# Dynamical properties of hard-core anyons in one-dimensional optical lattices

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# Dynamical properties of hard-core anyons in one-dimensional optical lattices

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We investigate the dynamical properties of anyons confined in one-dimensional optical lattice combined with a weak harmonic trap using the exact numerical method based on a generalized Jordan-Wigner transformation. The evolving density profiles, momentum distributions, occupation distributions, and occupations of the lowest natural orbital after quench of the harmonic trap, are obtained for different statistical parameters. The density profiles of anyons display the same behaviors irrespective of statistical parameter in the full evolving period. While the behaviors dependent on statistical property are shown in the momentum distributions and occupations of natural orbitals.

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## I. INTRODUCTION

According to the quantum statistical property, particle is generally classified as boson and fermion. Wave functions of identical bosons are symmetric under exchange while those of identical fermions are antisymmetric under exchange. As a natural generalization, physicist proposed that there exists anyon interpolating between Bose and Fermi statistics, which satisfies fractional statistics [1]. It has become an important concept in condensed matter physics [2–4] and has ever been used for successfully explaining the fractional quantum Hall effect (FQHE) [5]. Fractional statistics also play important roles in the theory for non-fermi liquid, Chern-Simon theory, and other aspects [6, 7]. Another potential application of anyons is to realize the topological quantum computation with non-abelian anyons [8, 9]. Besides the traditional two-dimensional electron system, low-dimensional cold atoms provide also a popular platform to realize the fractional statistics. It has been suggested that anyons can be created, detected and manipulated in rotating Bose-Einstein condensates (BECs) and cold atomic systems in optical lattices [10–12]. Researches on anyons in cold atomic systems are not restricted in the two-dimensional system, and the proposal to realize anyons in the one-dimensional (1D) optical lattice is also put forward recently by Keilmann et. al., who proposed to create anyons by controlling the occupation-dependent hopping amplitudes of bosons with assisted Raman tunnelling [13]. Particularly, the statistical parameter can be tuned by controlling the relative phase of external driving fields.

Since BECs are realized experimentally, great progress has been made in studying the cold atomic systems both experimentally and theoretically for their "purity" and high controllability comparing with traditional condensed matter systems. Due to the profound correlation effects in low-dimensional systems, the low-dimensional quantum gas has also attracted much attentions [14]. By loading the cold atoms in anisotropic magnetic traps or two-dimensional optical lattice potentials, the particle motion is tightly confined in two directions to zero point oscillations [15–17] and the strongly correlated Tonks-Girardeau (TG) regime can be achieved [16, 17]. By crossing the confinement-induced resonance (CIR) from the TG gas, the super TG (sTG) gas is also accessible [18]. The interaction between atoms can be tuned in the full interacting regime via the Feshbach resonance and the confinement-induced resonance by tuning the magnetic field. The excellent tunability of cold atoms makes the cold atomic system to be a promising candidate to realize fractional statistics.

Before the recent experimental proposal of Keilmann et. al. [13], the 1D anyon gas has been investigated theoretically in various 1D systems [19–22] including the Bose quantum gas with a special interaction potential. Particularly, Kundu proved that a 1D Bose gas interacting through $\delta$-function potential combined with double $\delta$-function potential and derivative $\delta$-function potential is equivalent to the anyon gas interacting via $\delta$-function potential [21]. This stimulated many research interests in $\delta$-anyon gas [22–32]. It turns out that the ground state density distribution of a $\delta$-anyon gas displays similar behaviors as that of Bose gas with the increasing interaction. In the strong interacting regime the density distribution shows the same behavior as the free fermion [33–37], which is irrelevant to the statistical parameter. The special property resulted from the fractional statistics exhibits in the reduced one body density matrices and the momentum distributions [38, 39]. The momentum distribution of anyon differs from fermion's oscillations and boson's single peak structure, which are symmetric about the zero momentum. The momentum distribution of anyons is asymmetric when the statistical parameters deviate from the Bose and Fermi limit [27–31]. This special behavior originates from that the reduced one body density matrix is a complex Hermitian one rather than a

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real one for bosons and fermions.

While most studies have focused on the static properties of the 1D anyonic gas, its dynamics remains to be investigated. The present paper shall study the evolving dynamics of hard-core anyons (HCAs) confined in the optical lattice with a weak harmonic trap after a sudden change of the harmonic trap. In Ref. [28], we have extended the exact numerical method originally used to treat hard-core bosons by Rigol and Muramatsu [40] to study the ground state properties of hard-core anyons in the optical lattice. Here, we further extend this method to investigate the dynamics of HCAs. By evaluating the exact time-dependent one-particle Green's function, we obtain the reduced one body density matrix (ROBDM) and thus the density profiles and the momentum distributions at arbitrary time. The dynamical properties induced by anyonic statistics shall be displayed in the momentum distribution. It is deserved to mention that although we only discuss the hard-core anyons which can be exactly treated in the present work, the general case with more than one anyons occupying the same site can be also considered by using the mean field theory, for which the corresponding ground state density profiles depend on the statistical parameter as observed in Ref. [13].

The paper is organized as follows. In Sec. II, we give a brief review of the 1D anyonic model and introduce the numerical method. In Sec. III, we present the density profiles, momentum distributions, occupation distributions, and the occupations of the lowest natural orbital for different statistics parameters. A brief summary is given in Sec. IV.

## II. FORMULATION OF MODEL AND METHOD

We consider $N$ hard-core anyons confined in an optical lattice of $L$ sites with a weak harmonic trap and the second quantized Hamiltonian can be formulated as

$$H_{\mathrm{HCA}} = -t\sum_{l=1}^{L}\left(a_{l+1}^{\dagger}a_l + H.C.\right) + \sum_{l=1}^{L} V_l a_l^{\dagger}a_l. \qquad (1)$$

Here the harmonic potential $V_l = V_0(l-(L+1)/2)^2$ with the strength of the harmonic trap $V_0$. The anyonic operator $a_l^{\dagger}$ ($a_l$) creates (annihilates) an anyon on site $l$ and satisfies the generalized commutation relations

$$\begin{aligned} a_j a_l^{\dagger} &= \delta_{jl} - e^{-i\chi\pi\epsilon(j-l)} a_l^{\dagger}a_j, \\ a_j a_l &= -e^{i\chi\pi\epsilon(j-l)} a_l a_j \end{aligned} \qquad (2)$$

for $j \neq l$, where the sign function $\epsilon(x)$ gives -1, 0 or 1 depending on whether $x$ is negative, zero, or positive and $\chi$ is the parameter related with fractional statistics. The generalized commutation relations reduce to fermionic commutation for $\chi = 0$ and reduce to bosonic commutation for $\chi = 1$, while for anyons satisfying fractional statistics $\chi$ changes in between them. The hard-core interactions between anyons restricts the additional condition $a_l^2 = a_l^{\dagger 2} = 0$ and $\left\{a_l, a_l^{\dagger}\right\} = 1$. In the Hamiltonian $t$ denotes the hopping amplitude between the nearest neighbor sites, which can be tuned by changing the strength of the optical lattice.

In order to solve the model of hard-core anyons, we extend the numerical method to investigate the hard-core bosons in optical lattices developed by Rigol and Muramatsu. We can map the above model into the polarized fermionic Hamiltonian using the generalized Jordan-Wigner transformation [28]

$$a_j = \exp\left(i\chi\pi\sum_{1\le s<j} f_s^{\dagger}f_s\right) f_j, \qquad (3)$$

$$a_j^{\dagger} = f_j^{\dagger}\exp\left(-i\chi\pi\sum_{1\le s<j} f_s^{\dagger}f_s\right), \qquad (4)$$

where $f_j^{\dagger}$ ($f_j$) is creation (annihilation) operator for fermions. The above Hamiltonian of system with $N$ anyons is transformed into a fermionic one with $N_F = N$ fermions

$$H_F = -t\sum_{l=1}^{L}\left(f_{l+1}^{\dagger}f_l + H.C.\right) + \sum_{l=1}^{L} V_l f_l^{\dagger}f_l, \qquad (5)$$

where the fermionic operator satisfy the Fermi anti-commutation relation

$$\left\{f_i, f_j^{\dagger}\right\} = \delta_{ij}, \{f_i, f_j\} = \left\{f_i^{\dagger}, f_j^{\dagger}\right\} = 0. \qquad (6)$$

The original question about anyons now can be investigated by solving the model on the polarized fermions in optical lattice. We can obtain the exact many body wavefunction of polarized fermions with the diagonalized method and therefore the ground state and interesting physical phenomena of hard-core anyons.

The equal-time Green's function for the hard-core anyons at time $\tau$ should be expressed as

$$\begin{aligned} G_{jl}(\tau) &= \left\langle \Psi_{\mathrm{HCA}}(\tau)\left| a_j a_l^{\dagger}\right| \Psi_{\mathrm{HCA}}(\tau)\right\rangle \qquad (7)\\ &= \langle \Psi_F(\tau)| \exp\left(i\chi\pi\sum_{\beta}^{j-1} f_{\beta}^{\dagger}f_{\beta}\right) f_j f_l^{\dagger} \\ &\quad \times \exp\left(-i\pi\sum_{\gamma}^{l-1} f_{\gamma}^{\dagger}f_{\gamma}\right)|\Psi_F(\tau)\rangle \\ &= \left\langle \Psi_F^A(\tau)\,|\Psi_F^B(\tau)\right\rangle \end{aligned}$$

with

$$\left\langle \Psi_F^A(\tau)\right| = \left(f_j^{\dagger}\exp\left(-i\chi\pi\sum_{\beta}^{j-1} f_{\beta}^{\dagger}f_{\beta}\right)|\Psi_F(\tau)\rangle\right)^{\dagger},$$

$$\left|\Psi_F^B(\tau)\right\rangle = f_l^{\dagger}\exp\left(-i\chi\pi\sum_{\gamma}^{l-1} f_{\gamma}^{\dagger}f_{\gamma}\right)|\Psi_F(\tau)\rangle\,.$$

Here $|\Psi_{\rm HCA}(\tau)\rangle$ is the wavefunction of hard-core anyons at time $\tau$ in a system with Hamiltonian $H_{\rm HCA}$, and $|\Psi_F(\tau)\rangle$ is the corresponding one for the equivalent polarized fermions. For the polarized fermions, the time evolution of their initial wavefunction $\left|\Psi_F^I\right\rangle$ is given by

$$|\Psi_F(\tau)\rangle = e^{-i\tau H_F/\hbar}\left|\Psi_F^I\right\rangle. \tag{8}$$

While the matrix representation of initial wavefunction can be expressed as

$$\left|\Psi_F^I\right\rangle = \prod_{n=1}^{N_f}\sum_{l=1}^{L} P_{ln}^I f_l^\dagger |0\rangle \tag{9}$$

so that

$$\begin{aligned}|\Psi_F(\tau)\rangle &= e^{-i\tau H_F/\hbar}\prod_{n=1}^{N_f}\sum_{l=1}^{L} P_{ln}^I f_l^\dagger |0\rangle \\ &= \prod_{n=1}^{N_f}\sum_{l=1}^{L} P_{ln}(\tau) f_l^\dagger |0\rangle .\end{aligned}$$

Thus the fermionic time-dependent wavefunction can be expressed as an $L\times N_f$ matrix $\mathbf{P}(\tau)$. The matrix $\mathbf{P}(\tau)$ can be evaluated as

$$e^{-i\tau H_F/\hbar}\mathbf{P}^I = Ue^{-i\tau D/\hbar}U^\dagger \mathbf{P}^I, \tag{10}$$

where $U$ is an unitary transformation diagonalizing the Hamiltonian $H_F$, *i.e.*, $U^\dagger H_F U = D$ with the diagonal matrix $D$. After an easy evaluation the state $\left|\Psi_F^A\right\rangle$ reads

$$\left|\Psi_F^A(\tau)\right\rangle = \prod_{n=1}^{N_f+1}\sum_{l=1}^{L} P_{ln}^{\prime A}(\tau) f_l^\dagger |0\rangle$$

with

$$\begin{aligned}P_{ln}^{\prime A}(\tau) &= \exp(-i\chi\pi) P_{ln}(\tau) \qquad \text{for } l\le j-1 \\ P_{ln}^{\prime A}(\tau) &= P_{ln}(\tau) \qquad\qquad\qquad \text{for } l\ge j\end{aligned}$$

for $n \le N_f$ and $P_{jN_f+1}^{\prime A}(\tau) = 1$ and $P_{lN_f+1}^{\prime A}(\tau) = 0$ $(l\neq j)$. The state $\left|\Psi_F^B\right\rangle$ has the same form with the replace of $j$ by $l$. The time-dependent Green's function is a determinant dependent on the $L\times(N_f+1)$ matrices $\mathbf{P}^{\prime A}(\tau)$ and $\mathbf{P}^{\prime B}(\tau)$

$$G_{jl}(\tau) = \left\langle\Psi_F^A(\tau)|\Psi_F^B(\tau)\right\rangle = \det\left[\left(\mathbf{P}^{\prime A}(\tau)\right)^T\mathbf{P}^{\prime B}(\tau)\right].$$

In the present paper we will focus on the time evolution of the density profile and momentum distribution for hard-core anyons with different statistical parameter $\chi$. The ROBDM can be evaluated by Green's function

$$\rho_{jl}(\tau) = \left\langle a_j^\dagger a_l\right\rangle = \delta_{jl}(1-G_{jl}(\tau)) - (1-\delta_{jl})e^{-i\chi\pi}G_{jl}(\tau).$$

The diagonal part of ROBDM is the density profile and its Fourier transformation is defined as the momentum distribution

$$n(k) = \frac{1}{2\pi}\sum_{j,l=1}^{L} e^{-ik(j-l)}\rho_{jl}(\tau). \tag{11}$$

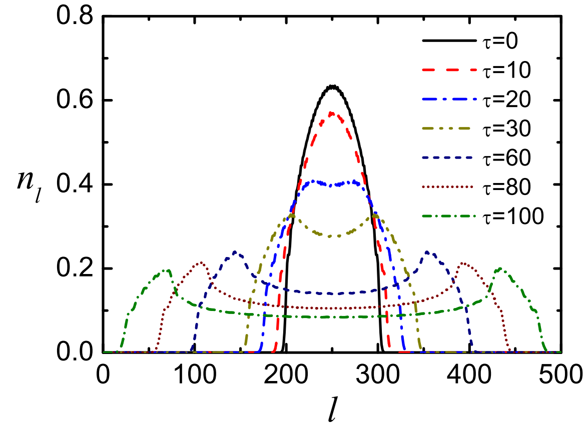


FIG. 1: (color online) The evolving of density distribution for 50 hard-core anyons in the optical lattice of 500 sites with $V_0^I = 1.0\times10^{-3}t$ and $V_0 = 1.0\times10^{-8}t$. Times $(\tau)$ are in units of $\hbar/t$.

The natural orbitals $\phi^\eta$ are defined as the eigenfunctions of the one-particle density matrix

$$\sum_{l=1}^{L}\rho_{jl}\phi_l^\eta = \lambda_\eta\phi_j^\eta, \quad j=1,2,...L, \tag{12}$$

and can be understood as the effective single-particle states with occupations $\lambda_\eta$.

## III. DYNAMICS OF DENSITY PROFILE AND MOMENTUM DISTRIBUTION

In the present paper we investigate the dynamics of hard-core anyons in the optical lattice. Initially we confine the anyons in optical lattice with a strong harmonic trap and then turn off the harmonic trap or reduce the strength of harmonic trap. The trapped anyons will evolve in the optical lattice. For convenience we set lattice constant $a$ as 1, the unit of $k$ is $1/a$ and the unit of time is $\hbar/t$.

In Fig. 1, 2, 3 and 4, we show the evolution of 50 anyons in the optical lattice of 500 sites which is initially trapped by an external harmonic trap with the strength of $V_0^I = 1.0\times10^{-3}t$ and then is released to a very weak trap with $V_0(0) = 1.0\times10^{-8}t$ at $\tau=0$. For the simplicity of calculation we use the very weak trap with strength of $1.0\times10^{-8}t$ rather than completely turnning off the trap. There are no essential difference between them in the present investigation. The evolution of density distribution is shown in Fig. 1. It turns out that the density distribution of anyons does not display any different dynamical property from those of bosons and fermions. We cannot distinguish the statistical properties by the density distribution in real space. Similar to hard-core bosons and polarized fermions, anyons expand in the optical lattice and gradually populate in the full lattice.

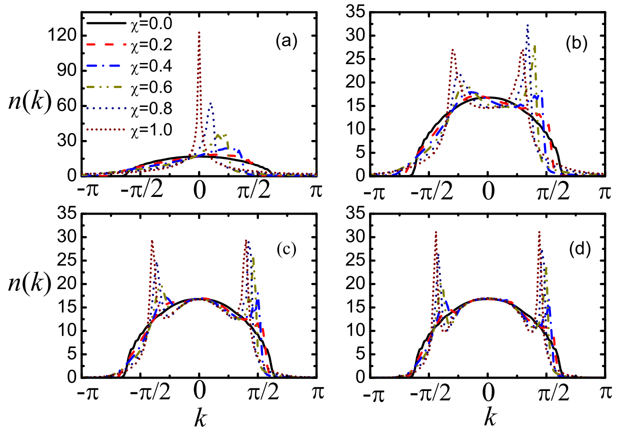


FIG. 2: (color online) The momentum distributions for 50 hard-core anyons in the lattice of 500 sites with $V_0^I = 1.0 \times 10^{-3}t$ and $V_0 = 1.0 \times 10^{-8}t$. (a) $\tau = 0$, (b) $\tau = 20$, (c) $\tau = 50$,(d) $\tau = 100$. Times ($\tau$) are in units of $\hbar/t$.

The density of anyons at the center reduces faster than the density at the border because anyons locating in sites of high density posses higher energy. As evolving time is long enough, anyons homogeneously distribute in the middle regime while its density distribution shows peaks at the border regime.

The corresponding momentum distribution is displayed in Fig. 2. Initially bosons and fermions distribute symmetrically about the zero momentum and anyons ($0 < \chi < 1$) exhibit the asymmetrical momentum distribution. After the harmonic trap is turned off, momentum distributions for anyons with different statistical parameters show different evolving properties. When $\chi = 0.0$, fermions do not show obvious change of momentum distribution during the time evolution (In fact the slight change happens according to the numerical data). As the statistical parameter deviates from $\chi = 0$, the momentum distribution of anyon evolves from the asymmetrical structure of a single peak to the structure similar to that of fermions but with two asymmetrical peaks. As statistical parameter increases, these two peaks become more and more obvious. For the case of $\chi = 1$ (hard-core bosons), the momentum distribution evolves to the structure of symmetrical double peaks. When the evolving time is long enough, the momentum distributions of anyons with different statistical parameters ($0 \leq \chi \leq 1$) exhibit identical behavior in the middle regime.

In Fig. 3 we show the occupation distribution of natural orbitals for the same system as above. In the full evolution period the occupation distribution of each orbital does not change qualitatively. In the Bose limit, hard-core bosons occupy the lower natural orbitals and the occupation distribution displays the single peak structure. In the Fermi limit, each fermion occupies one natural orbital and at any time the occupation distribution seems like a step-function. While for anyons in between these

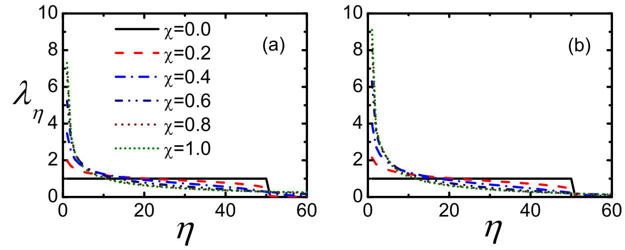


FIG. 3: (color online) The occupation distributions for 50 hard-core anyons in the lattice of 500 sites with $V_0^I = 1.0 \times 10^{-3}t$ and $V_0 = 1.0 \times 10^{-8}t$. (a) $\tau = 0$, (b) $\tau = 100$. Times ($\tau$) are in units of $\hbar/t$.

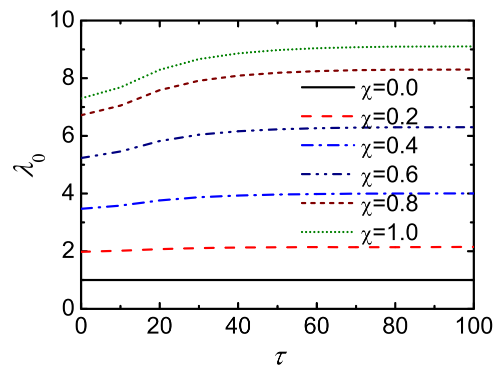


FIG. 4: (color online) The occupations of the lowest natural orbital for 50 bosons in lattice of 500 sites. $V_0^I = 1.0 \times 10^{-3}t$ and $V_0 = 1.0 \times 10^{-8}t$. Times ($\tau$) are in units of $\hbar/t$.

two limits, the occupations of higher natural orbitals increase as the statistical parameter evolves from the Bose limit to the Fermi limit. We also display the evolution of the occupation of the lowest natural orbital in Fig. 4. It is shown that the occupation is time independent in the Fermi limit and as deviating from the Fermi limit the occupation increases during the time evolution. The increase is more obvious for the bigger statistical parameter. When the evolving time is long enough, the occupation of the lowest natural orbital tends to a constant.

If the initially confined hard-core anyons in the optical lattice evolve under a sudden quench to a weaker harmonic trap rather than turning off the harmonic potential, the situation will be different. Anyons with different statistical parameters always exhibit the same density distributions, and we also cannot determine the statistical properties according to evolving properties of density profiles. In Fig. 5 we display the evolving density distributions of 50 anyons in the optical lattice of 300 sites combined with a weaker harmonic potential ($V_0 = 2.0 \times 10^{-4}t$). Initially, anyons distribute in the middle regime of the harmonic trap. After the harmonic potential becomes weak, anyons shall expand firstly. The

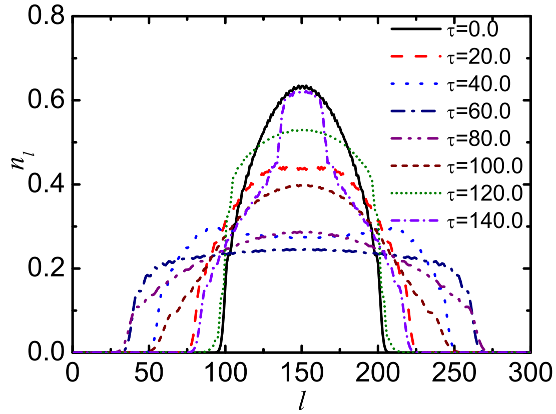


FIG. 5: (color online) The density distribution of 50 hard-core anyons in the optical lattice of 300 sites with $V_0^I = 1.0 \times 10^{-3}t$ and $V_0 = 2.0 \times 10^{-4}t$. Times ($\tau$) are in units of $\hbar/t$.

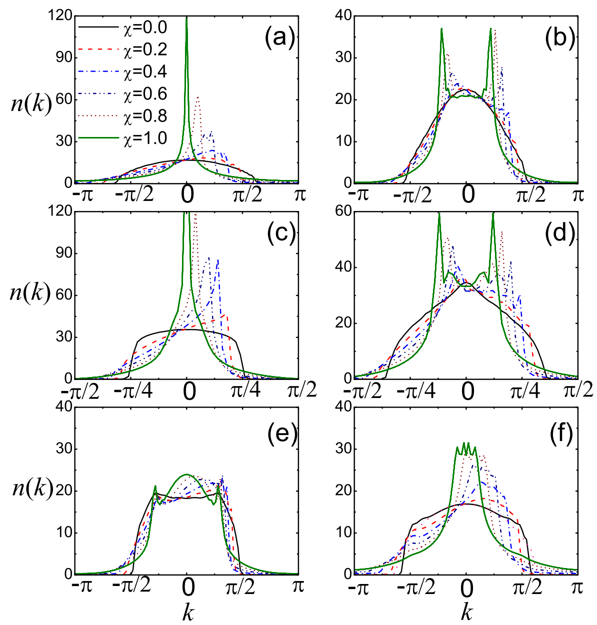


FIG. 6: (color online) The momentum distribution of 50 anyons in the optical lattice of 300 sites with $V_0^I = 1.0 \times 10^{-3}t$ and $V_0 = 2.0 \times 10^{-4}t$. (a) $\tau = 0$, (b) $\tau = 30$, (c) $\tau = 60$, (d) $\tau = 80$, (e) $\tau = 100$, (f) $\tau = 120$. Times ($\tau$) are in units of $\hbar/t$.

central density decreases faster than the boundary density such that the density profiles behave as a Fermi-like distribution at $\tau = 60$. Then anyons shall contract because of the confinement of harmonic trap. With the time evolution anyons redistribute in the middle regime of the trap.

During the expansion, momentum distributions show rich dynamical structures (Fig. 6). In the Bose limit,

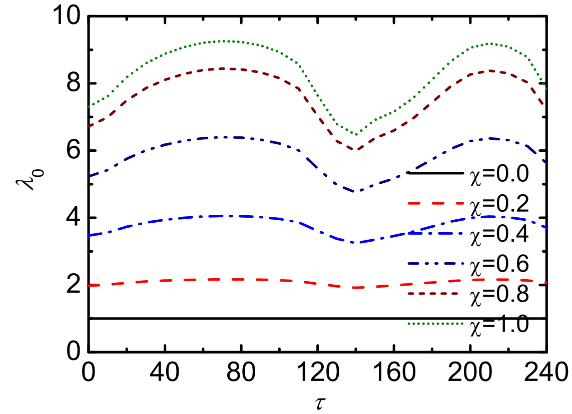


FIG. 7: (color online) The occupation of the lowest natural orbital for 50 anyons in the optical lattice of 300 sites with $V_0^I = 1.0 \times 10^{-3}t$ and $V_0 = 2.0 \times 10^{-4}t$. Times ($\tau$) are in units of $\hbar/t$.

the momentum distribution firstly evolves from the original structure of a single peak to the structure of double peaks, and then back to the structure of a single peak at $\tau = 60$. During the later period momentum distribution shall behave as the single peak and double peaks alternately. In the Fermi limit the momentum distribution does not keep its original profile as shown in Fig. 2 and shall exhibit the oscillating behavior. Contrary to the evolution of density distribution, the momentum distribution contracts firstly and at $\tau = 60$ (Fig. 6c) displays the step-function profile in the region of $-\pi/4 \leq k \leq \pi/4$. Then it gradually expands to the region of higher momentum and at $\tau = 120$ (Fig. 6f) momentum distribution almost recovers to the behavior at $\tau = 0.0$. For anyon gas in between ($0 < \chi < 1$), its asymmetric momentum distribution also shows the oscillating behavior. For the case of bigger statistical parameter asymmetric double peaks are displayed and for the case of smaller statistical parameter momentum distribution exhibits the behavior similar to those of fermions.

The evolution of occupation for anyons in the optical lattice with a weak harmonic potential is similar to the case only confined in the optical lattice. The occupation distribution always exhibits the same behaviors qualitatively as those at the initial time. But the occupation of the lowest natural evolves with time, which is displayed in Fig. 7. It is shown that in the Fermi limit the occupation is time-independent and as deviating from the Fermi limit it shall oscillate with the time evolution. It does not increase monotonously rather than fluctuates with the time evolution. The bigger the statistical parameter, the stronger the oscillation amplitude.

In order to investigate the dynamical properties of anyons in the Mott regime, we initially prepare a Mott state by superimposing the optical lattice with a tight harmonic potential and then turn off it at $\tau = 0.0$. The

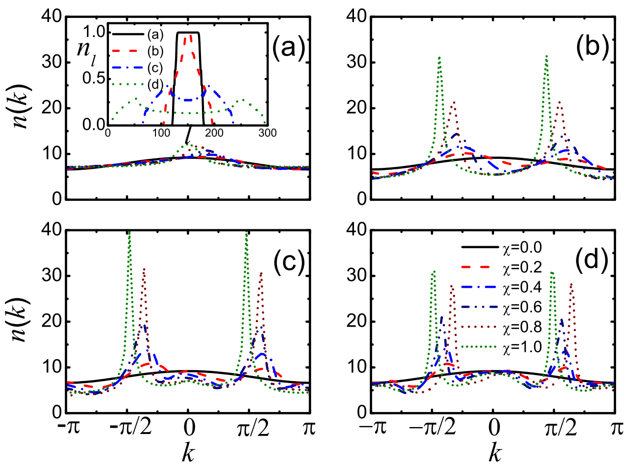


FIG. 8: (color online) The momentum distribution of 50 anyons in the lattice of 300 sites with $V_0^I = 1.0 \times 10^{-2}t$ and $V_0 = 1.0 \times 10^{-8}t$. (a) $\tau = 0$, (b) $\tau = 10$, (c) $\tau = 30$,(d) $\tau = 60$. Inset: Density distributions for the same system. Times ($\tau$) are in units of $\hbar/t$.

dynamical evolutions of the Mott state are displayed in Fig. 8. Initially, the momentum distributions for anyons with different statistical parameters exhibit behaviors similar to that of fermions. There are only tiny differences in the regime close to zero momentum. After the harmonic trap is turned off, the momentum distribution of anyons in the Fermi limit always preserves its initial profile. Anyons deviating this limit still display Fermi-like momentum distributions in the full momentum regime, but the structure of double peaks appears at a particular regime. In the full evolving period anyons in the Bose limit show the sharpest peaks, and as statistical parameter decreases (close to the Fermi limit) the peaks become smaller. It is same as before that anyons always exhibit the asymmetrical momentum distributions except of the Bose limit and the Fermi limit. We also display the evolving density profiles in the inset, which are independent of the statistical parameter. At the beginning anyons are confined in the central regime with an anyon per site. After the harmonic trap is turned off, anyons expand in the lattice with side peaks appearing.

## IV. CONCLUSIONS

In summary, in the present paper we have developed the exact numerical method to deal with the dynamics of hard-core anyons confined in the optical lattice superimposed with a weak harmonic potential. By evaluating the exact time-dependent one-particle Green's function, we obtain the density profiles, momentum distributions, occupation distribution, and the occupation of the lowest natural orbital in the full evolving period. It is shown that the evolving property of density profiles is independent on the statistical parameter of anyons. As the harmonic trap is turned off anyon shall expand in the optical lattice and distribute gradually in the full lattice. The dynamical properties of momentum distributions and occupation distributions of natural orbitals depend on statistical parameter of anyons. In the Bose limit, momentum distribution always displays the structure of double peaks. As deviating from the Bose limit, the double peaks become smaller and in the Fermi limit the double peaks disappear. When the evolving time is long enough, anyons display the same momentum distribution irrespective of the statistical parameter in the regime nearby the zero momentum. If the anyon gas is initially in the Mott regime, during the full evolving period it is always in the Mott regime, but at the particular momentum position double peaks appear. When the harmonic trap is replaced with a weaker one, the density profiles of anyons exhibit the breathing behavior. The momentum distributions of bosons alternatively display the structure of a single peak and double peaks, while fermions contract and expand in the $k$-space with the time evolution. For anyon interpolating between these two limit, momentum distribution also repeats its behavior at regular intervals dependent on the statistical parameter. The interval time is the longest for fermions and is the shortest for bosons. The occupation distributions of natural orbitals do not change qualitatively with the time evolution but the occupation of the lowest natural orbital evolves with time for anyon gas deviating the Fermi limit. When the harmonic trap is turned off, the occupation of the lowest natural orbital increases and gradually arrives at the biggest value. When the harmonic trap becomes weaker rather than being turned off, it shall fluctuate with the time evolution.


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