Detection efficiency in the loophole-free violation of Svetlichny's inequality

Yang Xiang, Hui-Xian Wang, and Fang-Yu Hong

Phys. Rev. A **86**, 034102 — Published 17 September 2012

DOI: 10.1103/PhysRevA.86.034102

Detection efficiency in the loophole-free violation of Svetlichny's inequality

Yang Xiang,[1,*] Hui-Xian Wang,[2] and Fang-Yu Hong[3]

[1] *School of Physics and Electronics, Henan University, Kaifeng, Henan 475004, People's Republic of China*

[2] *School of Physics and Engineering, Henan University of Science and Technology, Luoyang 471023, People's Republic of China*

[3] *Department of Physics, Center for Optoelectronics Materials and Devices, Zhejiang Sci-Tech University, Xiasha College Park, Hangzhou, Zhejiang 310018, People's Republic of China*

(Dated: September 4, 2012)

Abstract

Svetlichny's inequality (SI) is a Bell-like inequality, the violation of which can be used to confirm the existence of genuine multipartite correlations. However, the imperfection of detector's efficiency possibly cause the so-called detection loophole in actual Svetlichny's experiments. We derive a new SI to deal with the detection loophole in actual Svetlichny's experiments. If the experimental data can violate the new SI, it must result in the loophole-free violation of the original SI. We show that the minimum detection efficiency needed for a loophole-free violation of the tripartite SI is about 0.97. For the general case of n-particle, we give the analytic expression of the needed detection efficiency, and find that its value monotonously and rapidly approaches to 1 as the number of particles increases.

PACS numbers: 03.65.Ud, 03.65.Ta, 03.67.-a

[*] Email Address: njuxy@sohu.com

I. INTRODUCTION

An ubiquitous problem in Physics is to understand the correlation which is observed among different events, and quantum theory open a new world of non-classical correlations. Bell [1] demonstrate that quantum theory predicts that separated systems can produce outcomes whose correlation can not be explained by any local hidden variables theory. A more general version of Bell inequality for two qubits was given by Clauser, Horne, Shimony, and Holt [2] (CHSH inequality). Since then the experimental confirmation of Bell's prediction is one of the fundamental challenges of modern physics [3–7]. Besides causation and reality assumptions the derivation of Bell inequality require three additional assumptions (or say conditions): (i) The observers's measurement choices are not correlated with each other or hidden variable, and these choices are random [8–10]; (ii) Different observers's measurement events are spacelike separated; (iii) The fair sampling assumption, i.e., the detected data are representative of all those emitted from the source. So an incontrovertible experimental confirmation must simultaneously satisfy above three conditions, or one can't say that the experimental violation of Bell inequality demonstrate the downfall of local realistic theories. Any dissatisfaction of above three conditions means there exist loopholes of an experimental confirmation. Especially the third condition of the fair sampling assumption is unsatisfactory, why can't the data recorded by detectors be special? Actually soon after the discovery of Bell inequality, Pearle [11] has pointed out that if the detectors' efficiency is not perfect then it's possible to devise a local hidden variable model which can also produce the violation of Bell inequality. So if we abandon the fair sampling assumption and at the same time the detection efficiency is too low, the so-called detection loophole arises.

The detection efficiency is defined as the ratio between the numbers of detected particles and actually emitted particles by source. The minimum detection efficiency which is required to close the detection loophole, is called threshold efficiency η_{crit}. It's an interesting question of what the threshold efficiency η_{crit} is for any given scenario. To this day, the value of η_{crit} is known for many scenarios [12–25]. The value of η_{crit} for a loophole-free experiment based on Hardy's approach has also been deduced [26, 27]. In practice, one usually adopt photon experiments for Bell experiment. Though photon experiments are able to close the locality loophole [28], the optical detection efficiencies are still too low to close the detection loophole. Since the detection efficiency is the product of the transmission efficiency and the detector efficiency, the loss of photons in the transmission from the source to the observers' locations can drastically reduce the detection efficiency. Recently, a precertification technique has been proposed [29, 30], the key of the technique is to boost the transmission efficiency to one. There are some other proposals for closing experimental detection loophole in Ref. [31, 32].

This paper focuses on the issue of the threshold detection efficiency in actual Svetlichny's experiments. Svetlichny's inequality (SI) [33, 34] is a Bell-like inequality, the violation of which can be used to confirm the existence of genuine multipartite correlations. We will derive a new SI to deal with the detection loophole in actual Svetlichny's experiments. If the experimental data can violate the new SI, it must result in the loophole-free violation of the original SI. At the same time, we will give the threshold efficiency which is required for a loophole-free violation of SI. The general case of n-particle has also been addressed.

II. THRESHOLD DETECTION EFFICIENCY FOR THREE-PARTICLE SI

Three-particle SI [33] can be used to confirm the existence of genuine three-particle correlations which is essentially different from two-particle correlations. This means that one can find a violation of SI only if there exist genuine three-particle correlations in a three-particle setting. Consider three observers, Alice, Bob, and Carol, who share three entangled qubits. Each of them can choose to measure one of two dichotomous observables. We denote A_1 and A_2 as Alice's measurement results when she performs measurement a_1 and a_2 respectively, and similarly B_2, B_2, b_1, and b_2 (C_1, C_2, c_1, and c_2) for Bob's (Carol's), and the measurement results of all observables can be -1 or $+1$. Then SI is expressed as [33]

$$\begin{aligned}
&|E(A_1B_1C_1) + E(A_1B_1C_2) + E(A_1B_2C_1) \\
&+E(A_2B_1C_1) - E(A_1B_2C_2) - E(A_2B_1C_2) \\
&-E(A_2B_2C_1) - E(A_2B_2C_2)| \leq 4,
\end{aligned} \tag{1}$$

where $E(A_iB_jC_k)$'s represent the expectation value of the product of the measurement outcomes of the observables. The SI of Eq. (1) applies to the ideal case in which all experimental settings of the three observers would give results, and only in the ideal case one can assert that the violation of SI of Eq. (1) confirm the existence of genuine three-particle correlations. So in order to obtain a loophole-free violation of SI of Eq. (1), we essentially should get a violation of the following inequality,

$$\begin{aligned}
&|E(A_1B_1C_1|\Lambda_0) + E(A_1B_1C_2|\Lambda_0) \\
&+E(A_1B_2C_1|\Lambda_0) + E(A_2B_1C_1|\Lambda_0) \\
&-E(A_1B_2C_2|\Lambda_0) - E(A_2B_1C_2|\Lambda_0) \\
&-E(A_2B_2C_1|\Lambda_0) - E(A_2B_2C_2|\Lambda_0)| \leq 4,
\end{aligned} \tag{2}$$

where Λ_0 represent the ensemble where all measurements successfully give results, and $E(A_iB_jC_k|\Lambda_0)$ denote the expectation value of the product of $A_iB_jC_k$ in the ensemble Λ_0. The violation of Eq. (2) is a (detection) loophole-free confirmation of existence of genuine three-particle correlations.

However, $E(A_iB_jC_k|\Lambda_0)$ are unaccessible in actual experiment. The usual approach is to disregard these "no detected" events and only the coincident events contribute to estimation of $E(A_iB_jC_k)$. The coincident event is that Alice, Bob, and Carol all successfully obtain a measurement result in one trial. This approach is equal to that if a measurement is failed one will get the value 0 as the measurement result. So in actual experiment one essentially calculate the conditional correlations $E(A_iB_jC_k|\Lambda_{A_iB_jC_k})$, where $\Lambda_{A_iB_jC_k}$ represent the ensemble where all measurements a_i, b_j and c_k successfully give results -1 or1.

Here for convenience we define all notations which will be used frequently in the following text, these notations are borrowed from Ref. [24]. We use Λ_{A_i} denoting the ensemble where Alice's measurement setting a_i successfully gives result -1 or1, and similar for Λ_{B_j} and Λ_{C_k}; we use $\Lambda_{A_iB_j}$ denoting the ensemble where measurement setting a_i and b_j both give results -1 or1, and similar for $\Lambda_{A_iB_jC_k}$, etc. According to above notations, the ensemble Λ_0 can be expressed as $\Lambda_{A_1A_2B_1B_2C_1C_2}$. $P(\Lambda_{A_i})$ denote the probability of $A_i \neq 0$, and $P(\Lambda_{A_iB_j})$ denote the possibility that both A_i and B_j are nonzero, similar for $P(\Lambda_{A_iB_jC_k})$, etc; $P(\Lambda_{A_i}|\Lambda_{B_j})$ denote the conditional probability that $A_i \neq 0$ given that $B_j \neq 0$.

proposition 1 If we define

$$\delta_3 = \min_{ijk} P(\Lambda_0|\Lambda_{A_iB_jC_k}), \tag{3}$$

where $\min_{ijk}$ take over all measurements settings. From Eq. (2) and Eq. (3) we can obtain the inequality

$$|E(A_1B_1C_1|\Lambda_{A_1B_1C_1}) + E(A_1B_1C_2|\Lambda_{A_1B_1C_2})$$

$$
\begin{aligned}
&+E(A_1B_2C_1|\Lambda_{A_1B_2C_1})+E(A_2B_1C_1|\Lambda_{A_2B_1C_1})\\
&-E(A_1B_2C_2|\Lambda_{A_1B_2C_2})-E(A_2B_1C_2|\Lambda_{A_2B_1C_2})\\
&-E(A_2B_2C_1|\Lambda_{A_2B_2C_1})-E(A_2B_2C_2|\Lambda_{A_2B_2C_2})|\\
&\leq 4(2-\delta_3). \qquad (4)
\end{aligned}
$$

Proof. It's obvious that $\Lambda_0 \subset \Lambda_{A_iB_jC_k}$, the ensemble $\Lambda_{A_iB_jC_k}$ can be split into two disjointed sub-ensemble Λ_0 and its complement $\Lambda_0^c = \Lambda_{A_iB_jC_k} \backslash \Lambda_0$. We can get

$$
\begin{aligned}
& E(A_iB_jC_k|\Lambda_{A_iB_jC_k})-\delta_3 E(A_iB_jC_k|\Lambda_0)\\
\leq\ & |P(\Lambda_0^c|\Lambda_{A_iB_jC_k})E(A_iB_jC_k|\Lambda_0^c)|\\
& +|P(\Lambda_0|\Lambda_{A_iB_jC_k})E(A_iB_jC_k|\Lambda_0)-\delta_3 E(A_iB_jC_k|\Lambda_0)|\\
=\ & P(\Lambda_0^c|\Lambda_{A_iB_jC_k})|E(A_iB_jC_k|\Lambda_0^c)|\\
& +[P(\Lambda_0|\Lambda_{A_iB_jC_k})-\delta_3]|E(A_iB_jC_k|\Lambda_0)|\\
\leq\ & P(\Lambda_0^c|\Lambda_{A_iB_jC_k})E(|A_iB_jC_k||\Lambda_0^c)\\
& +[P(\Lambda_0|\Lambda_{A_iB_jC_k})-\delta_3]E(|A_iB_jC_k||\Lambda_0)\\
=\ & 1-\delta_3. \qquad (5)
\end{aligned}
$$

Combine Eq. (2) and Eq. (5), we obtain

$$
\begin{aligned}
& left\ hand\ side\ of\ Eq.(4)\\
\leq\ & \delta_3 \times left\ hand\ side\ of\ Eq.(2)+8(1-\delta_3)\\
\leq\ & 4\delta_3+8(1-\delta_3)\\
\leq\ & 4(2-\delta_3). \qquad (6)
\end{aligned}
$$

Hence the proof. □

In order to obtain the threshold detection efficiency required for loophole-free violation of SI, we must get the relation between δ_3 and the detection efficiency. For simplicity, we assume that the detection efficiencies of the three observers are equal and independent with each other, this means that $P(\Lambda_{A_i}) = P(\Lambda_{B_j}) = P(\Lambda_{A_i}|\Lambda_{B_j}) = \eta$.

proposition 2. $\delta_3 \geq 13 - \frac{12}{\eta}$

Proof. First, it's obvious that $P(\Lambda_{A_i}|\Lambda_{A_iB_jC_k}) = P(\Lambda_{B_j}|\Lambda_{A_iB_jC_k}) = P(\Lambda_{C_k}|\Lambda_{A_iB_jC_k}) = 1$. For $i^{'} \neq i$,

$$
P(\Lambda_{A_{i^{'}}}|\Lambda_{A_iB_jC_k})
$$

$$
\begin{aligned}
&= \frac{P(\Lambda_{A_{i'} A_i}|\Lambda_{B_jC_k})}{P(\Lambda_{A_i}|\Lambda_{B_jC_k})} \\
&= \frac{P(\Lambda_{A_{i'}}|\Lambda_{B_jC_k}) + P(\Lambda_{A_i}|\Lambda_{B_jC_k})}{P(\Lambda_{A_i}|\Lambda_{B_jC_k})} \\
&\quad - \frac{P(\Lambda_{A_i} \cup \Lambda_{A_{i'}}|\Lambda_{B_jC_k})}{P(\Lambda_{A_i}|\Lambda_{B_jC_k})} \\
&\geq \frac{2\eta - 1}{\eta}.
\end{aligned} \tag{7}
$$

In Eq. (7) we used that $P(\Lambda_{A_{i'}}|\Lambda_{B_jC_k}) = P(\Lambda_{A_i}|\Lambda_{B_jC_k}) = \eta$.

Now consider general $P(\Lambda_{A_{i'}B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k})$. We can find that if all equations of $i' = i, j' = j, k' = k$ are available $P(\Lambda_{A_{i'}B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k}) = 1$. If two of the three equations are available, $P(\Lambda_{A_{i'}B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k}) \geq \frac{2\eta-1}{\eta}$. For example

$$
\begin{aligned}
& P(\Lambda_{A_2B_2C_3}|\Lambda_{A_1B_2C_3}) \\
&= P(\Lambda_{A_2}|\Lambda_{A_1B_2C_3}) + P(\Lambda_{B_2C_3}|\Lambda_{A_1B_2C_3}) \\
&\quad - P(\Lambda_{A_2} \cup \Lambda_{B_2C_3}|\Lambda_{A_1B_2C_3}) \\
&\geq P(\Lambda_{A_2}|\Lambda_{A_1B_2C_3}) \\
&\geq \frac{2\eta - 1}{\eta}.
\end{aligned} \tag{8}
$$

If one of the three equations of $i' = i, j' = j, k' = k$ is available, $P(\Lambda_{A_{i'}B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k}) \geq 3 - \frac{2}{\eta}$. For example

$$
\begin{aligned}
& P(\Lambda_{A_2B_1C_3}|\Lambda_{A_1B_2C_3}) \\
&= P(\Lambda_{A_2}|\Lambda_{A_1B_2C_3}) + P(\Lambda_{B_1C_3}|\Lambda_{A_1B_2C_3}) \\
&\quad - P(\Lambda_{A_2} \cup \Lambda_{B_1C_3}|\Lambda_{A_1B_2C_3}) \\
&\geq P(\Lambda_{A_2}|\Lambda_{A_1B_2C_3}) + P(\Lambda_{B_1C_3}|\Lambda_{A_1B_2C_3}) - 1 \\
&\geq P(\Lambda_{A_2}|\Lambda_{A_1B_2C_3}) + P(\Lambda_{B_1}|\Lambda_{A_1B_2C_3}) - 1 \\
&\geq 3 - \frac{2}{\eta}.
\end{aligned} \tag{9}
$$

In the case of $i' \neq i$, $j' \neq j$, $k' \neq k$, $P(\Lambda_{A_{i'}B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k}) \geq 4 - \frac{3}{\eta}$, since

$$
\begin{aligned}
& P(\Lambda_{A_{i'}B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k}) \\
&\geq P(\Lambda_{A_{i'}}|\Lambda_{A_iB_jC_k}) + P(\Lambda_{B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k}) - 1 \\
&\geq 3 \times \frac{2\eta - 1}{\eta} - 2 = 4 - \frac{3}{\eta}.
\end{aligned} \tag{10}
$$

Finally we calculate $P(\Lambda_0|\Lambda_{A_iB_jC_k})$.

$$
\begin{aligned}
& P(\Lambda_0|\Lambda_{A_iB_jC_k}) \\
= \; & P(\cap_{\{i'j'k'\}}\Lambda_{A_{i'}B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k}) \\
\geq \; & \sum_{\{i'j'k'\}} P(\Lambda_{A_{i'}B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k}) - 7 \\
\geq \; & \left[1 + 3\times\left(2-\frac{1}{\eta}\right) + 3\times\left(3-\frac{2}{\eta}\right) + \left(4-\frac{3}{\eta}\right)\right] - 7 \\
= \; & 13 - \frac{12}{\eta}.
\end{aligned} \tag{11}
$$

From Eq. (3) and Eq. (11), we prove the proposition.□

Combining proposition 1 with proposition 2, we finally get a new SI which can be directly compared with experimental data

$$
\begin{aligned}
& |E(A_1B_1C_1|\Lambda_{A_1B_1C_1}) + E(A_1B_1C_2|\Lambda_{A_1B_1C_2}) \\
& +E(A_1B_2C_1|\Lambda_{A_1B_2C_1}) + E(A_2B_1C_1|\Lambda_{A_2B_1C_1}) \\
& -E(A_1B_2C_2|\Lambda_{A_1B_2C_2}) - E(A_2B_1C_2|\Lambda_{A_2B_1C_2}) \\
& -E(A_2B_2C_1|\Lambda_{A_2B_2C_1}) - E(A_2B_2C_2|\Lambda_{A_2B_2C_2})| \\
& \leq 4\left(\frac{12}{\eta} - 11\right).
\end{aligned} \tag{12}
$$

From the derivation of Eq. (12), we know that if the experimental data can violate the new SI, it must result in the violation of the SI of Eq. (2) which is a loophole-free violation of original SI. It was shown by Svetlichny [33] that the maximum value of the left hand side of Eq. (12) is $4\sqrt{2}$ allowed in quantum mechanics. So in order to get the violation of Eq. (12) the detection efficiency must fulfill that $\eta \geq \frac{12}{11+\sqrt{2}} \approx 0.9666$, we call the minimum efficiency (0.96666) required for the violation as the threshold efficiency η_{crit}.

III. THRESHOLD DETECTION EFFICIENCY FOR N-PARTICLE SI

Suppose n players who share n particles, each one of them performs dichotomous measurements on each of the n particles. The measurement settings are represented by x_1, x_2,...x_n, respectively, with possible values $0, 1$, and the corresponding measurement results are represented by A_{x_1}, A_{x_2},...A_{x_n}, respectively, and with possible values $-1, 1$. Then the

N-particle SI can be expressed as [34]

$$\left|\sum_{\{x_i\}} v(x_1, x_2, ..., x_n) E(A_{x_1} A_{x_2} \cdots A_{x_n})\right| \leq 2^{n-1}, \tag{13}$$

where $\{x_i\}$ stands for an n-tuple $x_1, ..., x_n$, $E(A_{x_1} A_{x_2} \cdots A_{x_n})$ represents the expectation value of the product of the measurement outcomes of observables $x_1, x_2, ..., x_n$, and $v(x_1, x_2, ..., x_n)$ is a sign function given by

$$v(x_1, x_2, ..., x_n) = (-1)^{[\frac{k(k-1)}{2}]}, \tag{14}$$

where k is the number of times index 1 appears in $(x_1, x_2, ..., x_n)$.

It's similar to the case of three-particle, only in the ideal case we can assert the violation of Eq. (13) is a confirmation of the existence of genuine n-particle correlations. So in order to obtain a loophole-free violation of SI of Eq. (13), we essentially should get a violation of the following inequality,

$$\left|\sum_{\{x_i\}} v(x_1, x_2, ..., x_n) E(A_{x_1} A_{x_2} \cdots A_{x_n} | \Lambda_0)\right| \leq 2^{n-1}, \tag{15}$$

where Λ_0 represent the ensemble where all measurements successfully give results. The violation of Eq. (15) is a loophole-free confirmation of existence of genuine n-particle correlations.

Similar to proposition 1 and proposition 2, we can obtain the following two propositions for the case of n-particle.

proposition 3. If we define

$$\delta_n = \min_{\{x_i\}} P(\Lambda_0 | \Lambda_{A_{x_1} A_{x_2} \cdots A_{x_n}}), \tag{16}$$

where $\min_{\{x_i\}}$ take over all measurements settings $(x_1, x_2, ..., x_n)$. From Eq. (15) and Eq. (16) we can obtain the inequality

$$\left|\sum_{\{x_i\}} v(x_1, x_2, ..., x_n) E(A_{x_1} A_{x_2} \cdots A_{x_n} | \Lambda_{A_{x_1} A_{x_2} \cdots A_{x_n}})\right| \\ \leq 2^{n-1} \times (2 - \delta_n), \tag{17}$$

Proof. Similar to the proof of proposition 1.

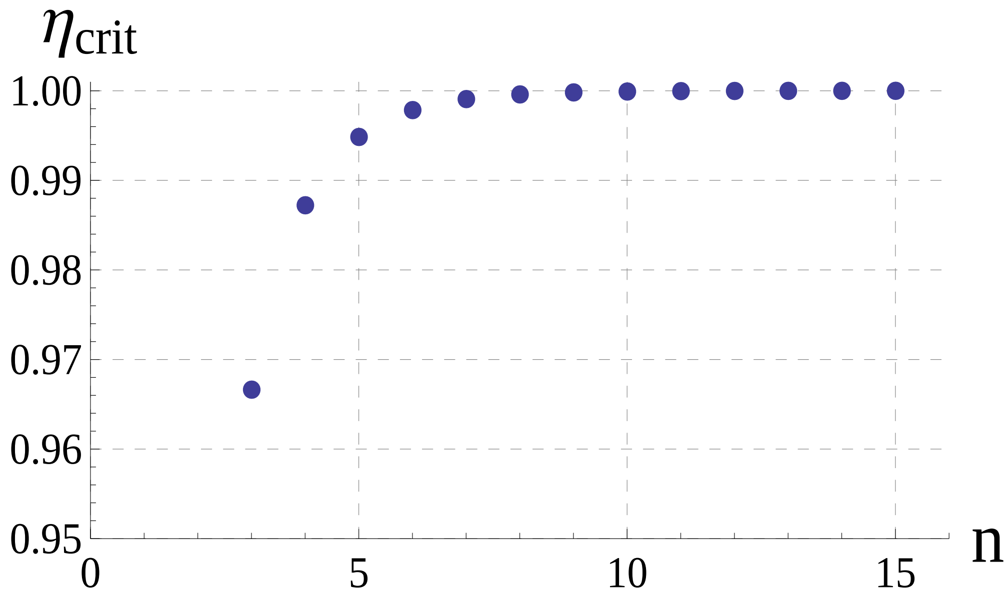

FIG. 1: The threshold efficiency η_{crit} for the cases of n ranging from 3 to 15. The value of η_{crit} monotonously and rapidly approaches to 1 as the value of n increases.

proposition 4. $\delta_n \geq 1 + n \times \left(2 - \frac{1}{\eta}\right) + \sum_{k=2}^{n} \binom{n}{k} \times \left[k \times \left(2 - \frac{1}{\eta}\right) - (k-1)\right] - (2^n - 1)$.

Proof. Similar to the proof of proposition 2.

$$\begin{aligned}
& P(\Lambda_0 | \Lambda_{A_{x_1} A_{x_2} \cdots A_{x_n}}) \\
= \; & P(\cap_{\{x_i'\}} \Lambda_{A_{x_1'} A_{x_2'} \cdots A_{x_n'}} | \Lambda_{A_{x_1} A_{x_2} \cdots A_{x_n}}) \\
\geq \; & \sum_{\{x_i'\}} P(\Lambda_{A_{x_1'} A_{x_2'} \cdots A_{x_n'}} | \Lambda_{A_{x_1} A_{x_2} \cdots A_{x_n}}) - (2^n - 1) \\
\geq \; & 1 + n \times \left(2 - \frac{1}{\eta}\right) \\
& + \sum_{k=2}^{n} \binom{n}{k} \times \left[k \times \left(2 - \frac{1}{\eta}\right) - (k-1)\right] \\
& -(2^n - 1).
\end{aligned} \tag{18}$$

From Eq. (16) and Eq. (18), we prove the proposition.□

The maximum value of the left hand side of Eq. (17) is $2^{n-1}\sqrt{2}$ allowed in quantum mechanics [34], combining it with proposition 4 we can obtain the threshold efficiency η_{crit} for the loophole-free confirmation of existence of genuine n-particle correlations for any n. We depict η_{crit} for the cases of n ranging from 3 to 15 in Fig.(1). We can find that as the value of n increases the value of η_{crit} monotonously and rapidly approaches to 1.

IV. CONCLUSION

The imperfection of detector's efficiency possibly cause the so-called detection loophole in actual Svetlichny's experiments. We derive a new SI to deal with the detection loophole in actual Svetlichny's experiments. If the experimental data can violate the new SI, it must result in the loophole-free violation of the original SI. We give the threshold detection efficiency which is required for a loophole-free violation of SI for the general case of n-particle. There is a remarkable contrast between our result of SI and the case of Mermin inequalities, where the threshold detection efficiency is 0.75 for three parties case and decreases to 0.5 as the number of parties tends to infinity [23]. The reason, we think, is that quantum mechanics allows a violation of Mermin inequalities that grows exponentially as the number of parties increases, while the increase of the number of parties does not contribute to magnify the violation of SI.

Acknowledgments

Y. X. is supported by National Nature Science Foundation of China under Grant No. 11005031. F.-Y. H. is supported by Zhejiang Provincial Natural Science Foundation of China under Grant No. Y6110314.

---

[1] J. S. Bell, Physics (Long Island City, N.Y.) **1**, 195 (1964).

[2] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. **23**, 880 (1969).

[3] A. Aspect *et al.*, Phys. Rev. Lett. **49**, 1804 (1982).

[4] W. Tittel *et al.*, Phys. Rev. Lett. **81**, 3563 (1998).

[5] G. Weihs *et al.*, Phys. Rev. Lett. **81**, 5039 (1998).

[6] M. A. Rowe *et al.*, Nature **409**, 791 (2001).

[7] D. N. Matsukevich *et al.*, Phys. Rev. Lett. **100**, 150404 (2008).

[8] J. Barrett and N. Gisin, Phys. Rev. Lett. **106**, 100406 (2011).

[9] Michael J.W. Hall, Phys.Rev.Lett. **105**, 250404 (2010).

[10] Michael J.W. Hall, Phys. Rev. A **84**, 022102 (2011).

[11] P. M. Pearle, Phys. Rev. D **2**, 1418 (1970).

[12] A. Garg and N. D. Mermin, Phys. Rev. D **35**, 3831 (1987).

[13] P. H. Eberhard, Phys. Rev. A **47**, R747 (1993).

[14] J.-Å. Larsson, Phys. Rev. A **57**, R3145 (1998).

[15] J.-Å. Larsson, Phys. Rev. A **59**, 4801 (1999).

[16] J.-Å. Larsson, Phys. Rev. A **57**, 3304 (1998).

[17] J.-Å. Larsson and J. Semitecolos, Phys. Rev. A **63**, 022117 (2001).

[18] A. Cabello and J.-Å. Larsson, Phys. Rev. Lett. **98**, 220402 (2007).

[19] N. Brunner, N. Gisin, V. Scarani, and C. Simon, Phys. Rev. Lett. **98**, 220403 (2007).

[20] S. Massar, Phys. Rev. A **65**, 032121 (2002).

[21] S. Pironio, Phys. Rev. A **68**, 062102 (2003).

[22] S. Massar and S. Pironio, Phys. Rev. A **68**, 062109 (2003).

[23] A. Cabello, D. Rodríguez, and I. Villanueva, Phys. Rev. Lett. **101**, 120402 (2008).

[24] A. Cabello, J.-Å. Larsson, and D. Rodríguez, Phys. Rev. A **79**, 062109 (2009).

[25] C. Branciard, Phys. Rev. A **83**, 032123 (2011).

[26] A. Garuccio, Phys. Rev. A **52**, 2535 (1995).

[27] G. Garbarino, Phys. Rev. A **81**, 032106 (2010).

[28] M. Genovese, Phys. Rep. **413** 319 (2005).

[29] A. Cabello and F. Sciarrino, Phys. Rev. X **2**, 021010 (2012).

[30] G. Cañas, J. F. Barra, E. S. Gómez, G. Lima, F. Sciarrino, and A. Cabello, arXiv:1206.2290v1.

[31] C. Simon and W. T. M. Irvine, Phys. Rev. Lett. **91**, 110405 (2003).

[32] R. Garcia-Patron *et al.*, Phys. Rev. Lett. **93**, 130409 (2004).

[33] G. Svetlichny, Phys. Rev. D **35**, 3066 (1987).

[34] M. Seevinck and G. Svetlichny, Phys. Rev. Lett. **89**, 060401 (2002).