Polarization property of the THz wave generated from a two-color laser-induced gas plasma

Dong Wang and Jinsong Liu

Phys. Rev. A **86**, 023833 — Published 20 August 2012

DOI: 10.1103/PhysRevA.86.023833

Polarization Property of THz Wave Generated from Two-color Laser-induced Gas Plasma

Dong Wang, Jinsong Liu*

Wuhan National Laboratory for Optoelectronics, School of Optoelectronic Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, China

**Corresponding author: jsliu4508@vip.sina.com*

Abstract: The recent experiments show that the polarization of THz wave generated from two-color laser-induced gas plasma is linear polarization, and the polarization angle of the THz wave can be controlled by the optical delay of the two laser beams. In this paper, the polarization property of THz wave is studied theoretically via a classical and a quantum model, respectively. An analytic expression is obtained to reflect the relationship between the polarization angle of the THz wave and the relative phase of the two color lasers.

1. Introduction

The most efficient process for terahertz (THz) generation using laser pulses is based on optical rectification and difference-frequency generation in second-order nonlinear media [1, 2]. However intensive THz radiation could be obtained from gas plasma channel induced by two-color femtosecond optical pulses. One pulse is the fundamental light (ω_1), and the other is the second harmonic light (ω_2) [3-9]. It is well known, that the process of the generation of the THz radiation in gases was described phenomenologically by a four-wave mixing process [6, 10] but the complete explanation of the process and the underling mechanism are described by the nonperturbative asymmetric field ionization models [11-14]. When the two light pluses are both circular polarization or one is circular polarization and the other is linear polarization, linear polarized THz wave is got and the azimuthal orientation of the THz polarization can be controlled by changing the optical delay of the two pulses ($\Delta\tau$), which bring a method for fast THz wave modulation and coherent control of nonlinear responses excited by intensive THz wave [15, 16]. These works about the controllable THz polarization are very successful and very interesting. They separately simulate the models that tunnel ionized electron wave packet dynamics [15] and one-dimensional transient current [16] to explain these experiment results. However, there is not an analytic expression to explain the relationship between the polarization angle of the THz wave and the optical delay. In this paper, based on the four-wave mixing process, by analyzing the function relation of θ_{THz} on $\Delta\varphi$, where θ_{THz} is the polarization angle of the THz wave and $\Delta\varphi$ is the relative phase, an analytic expression of θ_{THz} changing with $\Delta\varphi$ is obtained. The results demonstrate that θ_{THz} is equal to $-\Delta\varphi$ and $\Delta\varphi$ is in inverse proportion to $\Delta\tau$ when the pulses are both right-handed circular polarization. The cases for the two lasers being left circular or linear polarization are also discussed. Our results build a theoretical method for analysis of the polarization property of THz wave generated from two-color laser-induced gas plasma.

2. Problem formulation for the two color lasers being right circular polarization

Let $\varphi_{\omega 1}$, $\varphi_{\omega 2}$ and φ_{THz} are the phases of the ω_1, ω_2 light pulses and the THz wave, respectively, which can be expressed as $\varphi_{\omega 1} = \omega_1 t + \varphi_1$, $\varphi_{\omega 2} = \omega_2 t + \varphi_2$ and $\varphi_{THz} = \omega_{THz} t$, where

φ_1 and φ_2 are the initial phases. Because two ω_1 photons interact with one ω_2 photon in the plasma to generate one THz photon, i.e., $\hbar\omega_{THz} = 2\hbar\omega_1 - \hbar\omega_2$, we have the following relation

$$\Delta\varphi = 2\varphi_{\omega 1} - \varphi_{\omega 2} - \varphi_{THz} = 2\varphi_1 - \varphi_2 \quad (1)$$

Experimental results have shown that θ_{THz} is a function of the optical delay $\Delta\tau$ [13, 14]. The above-mentioned experiment has given the relationship $\Delta\tau = \Delta l(n_{\omega 2} - n_{\omega 1})$, while the relative phase due to the step translation stage is $\Delta\varphi = 2\varphi_1 - \varphi_2 = 2\pi\Delta l(2n_{\omega 1}/\lambda_{\omega 1} - n_{\omega 2}/\lambda_{\omega 2})$ $= 2\pi\Delta l/\lambda_{\omega 2}(n_{\omega 1} - n_{\omega 2}) \propto -\Delta\tau$, here Δl is the step translation stage, $n_{\omega 1}$ and $n_{\omega 2}$ are the refractive indices of the medium at ω_1 and ω_2, $\lambda_{\omega 1}$ and $\lambda_{\omega 2}$ are the wavelengths of the ω_1 light and ω_2 light, respectively. Therefore $\Delta\tau$ is in inverse proportion to $\Delta\varphi$. That means

$$\theta_{THz} = f(\Delta\varphi) \quad (2)$$

In order to determine the form of this function, we make an imaginative experiment. As shown in Fig. 1, x, y and z axes are the space coordinates. The arrows along the circles describe the helicity state of the photon, which is right circular polarization.

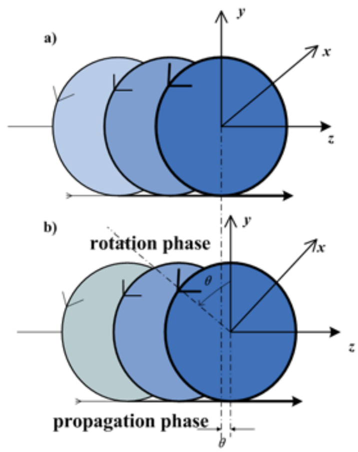

Fig. 1. (a) The photon before rotation, which is right circular polarization. (b) After rotation by θ the propagation phase is changed by the same angle.

At first, we rotate the whole experimental installation around the z axis with an angle θ, while we define the right circular rotation around the axis the positive angle. Such an operation creates a rotation phase that is equal to the propagation phase owing to the helicity state of photon. Thus the phase relationship after the rotation is

$$\begin{cases} \varphi_{\omega 1}' = \omega_1 t + \varphi_1 - \theta \\ \varphi_{\omega 2}' = \omega_2 t + \varphi_2 - \theta \\ \varphi_{THz}' = \varphi_{THz} = \omega_{THz} t \\ \Delta\varphi' = 2\varphi_{\omega 1}' - \varphi_{\omega 2}' - \varphi_{THz}' = 2\varphi_1 - \varphi_2 - \theta = \Delta\varphi - \theta \end{cases} \quad (3)$$

Note that the phase of the THz wave should stay the same because of the linear polarization. However, the azimuthal polarization orientation angle of the THz wave after the rotation is

$$\theta_{THz}' = f(\Delta\varphi') = f(\Delta\varphi - \theta) \quad (4)$$

On the other hand, in consideration of the isotropy of the space, θ_{THz}' relating to θ_{THz} would rotate by the same angle θ. Thus we have

$$\theta_{THz}{}' = \theta_{THz} + \theta \tag{5}$$

From Eqs. (2), (4) and (5), we can get

$$f(\Delta\varphi - \theta) = f(\Delta\varphi) + \theta \tag{6}$$

The derivation of the function $f(x)$ is

$$f^{(1)} = \lim_{\theta \to 0} \frac{f(\Delta\varphi - \theta) - f(\Delta\varphi)}{-\theta} = -1 \tag{7}$$

By integration we get the form of $f(x)$ as

$$f(x) = -x + x_0 \tag{8}$$

Assuming $\theta_{THz} = f(\Delta\varphi = 0) = 0$, we have the constant $x_0 = 0$. From Eqs. (3) and (8), we have the following relation

$$\theta_{THz} = -\Delta\varphi \tag{9}$$

This result represents that the azimuthal polarization orientation angle of the THz wave θ_{THz} is rigorous equal to the relative phase $-\Delta\varphi$.

3. Discussion for the two color lasers being left circular or linear polarization

The analysis above is suitable for the case that both light pulses are right-handed circular polarization, while for the left-handed case something is a little different. By using the method above we get

$$\theta_{THz} = \Delta\varphi = 2\varphi_1 - \varphi_2 \tag{10}$$

where both light pulses are left-handed circular polarization. This conclusion is in agreement with the previous experimental result [15]. By changed of π, θ_{THz} denotes the same polarization angle for the THz wave. Thus we have $\theta_{THz} \pm n\pi = -\Delta\varphi$ and $\theta_{THz} \pm n\pi = \Delta\varphi$ for the right- and left-handed cases, respectively, as shown in the Fig. 2, here n is integer.

The polarization of the THz wave $\hat{e}_{THz}$ can be expressed as the superposition of the two spherical unit vectors: the right circular polarization and the left circular polarization [17]. That is

$$\hat{e}_{THz} = \hat{e}_{+1} e^{-ir} \cos\left(\alpha - \frac{\pi}{4}\right) + \hat{e}_{-1} e^{ir} \sin\left(\alpha - \frac{\pi}{4}\right) \tag{11}$$

where $\hat{e}_{+1}$ and $\hat{e}_{-1}$ are the spherical unit vector and are correspond to the positive and negative helicity states, α reflects the type of the polarization, r is the azimuthal orientation of the polarization. We refer to $\hat{e}_{+1}$ as the right circular polarization and to $\hat{e}_{-1}$ as the left circular polarization. Consequently there should be two processes to generate THz wave in two-color laser-induced gas plasma. One is to generate right circular polarization, and the other is to generate left circular polarization. The probability amplitudes of the two processes are $e^{-ir}\cos(\alpha - \pi/4)$ and $e^{ir}\sin(\alpha - \pi/4)$, respectively. If $\alpha = 0$, the probability amplitudes of the processes are equal to each other. In this case, the THz wave is linear polarization with a azimuthal polarization orientation r. According to Eq. (9) we can specify that $r = -\Delta\varphi = -(2\varphi_1 - \varphi_2)$. So the Eq. (11) could be written as

$$\hat{e}_{THz} = P_I \hat{e}_+ e^{i\Delta\varphi} + P_{II} \hat{e}_- e^{-i\Delta\varphi} \tag{12}$$

where P_I and P_{II} are the probability amplitudes for right circular polarization process and left circular polarization process. In order to obtain linear polarization, $|P_I| = |P_{II}|$.

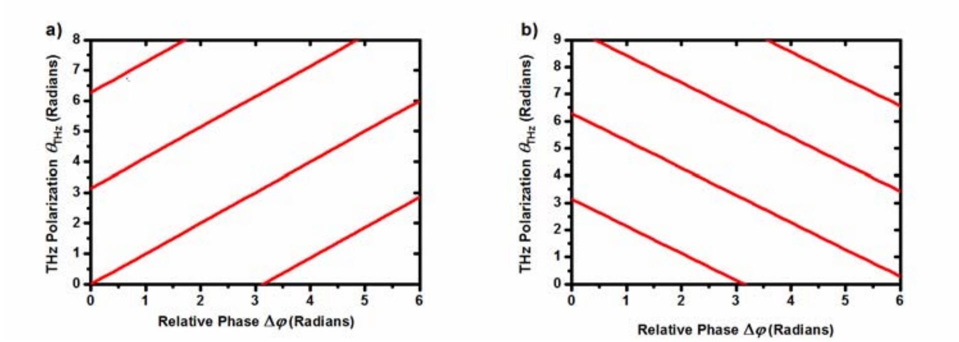

Fig. 2. The polarization angle of the THz wave θ_{THz} changes with the relative phase $\Delta\varphi$ for both the light pulses are (a) left- and (b) right-handed circular polarization.

The previous experimental results demonstrated that the polarization of the THz wave did not vary with the optical delay of the two pulses when the two pluses are both linear polarization [15, 16]. From the view above we can explain that the polarization angle of the THz wave doesn't change with the relative phase when the two lasers are linear polarization. Because the linear polarization could be expressed as the superposition of two spherical unit vectors, we should believe that the generation of the THz wave consist of two courses. One is that the two lasers are right-handed circular polarization, and the other is that they are left-handed circular polarization. For the first case, the polarization angle of the THz wave changes with $-\Delta\varphi = -(2\varphi_1 - \varphi_2)$ based on Eqs. (1) and (9), while for the second case, the polarization angle of the THz wave changes with $\Delta\varphi = 2\varphi_1 - \varphi_2$ based on Eq. (10). As a result, the polarization angle of the THz wave doesn't change with the relative phase.

When one laser is circular polarization and the other is linear polarization, THz wave can also be controlled by the relative phase. Since linear polarization could be expressed as superposition of two spherical unit vectors, it is the same case as that the two light pulses are both circular polarization. Noteworthily, our results predict that THz wave could not be obtained when one laser is left circular polarization and the other is right circular polarization.

4. Quantum theory

In this section, based on a seven-level system we build a quantum model to analyze the polarization property of THz wave generated from two-color laser-induced gas plasma. The THz generation is described by the four-wave mixing process, which is described by four-level system in quantum optics [18], and the nonlinear medium is the gas plasma induced by the two-color lasers. Energy levels of the system transform from the gas' ones into plasma's ones when the two-color laser pulses are incident on the medium. The latter ones must coincide with the energy and the spin of the two-color laser photons because the plasma is formed by the two-color laser pulses.

In order to describe the polarization of the THz photon, there are two four-level systems involved. One system is to generate right circular polarization THz photon, and the other is to generate left circular polarization THz photon. The polarization of the THz photon is superposition of the two processes. The two four-level systems share the same ground state, so altogether it is a seven-level system (Fig. 3). The coherence of the two four-level systems reflects the superposition of the right and left circular polarization THz photons and then reflects the polarization property of the THz photon generated from the two-color laser-induced gas plasma.

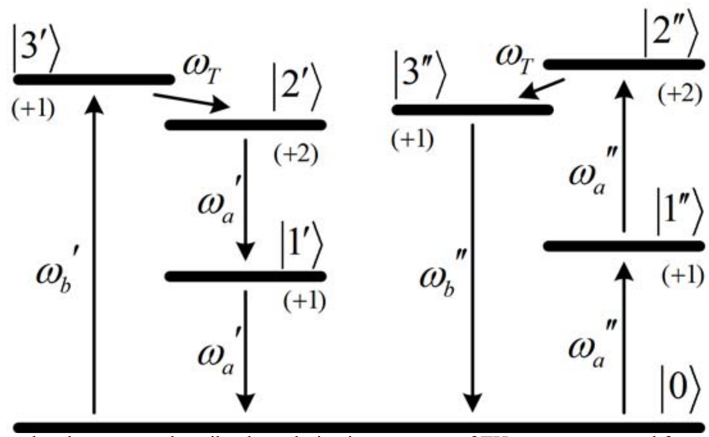

Fig. 3. The seven-level system to describe the polarization property of THz wave generated from two-color laser-induced gas plasma. $|0\rangle$, $|1'\rangle$, $|1''\rangle$, $|2'\rangle$, $|2''\rangle$, $|3'\rangle$, and $|3''\rangle$ describe the energy levels of the plasma excited by the two-color laser pulses. $+1$ and $+2$ describe the spin of the levels excited by the right circular polarization photons. The left four-level system is to generate left circular polarization THz photon; the right four-level system is to generate right circular polarization THz photon.

By treating the laser field as a classical electromagnetic wave, under the rotating wave approximation and the dipole approximation, the Hamilton of the seven-level system used to describe the THz photon generation is [18]

$$\begin{aligned} H/\hbar = {} & \omega_1'|1'\rangle\langle 1'| + \omega_1''|1''\rangle\langle 1''| + \omega_2'|2'\rangle\langle 2'| + \omega_2''|2''\rangle\langle 2''| + \omega_3'|3'\rangle\langle 3'| + \omega_3''|3''\rangle\langle 3''| \\ & + \omega_T a_{+1}^{+} a_{+1} + \omega_T a_{-1}^{+} a_{-1} \\ & + \frac{1}{2}\left[\begin{array}{l} \Omega_a' e^{i(\omega_a' t+\varphi_a)}\left(|0\rangle\langle 1'| + |1'\rangle\langle 2'|\right) + \Omega_b' e^{i(\omega_b' t+\varphi_b)}|0\rangle\langle 3'| \\ +\Omega_a'' e^{i(\omega_a'' t+\varphi_a)}\left(|0\rangle\langle 1''| + |1''\rangle\langle 2''|\right) + \Omega_b'' e^{i(\omega_b'' t+\varphi_b)}|0\rangle\langle 3''| \\ +\Omega_T a_{-1}^{+}|2'\rangle\langle 3'| + \Omega_T a_{+1}^{+}|3''\rangle\langle 2''| + h.c. \end{array}\right], \end{aligned} \tag{13}$$

where $|0\rangle$, $|1'\rangle$, $|1''\rangle$, $|2'\rangle$, $|2''\rangle$, $|3'\rangle$, and $|3''\rangle$ are the states of the seven-level system, $0, \hbar\omega_1'$, $\hbar\omega_1''$, $\hbar\omega_2'$, $\hbar\omega_2''$, $\hbar\omega_3'$, and $\hbar\omega_3''$ are respectively the eigenvalues of the corresponding states. Ω_a', Ω_a'', Ω_b', Ω_b'', and Ω_T are the Rabi frequencies, and ω_a', ω_a'', ω_b', ω_b'', and ω_T are the circular frequencies, for which the subscripts *a*, *b* and *T* denote the fundamental laser, the second harmonic laser, and the THZ wave, respectively. a_{+1}^{+}, a_{+1} and a_{-1}^{+}, a_{-1} are the creation and annihilation operators for right and left circular polarization photon. φ_a and φ_b are the phases of the pump lasers.

If the two-color lasers are both right circular polarization, the spin of the states $|1'\rangle$, $|1''\rangle$, $|3'\rangle$, and $|3''\rangle$ is $+1$, and the spin of the states $|2'\rangle$ and $|2''\rangle$ is $+2$, where we assume the spin of the state $|0\rangle$ is zero. Thus the spin of the THz photon generated from states $|2'\rangle$ and $|3'\rangle$ is -1, and the spin of the THz photon generated from states $|2''\rangle$ and $|3''\rangle$ is $+1$. It is worth noting that -1 denotes left circular polarization and $+1$ denotes right circular polarization.

If we assume that $\omega_1' = \omega_a' + \Delta_{a1}'$, $\omega_2' = 2\omega_a' + (\Delta_{a1}' + \Delta_{a2}')$, $\omega_3' = \omega_b' + \Delta_b'$, $\omega_1'' = \omega_a'' + \Delta_{a1}''$, $\omega_2'' = 2\omega_a'' + (\Delta_{a1}'' + \Delta_{a2}'')$ and $\omega_3'' = \omega_b'' + \Delta_b''$, the Hamilton in interaction picture with phase transformation becomes

$$\begin{aligned} H^I/\hbar &= \Delta_{a1}'|1'\rangle\langle 1'| + \Delta_{a1}''|1''\rangle\langle 1''| + (\Delta_{a1}' + \Delta_{a2}')|2'\rangle\langle 2'| \\ &+ (\Delta_{a1}'' + \Delta_{a2}'')|2''\rangle\langle 2''| + \Delta_b'|3'\rangle\langle 3'| + \Delta_b''|3''\rangle\langle 3''| \\ &+ \frac{1}{2}\begin{bmatrix} \Omega_a'(|0\rangle\langle 1'| + |1'\rangle\langle 2'|) + \Omega_b'|0\rangle\langle 3'| \\ +\Omega_a''(|0\rangle\langle 1''| + |1''\rangle\langle 2''|) + \Omega_b''|0\rangle\langle 3''| \\ +\Omega_T e^{i(\omega_T + 2\omega_a' - \omega_b')t} e^{i(2\varphi_a - \varphi_b)} a_{-1}^{+}|2'\rangle\langle 3'| + \Omega_T e^{i(\omega_T - 2\omega_a' + \omega_b')t} e^{i(\varphi_b - 2\varphi_a)} a_{+1}^{+}|3''\rangle\langle 2''| + h.c. \end{bmatrix}, \end{aligned} \tag{14}$$

where $\omega_T = \omega_b' - 2\omega_a' = 2\omega_a'' - \omega_b''$ is the THz frequency because of the conservation of energy. $\hbar\omega_1'$, $\hbar\omega_1''$, $\hbar\omega_2'$ and $\hbar\omega_2''$ are induced by the ω_1 light, while $\hbar\omega_3'$ and $\hbar\omega_3''$ are induced by the ω_2 light. Because the two-color lasers are femtosecond optical pulses, the ranges of their spectrum widths are quite large. That's why $\hbar\omega_j'$ is different from $\hbar\omega_j''\,(j = 1, 2, 3)$ although it is induced by the same pump light. Because we have assumed that the seven-level system is induced by the pump lasers, we can get $\varphi_a = \varphi_1$ and $\varphi_b = \varphi_2$. Thus Hamilton in Eq. (14) is

$$\begin{aligned} H^I/\hbar &= \Delta_{a1}'|1'\rangle\langle 1'| + \Delta_{a1}''|1''\rangle\langle 1''| + (\Delta_{a1}' + \Delta_{a2}')|2'\rangle\langle 2'| + (\Delta_{a1}'' + \Delta_{a2}'')|2''\rangle\langle 2''| \\ &+ \Delta_b'|3'\rangle\langle 3'| + \Delta_b''|3''\rangle\langle 3''| \\ &+ \frac{1}{2}\begin{bmatrix} \Omega_a'(|0\rangle\langle 1'| + |1'\rangle\langle 2'|) + \Omega_b'|0\rangle\langle 3'| \\ +\Omega_a''(|0\rangle\langle 1''| + |1''\rangle\langle 2''|) + \Omega_b''|0\rangle\langle 3''| \\ +\Omega_T e^{i\Delta\varphi} a_{-1}^{+}|2'\rangle\langle 3'| + \Omega_T e^{-i\Delta\varphi} a_{+1}^{+}|3''\rangle\langle 2''| + h.c. \end{bmatrix}. \end{aligned} \tag{15}$$

The motion of the wave function is described by the Schrödinger equation

$$i\frac{d|\Psi\rangle}{dt} = \frac{H^I}{\hbar}|\Psi\rangle. \tag{16}$$

The wave function is assumed as

$$\begin{aligned} |\Psi\rangle &= A_1|n+1, n, 3''\rangle + A_2|n, n, 2''\rangle + A_3|n, n, 1''\rangle \\ &+ A_4|n, n, 0\rangle + A_5|n+1, n, 0\rangle + A_6|n, n+1, 0\rangle \\ &+ A_7|n, n, 3'\rangle + A_8|n, n+1, 2'\rangle + A_9|n, n+1, 1'\rangle, \end{aligned} \tag{17}$$

where the first, second and third quantum numbers in the Dirac symbols denote the right-circular polarization THz photon, the left-circular polarization THz photon, and the energy levels in the seven-level system, respectively, and $A_j\,(j = 1-9)$ is superposition coefficient. The decay rates Γ_1, Γ_2, Γ_3, Γ_7, Γ_8 and Γ_9 can be incorporated in Eq. (16), and then we obtain

$$i\frac{dA_1}{dt} = \left(\Delta_b'' - i\Gamma_1\right)A_1 + \left(\sqrt{n+1}\frac{\Omega_T}{2}e^{-i\Delta\varphi}\right)A_2 + \left(\frac{\Omega_b''}{2}\right)A_5, \tag{18}$$

$$i\frac{dA_2}{dt}=\left(\sqrt{n+1}\frac{\Omega_T}{2}e^{i\Delta\varphi}\right)A_1+\left(\Delta_{a1}''+\Delta_{a2}''-i\Gamma_2\right)A_2+\left(\frac{\Omega_a''}{2}\right)A_3, \tag{19}$$

$$i\frac{dA_3}{dt}=\left(\frac{\Omega_a''}{2}\right)A_2+\left(\Delta_{a1}''-i\Gamma_3\right)A_3+\left(\frac{\Omega_a''}{2}\right)A_4, \tag{20}$$

$$i\frac{dA_5}{dt}=\left(\frac{\Omega_b''}{2}\right)A_1-i\Gamma_5A_5, \tag{21}$$

$$i\frac{dA_6}{dt}=\left(\frac{\Omega_a'}{2}\right)A_9-i\Gamma_6A_6, \tag{22}$$

$$i\frac{dA_7}{dt}=\left(\frac{\Omega_b'}{2}\right)A_4+\left(\Delta_b'-i\Gamma_7\right)A_7+\left(\sqrt{n+1}\frac{\Omega_T}{2}e^{-i\Delta\varphi}\right)A_8, \tag{23}$$

$$i\frac{dA_8}{dt}=\left(\sqrt{n+1}\frac{\Omega_T}{2}e^{i\Delta\varphi}\right)A_7+\left(\Delta_{a1}'+\Delta_{a2}'-i\Gamma_8\right)A_8+\left(\frac{\Omega_a'}{2}\right)A_9, \tag{24}$$

$$i\frac{dA_9}{dt}=\left(\frac{\Omega_a'}{2}\right)A_6+\left(\frac{\Omega_a'}{2}\right)A_8+\left(\Delta_{a1}'-i\Gamma_9\right)A_9, \tag{25}$$

$$|A_1|^2+|A_2|^2+|A_3|^2+|A_4|^2+|A_5|^2+|A_6|^2+|A_7|^2+|A_8|^2+|A_9|^2=1. \tag{26}$$

The steady-state solutions at the resonance situation are

$$A_1=\frac{i\Gamma_5\sqrt{n+1}\left(\frac{\Omega_a''}{2}\right)^2\left(\frac{\Omega_T}{2}\right)A_4e^{-i\Delta\varphi}}{(n+1)\Gamma_3\Gamma_5\left(\frac{\Omega_T}{2}\right)^2+\left[\Gamma_1\Gamma_5+\left(\frac{\Omega_b''}{2}\right)^2\right]\left[\left(\frac{\Omega_a''}{2}\right)^2+\Gamma_2\Gamma_3\right]}, \tag{27}$$

$$A_2=\frac{-\left(\frac{\Omega_a''}{2}\right)^2\left[\Gamma_1\Gamma_5+\left(\frac{\Omega_b''}{2}\right)^2\right]A_4}{(n+1)\Gamma_3\Gamma_5\left(\frac{\Omega_T}{2}\right)^2+\left[\Gamma_1\Gamma_5+\left(\frac{\Omega_b''}{2}\right)^2\right]\left[\left(\frac{\Omega_a''}{2}\right)^2+\Gamma_2\Gamma_3\right]}, \tag{28}$$

$$A_7=\frac{-i\left(\frac{\Omega_b'}{2}\right)\left\{\Gamma_8\left[\left(\frac{\Omega_a'}{2}\right)^2+\Gamma_6\Gamma_9\right]+\Gamma_6\left(\frac{\Omega_a'}{2}\right)^2\right\}A_4}{\Gamma_6\Gamma_7\left(\frac{\Omega_a'}{2}\right)^2+\left[\left(\frac{\Omega_a'}{2}\right)^2+\Gamma_6\Gamma_9\right]\left[(n+1)\left(\frac{\Omega_T}{2}\right)^2+\Gamma_7\Gamma_8\right]}, \tag{29}$$

$$A_8=\frac{-\sqrt{n+1}\left(\frac{\Omega_b'}{2}\right)\left(\frac{\Omega_T}{2}\right)\left[\left(\frac{\Omega_a'}{2}\right)^2+\Gamma_6\Gamma_9\right]e^{i\Delta\varphi}A_4}{\Gamma_6\Gamma_7\left(\frac{\Omega_a'}{2}\right)^2+\left[\left(\frac{\Omega_a'}{2}\right)^2+\Gamma_6\Gamma_9\right]\left[(n+1)\left(\frac{\Omega_T}{2}\right)^2+\Gamma_7\Gamma_8\right]}, \tag{30}$$

$$\langle\Psi|\left(a_{+1}^{+}|3''\rangle\langle 2''|\right)|\Psi\rangle = A_1^* A_2,$$
$$\langle\Psi|\left(a_{-1}^{+}|2'\rangle\langle 3'|\right)|\Psi\rangle = A_8^* A_7, \tag{31}$$

Here we use the average of the operators $a_{+1}^{+}|3''\rangle\langle 2''|$ and $a_{-1}^{+}|2'\rangle\langle 3'|$ to reflect the coherence of the two polarizations that right circular polarization and left circular polarization, as shown in Eq. (31). So the polarization of the THz photon can be expressed as

$$\hat{e}_{THz} = A_1^* A_2 \hat{e}_{+1} + A_8^* A_7 \hat{e}_{-1} = M_I e^{i\Delta\varphi} + M_{II} e^{-i\Delta\varphi}. \tag{32}$$

$$A_1^* A_2 = \frac{i\Gamma_5\sqrt{n+1}\left(\frac{\Omega_a''}{2}\right)^4\left(\frac{\Omega_T}{2}\right)\left[\Gamma_1\Gamma_5+\left(\frac{\Omega_b''}{2}\right)^2\right]A_4^2 e^{i\Delta\varphi}}{\left\{(n+1)\Gamma_3\Gamma_5\left(\frac{\Omega_T}{2}\right)^2+\left[\Gamma_1\Gamma_5+\left(\frac{\Omega_b''}{2}\right)^2\right]\left[\left(\frac{\Omega_a''}{2}\right)^2+\Gamma_2\Gamma_3\right]\right\}^2} = M_I e^{i\Delta\varphi}, \tag{33}$$

$$A_8^* A_7 = \frac{i\sqrt{n+1}\left(\frac{\Omega_T}{2}\right)\left(\frac{\Omega_b'}{2}\right)^2\left[\left(\frac{\Omega_a'}{2}\right)^2+\Gamma_6\Gamma_9\right]\left\{\Gamma_8\left[\left(\frac{\Omega_a'}{2}\right)^2+\Gamma_6\Gamma_9\right]+\Gamma_6\left(\frac{\Omega_a'}{2}\right)^2\right\}A_4^2 e^{-i\Delta\varphi}}{\left\{\Gamma_6\Gamma_7\left(\frac{\Omega_a'}{2}\right)^2+\left[\left(\frac{\Omega_a'}{2}\right)^2+\Gamma_6\Gamma_9\right]\left[(n+1)\left(\frac{\Omega_T}{2}\right)^2+\Gamma_7\Gamma_8\right]\right\}^2} = M_{II} e^{-i\Delta\varphi}, \tag{34}$$

The Eq. (32) has the same form as the Eq. (12), which means we can use the seven-level system to describe the two-color laser-induced gas plasma. This result demonstrates that the relative phase can control the coherence of the two cyclically coupled four-level system and then can control the polarization angle of the THz photon generated from the plasma. If Ω_a'' or Ω_b' is zero we can get left- or right- circular polarization THz wave, respectively.

5. Conclusion

We use a classical and a quantum model to describe the polarization property of THz waves generated from two-color laser-induced gas plasma. The conclusion obtained from the models can clearly explain that the polarization angle of THz wave is controllable by the relative phase of the two-color lasers. These models are promising for finding a way to obtain a THz wave with a wanted polarization state. We can use the quantum model to study the physical characteristic of the plasma, for example the nonlinear phenomenon.

Acknowledgments
The authors gratefully acknowledge support from the National Natural Science Foundation of China under Grant No. 10974063 and 61177095, Hubei Natural Science Foundation under grant No. 2010CDA001, Ph.D. Programs Foundation of Ministry of Education of China under grant No. 20100142110042, and the Fundamental Research Funds for the Central Universities, HUST: 2010MS041 and 2011TS001.

Reference

1. F. Blanchard, L. Razzari, H.C. Bandulet, G. Sharma, R. Morandotti, J. C. Kieffer, T. Ozaki, M. Reid, H. F. Tiedje, H. K. Haugen, and F. A. Hegmann, "Generation of 1.5 μJ single-cycle terahertz pulses by optical rectification from a large aperture ZnTe crystal," Opt. Express 15, 13212 (2007).
2. W. Shi, Y. J. Ding, N. Fernelius, and K. L. Vodopyanov, "An efficient, tunable, and coherent 0.18-5.27 THz source based on GaSe crystal," Opt. Lett. 27, 1454(2002).
3. E. T. J. Nibbering, P. F. Curley, G. Grillon, B. S. Prade, M. A. Franco, F. Salin, and A. Mysyrowicz, "Conical emission from self-guided femtosecond pulses in air", Opt. Lett. 21, 62 (1996).
4. G. Mechain, S. Tzortzakis, B. Prade, M. Franco, A. Mysyrow-icz, and B. Leriche, "Calorimetric detection of THz radiation from femtosecond filaments in air", Appl. Phys. B: Lasers Opt. 77, 707 (2003).
5. Hua Zhong, Nicholas Karpowicz, and X.-C. Zhang, "Terahertz emission profile from laser-induced air plasma", Applied Physics Letters 88, 261103 (2006).
6. X. Xie, J. Dai, and X.-C. Zhang, "Coherent control of THz wave generation in ambient air", Phys. Rev. Lett. 96, 075005 (2006).
7. J. Dai, X. Xie, and X.-C. Zhang, "Detection of broadband terahertz waves with laser-induced plasma in gases", Phys. Rev. Lett. 97, 103903 (2006).
8. J. Dai and X.-C. Zhang, "Terahertz wave generation from gas plasma using a phase compensator with attosecond phase-control accuracy", Appl. Phys. Lett. 94, 021117 (2009).
9. Zheng Chu, Jinsong Liu, Kejia Wang, and Jianquan Yao, "Four-wave mixing model solutions for polarization control of terahertz pulse generated by a two-color laser field in air", Chinese Optics Letters 8, 697 (2010).
10. D. J. Cook and R. M. Hochstrasser, "Intense terahertz pulses by four-wave rectification in air", Opt. Lett. 25, 1210 (2000).
11. K. Y. Kim, A. J. Taylor, J. H. Glownia and G. Rodriguez, "Coherent control of terahertz supercontinuum generation in ultrafast laser-gas interactions", Nature Photonics 2, 605 (2008).
12. Hui-Chun Wu, Jürgen Meyer-ter-Vehn and Zheng-Ming Sheng, "Phase-sensitive terahertz emission from gas targets irradiated by few-cycle laser pulses", New J. Phys. 10, 043001 (2008).
13. A. A. Silaev and N. V. Vvedenskii, "Residual-current excitation in plasmas produced by Few-cycle laser pulses", Phys. Rev. Lett. 102, 115005 (2009).
14. N. Karpowicz and X.-C. Zhang, "Coherent Terahertz Echo of Tunnel Ionization in Gases", Phys. Rev. Lett. 102, 093001 (2009).
15. J. Dai, N. Karpowicz and X.-C. Zhang, "Coherent polarization control of terahertz waves generated from two-color laser-induced gas plasma", Phys. Rev. Lett. 103, 023001 (2009).
16. H. Wen and A. M. Lindenberg, "Coherent terahertz polarization control through manipulation of electron trajectories", Phys. Rev. Lett. 103, 023902 (2009).
17. Keh-Ning Huang, "Theory of angular distribution and spin polarization of photoelectrons", Phys. Rev. A 22, 223 (1980).
18. M. O. Scully, M. S. Zubairy, Quantum Optics (Cambridge Univ. Press, Cambridge, England, 1997).