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Coherent propagation and energy transfer in low dimension nonlinear arrays

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We present a theory of coherent propagation and energy/power transfer in low dimension array of coupled nonlinear waveguides. It is demonstrated that in the array with non-equal cores (e.g. with the central core) the stable steady-state coherent multi-core propagation is possible only in the nonlinear regime - with a power controlled phase matching. The developed theory of energy/power transfer in nonlinear discrete systems is rather generic and has a range of potential applications including both high power fibre lasers and ultra-high-capacity optical communication systems.

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Nonlinear dynamics in discrete systems is an interdisciplinary research field that has links to a large number of areas of science and technology. A broad interest to studies of nonlinear discrete systems is based on their generic nature - a variety of different physical systems can be effectively described by the same mathematical model. Nonlinear discrete systems occur in a variety of phenomena in condensed matter, nonlinear optics, biology and other fields: from energy transport in molecular chains and protein molecules to light propagation in waveguide arrays (it is not possible to properly cite all important works in the field, see e.g. [1–15, 17–21] for particular examples relevant to the systems studied here). In this Letter we present a theory of coherent evolution and energy exchange in specific albeit generic low-dimension nonlinear discrete systems, using as a particular example practically important application - light propagation in multi-core fibre. We demonstrate novel features of coherent light transmission in such multi-core systems, that are different from properties previously studied in the infinite nonlinear discrete lattices [1, 6–15, 17], symmetric dimers [5] and directional couplers [2, 3, 20, 21].

Mathematical analysis of nonlinear dynamics in multi-core fibres and, in a more general mathematical formulation, the nonlinear evolution of the electromagnetic field in a small-number of interacting waveguides is directly relevant to the design of a new generation of fibre laser and telecommunication systems. Exponentially increasing demand for communication system capacity and the projected exhaustion of current infrastructure ("capacity crunch" [22]) is the driving force for introduction for spatial-division multiplexing using multi-core fibres. Multi-core fibre (MCF) technology enables the necessary scale-up in capacity per-fibre through spatial multiplexing where individual cores serve as independent channels [23]. The new important challenge here is space utilization efficiency and optimisation of capacity per unit area measured in $[bit/s/m^2]$. Interactions between the cores can be theoretically made small at the expense of space using large core separation. However, this decreases the spatial density of capacity. More efficient space utilization is achieved in the homogeneous MCF [24] (with more dense core spacing) making positive use of the the proximity of the cores to produce controlled linear core coupling. In the coherent optical communication most of the linear transmission effects can be undone at the receiver by digital signal processing. However, the coupling might be affected by nonlinear effects imposing limits on enhancing performance through increase of signal power (required to improve signal-to-noise ratio). The nonlinearity affects energy coupling between the cores that can result in information losses. It is important, therefore, to determine the fundamental threshold for the destructive energy transfer effects.

Similar mathematical problems arise in the field of powerful fibre lasers [25, 26]. The single mode fibre can transport only the power below certain threshold value determined by the nonlinear effects. The use of multi-core fibers is a promising way for the coherent combining to create high brightness sources. However, nonlinear interaction can destroy the mutual coherence. It is important, therefore, to know the limits imposed by the nonlinear interaction on maximum power transmitted through the MCF without loss of the final beam quality.

In this Letter we demonstrate that in arrays with non-equal cores (the most simple albeit general case is N-1 peripheral cores surrounding the central core; here N is not very large due to the geometrical and manufacturing restrictions) the phase matching and stable coherent propagation is possible only due to nonlinear effects for a certain power split between cores. We solve the stability problem of steady-state propagation and derive analytical conditions of the linear instability and energy transfer. This instability is an extreme discrete limit of the classical modulation instability in the continuous media and fibers arrays [12, 17, 27–29].

The basic model considered here is a low dimension version of the discrete nonlinear Schrödinger equation:

$$i\frac{\partial A_k}{\partial z} + \sum_{m=0}^{N} C_{km} A_m + 2\gamma_k |A_k|^2 A_k = 0;\ k = 0, ..., N \quad (1)$$

Here A_k is a field in the k-th core, with A_0 (when ap-

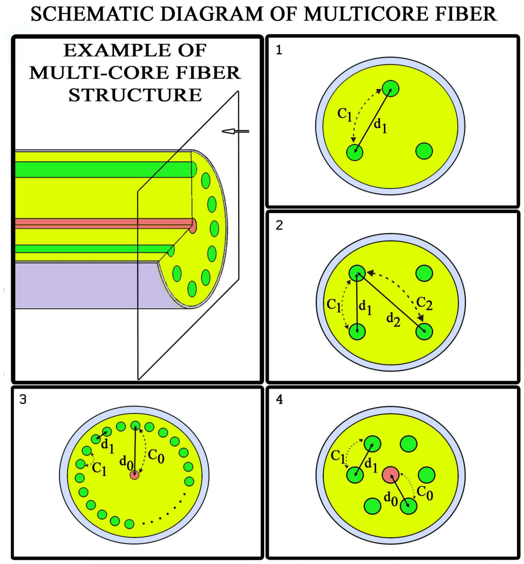

FIG. 1: The schematic depiction of the multi-core fibre.

plied) corresponding to the central core, $C_{km} = C_{mk}$ is the coupling coefficient between modes m and k; $C_{kk} = \beta_k$ are wave numbers in different cores that are not assumed to be the same. The phase matching and stable mutually coherent continuous wave (CW) propagation in arrays with non-equal cores (e.g. cases 3 and 4 in Fig. 1) is provided by the certain nonlinear phase shifts that we will determine below. The Eq. (1) governs all the designs shown in Fig.1:

$$(1): \; C_{mk} = C_1; \; (2): \; C_{k,k+1} = C_1, C_{k,k+2} = C_2;$$

$$(3,4): \; C_{k,k\pm1} = C_1 (k \neq 0), C_{k,0} = C_0;$$

Note that in general, e.g. for systems with the distinctive central core, nonlinear coefficients in different cores might be different. Consider first instability in the cases (1, 2) in Fig.1. Let $A_k = (\sqrt{P_k} + a_k + i b_k) e^{iqz}$, $a_k, b_k \ll \sqrt{P_k}$ where $P_k = P_0$. Cumbersome, but direct calculations of the dispersion relation for q show that for the case with three cores the instability occurs when $P_0 > P_{th}^{(3)} = 3C_1/(4\gamma)$. In the case of four cores (2) the instability threshold is: $P_0 > P_{th}^{(4)} = (C_1 + C_2)/(2\gamma)$. When propagation constants are different or in the case of multiple peripheral cores surrounding a central one even the existence of steady state solution is nontrivial and we look at it in more detail. In the main order, dynamics in systems with similar peripheral cores can be reduced (assuming $A_k = A_1, k = 1, ..., N$) to analysis of an effective two-core model that is a symmetric limit of multi-core systems:

$$i \frac{\partial U_0}{\partial z} = -U_1 - \frac{2N\gamma_0}{\gamma_1} |U_0|^2 U_0 = \frac{\partial H}{\partial U_0^*}, \tag{2}$$

$$i \frac{\partial U_1}{\partial z} = -\kappa U_1 - U_0 - 2|U_1|^2 U_1 = \frac{\partial H}{\partial U_1^*}, \tag{3}$$

Here we introduced normalized variables:

$$A_{0,1} = \sqrt{P_{0,1}}\, U_{0,1} e^{i\beta_0 L z}; \; z' = z/L; \; L = \frac{1}{C_0 \sqrt{N}}, \tag{4}$$

$$P_0 = N P_1 = N^{3/2} C_0/\gamma_1, \kappa = \frac{(\beta_1 - \beta_0) + 2C_1}{C_0 \sqrt{N}}. \tag{5}$$

The system (2, 3) is a Hamiltonian one (as well as (1)) with the following conserved quantities: total (normalized) power P_t and the Hamiltonian H:

$$P_t = N(|U_0|^2 + |U_1|^2), \tag{6}$$

$$H = -\kappa |U_1|^2 - (U_0^* U_1 + U_1^* U_0) - |U_1|^4 - \frac{N\gamma_0}{\gamma_1} |U_0|^4. \tag{7}$$

We would like to stress that despite simple appearance, even the stationary, steady state solution of the system (2, 3) is non-trivial anymore (compared e.g. to the symmetric dimer [5]). To provide for coherent light evolution in multiple cores, difference in propagation constants has to be compensated by the nonlinear phase shifts:

$$\{U_0, U_1\} = \{A, B\} \times e^{i\lambda z}, \Gamma = \frac{B}{A}, \tag{8}$$

$$|A|^2 = \frac{P_t}{N(1+\Gamma^2)}, \; \lambda = \Gamma + \frac{2\gamma_0 P_t}{\gamma_1 (1+\Gamma^2)}. \tag{9}$$

$$\Gamma^4 - \left(\kappa + \frac{2P_t}{N}\right) \Gamma^3 - \left(\kappa - \frac{2\gamma_0 P_t}{\gamma_1}\right) \Gamma - 1 = 0. \tag{10}$$

The steady state solutions and their stability for more general situation including gain and attenuation have been considered numerically in [16]. In a dissipative system only numerical evaluation for some specific parameters is possible and the the emphasis in [16] was on formation of localized structures. Here we are interested mainly in energy/power transfer between the cores. The relatively simple mathematical result (8 - 10) leads to quite nontrivial physical consequences. Namely, steady state dynamics in such system is possible only with a certain imbalance (given by factor Γ^2) between powers propagating in different cores. The physics is rather transparent - this power split is due to nonlinear phase shift contribution to the phase matching condition required for coherent propagation in multiple cores. Surprisingly, there are several power distributions (between central and peripheral cores) that can provide for a coherent steady state propagation of light. The amount of

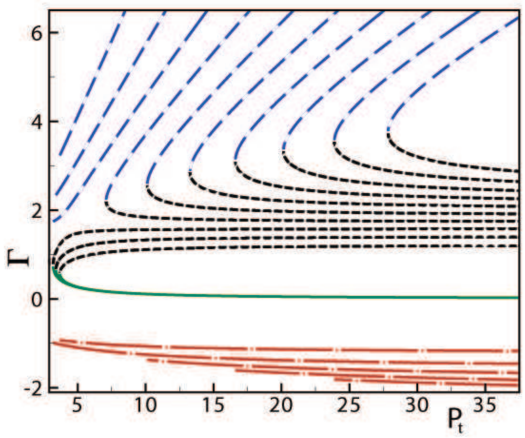

FIG. 2: Four values of Γ corresponding to different power splits between cores as functions of total input power; here $\gamma_0/\gamma_1 = 0.5$ and $\kappa = 1$. Blue long dashed, green solid and red dash-dotted branches are stable while the black short-dashed one is unstable. Here different curves for each branch correspond to N varying from 3 to 12 (from the bottom to the top). For red short dashed curve only odd N are shown.

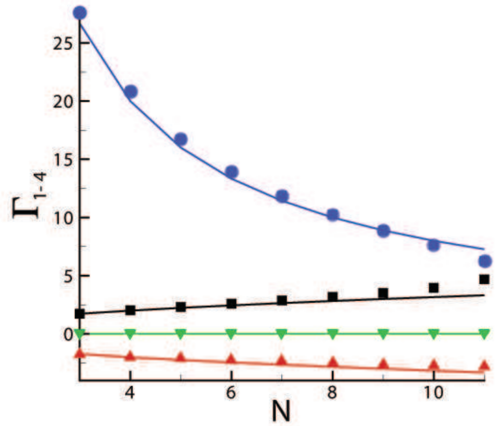

FIG. 3: Dependence of the four solutions of Eq. (10) (shown by squares) on N. Here $P_t = 40$, $\gamma_0/\gamma_1 = 1$ and $\kappa = 1$. Solid lines are for the analytical solutions valid in the limit $P_t \gg 1$. Blue circles curve: $\Gamma_1 = 2P_t/N$; black squares line: $\Gamma_2 = \sqrt{\gamma_1 N/\gamma_0}$; green triangles line: $\Gamma_3 = \gamma_1/(2\ \gamma_0\ P_t)$; and red inverse triangle line: $\Gamma_4 = -\sqrt{\gamma_1 N/\gamma_0}$.

power that has to be coupled to each core for steady state evolution given by solutions of (10) depends on four parameters: (i) N, (ii) input power P_{in} (or total power P_t), (iii) linear phase mismatch κ, and (iv) the ratio between the nonlinear coefficients γ_0/γ_1. To get idea of solution structure consider practically important case $P_t >> 1$. In this case from (10) we will get four families of solutions. In the $\Gamma_1 = 2P_t/N$ and $\Gamma_3 = \gamma_1/(2\ \gamma_0\ P_t)$ most of the energy propagates in the ring or central core, correspondingly. For $\Gamma_{2,4} = \pm\sqrt{\gamma_1 N/\gamma_0}$ the ratio of energy in ring and central core is independent of propagating power. Negative Γ means out-of phase fields in the central and peripheral cores. Figure 3 shows an excellent applicability of analytical results.

Consider now stability of steady state solutions of (8-10) - analogue of the modulation instability for low dimension discrete system. The small amplitude disturbance is taken in a standard form $\{U_0, U_1\} = \{A + a + ib, B + c + id\} \times e^{i\lambda z}$, for perturbations proportional to $\exp[pz]$ the growth rate of instability is:

$$p^2 + 2 = -\frac{1}{\Gamma}\left(\frac{1}{\Gamma} - 4B^2\right) - \Gamma\left(\Gamma - \frac{4N\gamma_0 A^2}{\gamma_1}\right). \quad (11)$$

In the limit $P_t >> 1$ only mode Γ_2 is instable. Instability results in periodic oscillations of energy between cores with amplitude of modulations depending on total power, i.e. the relative modulation depth decreases with growing input power. The most important consequences of the instability is that it makes control of power dynamics hardly possible. For system with more than three cores the instability, in general, produces stochastic modulation breaking the mutual coherence in the cores. The energy exchange oscillations can be produced not only as a result of the instability, but also as a result of initial conditions (in case of arbitrary input powers).

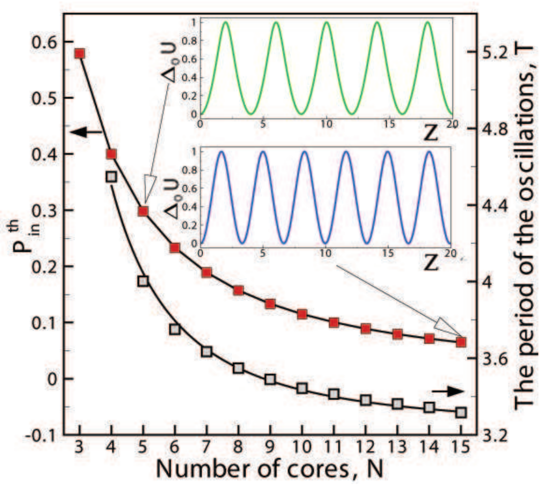

FIG. 4: Y-axis (left): The comparison of numerically calculated threshold for energy/power transfer (red markers) and the analytical formula (12) (solid line). Y-axis (right): numerically calculated period of the power oscillations (gray markers) and analytical approximation: $3.23{+}2.04/N^2$ (solid line). Insets: energy/power transfer with distance. The complete transfer occurs only at certain distances.

The Hamiltonian structure of the equations and the additional conserved quantity greatly restricts dynamics in the considered low dimension dynamic system imposing constraints on the evolution of the waves and the energy exchange between cores. For instance, considering evolution of initial powers equally distributed between all cores $|U_0|^2 = P_{in}/N, |U_1|^2 = P_{in}$, using the connections between the fields imposed by $dH/dz = 0$ it is easy to show that the complete energy transfer from the outer cores to the central one is possible only for one specific value of input power (and at specific propagation length):

$$P_{in} = P_{in}^{th} = \frac{\kappa + 2N^{-1/2}}{\gamma_0(N+2)/\gamma_1 - 1}. \qquad (12)$$

The observed effect - localization of all initially evenly distributed power into the central core can be considered as an ultimate discrete version of the self-focusing of light.

Figure 4 shows comparison of the analytical result (12) and numerically calculated threshold of an energy transfer given by $\Delta_0 U = (N|U_0|^2 - |U_1|^2)/P_t$ ($P_t = (N+1)P_{in}$). Here $\gamma_0 = \gamma_1; C_0 = C_1, \beta_0 = \beta_1$. The period of the energy exchanges decays with N as N^{-2}.

Note that the presented theory can be easily generalized to pulse propagation and nonlinear temporal dynamics having numerous applications. In the recent important work [30] it has been studied the efficiency of nonlinear matching of optical fibers through a fundamental soliton coupling from one fiber into another opening a range of engineering applications, e.g. optimized Raman red-shift and supercontinuum generation.

To conclude, in this Letter we have presented a theory of energy/power transfer in low dimension arrays of coupled nonlinear waveguides. The developed theory is rather generic and has a range of potential applications. Without loss of generality, particular emphasis in the analysis is made on multi-core fibre technology, important in the fields of both high power fibre lasers and ultra-high-capacity optical communication systems. We have derived for the array with non-equal cores the nonlinear phase matching conditions that provide for stable coherent steady-state propagation in multiple cores. We solved the stability problem and found an exact analytical condition of complete energy transfer from peripheral to the central core - ultimate discrete analogy of the self-focusing effect.

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