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Continuous change of the propagation direction of a classical control field in the process of its off-resonant Raman interaction with a weak signal field in a three-level atomic medium is suggested for quantum storage of a single-photon wave packet. It is shown that due to phase matching condition such an angular control allows one to reversibly map the single-photon wave packet to the Raman spatial coherence grating. Thus, quantum storage and retrieval can be realized without using inhomogeneous broadening of the atomic transitions or manipulating the amplitude of the control field. Under some conditions the proposed scheme proves to be mathematically analogous to the quantum storage scheme based on controlled reversible inhomogeneous broadening of the atomic states.

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I. INTRODUCTION

Manipulating single-photon states of the electromagnetic field is an important part of implementation of various quantum information protocols. In particular, storage and retrieval of single-photon wave packets is of great importance. Optical quantum memory [1–3] is at the heart of linear optical quantum computing [4] and long-distance quantum cryptography with quantum repeaters [5]. It also may be useful for making heralded single-photon sources deterministic [6]. An efficient storage and retrieval of single photons has been demonstrated recently in gases [7] and rare-earth-ion doped solids [8] using controlled reversible inhomogeneous broadening (CRIB) of resonant atomic transitions. Significant progress has also been achieved in demonstration of quantum storage using atomic frequency comb [9–14], electromagnetically induced transparency [15–18] and off-resonance Raman interaction [19, 20].

It is usually assumed that to store and recall optical pulses one needs an inhomogeneous broadened atomic transition (tailored or controlled) or a modulated control field amplitude matched an input pulse (see reviews [1–3] and recent experiments mentioned above). In the present work, we develop another approach which requires neither inhomogeneous broadening nor temporal modulation of the control field amplitude, but resorts to continuous phase-matching control in an extended resonant medium. We consider off-resonant Raman interaction of a single-photon wave packet and a classical control field in a three-level atomic medium. Under such conditions the phase-matching control can be achieved by modulating refractive index of the resonant medium [21] or by modulating the direction of propagation of the control field, which is studied in the present work. Another possibility is to use a frequency chirp of the control field, which requires synchronous modulation of the atomic transition frequency in order to keep zero two-photon detuning [22]. In any case, a continuous change of the wave vector of the control field during the interaction leads to the mapping of a single photon state to a superposition of atomic collective excitations (spin waves) with different wave vectors and vice versa. In comparison with [21], where the cavity-model of quantum memory was considered, here we discuss a free-space model. We show that under some conditions the proposed scheme proves to be mathematically analogous to the longitudinal CRIB-based quantum storage, just as a two-level scheme with refractive index control does [23]. The angular control scheme, contrary to CRIB, allows one to use materials which cannot be controlled by external dc electric or magnetic fields. Generally speaking, the angular control scheme is also much easier for implementation than the refractive index modulation. The problem for the latter is that it needs to be achieved without modulation of atomic levels. It may be done in some specific materials like Tm:$LiNbO_3$ [23]. But in general case such scenario is rather difficult to implement as it was discussed in [24].

The paper is organized as follows. In Sec. II, the model of the quantum memory is presented. In Sec. III, we analyze the storage and retrieval of single-photon wave packets with transverse propagation of the control field. In Sec. IV, a more general situation is considered in the limit of a large Fresnel number of interaction volume. In Sec. V, we consider some implementation issues. Sec. VI concludes the paper with final remarks.

II. THE MODEL AND BASIC EQUATIONS

We consider a system of identical three-level atoms interacting with a weak quantum field (single-photon wave packet) to be stored and with a strong classical control field. The atoms have a Λ-type level structure, and the fields are Raman resonant to the lowest (spin) transition, see Fig. 1(a). We assume that the atoms are stationary-like impurities embedded into a solid-state material or cold atoms in an optical lattice. The sample is approximated by a parallelepiped with cross section $L_x \times L_y$ and length L. The coordinate system is originated at the center of the medium. The single-photon wave packet is x polarized and propagates in z direction. It is assumed that the transverse spatial profile of this quantum field completely goes into the sample cross section. The control field is supposed to be polarized along axis y and propagated in any direction in (x, z)-plane. The duration of storage and retrieval processes is equal to T. During the storage, $t \in (-T, 0)$, the propagation direction of the control field changes so that at different moments, the signal field interacts with the control field of different directions, thus creates coherence between $|1\rangle$ and $|3\rangle$ (spin wave) with different spin wave vectors, see Fig. 1(b). During retrieval, $t \in (0, T)$, the interaction between the spin wave and the control field reconstructs the signal field, as illustrated in Fig. 1(c). The storage time between the end of the storage and the beginning of the retrieval is subjected a free decay of the spin wave, which will be neglected through out this paper.

The signal field corresponding to the single-photon wave packet of average frequency ω_s and wave vector k_s is considered in the paraxial approximation and written as

$$E_s(\mathbf{r}, t) = \frac{i}{n}\sqrt{\frac{\hbar\omega_s}{2\varepsilon_0 c}}a(\mathbf{r}, t)\, e^{i(k_s z - \omega_s t)} + \text{H.c.}, \tag{1}$$

where n and c are the refractive index and phase velocity of the quantum field inside the medium, $a(\mathbf{r}, t)$ is the slowly

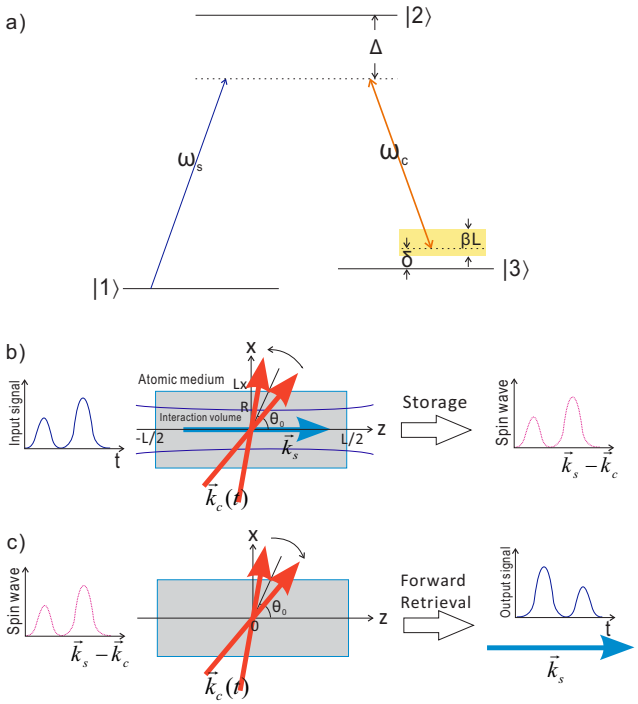

FIG. 1. (Color online) (a) Energy diagram of the Raman interaction in a three-level Λ system; (b) During storage, the temporal profile of the signal field is mapped into the spin wave distributed over different spin wave vectors; (c) During forward retrieval, the spin wave profile is mapped back into the output signal field.

varying annihilation operator. Expression (1) is written in the approximation of the equal phase and group speeds of light in the medium (see, e.g., [25]) which is made for simplicity.

The classical control field is supposed to be a monochromatic plane wave subjected to a phase modulation due to rotation of its wave vector on a small angle. Since most of the atoms remain in the ground state during the interaction, we neglect the control field absorption so that the field can be written as

$$E_c(\mathbf{r},t) = E_0\, e^{i(\bar{\mathbf{k}}_c\mathbf{r}-\omega_c t+\phi(\mathbf{r},t))} + \text{c.c.}\,, \tag{2}$$

where E_0 is a constant amplitude of the plane wave, $\bar{\mathbf{k}}_c$ is an average wave vector of $\mathbf{k}_c(t)$ during rotation, and $\phi(\mathbf{r},t)$ is a phase shift due to the rotation, which is specified later in this section.

The collective atomic operators are defined as the mean values of the single-atom operators

$$\sigma_{mn}(\mathbf{r},t) = \frac{1}{N}\sum_j |m_j\rangle\langle n_j|\,\delta^{(3)}(\mathbf{r}-\mathbf{r}_j)\,, \tag{3}$$

where N is the atomic number density, which is supposed to be constant in space, and $|n_j\rangle$ is the nth state ($n=1,2,3$) of jth atom with the energy $\hbar\omega_n$ ($\omega_1 = 0 < \omega_3 < \omega_2$). The slowly varying amplitude describing coherence on the Raman transition, $s(\mathbf{r},t)$, is then introduced as

$$\sigma_{13}(\mathbf{r},t) = s(\mathbf{r},t)\, e^{i(\mathbf{k}_s-\bar{\mathbf{k}}_c)\mathbf{r}-i(\omega_s-\omega_c)t}\,. \tag{4}$$

Off-resonant Raman interaction is described by the following equations:

$$\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right) a(\mathbf{r},t) = \frac{i}{2k_s}\Delta_\perp a(\mathbf{r},t) + \\ + g^* N s(\mathbf{r},t)\, e^{i\phi(\mathbf{r},t)}\,, \tag{5}$$

$$\frac{\partial}{\partial t} s(\mathbf{r},t) = (-\gamma + i\delta)\, s(\mathbf{r},t) - g a(\mathbf{r},t)\, e^{-i\phi(\mathbf{r},t)}\,, \tag{6}$$

where $\Delta_\perp = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $g = \frac{1}{n}\sqrt{\frac{\omega_s}{2\varepsilon_0\hbar c}}\frac{d_{21}\Omega}{\Delta}$ is the coupling constant between the atoms and the weak quantized field, $\Omega = d_{32}E_0/\hbar$ is the Rabi frequency of the classical control field, d_{ij} is the dipole moment of the transition between $|i\rangle$

and $|j\rangle$, $\Delta = \omega_2 - \omega_s$ is one-photon detuning, γ is the rate of dephasing of the spin coherence, which in the general case includes both homogeneous and inhomogeneous broadening of the Raman transition, and δ is two-photon detuning. When writing Eqs. (5) and (6), it is assumed that 1) the time of propagation of photons through the system L/c is negligibly short compared to the evolution time of the slowly time-varying envelopes; 2) all the atoms are initially in the ground state $|1\rangle$ and most of them remain in the ground state so that Langevin noise atomic operators are not included; 3) the frequency shift $|\Omega|^2/\Delta$ of the Raman transition induced by the coupling field is taken into account by redefining field frequency ω_c; 4) the refractive index change of the medium due to the atoms is incorporated into the value of n.

It is convenient to expand the slowly varying atomic and field operators on the transverse mode basis

$$a(\mathbf{r},t) = \sum_{mn} u^*_{mn}(x,y) a_{mn}(z,t)\,, \tag{7}$$

$$s(\mathbf{r},t) = \sum_{mn} u^*_{mn}(x,y) s_{mn}(z,t)\,, \tag{8}$$

where the transverse mode functions $u_{mn}(x,y)$ satisfy the conditions of completeness and orthogonality:

$$\sum_{mn} u^*_{mn}(x,y)\, u_{mn}(x',y') = \delta(x-x')\,\delta(y-y')\,, \tag{9}$$

$$\iint dx\, dy\, u^*_{mn}(x,y)\, u_{m'n'}(x,y) = \delta_{mm'}\,\delta_{nn'}\,. \tag{10}$$

In what follows, we use the set of paraxial plane waves

$$u_{mn} = \frac{1}{\sqrt{L_x L_y}}\, e^{-i\mathbf{q}_{mn}\boldsymbol{\rho}}, \tag{11}$$

where $\boldsymbol{\rho} = (x,y)$, $\mathbf{q}_{mn} = (2\pi m/L_x, 2\pi n/L_y)$, $m,n \in \mathbb{Z}$, and L_x, L_y are transverse linear sizes of the sample of a rectangular cross section. This definition corresponds to periodic boundary conditions in the x,y-plane.

In order to write down the equations in the transverse reciprocal space, the function $\phi(\mathbf{r},t)$ needs to be specified. Let us suppose that the wave vector of the control field is rotated around some average direction on a small angle so that the induced phase shift may be considered as a linear function of the rotation angle. In such linear regime, we can write

$$\phi(\mathbf{r},t) = \beta_x(x-x_0)t + \beta_y(y-y_0)t + \beta_z(z-z_0)t + \phi_0(\mathbf{r}), \tag{12}$$

where β_i is the rate of change of the component $\mathbf{k}_c$ along ith axis, and the coordinates x_0, y_0, z_0 correspond to a phase stationary point where the phase $\phi(t)$ of the control field remains constant during the rotation. $\phi_0(\mathbf{r})$ is a time-independent phase factor which can be incorporated into $s(\mathbf{r},t)$.

In addition, we extract factors describing the phase due to transverse momentum by introducing new variables

$$A_{mn}(z,t) = a_{mn}(z,t) w_{mn}(z)\,, \tag{13}$$

$$S_{mn}(z,t) = s_{mn}(z,t) w_{mn}(z)\,, \tag{14}$$

where $w_{mn}(z) = \exp\left[i\frac{q^2_{mn}}{2k_s}(z-z_w)\right]$, and z_w corresponds to the position of the beam waist. Then the set of Eqs. (5) and (6) in the reciprocal space take the form

$$\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right) A_{mn}(z,t) = \\ = g^* N \sum_{m'n'} S_{m'n'}(z,t)\, B^+_{m'n',mn}(z,t)\,, \tag{15}$$

$$\frac{\partial}{\partial t} S_{mn}(z,t) = (-\gamma + i\delta)\, S_{mn}(z,t) \\ - g \sum_{m'n'} A_{m'n'}(z,t)\, B^-_{m'n',mn}(z,t)\,, \tag{16}$$

where

$$B^{\pm}_{m'n',mn}(z,t) = \mathrm{sinc}\left[\frac{(\pm\beta_x t + q_{m'n',x} - q_{mn,x})L_x}{2}\right] \times \mathrm{sinc}\left[\frac{(\pm\beta_y t + q_{m'n',y} - q_{mn,y})L_y}{2}\right] \times w^*_{m'n'}(z)\, w_{mn}(z)\, e^{\pm i(\beta_z z - \beta_z z_0 - \beta_x x_0 - \beta_y y_0)t}\,. \quad (17)$$

Without rotating the control field the factors $B^{\pm}_{m'n',mn}(z,t)$ become identity matrices, and each transverse mode can be considered independently in the framework of one-dimensional model. However, even in the case of rotating control field, the decoupling is also possible if $\beta_x t$ and $\beta_y t$ remain small with respect to the interval between the modes $2\pi/L_x$ and $2\pi/L_y$, respectively, while $\beta_z t$ being larger than $2\pi/L$. In such a case, different Fourier transverse modes evolve independently on each other. The quantum storage can then be considered in single-mode approximation, which is the subject of Sec. III. Another simplification is possible in the limit of a large Fresnel number F of the excitation volume. In this situation, diffraction spreading may be neglected, and Eqs. (5) and (6) allow one to consider each point (x, y) independently. Such a geometrical optics approximation is considered in Sec. IV. Regarding the excitation geometry, it should be noted that $\beta_x x_0 + \beta_y y_0 + \beta_z z_0$ becomes zero when average control wave vector $\bar{\mathbf{k}}_c$ is directed from stationary phase point (x_0, y_0, z_0) to the center of the sample. We will see in Sec. IV that this is the most convenient excitation geometry, which needs a zero two-photon detuning δ and requires no manipulation with δ during storage and retrieval.

The main figures of merit those should be calculated for the quantum storage are total efficiency η and fidelity $\mathscr{F}$. Since the input and output faces of the sample are located at the points $z = -L/2$ and $z = L/2$, respectively, the input and output single-photon wave packets are described by the operators $A_{mn,\mathrm{in}}(t) = A_{mn}(-L/2, t)$ and $A_{mn,\mathrm{out}}(t) = A_{mn}(L/2, t)$. The total efficiency is defined as

$$\eta = \frac{N_{\mathrm{out}}}{N_{\mathrm{in}}}\,, \quad (18)$$

where

$$N_{\mathrm{in}} = \sum_{mn}\int_{-\infty}^{0} dt \langle A^{\dagger}_{mn,\mathrm{in}}(t) A_{mn,\mathrm{in}}(t)\rangle\,, \quad (19)$$

$$N_{\mathrm{out}} = \sum_{mn}\int_{0}^{\infty} dt \langle A^{\dagger}_{mn,\mathrm{out}}(t) A_{mn,\mathrm{out}}(t)\rangle\,, \quad (20)$$

since we assume that the storage process terminates at the moment $t = 0$, while retrieval process begins at this moment of time.

For the time-reversed output pulse the fidelity is usually defined as

$$\mathscr{F} = \eta\mathscr{F}'\,, \quad (21)$$

where

$$\mathscr{F}' = \frac{\left|\sum_{mn}\int_0^{\infty} dt \langle A^{\dagger}_{mn,\mathrm{out}}(t) A_{mn,\mathrm{in}}(\bar{t} - t)\rangle\right|^2}{N_{\mathrm{in}} N_{\mathrm{out}}} \quad (22)$$

is a measure of pulse preservation independent of the total efficiency, and $\bar{t}$ is the delay that maximizes $\mathscr{F}'$. In what follows, it is the latter quantity $\mathscr{F}'$ that is used for characterizing quantum storage together with the efficiency. The definitions of η and $\mathscr{F}'$ can also be done in a similar way in terms of the spatial variables $a_{\mathrm{in}}(\mathbf{r}, t)$ and $a_{\mathrm{out}}(\mathbf{r}, t)$. In real space $N_{\mathrm{in}} = \int d^2\rho \int_{-\infty}^{0} dt\, \langle a^{\dagger}_{in}(\boldsymbol{\rho}, t) a_{in}(\boldsymbol{\rho}, t)\rangle$, and $N_{\mathrm{out}} = \int_0^{\infty} dt \int d^2\rho\, \langle a^{\dagger}_{\mathrm{out}}(\boldsymbol{\rho}, t) a_{\mathrm{out}}(\boldsymbol{\rho}, t)\rangle$, while $\mathscr{F}' = \frac{1}{N_{\mathrm{in}} N_{\mathrm{out}}}\left|\int_0^{\infty} dt \int d^2\rho\, \langle a^{\dagger}_{\mathrm{in}}(\bar{t} - t) a_{\mathrm{out}}(\boldsymbol{\rho}, t)\rangle\right|^2$.

III. SINGLE-MODE APPROXIMATION

For simplicity, we assume in what follows that the wave vector of the control field is rotated in (x, z)-plane around an average polar angle θ_0, as shown in Fig. 2, so that $\bar{\mathbf{k}}_c = k_c \sin\theta_0 \mathbf{x} + k_c \cos\theta_0 \mathbf{z}$, where $k_c = \frac{\omega_c}{c}$. This also ensures

that the control field polarization is unchanged during the interaction. In this case, we have $\beta_x = \beta\cos\theta_0$, $\beta_y = 0$ and $\beta_z = -\beta\sin\theta_0$. The total angle of rotation $2\Delta\theta$ during the storage or retrieval process is $2\Delta\theta = \beta T/k_c = \beta T\lambda_c/2\pi \ll 1$.

Now we consider the case when the control field propagates perpendicular to the signal field, i.e. $\theta_0 = \pi/2$, which corresponds to a transverse control field. Retrieval is done in a forward way. The transverse size of our sample is assumed to be smaller than the sample length. In this case, as pointed out in Sec. II, it is possible to switch between different longitudinal modes without switching between transverse modes when rotating the control field. The angle of rotation $\Delta\theta$ should satisfy the following condition in order to stay within a single transverse mode: $\Delta\theta < \sqrt{\frac{2T}{\Delta t}\frac{\lambda_c}{L_x}}$, where Δt is the duration of the signal pulse. In such a case, Eqs. (15) and (16) are decoupled, and w_{mn} gives only diffraction effect for each of the transverse mode of the signal field. Thus it is sufficient to consider the evolution of a single mode. Let us define $S'_{mn} = S_{mn}e^{i\beta_z(z-z_0)t}$ and go to co-moving frame $\tau = t - z/c$. Since $\beta_x = 0$ and $\beta_z = -\beta$, from Eqs. (15) and (16) we have

$$\begin{cases} \dfrac{\partial}{\partial z}A_{mn}(z,\tau) = g^* N S'_{mn}(z,\tau)\,, & (23) \\ \dfrac{\partial}{\partial \tau}S'_{mn}(z,\tau) = \left(-\gamma + i\delta - i\beta(z-z_0)\right)S'_{mn}(z,\tau) - \\ \qquad\qquad -gA_{mn}(z,\tau)\,. & (24) \end{cases}$$

Then one recognizes that in such a regime, equations describing the system are the same as those for longitudinal CRIB scheme(which is also referred to as gradient echo memory scheme) [26]. In longitudinal CRIB scheme, a space dependent absorption line along the medium is created. While the signal field propagates through the medium, different frequency components get absorbed by different absorption lines at different longitudinal positions, resulting in a space dependent coherence in the sample. During retrieval, such coherence is mapped back onto the output signal. We can understand our scheme in exactly the same way: Since the rotation of the control field yields a factor $e^{i\phi(\mathbf{r},t)} = e^{i\beta_z t(z-z_0)}$, thus on the one hand, there is a time dependent wave vector βt which is responsible for the writing of the spin waves with different wave vectors and the recording of the temporal profile of the signal field. On the other hand, this term can be viewed as a space dependent absorption line at frequency βz, which absorbs different frequency components of the signal at different positions of the medium. The medium opens an absorption window of width βL (Fig. 1(a)) along longitudinal direction. The absorption of the central frequency of the signal field happens at $z_p = z_0 + \delta/\beta$. It is necessary for z_p to be inside the medium, and better at the middle of the sample. This means that if $z_0 = 0$, i.e. the longitudinal position of the phase stationary point of the control field corresponds to the center of the sample, then the two-photon detuning δ should be equal to zero. Otherwise, one can take advantage of the two-photon detuning to shift the phase stationary point longitudinally to the medium center. Although the equations of motion can be reduced to the same as those of CRIB, the underlying physics of our scheme is quite different. Instead of controlling inhomogeneous broadening, we realize quantum storage by continuously creating spin wave vectors of different values to record the temporal information of the incoming single-photon wave packet. In other words, we achieve the same storage effect as in CRIB via a specific phase control of the control field which results in the same effect as the spatially depended frequency control of the atomic levels.

In order to have good storage efficiency, the absorption window width βL should cover the input pulse spectrum width: $\beta L > 2\pi/\Delta t$. So

$$2\Delta\theta > \frac{T}{\Delta t}\frac{\lambda_c}{L}\,. \tag{25}$$

This condition is the same as that resulted from switching between different longitudinal modes during the storage of the signal pulse. On the other hand, it is known that the "optical density" for each spectral component of the signal field in Fourier space for longitudinal CRIB [27] is $2\pi\left|g\right|^2 N/\beta$. This quantity needs to be larger than unity, so

$$2\Delta\theta < T\lambda_c\left|g\right|^2 N\,. \tag{26}$$

Thus one needs to keep a balance between the absorption window width and the "optical density". The bigger β is, the wider the absorption window opens while the smaller the "optical density" becomes, and vice versa. Besides, we choose the parameters to avoid significant storage phase factor [27] throughout the paper. The retrieval of the signal is done by switching the sign of β (and the sign of δ, if a non-vanishing δ is used to shift the absorption center), corresponding to a reversal scan of the control field. The retrieval field experiences a phase modulation [28], which can strongly decrease the fidelity defined as (22), and has to be taken into account beyond conditions (25) and (26). The performance of the storage and retrieval is then equivalent to that of longitudinal CRIB, which has been well studied and understood in many papers [28–31]. However, it is worth noting that a retrieval signal can also be generated

without switching the scanning direction of the control field (thus the sign of β), as opposed to longitudinal CRIB scheme. Yet in this paper, we always perform a reversal scan during retrieval.

We note here that Eqs. (23) and (24) with $z_0 = -L/2$ are also the equations describing quantum memory scheme via refractive index control [21] in free space, for which the above discussions are applicable. In this case, a non-vanishing two-photon detuning $\delta = \beta L/2$ is required to shift the position of the absorption line of the central frequency of the signal field to the center of the medium.

IV. GEOMETRICAL OPTICS APPROXIMATION

In the previous section, we consider a special case when the propagation directions of the signal and the control fields are perpendicular to each other. This allows us to decouple equations in the reciprocal space and make use of single-mode approximation. However, the transverse excitation requires more power because of the large control beam cross section. In this section we consider control field propagating at an arbitrary angle with respect to the z axis. But we make use another approximation when the Fresnel number F of interaction volume is much bigger than unity. Then the second order transverse derivative in Eqs. (5) can be dropped so that each point in the transverse plane (x, y) can be considered independently. The present case corresponds to geometrical optics when the signal field is described as a series of rays propagating along the axis z. The transverse profile of the signal field is assumed to be smooth enough and treated as featurelessness within each of the Fresnel zones.

In the reciprocal space, the second order transverse derivative term $\frac{i}{2k_s}\Delta_\perp a(\mathbf{r},t)$ gives $-iq_{mn}^2/(2k_s)\, a_{mn}(z,t)$. This term describes phase shift due to transverse momentum, which can be ignored when $q_{mn}^2/k_s \ll \pi$. The latter corresponds to a very directional propagation of the signal field when Fresnel number of interaction volume is large. However, when the control field is rotated and different spin waves are created in the medium during the Raman interaction, the signal beam can be subject to spreading in transverse direction due to the scattering of the control field on the spin waves with different wave vectors. Therefore, in order to neglect the transverse derivative in the equations, we need not only large Fresnel number of the interaction volume, but also sufficiently small angle of control field rotation $\Delta\theta$. The upper limit of $\Delta\theta$ can be estimated in the following way. Consider the interaction between signal field of wave vector $\mathbf{k}_s$ and control field of wave vector $\mathbf{k}_c(t)$ (Fig. 2). While $\mathbf{k}_c$ changes from $\mathbf{k}_c(t_1)$ (at an angle $\theta_0 - \Delta\theta$) to $\mathbf{k}_c(t_2)$ (at an angle $\theta_0 + \Delta\theta$), it continuously creates a set of spin wave vectors $\mathbf{k}_s - \mathbf{k}_c(t)$ depicted in Fig. 2. As a result the signal wave vector can spread over an angle φ. For $k_s \approx k_c$, $\Delta\theta \ll 1$, one can estimate $\varphi < 2\Delta\theta$. We need $\left(\Delta q_x^2 + \Delta q_y^2\right) L/(2k_s) \ll \pi$. Since $\Delta q_x \sim k_s \sin\varphi \leqslant 2\Delta\theta k_s$, $\Delta q_y = 0$, we arrive eventually at the following limit: $\Delta\theta \ll \Delta\theta_{\max} = \frac{1}{2}\sqrt{\frac{\lambda_s}{L}} \approx \frac{1}{2}\sqrt{F}\theta_d$, where θ_d is the diffraction angle. It should be noted that this inequality can be considered as just a sufficient condition for neglecting second derivatives and geometrical optics approximation, but not necessary for the considered quantum storage protocol.

Now, by defining a new variable $s'(\mathbf{r},t) = s(\mathbf{r},t)\cdot\, e^{i[\beta_x(x-x_0)t+\beta_z(z-z_0)t]}$, we have the following equations in the co-moving frame:

$$\begin{cases} \dfrac{\partial}{\partial z}a(\mathbf{r},\tau) = g^* N s'(\mathbf{r},\tau)\,, & (27) \\ \dfrac{\partial}{\partial \tau}s'(\mathbf{r},\tau) = \{-\gamma + i\delta + i[\beta\cos\theta_0\,(x-x_0) - \\ \qquad -\beta\sin\theta_0\,(z-z_0)]\}s'(\mathbf{r},\tau) - g a(\mathbf{r},\tau)\,. & (28) \end{cases}$$

Taking the same argument as in Sec. III, we arrive at the following condition for efficient quantum storage:

$$\frac{T}{\Delta t}\frac{\lambda_c}{L} < 2\Delta\theta\sin\theta_0 < T\lambda_c\left|g\right|^2 N. \tag{29}$$

Beyond this, we need to confine the value of the average polar angle θ_0. The central absorption line appears at the position $z_p = \frac{\delta}{\beta\sin\theta_0} + z_0 + \cot\theta_0\,(x - x_0)$. z_p should be inside the medium, so

$$-L/2 < z_p < L/2. \tag{30}$$

Again, one can engineer the control field to make the location of the phase stationary point $(x_0, z_0) = (0, 0)$ and set two-photon detuning $\delta = 0$. However, this condition can be relaxed in a way that the frequency shift introduced by z_0 is canceled by the shift introduced by x_0 and δ, namely, $z_0 - x_0\cot\theta_0 + \delta/(\beta\sin\theta_0) = 0$. So the phase stationary point does not need to be exactly at the center of the sample, but may be situated anywhere on the line

$$x_0 = z_0\tan\theta_0 + \delta/(\beta\cos\theta_0)\,. \tag{31}$$

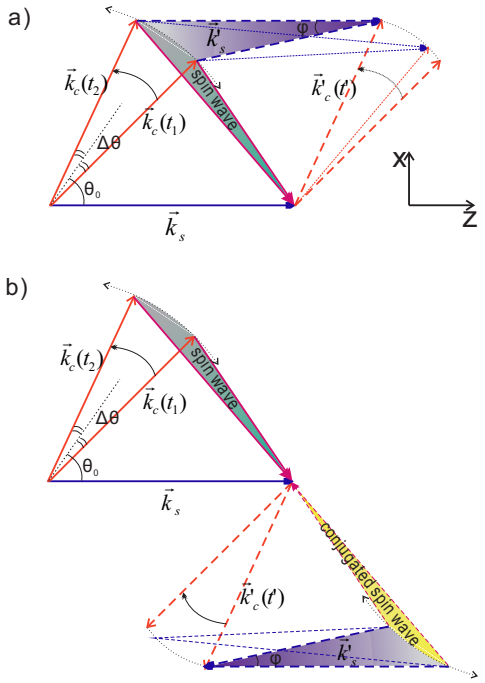

FIG. 2. (Color online) Diagram of the signal wave vector, control wave vector and spin wave vector for (a) forward retrieval and (b) backward retrieval. A writing control wave vector changes direction from $\mathbf{k}_c(t_1)$ to $\mathbf{k}_c(t_0)$. For $t_1 = -T$, $t_2 = 0$, the control wave vector sweeps an angle $2\Delta\theta$ around an average polar angle θ_0. When a reading control field is applied at time t', the retrieved signal field spreads over an angle φ. In backward retrieval, the spin waves need to be flipped to their conjugated components in the time interval between the end of the storage and beginning of the retrieval.

When $\delta = 0$, this is just the bisector of the rotation. If there exists any difficulty to put (x_0, z_0) on the bisector, a non-vanishing δ can be used to shift the position of the absorption lines. In the following we will assume this condition is fulfilled. Then (30) gives $-L/2 < x\cot\theta_0 < L/2$. Replace x by the radius of the excitation volum R, we have $|\cot\theta_0| < \frac{L}{2R}$. Define geometry angle $\theta_g = \arctan\frac{2R}{L}$, then the condition on the average polar angle θ_0 can be written as

$$\theta_0 > \theta_g = \arctan\frac{2R}{L}\,. \tag{32}$$

If (31) is not fulfilled, the angle θ_0 needs to be even bigger. Since $\theta_g \approx F\theta_d$ and $\Delta\theta_{\max} \approx \frac{1}{2}\sqrt{F}\theta_d$, for Fresnel number $F \gg 1$ condition (32) automatically ensures that $\theta_0 \gg \Delta\theta$, which is the condition we have been using through out this paper.

A. Forward retrieval

Under the condition (31), the Eqs. (27) and (28) become

$$\begin{cases} \dfrac{\partial}{\partial z}a(\mathbf{r},\tau) = g^* N s'(\mathbf{r},\tau)\,, & (33) \\ \dfrac{\partial}{\partial \tau}s'(\mathbf{r},\tau) = [-\gamma + i\beta(x\cos\theta_0 - \\ \qquad -z\sin\theta_0)]s'(\mathbf{r},\tau) - g a(\mathbf{r},\tau)\,. & (34) \end{cases}$$

The term $\beta x\cos\theta_0 s'(\mathbf{r},t)$ has no positive contribution to the storage and retrieval of the signal field. It appears as a side effect due to the change of $\mathbf{k}_c$ on transverse direction. For forward retrieval, this term could introduce some transverse profile to the retrieved signal. This transverse profile reveals itself as a transversal space dependent time shift (ahead or behind) of the forward retrieved signal, as seen in Fig. 3 (d). The reason of the time delay is as follows: the signal field in the medium excites spin wave and evolves in the form of polariton, which is the combination of the signal field and the excited spin wave [29]. The polariton, after storage, freezes inside the medium at the position of

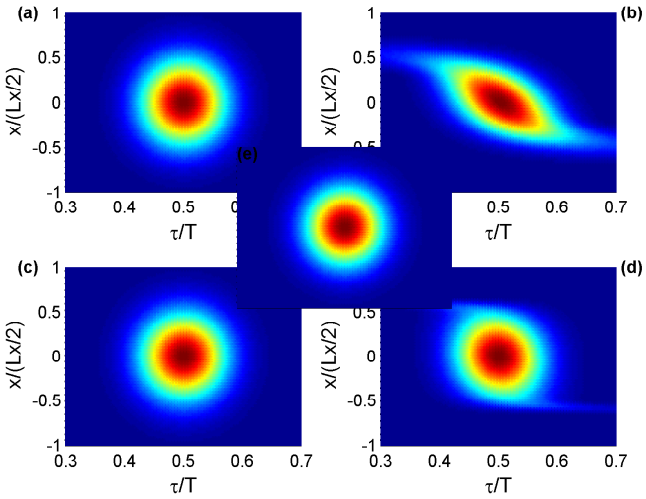

FIG. 3. (Color online) Forward retrieval fields for $\theta_0 = \pi/2$ in (a) & (c) and $\theta_0 = \pi/9$ in (b) & (d). (e) shows the signal field to be stored. The vertical axis $x/(L_x/2)$ is the transverse dimension of the sample normalized by the half transverse size $L_x/2$; the horizontal axis τ/T is the local time normalized by the duration of the storage(retrieval) process T. The color indicates the signal field amplitude. (a, b) Retrieval fields for constant $\Delta\theta$. The field is distorted for small θ_0 because (1): the absolute value of $\beta_z = -\beta\sin\theta_0$ becomes smaller such that it approaches the lower limit of (29); (2): $\beta_x = \beta\cos\theta_0$ becomes large enough to lead a spacial-temporal distortion; (c, d) Retrieval fields for constant $\Delta\theta\sin\theta_0$. There is transverse distortion of the retrieval field due to the non-vanishing β_x. The figures are generated under the following parameters: $\lambda_s \approx \lambda_c = 1550$ nm(fiber-optic communication band), $|g|^2 N = 8.3\times10^{10}$ /(s· m), $\Delta t/T = 1/20$, $T = 1000$ ns, $2R/L_x = 1/6$, where R is considered to be the transverse spatial half width of the input signal, $L_x = 0.6$ cm, $L = 1$ cm, $\Delta\theta = 8\times10^{-3}$ rad in (a) & (b), $= 8\times10^{-3}/\sin\theta_0$ rad in (c) & (d).

central absorption line $z_p = x\cot\theta_0$. It is now clear that the condition (30) is to place the polariton just inside the medium. Otherwise if no polariton is created, the signal field can not be stored. During forward retrieval, the control field picks up the polariton at z_p and generates retrieval signal at $z = L/2$. So the group velocity of the signal field reduces from c to 0 while $z = -L/2 \to z_p$, and recovers from 0 to c while $z = z_p \to L/2$. As a result it is obvious that if z_p is not exactly equal to 0, there must be a time shift going along with the forward retrieved signal. This time shift is avoided in the case of $\theta_0 = \pi/2$ because there z_p is independent on x (transversely homogeneous). Even if z_p is not exactly equal to 0, there would be only a time shift as a whole for the recalled signal. However, for $\theta_0 \neq \pi/2$, at $x \neq 0$ this time shift is unavoidable for forward retrieval, and moreover, it must introduce a transverse distortion of the signal. For z_p not deviating much from 0 and $4c|g|^2 N/(\beta^2 L^2\sin^2\theta_0) \gg 1$, the group velocity of the signal field near the ends of the medium is [28] $v_g(z)|_{|z\pm L/2|\ll L/2} \sim \beta^2(z-z_p)^2/\left(|g|^2 N\right)$. Since the delay is due to the propagation between $z = L/2 + 2z_p$ and $z = L/2$, from $dz = v_g(z)dt$, the delay time can be estimated as

$$t_d \sim \frac{|g|^2 N}{\beta^2\sin^2\theta_0}\left(\frac{1}{L/2+z_p} - \frac{1}{L/2-z_p}\right). \tag{35}$$

Taking $z_p = \pm R\cot\theta_0$, one estimates the temporal broadening of the forward retrieved signal with respect to the input signal as

$$\Delta t_{br} \sim \frac{|g|^2 N}{\beta^2\sin^2\theta_0}\frac{4R\cot\theta_0}{(L/2)^2 - R^2\cot^2\theta_0}, \tag{36}$$

which is finite and positive considering the condition (32).

Fig. 4 shows how the efficiency and fidelity depend on θ_0. Although a smaller θ_0 requires less control field power, it gives less fidelity due to the transversal distortion for a given $\Delta\theta$. Alteratively, it is necessary to increase $\Delta\theta$ when θ_0 approaches θ_g from above, in order to maintain high fidelity of quantum storage, as shown in Fig. 4 (c, d).

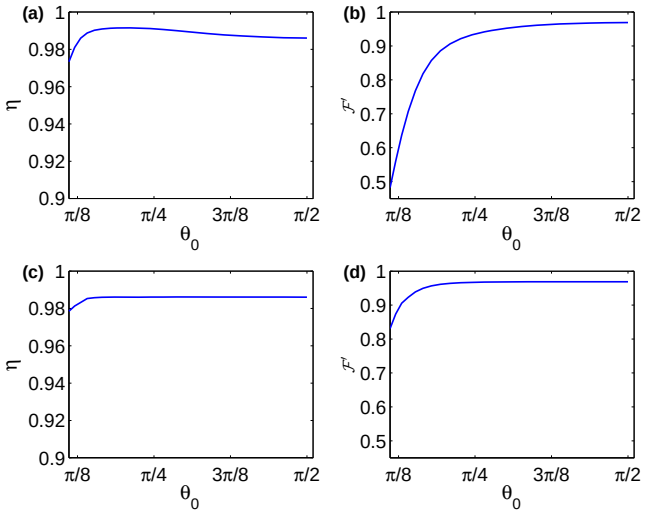

FIG. 4. (Color online) Forward retrieval Efficiency η and Fidelity $\mathscr{F}'$ v.s. the average polar angle θ_0. (a, b) Efficiency(a) and Fidelity(b) for forward retrieval with $\Delta\theta$ kept as constant; (c, d) Efficiency(c) and Fidelity(d) for forward retrieval with $\Delta\theta \sin\theta_0$ kept as constant. The parameters are the same as described in Fig. 3.

B. Backward retrieval

In the case of forward retrieval, the above transverse distortion decreases the fidelity of quantum storage when the average polar angle θ_0 becomes small. This can be avoided in a backward retrieval, wherein the time shift at each transverse point is exactly compensated during backward propagation. As a result, the fidelity remains high for all average angles θ_0 of interest. However, due to the phase matching condition, it is not enough to merely switch the propagation direction of the control field to the opposite. In general, it requires phase conjugation of the spin waves. Under considered geometrical optics approximation, it is sufficient to flip the direction of the spin wave vector along direction $\mathbf{K} = \mathbf{k}_s - \mathbf{k}_c(t = 0)$ before the retrieval. This can be done by applying two non-colinearly propagating π pulses one followed by another [32, 33], both on resonance with level $|3\rangle$ and another energy level, say, $|4\rangle$. The transition frequency ω_{34} needs to be bigger than $c|\mathbf{K}|$. The directions of the two π pulses are arranged in such a way that the wave vectors difference between them is along $-\mathbf{K}$. Another possibility is to generate spin wave vectors perpendicular to the signal wave vector [34]. In this case, k_c and k_s needs to differ from each other significantly.

In the case of backward retrieval, the input signal enters at $z = -L/2$ and creates polariton at $z = z_p$, while the retrieved signal starts from $z = z_p$ and emits at $z = -L/2$. As a result, there is no delay of the output field for any value of $z_p \in (-L/2, L/2)$.

Generally speaking, in either forward or backward retrieval, the scanning of the control field can be implemented in different ways. The control of the scanning order and scanning rates offers the possibility of the manipulation of the retrieval pulses, for instance, recalling different temporal parts of the input pulses in different orders. Such a manipulation in the angular-time domain may be thought of as counterpart of that in the frequency-spatial domain using CRIB [35].

V. IMPLEMENTATION ISSUES

Let us discuss possible experimental conditions under which the storage and retrieval of weak optical pulses via angular scanning of the control field is possible.

In Fig. 3 and 4 we demonstrate the performance of our quantum memory scheme with signal field frequency around fiber-optic communication band. Yet experimentally it is preferable to store and recall the signal field with small total angle of rotation $2\Delta\theta$ and rotation rate $2\Delta\theta/T$. If we take $\lambda_s \approx \lambda_c = 700$ nm, according to the numerical simulations, a single Gaussian pulse of duration $\Delta t = T/20$ with $T = 500$ ns can be stored and recalled with high efficiency and fidelity using transverse control field if the angle of rotation is of the order of $10\lambda/L$, which gives $\Delta\theta/T \sim 10^3$ rad/s for $L = 1$ cm. Such rate of beam deviation can be achieved by commercial equipment. Regarding promising storage materials, two points must be kept in mind in the context of the present scheme: 1) off-resonant Raman interaction needs systems with relatively strong optical transitions, especially in the case of transverse control field; 2) exploiting

coherent spatial grating needs atoms to be stationary in space. From this point of view, a system of cold atoms trapped in an optical lattice seems to be one of the promising candidates [36, 37]. The influence of regular atomic structure on the efficiency of quantum memory in such a system was analyzed recently in [38]. We note only that the maximum length of the wave vector of a spin wave created via non-collinear Raman interaction is limited by the minimum interatomic distance, which means that a blue detuned optical lattice should be used for the case of transverse control field. Another promising material is an ensemble of defect centers in diamond such as nitrogen-vacancy centers [39, 40]. The existence of lambda type optical transitions in such a system was demonstrated via electromagnetically induced transparency [41] and coherent population trapping [42]. The effect of inhomogeneous broadening of the Raman transition may be removed through the use of a spin-echo pulse sequence, and long dephasing times are achieved by dynamic decoupling of spin qubits from their local environment [43–45]. Moreover, transfer of quantum states between the electron spins and nuclear spins is possible [46], which in combination with dissipative decoupling schemes allows long-lived quantum storage [47].

VI. CONCLUSION

We propose a new method to store and retrieve weak pulses such as single-photon wave packets based on off-resonant Raman interaction. By changing the propagation direction of the strong classical control field, the temporal profile of the signal field is mapped into the spatial grating of Raman coherence. If the wave vector of the control field is perpendicular to that of the signal field, the quantum storage can be described in single-mode approximation so that the proposed scheme proves to be equivalent to longitudinal CRIB scheme. When the control field approaches longitudinal one, in the case of forward retrieval there maybe additional spatial-temporal distortion of the output field with respect to the input signal due to the change of the control wave vector on transverse direction. Such distortion can be avoided in backward retrieval. The proposed scheme has the advantage of longitudinal CRIB in that high efficiency can be achieved without backward retrieval. Besides unlike CRIB, this scheme does not require a direct control of atomic levels, thus potentially reduces decoherence in the system. The scheme can be implemented in resonant media which do not demonstrate linear Stark or Zeeman effects and allows one to combine Raman-interaction-based and CRIB-based approaches solely in the framework of the former.

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