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# Atomic and photonic entanglement concentration via photonic Faraday rotation

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We propose two alternative entanglement concentration protocols (ECPs) using the Faraday rotation of photonic polarization. Through the single-photon input-output process in cavity QED, it is shown that the maximally entangled atomic (photonic) state can be extracted from two partially entangled states. The distinct feature of our protocols is that we can concentrate both atomic and photonic entangled states via photonic Faraday rotation, and thus they may be universal and useful for entanglement concentration in the experiment. Furthermore, as photonic Faraday rotation works in low-Q cavities and only involves virtual excitation of atoms, our ECPs are insensitive to both cavity decay and atomic spontaneous emission.

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Entanglement is the key resource in quantum information processing (QIP), such as quantum teleportation [1], quantum key distribution [2] and quantum dense coding [3]. In order to complete such QIP protocols perfectly, the maximally entangled states are usually required. However, the entanglement will inevitably degrade in the process of distribution and storage due to the interaction between system and its external environment. To overcome the dissipation and decoherence, Bennett *et al.* proposed the protocols of entanglement purification [4] and entanglement concentration [5]. By use of entanglement purification protocols (EPPs), one can distill a set of mixed entangled states into a subset of highly entangled states with local operation and classical communication [4]. However, EPPs can only improve the quality of the mixed state and can not get the maximally entangled state. On the other hand, entanglement concentration protocols (ECPs) [5] can be used to convert the partially entangled pairs to the maximally entangled ones. In the early days, many efforts have been devoted to photonic ECPs with linear [6, 7] or nonlinear [8] optical elements. Recently, ECPs of solid state qubits (such as atomic [9–11] or electric qubits [12]) have also been investigated frequently.

Cavity quantum electrodynamics (QED) system [13] is an excellent platform for understanding the fundamental principle of quantum mechanics and investigating QIP. In most of QIP protocols based on cavity QED, they usually require that atoms strongly interact with high-Q cavity field, which guarantees not only entanglement preparation but also further implementation of QIP tasks. However, as the high-Q cavity is well isolated from the environment, it seems unsuitable for efficiently accomplishing the input-output process of photons, which is the key step to implement long-distance QIP in a scalable fashion. Recently, An *et al.* [14] proposed a novel scheme to implement QIP with a single photon by an input-output process with respect to low-Q cavities. It is shown that the different polarized photon can gain different phase shift when it interacts with the atom trapped in the low-Q cavity, which is known as Faraday rotation [15]. Due to the fact that photonic Faraday rotation works in low-Q cavities and only involves virtual excitation of atoms, it is insensitive to both cavity decay and atomic spontaneous emission. Following this scheme, various works including entanglement generation [16], quantum logic gate [17] and quantum teleportation [18] have been presented. To our best knowledge, ECPs in cavity QED mainly focused on atomic entanglement concentration [9–11], but there is no report on concentration of photonic entanglement. The main reason is that in most cases we are only interested in high-Q cavities not the low-Q ones, and in some cases the cavity mode is even adiabatically eliminated and thus has no contribution to the system evolution in the case of large detuning between the cavity field and atoms [9, 19].

Inspired by Ref. [14], we investigate ECPs using the Faraday rotation of photonic polarization. The low-Q cavity and single-photon pulse (three-level atom) are introduced to assist concentration of atomic (photonic) entangled state. Through the single-photon input-output process in cavity QED, we can extract the maximally entangled atomic (photonic) state from two partially entangled states. The distinct feature of our proposals is that we can concentrate both atomic and photonic entangled states via photonic Faraday rotation, and thus they may be universal and useful in the experiment. Furthermore, as our ECPs work in low-Q cavities and only involve virtual excitation of atoms, they are insensitive to both cavity decay and atomic spontaneous emission, and may be feasible with current technology.

Firstly, we briefly review photonic Faraday rotation. Consider a three-level atom interacting with a low-Q cavity (one-side) driving by an input photon pulse, as shown in Fig. 1. The atom has two degenerate ground states ($|g_L\rangle$ and $|g_R\rangle$) and an excited state ($|e\rangle$). The tran-

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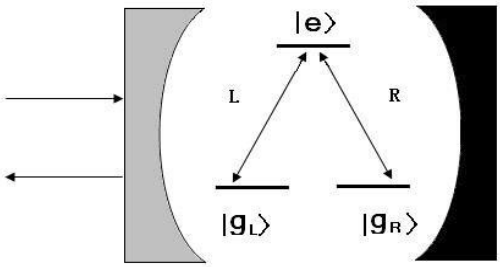


FIG. 1: The interaction between three-level atom and a single-photon pulse propagating input-output the low-Q cavity.

sitions $|g_L\rangle\leftrightarrow|e\rangle$ and $|g_R\rangle\leftrightarrow|e\rangle$ for the atom are assisted respectively by left-circularly ($L$) and right-circularly ($R$) polarized photons and the transition frequency is $\omega_0$. We consider the low-Q cavity limit and the weak excitation limit, then can solve the Langevin equations of motion for cavity and atomic lowering operators analytically. Adiabatically eliminating the cavity mode, we obtain the reflection coefficient for the atom-field system as follows [14]

$$r_j(\omega_p)=\frac{[i(\omega_c-\omega_p)-\frac{\kappa}{2}][i(\omega_0-\omega_p)+\frac{\gamma}{2}]+g^2}{[i(\omega_c-\omega_p)+\frac{\kappa}{2}][i(\omega_0-\omega_p)+\frac{\gamma}{2}]+g^2},(j=L,R) \quad (1)$$

where $\omega_c$ and $\omega_p$ are the frequencies of the cavity and photon pulse, $\kappa$ and $\gamma$ are the cavity damping rate and atomic decay rate respectively, and $g$ is the atom-cavity coupling strength. Due to the large damping rate of cavity, the absolute value of $r_j(\omega_p)$ is verified to be close to unity [14]. This implies that the photon experiences a very weak absorption, and thereby we may approximately consider that the output photon only experiences a pure phase shift, i.e., $r_j(\omega_p) = e^{i\phi}$, without any absorption. On the other hand, considering the case $g$=0 (the atom uncoupled to the cavity or an empty cavity) we have $r_{j0}(\omega_p) = \frac{i(\omega_c-\omega_p)-\frac{\kappa}{2}}{i(\omega_c-\omega_p)+\frac{\kappa}{2}}$ which can be rewritten as a pure phase shift, i.e., $r_{j0}(\omega_p)=e^{i\phi_0}$.

If the parameters of atom-field system satisfy $\omega_0=\omega_c$, $\omega_p=\omega_c-\kappa/2$ and $g=\kappa/2$, we can obtain $\phi=\pi$ and $\phi_0=\pi/2$, corresponding to the evolution of atom and photon as

$$\begin{aligned} &|L\rangle|g_L\rangle \rightarrow -|L\rangle|g_L\rangle, \quad |R\rangle|g_L\rangle \rightarrow i|R\rangle|g_L\rangle, \\ &|L\rangle|g_R\rangle \rightarrow i|L\rangle|g_R\rangle, \quad |R\rangle|g_R\rangle \rightarrow -|R\rangle|g_R\rangle. \end{aligned} \quad (2)$$

In the following ECPs, we will straightforwardly utilize the evolution as shown in Eq. (2) without further illustration.

We now discuss concentration of atomic entangled states via photonic Faraday rotation and the schematic setup is sketched in Fig. 2. Assume that there are two pairs of non-maximally entangled three-level atoms 1, 2 and 3, 4 as follows

$$\begin{aligned} |\psi\rangle_{12} &= a_1|g_L\rangle_1|g_R\rangle_2 + b_1|g_R\rangle_1|g_L\rangle_2, \\ |\psi\rangle_{34} &= a_2|g_L\rangle_3|g_R\rangle_4 + b_2|g_R\rangle_3|g_L\rangle_4, \end{aligned} \quad (3)$$

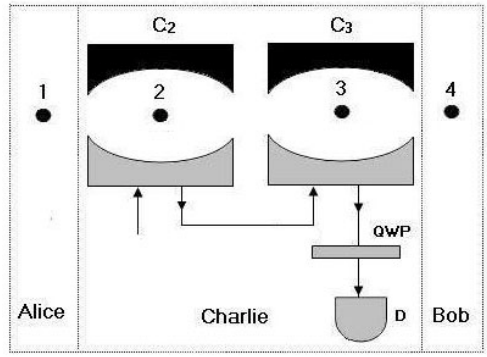


FIG. 2: The schematic of atomic ECP. Atoms 2 and 3 are trapped in low-Q cavities $C_2$ and $C_3$, respectively. A single-photon pulse is sent into cavities $C_2$, $C_3$ and a quarter-wave plate, then detected by photon detector $D$.

where $a_i$ and $b_i$ ($i$=1, 2) are the normalized coefficients such that $|a_i|^2+|b_i|^2=1$, and we assume that they are all real numbers without loss of generality. In principle, the entangled states $|\psi\rangle_{12}$ and $|\psi\rangle_{34}$, which are prepared by the same experimental setup, have the identical amount of entanglement, i.e., $a_1=a_2$ and $b_1=b_2$. However, the experimental imperfections or the effect of communication channels in the preparation and distribution processes will lead to a tiny deviation between $a_1(b_1)$ and $a_2(b_2)$. For simplicity, we firstly omit the deviation and will discuss its effect to the fidelity of our ECP later. We assume that three spatially separate users, say Alice, Bob and Charlie, share entangled states $|\psi\rangle_{12}$ and $|\psi\rangle_{34}$ where atoms 1, 4 are in the hands of Alice and Bob respectively, and atoms 2, 3 are all in the hand of Charlie.

To extract maximally entangled state from the pair of non-maximally entangled states via photonic Faraday rotation, two low-Q cavities $C_2$ and $C_3$, where atoms 2 and 3 are trapped respectively, are introduced at Charlie's station. A single-photon pulse with the initial state $|\psi\rangle_p=\frac{1}{\sqrt{2}}(|L\rangle+|R\rangle)$ will be sent through the cavities $C_2$ and $C_3$ sequentially. Then Charlie performs the Hadamard operation on atoms 2, 3 and photon respectively. Note that atomic Hadamard gate can be implemented by driving the atom with an external classical field (polarized lasers), and the quarter-wave plate (QWP) acts as the role of photonic Hadamard gate. To be concrete, atomic and photonic Hadamard operations can be expressed as $|g_L\rangle\rightarrow\frac{1}{\sqrt{2}}(|g_L\rangle + |g_R\rangle)$, $|g_R\rangle\rightarrow\frac{1}{\sqrt{2}}(|g_L\rangle-|g_R\rangle)$, $|L\rangle\rightarrow\frac{1}{\sqrt{2}}(|L\rangle+|R\rangle)$ and $|R\rangle\rightarrow\frac{1}{\sqrt{2}}(|L\rangle-|R\rangle)$. After this evolution process, the quantum state of whole system is

$$\begin{aligned} \sum_{j,k=L,R} \frac{1}{2}[&\mp i|L\rangle|g_j\rangle_2|g_k\rangle_3(a_1a_2|g_L\rangle_1|g_R\rangle_4\pm b_1b_2|g_R\rangle_1|g_L\rangle_4) \\ &+|R\rangle|g_j\rangle_2|g_k\rangle_3(\mp a_1b_2|g_L\rangle_1|g_L\rangle_4+b_1a_2|g_R\rangle_1|g_R\rangle_4). \end{aligned} \quad (4)$$

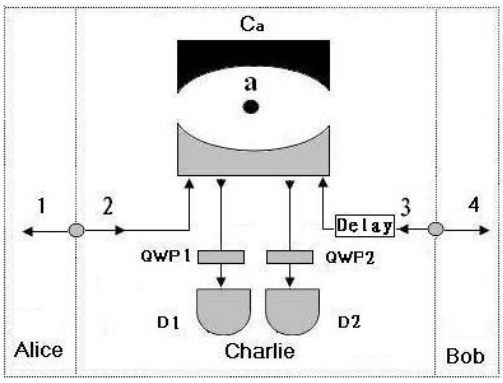


FIG. 3: The schematic of photonic ECP. Atoms $a$ trapped in low-Q cavity $C_a$, Quarter-Wave plate QWP1, QWP2, and photon detectors $D_1$, $D_2$ are introduced at Charlie's station.

Finally, Charlie performs measurement on the states of photon and atoms at his side, and thus the atomic state at Alice's and Bob's sides will collapse into one of the corresponding components in Eq. (4). To be explicit, if Charlie's measurement outcome is $|L\rangle|g_j\rangle_2|g_k\rangle_3$ $(j,k{=}L,R)$, the quantum state of atoms 1, 4 will be $|\psi\rangle_{14}{=}a_1a_2|g_L\rangle_1|g_R\rangle_4{\pm}b_1b_2|g_R\rangle_1|g_L\rangle_4$ (unnormalized). On the other hand, if Charlie's measurement outcome is $|R\rangle|g_j\rangle_2|g_k\rangle_3$ $(j,k{=}L,R)$, the quantum state of atoms 1, 4 will be $|\psi'\rangle_{14}{=}\mp a_1b_2|g_L\rangle_1|g_L\rangle_4 + b_1a_2|g_R\rangle_1|g_R\rangle_4$. Obviously, $|\psi'\rangle_{14}$ are maximally entangled under the previous condition of the initial states, and thus the total successful probability of our ECP is $P{=}2a_1^2(1{-}a_1^2)$, which is the same as that in Ref. [9].

In this atomic ECP, two atoms 1, 4, which never interacted with each other before, are left in a pure maximally entangled state after the whole operation process. In other words, Alice and Bob are completely passive in the whole concentration process. Starting from this point of view, we can generalize this ECP to reconstruct multi-atom Greenberger-Horne-Zeilinger (GHZ) state from the partially entangled atomic GHZ-class states as follows

$$\begin{aligned}|\Psi\rangle_1 &= a_1|g_Lg_R\cdots g_R\rangle_{C_1A_1\cdots A_N}{+}b_1|g_Rg_L\cdots g_L\rangle_{C_1A_1\cdots A_N},\\ |\Psi\rangle_2 &= a_2|g_Lg_R\cdots g_R\rangle_{C_2B_1\cdots B_N}{+}b_2|g_Rg_L\cdots g_L\rangle_{C_2B_1\cdots B_N},\end{aligned} \tag{5}$$

where the subscripts $A_j$, $B_j(j{=}1,...,N)$, $C_1$ and $C_2$ represent atoms held by Alice, Bob and Charlie, respectively. If we define $|g'_L\rangle_A{=}|g_L\cdots g_L\rangle_{A_1\cdots A_N}$, $|g'_R\rangle_A{=}|g_R\cdots g_R\rangle_{A_1\cdots A_N}$, $|g'_L\rangle_B{=}|g_L\cdots g_L\rangle_{B_1\cdots B_N}$ and $|g'_R\rangle_B{=}|g_R\cdots g_R\rangle_{B_1\cdots B_N}$, the GHZ-class states can be rewritten as $|\Psi\rangle_1 = a_1|g_L\rangle_{C_1}|g'_R\rangle_A{+}b_1|g_R\rangle_{C_1}|g'_L\rangle_A$ and $|\Psi\rangle_2 = a_2|g_L\rangle_{C_2}|g'_R\rangle_B{+}b_2|g_R\rangle_{C_2}|g'_L\rangle_B$. By inspection, they just have the same forms as the entangled states in Eq. (3). Thus, we can adopt the same procedure as that in the case of two-atom entangled state, and reconstruct $2N$-atom GHZ state between Alice and Bob with the successful probability $P{=}2a_1^2(1{-}a_1^2)$.

Benefitting from the single-photon input-output process in the cavity QED system, we show that photonic Faraday rotation can also be used to concentrate photonic entangled states. The concrete schematic setup for photonic ECP is depicted in Fig. 3. We assume that there are two pairs of partially entangled photonic states as follows

$$\begin{aligned}|\phi\rangle_{12}{=}a_1|L\rangle_1|R\rangle_2{+}b_1|R\rangle_1|L\rangle_2,\\ |\phi\rangle_{34}{=}a_2|L\rangle_3|R\rangle_4{+}b_2|R\rangle_3|L\rangle_4,\end{aligned} \tag{6}$$

where photons 1, 4 are in the hands of Alice and Bob respectively, and Charlie holds photons 2 and 3. To implement photonic ECP, a three-level atom (labeled as $a$), which is trapped in a low-Q cavity $C_a$, is introduced at Charlie's station. The initial state of atom is $|\phi\rangle_a{=}\frac{1}{\sqrt{2}}(|g_L\rangle_a{+}|g_R\rangle_a)$. Charlie guides photons 2, 3 into the cavity sequentially, and then lets them pass QWP1 and QWP2 respectively after leaving the cavity $C_a$. By performing the Hadamard operation on atom $a$, the quantum state after this evolution process will be

$$\begin{aligned}\sum_{j,k=L,R}\frac{1}{2}[\mp i|j\rangle_2|k\rangle_3|g_L\rangle_a(a_1a_2|L\rangle_1|R\rangle_4{+}b_1b_2|R\rangle_1|L\rangle_4)\\ +|j\rangle_2|k\rangle_3|g_R\rangle_a(a_1b_2|R\rangle_1|R\rangle_4{\mp}b_1a_2|L\rangle_1|L\rangle_4)].\end{aligned} \tag{7}$$

Charlie then measures the quantum states of photons and atom at his side, followed by the collapse of photonic states at Alice's and Bob's side to one of the corresponding components in Eq. (7). In detail, the quantum state of photons 1, 4 will be $|\psi\rangle_{14}{=}a_1a_2|L\rangle_1|R\rangle_4{+}b_1b_2|R\rangle_1|L\rangle_4$ or $|\psi'\rangle_{14}{=}a_1b_2|R\rangle_1|R\rangle_4{\mp}b_1a_2|L\rangle_1|L\rangle_4$ corresponding to the measurement outcomes $|j\rangle_2|k\rangle_3|g_L\rangle_a$ or $|j\rangle_2|k\rangle_3|g_R\rangle_a$ $(j,k{=}L,R)$, respectively. Similar to atomic ECP, $|\psi'\rangle_{14}$ are maximally entangled and then the successful probability of our photonic ECP is $P{=}2a_1^2(1{-}a_1^2)$.

In this photonic ECP, Charlie needs to strictly control the time interval of photons 2 and 3 passing the low-Q cavity, in order to avoid the case that both of photons interact with the atom simultaneously. Otherwise, the ECP will fail. However, the order of photon (2 or 3) interacting with atom $a$ will not affect the final results of ECP because the situation of photon 2 and 3 is completely equivalent. Similar to the atomic ECP, we can also generalize the photonic ECP to reconstruct multi-photon GHZ state from the partially entangled photonic GHZ-class states $|\Phi\rangle_1{=}a_1|LR\cdots R\rangle_{C_1A_1\cdots A_N}{+}b_1|RL\cdots L\rangle_{C_1A_1\cdots A_N}$ and $|\Phi\rangle_2{=}a_2|LR\cdots R\rangle_{C_2B_1\cdots B_N}{+}b_2|RL\cdots L\rangle_{C_2B_1\cdots B_N}$ with the same successful probability. It is noted that our ECPs for atomic and photonic states may be universal as the entanglement can be concentrated whenever $a_i{<}b_i$ or $a_i{>}b_i$ for every $i{=}1,2$.

We briefly discuss the experimental feasibility of our protocols. Consider a $^{87}$Rb atom trapped in the fiber-based Fabry-Perot cavity [21]. The states $|\mathrm{F}{=}2, m_\mathrm{F}{=}{\pm}1\rangle$ of level $5S_{1/2}$ correspond to degenerate ground states $|g_L\rangle$

and $|g_R\rangle$ respectively, the state $|\mathrm{F}'=3, m_\mathrm{F}=0\rangle$ of level $5P_{3/2}$ is chosen as the excited state $|e\rangle$ and the corresponding transition frequency $\omega_0=2\pi c/\lambda$ with $\lambda$=780nm ($\mathrm{D}_2$ line). In Ref. [21], the cavity length $L$=38.6$\mu$m, waist radius $w_0$=3.9$\mu$m and finesse $\mathcal{F}$=37000, which correspond to longitudinal mode number $n$=99, the cavity decay rate $\kappa=2\pi\times$53MHz (the relevant Q factor $Q=\omega_c/(2\kappa)=3.63\times10^6$) and the maximal coupling strength $g_0=2\pi\times$215MHz. In our protocols, the atom-cavity coupling strength $g=g_0\cos(2\pi x/\lambda)$ should be matched with cavity decay rate ($g=\kappa/2$), which can be satisfied by adjusting the appropriate atomic longitudinal coordinate ($x=n\frac{\lambda}{2}$+179nm). Meanwhile, the input photon can be tuned to be nearly resonant with the atom-cavity system, i.e., $\omega_p=\omega_c-\kappa/2$. Therefore, based on present experiment technology in cavity QED [13, 21], the required atom-cavity parameters can be tuned to control the reflectivity of the input photon for obtaining the desired phase shifts. In the following, we consider the possible realization of our ECPs in the context of low-Q cavity. In the experiment, the cavity Q factor and the decay rate are closely related with the cavity finesse which depends solely on the intensity transmission and loss of cavity mirror. In Ref. [21], if the atom is located at the antinode of the cavity field ($x=n\frac{\lambda}{2}$), we can obtain the maximal atom-cavity coupling ($g=g_0=2\pi\times$215MHz). Consider the transmission of cavity mirror $\mathcal{T}$=666ppm (i.e., the cavity finesse $\mathcal{F}$=4510), the practical Q factor of cavity reduces to only $Q=4.47\times10^5$ and then the decay rate satisfies $\kappa=2g_0$. As to the ultra low-Q cavity (high decay rate of cavity $\kappa$), the condition $g=\kappa/2$ may also be satisfied by obtaining the large enough coupling between atom and cavity field.

However, there are still some imperfections in the realistic experiment. For instance, the cavity resonance frequency may be deviated from the atomic eigenfrequency due to the tiny change of cavity length, and the coupling strength may be not strictly matched with the cavity decay rate because of the variation of atomic position in the cavity. The slight deviation of resonance ($\omega_c\sim\omega_0$) and mismatch of coupling strength ($g\sim\kappa/2$) will not change the reflection amplitudes but phase shifts $\phi(\phi_0)$. In the case of $\omega_c-\omega_0\approx\kappa/10$, the phase shifts $\phi\approx2.75$ and $\phi_0\approx1.36$. The fidelity of obtaining atomic and photonic states $|\psi\rangle_{14}$ is about $F=\frac{1}{2}[1-\cos2(\phi-\phi_0)]\approx0.955$. If the coupling strength satisfies $g\approx3\kappa/5$, we can obtain the phase shift $\phi\approx2.31$ and the fidelity of $|\psi\rangle_{14}$ $F\approx0.455$. Interestingly, the fidelity of quantum state $|\psi'\rangle_{14}$ is just 1 as it is independent of the Faraday rotation angle. Thus, our atomic and photonic ECPs are immune to the experimental imperfections as discussed above.

In our atomic and photonic ECPs, we have assumed that the initial condition of the entangled states satisfies $a_1=a_2$ and $b_1=b_2$. But in practice, there may be imperfections in the entanglement preparation and distribution processes, which lead the entangled states into less entangled pure or even mixed ones. Here, we consider that the entanglement preparation process is near perfect and the communication channels between Alice (Bob) and Charlie are of high quality. Then the resulting entangled states, after the entanglement preparation and distribution processes, may have a tiny deviation to the ideal ones, i.e., $a_2=a_1+ka_1$ with $k$ being a small constant. In this case, the fidelity of obtaining the desired state $|\psi'\rangle_{14}$ is $F(a_1,k)=\frac{[\sqrt{1-a_1^2(1+k)^2}+(1+k)\sqrt{1-a_1^2}]^2}{2[1+(1+k)^2-2a_1^2(1+k)^2]}$ [9]. If we consider $a_1\in(0,0.7)$ and $k=\pm0.1$, the minimal fidelity $F$=0.989, 0.991 for $a_1$=0.7 and $k=\pm0.1$, which indicates that the small deviation of coefficients, due to the effect of imperfections described above, only affects the fidelity of the result state slightly.

In the following, we make comparison with the previous ECPs. Note that ECPs involving a pair of partially entangled states can be realized via entanglement swapping [20]. The crucial step of entanglement swapping is the implementation of joint Bell-state measurement, which is also at the heart of other QIP tasks such as quantum teleportaion [1] and dense coding [3]. In our atomic and photonic ECPs, we have introduced low-Q cavities, three-level atom and single-photon pulse, and can implement entanglement swapping without joint Bell-state measurement, only by detecting the quantum state of atoms and photons separately.

For concentration of atomic entanglement, the distinct advantage of our atomic ECP is that we only need low-Q optical cavity while the high-Q cavity is usually required in Refs. [9–11]. In Refs. [9, 11], the atomic state is used as the flying qubit, but it is actually suitable for acting as stationary qubit which will be feasible in experiment. In Ref. [10], Cao *et al.* proposed atomic ECP through cavity decay which relies on two leaking photon reaching the beam splitter simultaneously, as well as the high efficiency of two photon detectors. Due to the large inefficiency of photon detector, our atomic ECP may be more efficient than Ref. [10] as only a single-photon detector is involved for ours. Furthermore, we can use the coherent input pulse to replace the single-photon pulse as shown in Ref. [17], and also implement atomic ECP with homodyne detection of coherent light, which can greatly relaxes the experiment requirement for photon source and reduce measurement difficulties. On the other hand, we propose to concentrate photonic entanglement via single-photon input-output process in cavity QED for the first time. With the assistance of three-level atom trapped in the low-Q cavity, two-photon and multi-photon maximally entangled states can be reconstructed with the same efficiency (successful probability) as that in Ref. [6]. In fact, the low-Q cavity and three-level atom function as photonic phase-shift controller in photonic Faraday rotation, which is similar to the cross-kerr nonlinearity [23]. Therefore, we can also construct photonic parity gate via photonic Faraday rotation and then implement photonic ECP as Ref. [8].

In conclusion, we have proposed to concentrate atomic and photonic entanglement via photonic Faraday rotation. Through the single-photon input-output process in

cavity QED, it is shown that the maximally entangled atomic and photonic state can be extracted from two partially entangled states. In our ECPs, we only need the low-Q cavity, three-level atom and the basic optical elements such as QWP and photon detector to complete entanglement concentration, and they may be feasible with current cavity QED and quantum optics technology. The essential idea in our ECPs is the single-photon input-output process in cavity QED, and thus it may be worth studying entanglement purification and concentration using other similar cavity QED schemes [24] in the future.

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