S\alpha|^2 + \mathrm{Tr}\left(\mathbf{1} - SS^\dagger\right) \right\} \\ &\quad + [f(T)]^2\, \mathrm{Tr}\left[\left(\mathbf{1} - SS^\dagger\right)^2 \right]. \end{aligned} \qquad (7)$$

For $T \to 0$, $f(T) \to 0$, (6) and (7) reduce to the Poissonian result

$$\langle \mathcal{N} \rangle = \left\langle \delta\mathcal{N}^2 \right\rangle = |S\alpha|^2. \qquad (8)$$

The CPA condition is achieved when the index of refraction of the cavity, $n(\vec{r})$, is chosen such that there exists an eigenvector of the S-matrix with eigenvalue zero at a specific input frequency ω_0. If α is chosen to be this eigenvector, then we see from Eq. (8) that the mean outgoing photon flux vanishes (as it should be from the classical CPA theorem), and so does its variance (the shot noise). This confirms the statement that the CPA effect is unaffected by quantum fluctuations at zero temperature for a purely monochromatic input.

For $T > 0$, $\langle \mathcal{N} \rangle$ and $\left\langle \delta\mathcal{N}^2 \right\rangle$ are unequal, and differ by a term arising from the beating between the thermal emission and the scattered flux:

$$\left\langle \delta\mathcal{N}^2 \right\rangle - \langle \mathcal{N} \rangle = f(T) \left[2|S\alpha|^2 - 2|S^\dagger S\alpha|^2 \right] + O(f^2). \qquad (9)$$

This difference can be shown to be strictly positive for any input field. The term linear in $f(T)$ vanishes when the input field, α, corresponds to the zero eigenvector, so thermal noise in the CPA is very small for $T \lesssim \hbar\omega_0/k_B$.

We will discuss the $T > 0$ case further at the end of the paper.

To estimate the sensitivity of a CPA as a detector, we study its behavior for frequencies near the perfect absorption resonance, ω_0. For now, we consider $T = 0$. In the absence of loss, the S-matrix has a sequence of resonances, associated with poles in the lower half of the complex ω plane and symmetrically placed zeros in the upper half plane [1], at discrete frequencies $\omega = \omega_0 \mp i\gamma_c/2$. Near ω_0, one of the S-matrix eigenvalues takes the following approximate form [18]:

$$s(\omega) \approx e^{i\varphi(\omega)}\, \frac{\omega - \omega_0 - i\gamma_c/2}{\omega - \omega_0 + i\gamma_c/2}, \tag{10}$$

where φ is an irrelevant phase factor and γ_c is the cavity lifetime. Eq. (10) is a "single resonance approximation" which ignores the presence of other nearby poles and zeros. It satisfies the requirements that $|s| = 1$ when $\omega \in \mathbb{R}$ (the lossless S-matrix has unimodular eigenvalues), and that s goes to zero and infinity at $\omega = \omega_0 \pm i\gamma_c/2$. Adding absorption pushes the zero and pole down in the complex frequency plane, as shown in Fig. 1(a); to lowest order, they move down by equal amounts. To achieve the CPA condition, exactly enough absorption is added to push the zero down to the real ω axis. Then the eigenvalue of the absorbing cavity is

$$s(\omega) \approx e^{i\varphi(\omega)}\, \frac{\omega - \omega_0 + i\,\delta\gamma}{\omega - \omega_0 + i\gamma_c}, \tag{11}$$

where the parameter $\delta\gamma \ll \gamma_c$ represents a small detuning of the material loss from the perfect absorption resonance. Note that the resonance of the absorbing cavity has twice the width of the passive cavity resonance ($\gamma_c/2 \to \gamma_c$). This is due to the "critical coupling" condition that the absorption loss rate is equal to the scattering loss rate.

In this case, if the input mode, α, corresponds to the zero eigenvector of the S-matrix, then (at $T = 0$):

$$\langle \mathcal{N} \rangle = \langle \delta\mathcal{N}^2 \rangle \approx \frac{(\omega-\omega_0)^2 + \delta\gamma^2}{(\omega-\omega_0)^2 + \gamma_c^2}\, \frac{P_{\rm in}(\omega)}{\hbar\omega\Delta_{\rm in}}. \tag{12}$$

Here we have expressed the input photon flux per unit frequency, $|\alpha|^2$, in terms of the input power

$$P_{\rm in}(\omega) = |\alpha(\omega)|^2 \hbar\omega\Delta_{\rm in}. \tag{13}$$

This power is averaged over some bandwidth $\Delta_{\rm in}(\omega)$, which is taken to be the frequency resolution of either the input state or the output detector/spectrometer, whichever is smaller; ω is the center frequency of the bandwidth window,

According to Eq. (12), as $\delta\gamma \to 0$ and $\omega \to \omega_0$, the mean and variance of the photon flux behaves as

$$\langle \mathcal{N} \rangle = \langle \delta\mathcal{N}^2 \rangle \approx \frac{(\omega-\omega_0)^2}{\gamma_c^2}\, \frac{P_{\rm in}(\omega)}{\hbar\omega\Delta_{\rm in}}. \tag{14}$$

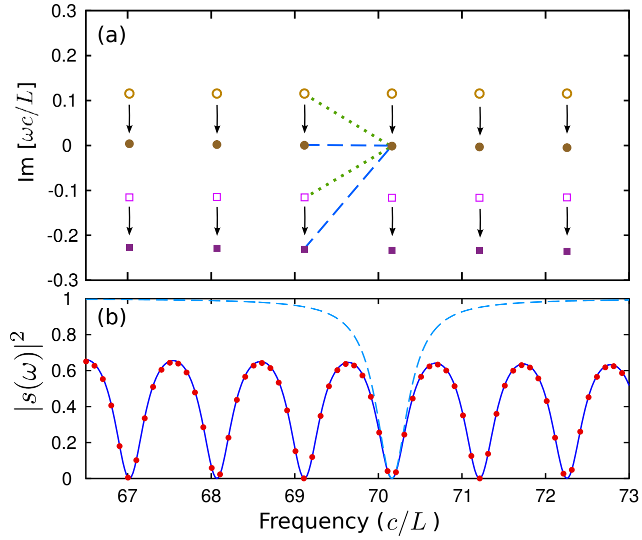

FIG. 1. (color online) Scattering properties of a one-port coherent perfect absorber (critically-coupled resonator) consisting of a one-dimensional uniform dielectric slab of length L, with a perfect mirror on one side and vacuum on the other. (a) Location of poles (squares) and zeros (circles) in the complex frequency plane. Open symbols show the poles and zeros for the passive cavity with refractive index $n = 3$, and filled symbols for an absorbing cavity with refractive index $n = 3 + 0.005i$. The dashed and dotted lines are guides to the eye for the geometric interpretation of the Petermann factor given in Eq. (15). Without absorption, $|\omega - \omega^z| = |\omega - \omega^p|$, where $\omega \in \mathbb{R}$ and ω^z and ω^p are the frequencies of a neighboring pair of zeros and poles (green dots). With very narrow band absorption, near a CPA resonance (in this case $\omega \approx 70.1c/L$), this remains approximately true. However, for broadband absorption, $|\omega - \omega^z| < |\omega - \omega^p|$ (blue dashes). (b) Plot of $|s(\omega)|^2$, where $s(\omega)$ is the eigenvalue of the scattering matrix (i.e. in this one-port case the reflection coefficient). The red circles show exact numeric results obtained by the transfer matrix method, while the solid blue curve shows Eq. (15), with the product taken over the 20 pairs of poles and zeros nearest to $\omega = 70L/c$. The dashed curve shows the single-resonance approximation, Eq. (11), using the pole and zero nearest to $\omega = 70L/c$.

The CPA resonance has a quadratic zero at ω_0 (Fig. 2), and, in the single resonance approximation, its width is γ_c, the critically-coupled cavity linewidth. (We assume that γ_c is much less than the free spectral range of the resonator).

However, adding a lossy medium to the resonator affects the S-matrix eigenvalue beyond the single-resonance approximation, and the corrections to this approximation generically increase the width. This is the exact analog of the Petermann factor in lasers, which increases the ST linewidth [7, 8]. This is most easily seen in the one-port case, where we can express the S-matrix eigenvalue with a zero on the real axis as

$$s(\omega) = e^{i\varphi(\omega)}\, \frac{\omega - \omega_0}{\omega - \omega_0 + i\gamma_c} \prod_n \frac{\omega - \omega_n^z}{\omega - \omega_n^p}, \tag{15}$$

where n indexes the other pairs of zeros and poles, which have frequencies ω_n^z and ω_n^p respectively. This ansatz agrees very well with exact numerical calculations of $s(\omega)$, as shown in Fig. 1(b). Due to time-reversal symmetry, in the absence of loss, each factor in the product would be unimodular, since the zeros and poles are symmetrically distributed around the real axis. The introduction of loss breaks this symmetry for all the poles and zeros, not just the zero at ω_0; this causes the zeros to move towards the real axis and the poles away, so that $|\omega - \omega_n^z| < |\omega - \omega_n^p|$ for any real ω, as shown in Fig. 1(a). This effect is neglected in the single resonance approximation of Eq. (14). Including it leads to

$$\langle \mathcal{N} \rangle = \langle \delta \mathcal{N}^2 \rangle \approx \frac{(\omega - \omega_0)^2}{K\gamma_c^2} \, \frac{P_{\rm in}(\omega)}{\hbar\omega\Delta_{\rm in}}, \tag{16}$$

where $K = \prod_n |(\omega - \omega_n^p)/(\omega - \omega_n^z)| > 1$ for the one-channel case. Calculating the Petermann factor for the time-reversed counterpart of this cavity, which contains gain instead of loss and sits at the first lasing threshold, would yield exactly the same value of K [19]. This equivalence follows from the properties of the S-matrix under time-reversal, as does the CPA effect itself. For a more complex, multichannel cavity, the explicit calculation of the S-matrix eigenvalues is more complicated, but the symmetry that leads to the factor $1/K$ in the eigenvalue still holds. In other words, adding loss to the cavity so as to reach the CPA condition increases the critically-coupled cavity resonance linewidth from $\gamma_c \to \sqrt{K}\gamma_c$.

In calculating this broadening effect, one could assume that the loss of the cavity is broadband, affecting all relevant poles and zeros equally; a similar assumption of broadband gain is made in standard calculations of the Petermann factor. When this is not the case, both for CPAs and lasers the Petermann correction is reduced, and needs to be calculated using the frequency-dependent refractive index of the cavity [19]. For the subsequent analysis, we will assume that the K occurring in Eq. (16) is a given parameter.

Exactly at ω_0, both signal and noise vanish; since the the shot noise $[\langle \delta\mathcal{N}^2 \rangle]^{1/2}$ is the square root of the signal $\langle \mathcal{N} \rangle$, the noise will dominate the signal when $\langle \mathcal{N} \rangle = 1$, which will occur at a crossover frequency near ω_0. The crossover frequency scale is

$$\delta\omega_{\rm x} \sim \frac{\sqrt{K}\gamma_c}{|\alpha(\omega_0)|} = \left[\frac{\hbar\omega_0\Delta_{\rm in}}{P_{\rm in}}\right]^{1/2} \sqrt{K}\,\gamma_c. \tag{17}$$

The measured values of the signal and noise must, however, be obtained by averaging (16) over the bandwidth $\Delta_{\rm in}$ at each frequency, as indicated by the dashed curves in Fig. 2. These averaged values do not vanish at $\omega = \omega_0$. Up to a factor of order unity depending on the averaging procedure, their residual values are

$$\overline{\langle \mathcal{N} \rangle}_{\omega_0} = \overline{\langle \delta\mathcal{N}^2 \rangle}_{\omega_0} \approx \frac{P_{\rm in}(\omega_0)\,\Delta_{\rm in}}{12\hbar\omega_0 K\gamma_c^2} \sim \left[\frac{\Delta_{\rm in}}{\delta\omega_{\rm x}}\right]^2. \tag{18}$$

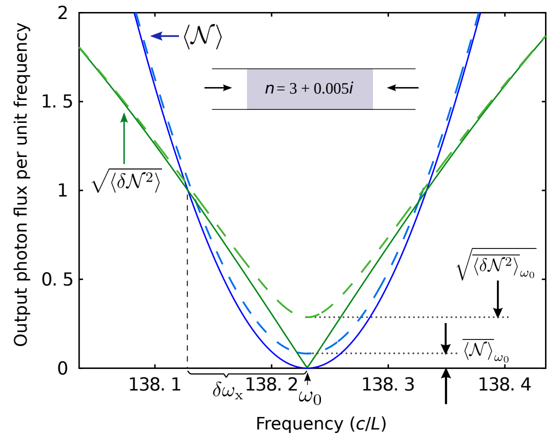

FIG. 2. (color online) Plot mean output power ($\mathcal{N}$) and shot noise power ($\sqrt{\delta\mathcal{N}^2}$), as a function of frequency near a CPA zero, for $T = 0$. The cavity is a two-sided uniform dielectric slab with refractive index $3 + 0.005i$ and length L (see schematic). For this case the CPA eigenmodes are simply incoming coherent plane waves of equal amplitude from each direction, with either even or odd parity. Results for an even CPA mode at frequency $\omega \approx 138.23\,c/L$ are shown. We have chosen the input intensity $|\alpha|^2 = 25$. The solid curves are obtained from Eq. (8), using the transfer matrix method to find $S(\omega)$. The dashed curves are obtained by averaging these values over a bandwidth $\Delta_{\rm in} = 0.1\,c/L$, corresponding to a finite spectrometer resolution. Inset: schematic of the system.

In particular, we can regard $[\overline{\langle \delta\mathcal{N}^2 \rangle}_{\omega_0}]^{1/2}$ as the effective shot noise level at the absorption resonance. It dominates over the bandwidth-averaged signal if $\Delta_{\rm in} \ll \delta\omega_{\rm x}$.

The crossover frequency scale $\delta\omega_{\rm x}$ is related to the sensitivity of the output to the loss detuning parameter $\delta\gamma$. From Eq. (12), the change in the output signal at $\omega = \omega_0$ resulting from $\delta\gamma \neq 0$ is

$$\langle \Delta\mathcal{N} \rangle = \left[\frac{\delta\gamma}{\delta\omega_{\rm x}}\right]^2. \tag{19}$$

(To lowest order, this quantity is unaffected by bandwidth-averaging.) The sensitivity of the CPA as a low-background detector is given by the minimum $\delta\gamma$ for which $\langle \Delta\mathcal{N} \rangle$ is distinguishable from the effective noise level. Comparing (19) to (18), we obtain the result

$$|\delta\gamma| \gtrsim \sqrt{\delta\omega_{\rm x}\Delta_{\rm in}}. \tag{20}$$

Since a CPA can serve as an absorbing interferometer/detector, it is useful to compare it to a lossless interferometer such as a Mach-Zender interferometer (MZ). As shown in Fig. 3, an analogy can be made between a CCR (single-channel CPA) and a MZ. The input photon operator a and the reservoir operator c, from the one-channel version of Eq. (1), map onto two input photon operators for the MZ; meanwhile the output photon operator b maps onto one of the MZ outputs. The fluctuation-dissipation relation, Eq. (2), is equivalent to the relation for transmission into the *observed port* of the

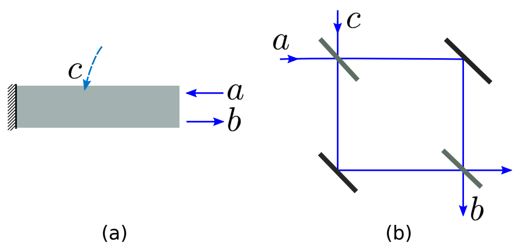

FIG. 3. (color online) (a) Schematic of a one-channel absorbing cavity, where the output amplitude b is a superposition of the input amplitude a and the reservoir operator c. (b) The corresponding Mach-Zender interferometer, with a and c entering in the two input ports and b being one of the outputs.

MZ, S_b, (which follows from the unitary scattering matrix for both ports):

$$S_b = \frac{1}{2}\left(e^{i\theta} - 1\right), \quad |U^2| = 1 - |S_b|^2, \tag{21}$$

where θ is the phase difference between the two arms of the interferometer. Perfect absorption in the cavity corresponds to $\theta = 0$, so that the a input is directed entirely into the second, unmonitored output port, leaving the b port empty. Quantum fluctuations in the empty c input contribute to the shot noise. Note that in the CCR, the single input beam interferes with itself and it does not function as an optical interferometer, i.e. it does not measure the relative phase of two input beams. The multi-channel CPA does, however, act as an interferometer. In the simplest case of two input channels and a cavity with parity symmetry, the two S-matrix eigenmodes have even and odd symmetry respectively, and only one of these are perfectly absorbed at a given absorption resonance. The interferometry is performed by making small phase changes between the two input beams, moving the system slightly away from the eigenchannel. For $N > 2$ and/or absent parity symmetry, both the magnitude and phase of each input amplitude must be tuned in order to reach perfect absorption; this is analogous to a multi-input MZ-like interferometer whose input amplitudes and phases may be tuned to send all the output into a single port, playing the role of the absorbing channel.

The main difference between the CPA and the MZ lies in the frequency dependence. For the latter, S_b varies sinusoidally in frequency with free spectral range $\sim c/L$, where L is the dimension of the system. Absorbing cavities, however, are described by Eq. (11), and for high-Q cavities the absorption resonances are much narrower than c/L. Since the signal-to-noise ratio depends inversely on $\gamma_c, 1/L$ in the two cases, for a given value of $\omega-\omega_0$, the CPA would have a better signal to noise ratio. This resonant enhancement of the sensitivity of a CCR has been pointed out in a different context in Ref. [10]. A similar effect could be achieved in a MZ by adding resonant cavities along each arm to effectively increase the optical path length.

Finally, we make some comments about the $T > 0$ case. In Eq. (6), we see that $\langle \mathcal{N} \rangle$ is written as the sum of the classical scattered flux and the gray-body thermal emission, $f(T)\, N[1-\bar{\sigma}]$, $\bar{\sigma}$ being the mean scattering strength per channel [12]. For a CCR (one-channel CPA), $\bar{\sigma} = 0$ at the operating frequency of the absorption resonance, so the thermal emission has the black-body value. In the MZ analogy, Fig. 3(b), this is equivalent to connecting the c input port to a black-body source. Since the MZ is tuned so that the a input is completely directed into the unmonitored output port, the black-body emission into c is directed into the monitored output port b.

For a multi-channel CPA, the thermal emissivity is less than the black-body value, even at the operating frequency of the absorption resonance. This is because only one of the N scattering strengths vanishes; the other $N-1$ scattering strengths are nonzero, so that $\bar{\sigma} > 0$. From Eq. (9), the same is true of the thermal contribution to the output noise $\langle \mathcal{N}^2 \rangle$. Furthermore, for $N \gg 1$ a "hidden black" scenario is possible, in which the CPA perfectly absorbs the input field, α, but emits a negligible amount of thermal radiation [17]. More specifically, for a weakly absorbing system it is possible that the mean albedo (reflectivity) can be large, $\bar{\sigma} \to 1$, indicating an almost white body, and implying that the thermal contributions to $\langle \mathcal{N} \rangle$ and $\langle \mathcal{N}^2 \rangle$ vanish, while nonetheless the system is perfectly absorbing (down to the quantum noise floor) if the correct input field is supplied. Elsewhere, two of the authors have shown that this effect can be generalized beyond the case of a perfect CPA at resonance, to a disordered scattering medium with weak absorption over a large range of frequency and absorptivity [17].

The presence of thermal noise also affects the sensitivity of the CPA as a detector. From Eq. (7), the noise level at $\omega = \omega_0$ in the thermal noise dominated limit is

$$\left\langle \delta\mathcal{N}^2 \right\rangle_{\omega_0}^{\text{thermal}} \equiv g(T) \approx f(T)\, \mathrm{Tr}\left(\mathbf{1} - SS^\dagger\right). \tag{22}$$

Comparing this to the signal (19), we obtain the sensitivity limit

$$|\delta\gamma| \gtrsim g(T)^{1/4}\, \delta\omega_{\mathrm{x}}. \tag{23}$$

The crossover between the bandwidth-dominated and thermal-dominated noise regimes occurs at

$$g(T) \sim \left[\frac{\Delta_{\mathrm{in}}}{\delta\omega_{\mathrm{x}}}\right]^2. \tag{24}$$

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