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Nonlocal solitons in dual-periodic PT symmetric optical lattices

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We numerically study the nonlocal solitons in dual-periodic parity-time (PT) symmetric optical lattices built into a nonlocal self-focusing medium. We state the existence, stability, and propagation dynamics of such PT gap solitons in detail. Simulated results show that there exist stable gap solitons. The energy flow density and the stable region of the PT gap solitons in both the propagation constant and the degree of nonlocality are also examined.

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Self-action of light in periodic photonic crystals with periodic modulation of the refractive index generates rich optical phenomena [1]. The photonic crystals can provide efficient control of the transmission and localization of light, and they open the ways to tailor the diffraction and the route of the electromagnetic waves [2]. Quite a lot of nonlinear optical phenomena in nonlinear photonic crystals have been reported, including the localized self-trapped modes in the form of gap solitons [3] which was also studied in other materials, layered microstructures, fiber Bragg gratings, Bose-Einstein condensates, waveguide arrays, and optically induced lattices. Periodic domain structure has wide applications not only for second-harmonic generation, sum-frequency generation, difference-frequency generation, and optical parametric oscillators but also for new fields such as generation of squeezed light for optical communication and information processing and optical solitons [4]. A novel structure introduced by by Liu et al., a dual-periodic structure, in which two optical parametric interactions are coupled into a single superlattice crystal [5]. Compared with other structures, this one provides clearer physical sight. In addition, they designed and fabricated such a dual-periodic domain-reversal structure in a $LiTaO_3$ crystal, and the experimental result was presented at the same time. Nonlocal effects come to play an important role as the characteristic correlation radius of the medium's response function becomes comparable to the transverse width of the wave packet [6]. Nonlocal nonlinear response may drastically modify the conditions necessary for the existence of gap solitons [1].

Recently, the solitons in synthetic optical media with parity-time (PT) symmetries have caught much attention [7–13, 15–17, 20–29]. Musslimani and the cooperator firstly discovered that a novel class of nonlinear self-trapped modes exist in optical PT synthetic lattices [7] and PT periodic structures exhibit unique characteristics stemming from the nonorthogonality of the associated Floquet-Bloch modes [8]. The behavior of a PT optical coupled system judiciously involving a complex index potential was observed in the experiment in 2010 [9, 10]. It was also stated the analytical solutions to a class of nonlinear Schrödinger equations with PT-like potentials [11], the stable dissipative defect modes in both focusing and defocusing media where periodic optical lattices were imprinted in the cubic nonlinear media with strong two-photon absorption [12], and the defect solitons in parity-time periodic potentials [13, 14]. We also reported the gray solitons in PT symmetric potentials [15] and the gap solitons in PT complex periodic optical lattices with the real part of superlattices [16]. However, thus far all studies focus on the local nonlinear media with the PT symmetry potentials, and the solitons supported by the nonlocal nonlinear media with the PT symmetry optical lattices are hardly reported.

In this paper, the gap solitons in the dual-periodic PT symmetric optical lattices built into a nonlocal self-focusing medium are studied. We state the existence, stability, and propagation dynamics of such PT gap solitons in detail. Simulated results show that there exist stable gap solitons. In addition, we find that the degree of nonlocality can influence the soliton power, and the region where the stable PT gap solitons can exist. The results may make us find a new class of PT-synthetic materials with unexpected properties. Based on the dual-periodic PT symmetric optical lattices, we can also find the applications of the the multi-cycle structure on the PT spatial optical solitons. In Ref. [14], Hu et al. also introduced a PT symmetric optical lattices with nonlocal nonlinearity. However, the super-lattice is designed to study the influences of the introduction of a perturbation in PT symmetric periodic potential on spatial solitons in their paper. Importantly, the physical mechanisms of the formation of the gap solitons are different. In our paper, the formation of the gap solitons is determined by the interaction of the penetration of wave energy between the adjacent waveguides and waveguides nonlinearity [30]. When discrete diffraction effect and self-focusing effect cancel out each other, the gap solitons can be generated. While the physical mechanism of defect solitons in the paper of Hu is the interaction of the attractive and repulsive forces caused by the discrete

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diffraction effects and lattice defects and self-focusing effect. In addition, the influences of the perturbation of the PT periodic symmetric potential and the depth of defect on the existence and stability of the PT spatial optical solitons were studied in detail. They did not focus on the influence of degrees of nonlocality on the PT solitons in Hu's article. So, the key conclusions between two articles are different.

In optics, spatial diffraction and temporal dispersion are known to obey equations that are formally equivalent to that of Schrödinger in quantum mechanics. Here, we pay our attention to the optical beam propagation in PT-symmetric complex potentials. In fact, such PT optical potentials can be realized through a judicious inclusion of index guiding and gain/loss regions [7, 8]. If the complex refractive index distribution $n(x) = n_0 + n_R(x) + in_i(x)$ where n_0 represents the background refractive index, $n_R(x) + in_i(x)$ represent the PT symmetric optical lattices and satisfy the conditions $n_R(x) = n_R(-x)$ and $n_i(x) = -n_i(-x)$, and x is the normalized transverse coordinate [7, 8]. Under these conditions, in a nonlocal self-focusing medium with PT symmetric optical lattice, the one-dimensional optical wave propagation can be described by the normalized nonlinear Schrödinger (NLS)-like equation

$$i\frac{\partial q}{\partial z} + \frac{\partial^2 q}{\partial x^2} + Rq + q\int_{-\infty}^{+\infty} g(x-\lambda)|q(\lambda)|^2 d\lambda = 0, \quad (1)$$

where q is the complex dimensionless light field amplitude, z is the normalized longitudinal coordinate, and g is the nonlocal response function. $R = V(x) + iW(x)$ is the PT symmetric optical lattice, and $V(x)$ and $W(x)$ are its real and imaginary components, respectively. We further search for a stationary soliton solution of Eq. (1) in the form of $q(x,z) = u(x)e^{ibz}$, where u is a complex function and b is the propagation constant of spatial solitons [18]. Thus, Eq. (1) can be changed into

$$\frac{\partial^2 u}{\partial x^2} + Ru - bu + u\int_{-\infty}^{+\infty} g(x-\lambda)|u(\lambda)|^2 d\lambda = 0. \quad (2)$$

Here, the nonlocality of the materials is supposed to be ruled with an exponential response function $g(x) = 1/(2d^{1/2})\exp(-|x|/d^{1/2})$ (as in liquid crystals), where d is the degree of the nonlocality. We assume a PT symmetric optical lattices in which $V(x) = V_0\{\cos[2\pi\cos(x)] + 1\}/2$ and $W(x) = W_0 \sin(2x)$, and V_0 and W_0 are the amplitudes of the real and imaginary parts. Here, $V(x)$ is a non-uniform distribution dual-periodic optical lattice which matches the conditions of PT potential. It can influence the band structure so that PT solitons have different features from the PT solitons in the uniform lattices. Although the PT symmetric optical lattice has crossed the phase transition point, the solitons still exist because the amplitude of the refractive index distribution can be altered by the beam itself through the optical nonlocal nonlinearity. The parity-time symmetry will remain broken if it cannot be nonlocal nonlinearly restored [7].

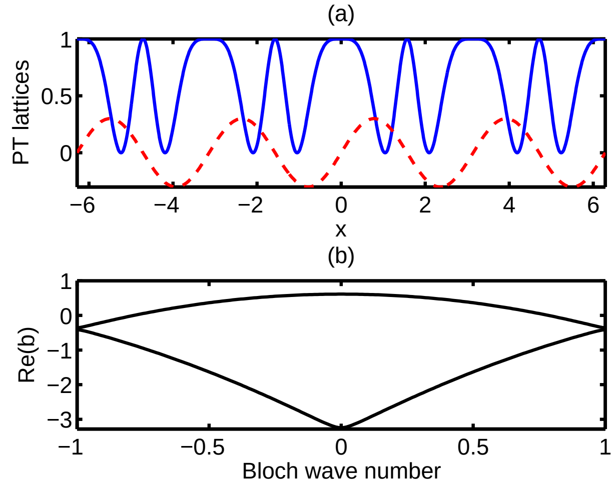

FIG. 1. (Color online) (a) PT complex periodic optical lattices when $V_0 = 1$, $W_0 = 0.30$. The solid blue and dashed red curves represent the real part and the imaginary part of the optical lattices, respectively. (b) The band structures corresponding to (a). All quantities are plotted in arbitrary dimensionless units.

The linear version of Eq.(2) is

$$\frac{\partial^2 u}{\partial x^2} + Ru - bu = 0, \quad (3)$$

where b now represents the propagation constant in the PT symmetric optical lattices. The Bloch theorem tells us that the eigenfunctions of Eq.(3) are in the form of $u = F_k exp(ikx)$, where k is the Bloch wave number, and F_k is a periodic function of x with the same period as the lattices R. Substituting the Bloch solution into Eq.(3), we can get the eigenvalue equation and then obtain the band structure using the plane wave expansion method [7, 15]. The PT symmetric optical lattice is shown in Fig. 1 (a). We find that the purely real bands are possible in the range $0 \leq W_0 \leq 0.30$, and the region of the semi-infinite gap is $b \geq 0.62$ when $W_0 = 0.30$ in Fig. 1(b) [7].

Based on the band-gap structure, we get the soliton solutions by solving Eq.(2) numerically using the modified squared-operator method [19]. We find a family of localized solutions with real eigenvalues located within the semi-infinite gap ($b \geq 0.62$). The typical cases of these gap solitons are shown in Fig. 2, where it is depicted in (a) the field of the stable gap soliton at $d = 0.5$ and $b = 2$. To shed more light on the properties of the stable solitons, we study the parameter $S = (i/2)(uu_x^* - u^*u_x)$ associated with the transverse power flow density or Poynting vector across the beam [7]. The transverse power flow density S is shown in Fig. 2(c), where one can find that S is not positive everywhere and across the lattice [7].

To check the stability of the solitons with the method of linear stability analysis, we assume $q(x,z) = u(x)e^{ibz} + \epsilon[F(x)e^{i\delta z} + G^*(x)e^{i\delta^* z}]e^{ibz}$, where $\epsilon \ll 1$, F and G are

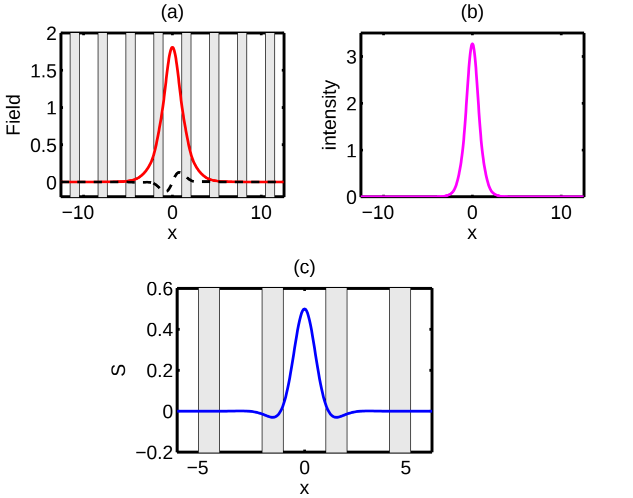

FIG. 2. (Color online) (a) The Field and (b) intensity of the soliton with $b = 2$ at $d = 0.5$. The solid red curves and the dashed black curves represent the real part and the imaginary part, respectively. (c) Transverse power flow of the solitons shown in (a). The potential parameters are $V_0 = 1$ and $W_0 = 0.3$. All quantities are plotted in arbitrary dimensionless units.

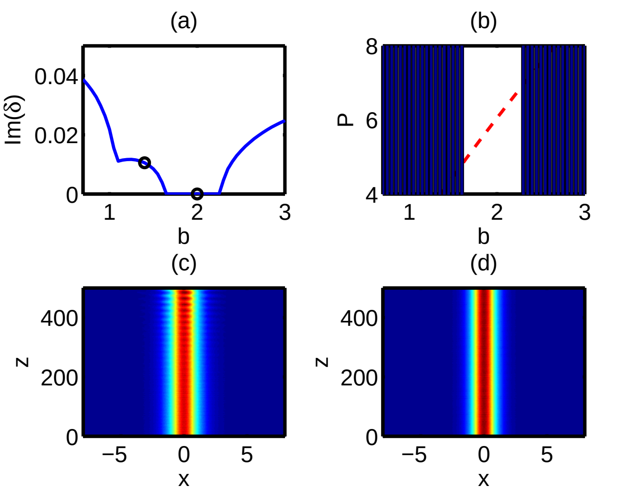

FIG. 3. (Color online) (a) Im(δ) versus b. Points marked with circle correspond to the cases shown in (c) and (d). (b) P versus b. Simulated propagation of the solitons with 5% random noise for (c) $b = 1.40$ and (d) $b = 2$, respectively. The other parameters are $V_0 = 1$, $W_0 = 0.3$ and $d = 0.5$. All quantities are plotted in arbitrary dimensionless units.

the perturbation eigenfunctions, and δ is the growth rate of the perturbation [7]. By linearizing Eq.(1), we gain

$$\delta F = \frac{\partial^2 F}{\partial x^2} + (V + iW)F + nF - bF + u\Delta n, \quad (4)$$

$$\delta G = -\frac{\partial^2 G}{\partial x^2} + (-V + iW)G - nG + bG - u^* \Delta n, \quad (5)$$

where $n = \int_{-\infty}^{+\infty} g(x-\lambda)|u(\lambda)|^2 d\lambda$, and $\Delta n = \int_{-\infty}^{+\infty} g(x-\lambda)[G(\lambda)u(\lambda) + F(\lambda)u^*(\lambda)]d\lambda$. The gap solitons are linearly unstable when δ has an imaginary component, on the contrary, they are stable when δ is real. In Fig. 3 (a), there is a stability window in b, where Im(δ), the imaginary component of δ, is zero. The power of solitons is defined as $P = \int_{-\infty}^{+\infty} |u|^2 dx$, and P is a monotonically-increasing function of the propagation constant b, as is shown in Fig. 3(b). To examine further the robustness of these PT lattice self-strapped modes, we also simulate the propagation of beams under different conditions, as presented in Fig. 3 (c) and (d), which support the above conclusion about the stability of nonlocal gap solitons as well.

We next investigate the nonlocality effects on solitons properties with $V_0 = 1$, $W_0 = 0.3$ and $b = 5$. The two cases of these gap solitons are shown in Fig. 4, where it is depicted in (a) and (b) the field of the stable gap soliton at $d = 2$ and $d = 5$. Evidently, the real part amplitudes of the solitons are influenced by the degree of the nonlocality d and the imaginary part amplitudes of the solitons are not. The intensity of the solitons also are weakly affected by d because the width of the convolution results is proportional to d when the width of the solitons intensity is a constant, see Fig. 4 (c).

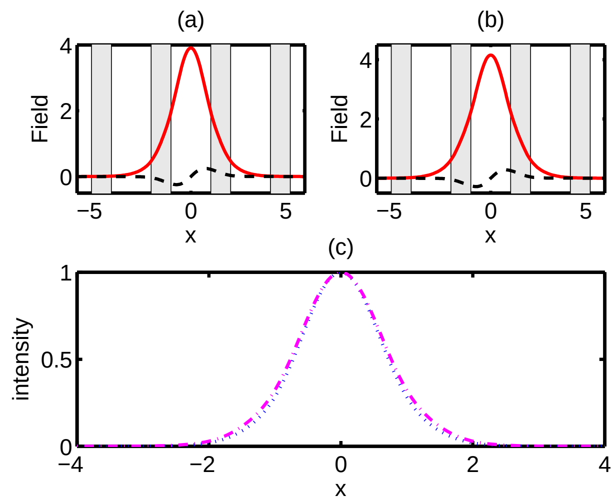

FIG. 4. (Color online) (a) and (b) The Fields of the solitons with $d = 2$ and $d = 5$, respectively. (c) The intensity of the solitons. The dotted blue curve and the dot-dashed magenta curves represent (a) and (b), respectively. The other parameters are $V_0 = 1$, $W_0 = 0.3$ and $b = 5$. All quantities are plotted in arbitrary dimensionless units.

In Fig. 5 (a), there is a stability window in d, where Im(δ), the imaginary component of δ, is zero. So, there exists a stable region of the degree of the nonlocality d for the gap solitons. As depicted in Fig. 3 (a), the propagation constant b of soiltons exists in a region when the solitons are stable in optical lattices, and large and small b leads to the instability based on the band-gap structures. Because the propagation constant b has relations with the nonlinear effect affected by the degree of the nonlocality d, the degree of the nonlocality d should also

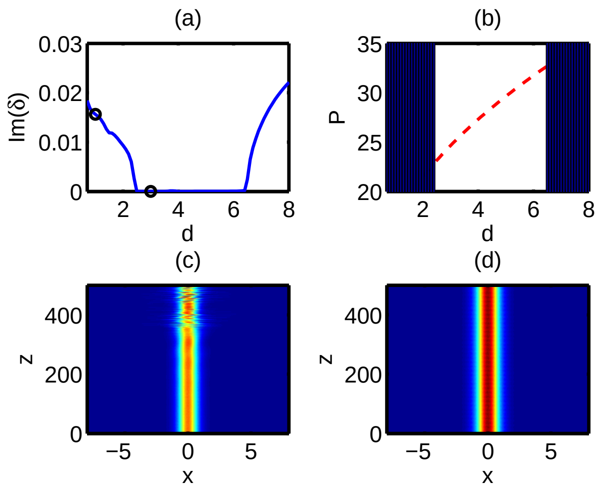

FIG. 5. (Color online) (a) $\mathrm{Im}(\delta)$ versus d. (b) P versus d. Points marked with circle correspond to the cases shown in (c) and (d). Simulated propagation of the solitons with 5% random noise for (c) $d = 1$ and (d) $d = 3$, respectively. The other parameters are $V_0 = 1$, $W_0 = 0.3$ and $b = 5$. All quantities are plotted in arbitrary dimensionless units.

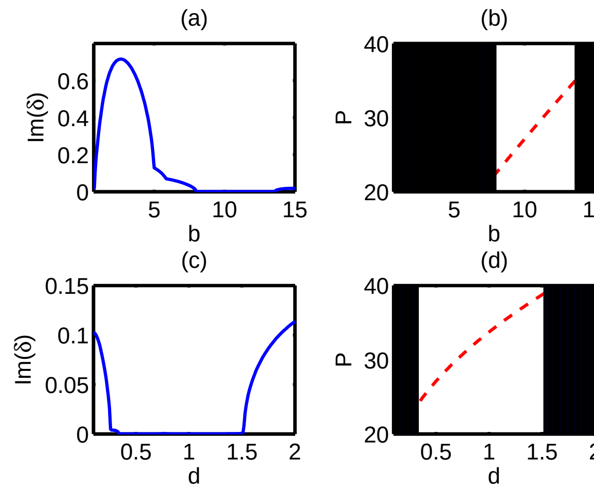

FIG. 6. (Color online) (a) $\mathrm{Im}(\delta)$ versus b at $d = 0.5$. (b) P versus b at $d = 0.5$. (c) $\mathrm{Im}(\delta)$ versus d at $b = 10$. (d) P versus d at $b = 10$. The other parameters are $V_0 = 1$, $W_0 = 0.3$. All quantities are plotted in arbitrary dimensionless units.

exist in a region, and large and small d leads to the instability of the solitons. To examine further the robustness of these PT lattice self-strapped modes, we simulate the propagation of beams under different conditions, as presented in Fig. 5(c) and (d) with $d = 1$ and $d = 3$, which support the above conclusion about the stability of nonlocal gap soiltons as well.

Finally, we exchange the position of the dual-periodic parity-time symmetric optical lattices with $V_0 = 1$, $W_0 = 0.3$, $V(x) = V_0\{\cos[2\pi \sin(x)] + 1\}/2$ and $W(x) = W_0 \sin(2x)$. The band-gap structure is as the same as above. The simulated results show that the region and numerical value of b for the stable solitons is bigger at $d = 0.5$ when the period of the middle waveguide is smaller in Fig. 6(a). The curve of power versus b is a monotonically-increasing function from Fig. 6(b). Here, the reason is that the solitons can be formed by stronger nonlinear effect, so b corresponding to the nonlinear effect shows the phenomena. Moreover, with the results of b of contrast, the region and numerical value of d for the stable solitons is smaller at $d = 0.5$, see Fig. 6(c) and (d).

In conclusion, we investigate the gap solitons in the dual-periodic PT symmetric optical lattices built into a nonlocal self-focusing medium. The existence, stability, and propagation dynamics of such PT gap solitons are stated in detail. Simulated results show that there exist stable gap solitons, and the degree of nonlocality can influence the soliton power and the region where stable PT gap solitons can exist.

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