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Generation of GHZ entangled states of photons in multiple cavities via a superconducting qutrit or an atom through resonant interaction

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Abstract

We propose an efficient method to generate a GHZ entangled state of n photons in n microwave cavities (or resonators) via resonant interaction to a single superconducting qutrit. The deployment of a qutrit, instead of a qubit, as the coupler enables us to use resonant interactions exclusively for all qutrit-cavity and qutrit-pulse operations. This unique approach significantly shortens the time of operation which is advantageous to reducing the adverse effects of qutrit decoherence and cavity decay on fidelity of the protocol. Furthermore, the protocol involves no measurement on either the state of qutrit or cavity photons. We also show that the protocol can be generalized to other systems by replacing the superconducting qutrit coupler with different types of physical qutrit, such as an atom in the case of cavity QED, to accomplish the same task.

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I. INTRODUCTION

Entanglement is one of the most fascinating features of quantum mechanics and plays an important role in quantum communication and quantum information processing (QIP). During the past decade, experimental preparation of entanglement with eight photons via *linear optical devices* [1], eight ions [2], three spins [3], two atoms in microwave cavity QED [4], two atoms plus one cavity mode [5], or two excitons in a single quantum dot [6] has been reported.

Over the past ten years, there has been much interest in quantum information processing with superconducting qubits. By having qubits coupled through capacitors, entangling two [7] or three superconducting qubits [8] has been experimentally demonstrated. In addition, a tripartite entanglement consisting of a superconducting qubit and two microscopic two-level systems has been reported recently [9].

On the other hand, physical systems composed of cavities and superconducting qubits such as transmon and phase qubits are considered as one of the most promising candidates for quantum information processing. For the sake of simplicity, hereafter the term cavity refers to either a three-dimensional cavity or any other types of resonant structure such as a coplanar waveguide (CPW) resonator, a microstrip resonator, or even a lumped circuit LC resonator. In this circuit QED approach, a cavity acts as a quantum bus which can mediate long-distance, fast interaction between distant superconducting qubits [10-14]. Theoretically, it was predicted earlier that the strong coupling limit can readily be achieved with superconducting flux qubits [15] or charge qubits [12] coupled to resonant cavities, which has been experimentally demonstrated soon after [16,17]. Based on circuit QED, a large number of theoretical schemes for creating entangled states with superconducting qubits in single cavities have been proposed [10,15,18-25]. In addition, various two-qubit or three-qubit entangled states have been experimentally demonstrated with superconducting qubits coupled to single cavities [26-30]. All of these theoretical and experimental works are focused primarily on entanglement of superconducting qubits coupled to a single cavity, which has paved the way for fundamental tests of quantum entanglement and made superconducting qubit circuit QED very attractive for quantum information processing.

Recently, attention has been progressed to entanglement generation of qubits or photons resided in multiple cavities because of its importance to scalable QIP. Within circuit QED,

several theoretical proposals for generation of entangled photon Fock states of *two* resonators have been presented [31,32]. Moreover, by using a superconducting phase qubit coupled to two resonators, recent experimental demonstration of an entangled NOON state of photons in *two* superconducting microwave resonators has been reported [33].

In this paper, we focus on the preparation of GHZ (Greenberger-Horne-Zeilinger) entangled states of photons in *multiple* cavities. The GHZ entangled states are of great interest to the foundations of quantum mechanics and measurement theory, and are an important resource for quantum information processing [34], quantum communication (e.g., cryptography) [35-37], error correction protocols [38], and high-precision spectroscopy [39].

In the following, we propose an efficient method to generate a GHZ entangled state of n photons distributed over n microwave cavities that are coupled by a superconducting *qutrit* (a.k.a. coupler) through *resonant* interaction. By local operations on a qubit (e.g., an atom etc.) placed in each cavity, the created GHZ states of photons can be transferred to qubits for a long time storage and then can be transferred back to the photons once they are needed to be sent through quantum channels for implementing quantum communication or quantum information processing in a network.

As shown below, this proposal does not require measurement on the states of the coupler qutrit or the cavity-mode photons for each cavity, and only requires resonant qutrit-cavity interaction and resonant qutrit-pulse interaction for each step of the operations. Thus, it is relatively straightforward to implement the method in experiments. Furthermore, the result of numerical simulation with realistic circuit parameters indicates that by careful design and optimization high fidelity GHZ states of multiple cavity photons are within the reach of present day technology.

We emphasize that this proposal is quite general, and can be used to create GHZ states of photons in multiple cavities with different types of physical qutrit, such as a Rydberg atom or a quantum dot, as the coupler. Finally, we show how to apply the method to generate a GHZ state of photons in multiple cavities using an atom as an example.

The paper is organized as follows. In Sec. II, we show how to generate a GHZ state of n photons in n cavities coupled by a superconducting qutrit. In Sec. III, we discuss how to extend the method to prepare a GHZ state of n photons in the n cavities using an atom. A concluding summary is given in Sec. IV.

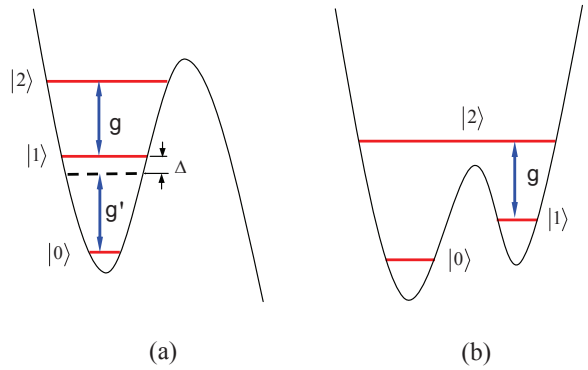

FIG. 1: (Color online) Illustration of qutrit-cavity resonant interaction. The cavity mode is resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition of the qutrit. g is the coupling constant between the cavity mode and the $|1\rangle \leftrightarrow |2\rangle$ transition. In (a), the cavity mode is decoupled from the $|0\rangle \leftrightarrow |1\rangle$ transition of a phase qutrit as long as the large detuning condition $\Delta \gg g'$ is satisfied. Here, Δ is the detuning between the cavity mode frequency and the $|0\rangle \leftrightarrow |1\rangle$ transition frequency, g' is the coupling constant between the cavity mode and the $|0\rangle \leftrightarrow |1\rangle$ transition. In (b), the dipole matrix element between $|0\rangle$ and $|1\rangle$ can be made much weaker than that between $|1\rangle$ and $|2\rangle$ by increasing the barrier height of the double well potential. Thus the coupling between $|0\rangle$ and $|1\rangle$ via the cavity mode is negligible. Note that the coupling strength g may vary when the qutrit couples with different cavities or resonators. Thus, g is replaced by g_i to denote the coupling strength between the qutrit and cavity i $(i = 1, 2, ..., n)$.

II. GENERATION OF A N-PHOTON GHZ STATE IN THE N CAVITIES VIA A SUPERCONDUCTING QUTRIT

In this section, we show how to create a n-photon GHZ state in n cavities via a superconducting qutrit, estimate the fidelity of the prepared GHZ state for $n = 2$, 3 and 4, and then end with a brief discussion.

A. Generation of n-photon GHZ states in n cavities

Consider a superconducting qutrit A, which has three levels as depicted in Fig. 1. The three-level structure in Fig. 1(a) applies to superconducting phase qutrits [7,33,40] and transmon qutrits [41], while the one in Fig. 1(b) applies to flux qutrits [42]. In addition, the three-level structure in Fig. 1(a) or Fig. 1(b) is also available in atoms. The coupler

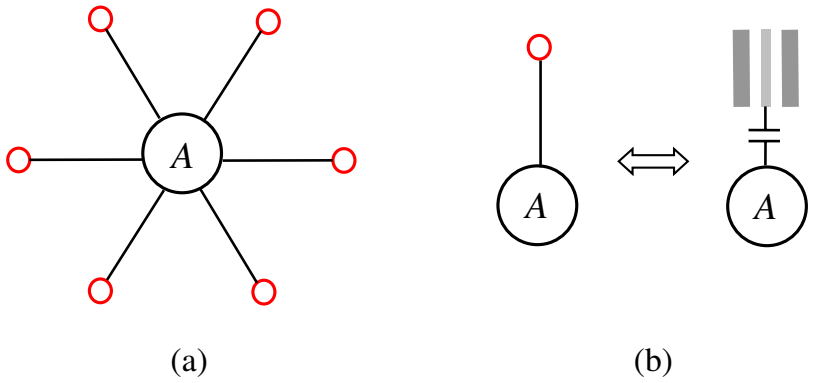

FIG. 2: (Color online) (a) Diagram of a superconducting qutrit A (a circle at the center) and n cavities. Each red dot represents a one-dimensional coplanar waveguide resonator which is capacitively coupled to the coupler qutrit A, as shown in (b). (b) The diagram on the left side is equivalent to the diagram on the right side.

qutrit A shall have the following properties: (i) for the three-level structure depicted in Fig. 1(a), transition between the two lowest levels is highly detuned (decoupled) from the mode of each cavity by prior adjustment of the level spacings of the qutrit; and (ii) for the three-level structure depicted in Fig. 1(b), the dipole interaction (i.e., matrix element) between the two lowest levels is weak by increasing the potential barrier between the two levels $|0\rangle$ and $|1\rangle$ [43-45]. Note that for superconducting qutrits, the level spacings can be rapidly adjusted by varying external control parameters (e.g., magnetic flux applied to phase, transmon, or flux qutrits, see e.g. [43-46]).

Let us now consider n cavities $(1, 2, ..., n)$ each coupled to a superconducting coupler qutrit A (Fig. 2). Initially, qutrit A is in its ground state $|0\rangle$ and decoupled from all cavities $(1, 2, ..., n)$ by prior adjustment of each cavity's frequency; next, qutrit A is transformed by a $\pi/2$-microwave pulse to the state $(|0\rangle + |2\rangle)/\sqrt{2}$ (hereafter, the three states of qutrit A are denoted by $|0\rangle, |1\rangle$, and $|2\rangle$ respectively without subscripts) while each cavity i $(= 1, 2, ..., n)$ remains in its vacuum state $|0\rangle_{c,i}$.

To begin with, we define ω_{21} (ω_{20}) as the $|1\rangle \leftrightarrow |2\rangle$ ($|0\rangle \leftrightarrow |2\rangle$) transition frequency of qutrit A and Ω_{21} (Ω_{20}) as the pulse Rabi frequency of the coherent $|1\rangle \leftrightarrow |2\rangle$ ($|0\rangle \leftrightarrow |2\rangle$) transition. In addition, the frequency, initial phase, and duration of the microwave pulse are denoted as $\{\omega, \varphi, t'\}$ in the rest of the paper. The operations for realizing a GHZ state

of n photons in the n cavities are described below:

Step i ($i = 1, 2, ..., n-2$): Adjust the frequency $\omega_{c,i}$ of cavity i, which will be referred to as the *active* cavity hereafter, such that it is resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition of qutrit A (i.e., $\omega_{c,i} = \omega_{21}$). After an interaction time $t_i = \pi/(2g_i)$, the state $|0\rangle |0\rangle_{c,i}$ remains unchanged while the state $|2\rangle |0\rangle_{c,i}$ changes to $-i |1\rangle |1\rangle_{c,i}$. Then, adjust the frequency of the active cavity away from ω_{21} to decouple it from qutrit A. Finally, a microwave pulse of $\{\omega_{21}, \pi, \pi/(2\Omega_{21})\}$ is applied to qutrit A to transform its state from $|1\rangle$ to $i |2\rangle$.

After executing step 1 to step $n-2$, the initial state $(|0\rangle + |2\rangle) \prod_{i=1}^{n} |0\rangle_{c,i}$ of the whole system is transformed to (here and below a normalization factor is omitted for simplicity)

$$\left(|0\rangle \prod_{i=1}^{n-2} |0\rangle_{c,i} + |2\rangle \prod_{i=1}^{n-2} |1\rangle_{c,i} \right) |0\rangle_{c,n-1} |0\rangle_{c,n} . \tag{1}$$

Step $n-1$: Adjust the frequency $\omega_{c,n-1}$ of cavity $n-1$ to have $\omega_{c,n-1} = \omega_{21}$ for an interaction time $t_{n-1} = \pi/(2g_{n-1})$. As a result, the state $|0\rangle |0\rangle_{c,n-1}$ remains unchanged while the state $|2\rangle |0\rangle_{c,n-1}$ changes to $-i |1\rangle |1\rangle_{c,n-1}$Then, adjust the frequency of cavity $n-1$ to decouple it from qutrit A. Next, apply a pulse of $\{\omega_{20}, -\pi/2, \pi/(2\Omega_{20})\}$ to qutrit A to transform its state from $|0\rangle$ to $|2\rangle$; finally a pulse of $\{\omega_{21}, \pi/2, \pi/(2\Omega_{21})\}$ is applied to qutrit A to transform the state $|1\rangle$ to $-|2\rangle$ and the state $|2\rangle$ to $|1\rangle$.

It is easy to verify that after completing the $n-1$ steps prescribed above, we obtain the state transformation $|0\rangle |0\rangle_{c,n-1} \rightarrow |1\rangle |0\rangle_{c,n-1}$ and $|2\rangle |0\rangle_{c,n-1} \rightarrow i |2\rangle |1\rangle_{c,n-1}$, which propagates state (1) to

$$\left(|1\rangle \prod_{i=1}^{n-1} |0\rangle_{c,i} + i |2\rangle \prod_{i=1}^{n-1} |1\rangle_{c,i} \right) |0\rangle_{n} . \tag{2}$$

Step n: Adjust the frequency $\omega_{c,n}$ of cavity n to resonate with ω_{21} for an interaction time $t_n = \pi/(2g_n)$, so that the state $|2\rangle |0\rangle_{c,n}$ changes to $-i |1\rangle |1\rangle_{c,n}$ while the state $|1\rangle |0\rangle_{c,n}$ remains unchanged. Then, adjust $\omega_{c,n}$ to decouple cavity n from qutrit A.

It can be seen that after this step of operation, state (2) becomes

$$|1\rangle \left(\prod_{i=1}^{n} |0\rangle_{c,i} + \prod_{i=1}^{n} |1\rangle_{c,i} \right) . \tag{3}$$

The result (3) shows that the n cavities are prepared in a n-photon GHZ state $\prod_{i=1}^{n} |0\rangle_{c,i} + \prod_{i=1}^{n} |1\rangle_{c,i}$, while the qutrit A is disentangled from all cavities, after the above n-step operation.

It should be noticed that rapid tuning of cavity frequencies required by the proposed protocol has been demonstrated recently in superconducting microwave cavities (e.g., in less than a few nanoseconds for a superconducting transmission line resonator [47]). Alternatively, the method can also be implemented with cavities of different resonant frequencies by rapid tuning of level spacing ω_{21} of the coupler qutrit.

Let us now discuss issues which are most relevant to the experimental implementation of the method. For the method to work the primary considerations shall be given to:

(a) The total operation time τ, given by

$$\tau = \sum_{i=1}^{n} \pi/(2g_i) + (n-1)\,\pi/\left(2\Omega_{21}\right) + \pi/\left(2\Omega_{20}\right) + 2nt_d \tag{4}$$

(where t_d is the typical time required for adjusting the cavity mode frequency), needs to be much shorter than the energy relaxation time T_1 ($T_1^{'}$) and dephasing time T_2 ($T_2^{'}$) of the level $|2\rangle$ ($|1\rangle$) of qutrit A, such that decoherence caused by energy relaxation and dephasing of qutrit A is negligible for the operation. Note that $T_1^{'}$ and $T_2^{'}$ of qutrit A are comparable to T_1 and T_2, respectively. For instance, $T_1^{'} \sim \sqrt{2}T_1$ and $T_2^{'} \sim T_2$ for phase qutrits.

(b) For cavity i ($i = 1, 2, ..., n$), the lifetime of the cavity mode is given by $T_{cav}^{i} = \left(Q_i/2\pi\nu_{c,i}\right)/\overline{n}_i$, where Q_i and $\overline{n}_i$ are the (loaded) quality factor and the average photon number of cavity i, respectively. For n cavities, the lifetime of the cavity modes is given by

$$T_{cav} = \frac{1}{n}\min\{T_{cav}^{1}, T_{cav}^{2}, ..., T_{cav}^{n}\}, \tag{5}$$

which should be much longer than τ, such that the effect of cavity decay is negligible for the operation.

(c) For step i ($i = 1, 2, ..., n$) of the operation, there exists a qutrit mediated interaction (crosstalk) between the active cavity and each of the remaining $n-1$ *idling* cavities (which are not intended to be involved in the operation). When qutrit A is in the state $|2\rangle$, the probability of exciting an idling cavity $j \neq i$ from the vacuum state $|0\rangle_{c,j}$ to $|1\rangle_{c,j}$, after the completion of step i, is given approximately by

$$p_j \approx \frac{1}{2}\left(1 - \cos\frac{\pi\sqrt{4\widetilde{g}_j^2 + \Delta_j^2}}{2g_i}\right)\left(1 - \frac{\Delta_j^2}{4\widetilde{g}_j^2 + \Delta_j^2}\right), \tag{6}$$

where $\widetilde{g}_j$ is the off-resonant coupling constant between cavity j and the $|1\rangle \leftrightarrow |2\rangle$ transition of qutrit A, and $\Delta_j = \omega_{21} - \widetilde{\omega}_{c,j}$ is the detuning of the frequency of cavity j with the $|1\rangle \leftrightarrow |2\rangle$

transition frequency. Hereafter, $\widetilde{\omega}_{c,j}$ represents the frequency of cavity j when idling [see Fig. 3(a)].

It can be seen from Eq. (6) that p_j is negligibly small when $\Delta_j \gg \widetilde{g}_j$. Hence, as long as the large detuning condition is satisfied for all of the idling cavities, crosstalk caused error can be suppressed to a tolerable level.

(d) For step i ($i = 1, 2, ..., n$) of the operation, there also exists an inter-cavity cross coupling which is determined mostly by the coupling capacitance C_c and the qutrit's self capacitance C_q, because field leakage through space is extremely low for high-Q cavities as long as inter-cavity distances are much greater than transverse dimension of the cavities - a condition easily met in experiments for $n \leq 8$. Furthermore, as the result of our numerical simulation shown below (see Fig. 4), the effects of these inter-cavity couplings can however be made negligible as long as $g_{kl} \leq 10^{-2} g_i$, where g_{kl} is the corresponding inter-cavity coupling constant between cavities k and l.

B. Fidelity

The proposed protocol for creating the n-photon GHZ state described above involves three basic types of transformation:

(i) The first one requires that during step i ($i = 1, 2, ..., n$) of the operation, cavity i is tuned to resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition of qutrit A while other cavities are decoupled from qutrit A. In the interaction picture (the same without mentioning hereafter), the interaction Hamiltonian governing this basic transformation is given by

$$\begin{aligned} H_{I,1} \;=\; & g_i \left(a_i S_{12}^{+} + h.c.\right) + g_i' \left(e^{i\Delta t} a_i S_{01}^{+} + h.c.\right) \\ & + \sum_{j\neq i, j=1}^{n} \widetilde{g}_j \left(e^{i\Delta_j t} a_j S_{12}^{+} + h.c.\right) + \sum_{j\neq i, j=1}^{n} \widetilde{g}_j' \left(e^{i\Delta_j' t} a_j S_{01}^{+} + h.c.\right) \\ & + \sum_{k\neq l; k,l=1}^{n} g_{kl} \left(e^{i\Delta_{kl} t} a_k a_l^{+} + h.c.\right). \end{aligned} \tag{7}$$

where $S_{12}^{+} = |2\rangle\langle 1|$, $S_{01}^{+} = |1\rangle\langle 0|$, and $a^{+}(a)$ is the cavity photon creation (annihilation) operator. The first term describes the resonant coupling between cavity i and the $|1\rangle \leftrightarrow |2\rangle$ transition of qutrit A with a coupling constant g_i [Fig. 3(a)] while the second term represents the off-resonant coupling between cavity i and the $|0\rangle \leftrightarrow |1\rangle$ transition with a coupling constant g_i' and detuning $\Delta = \omega_{10} - \omega_{c,i}$ [Fig. 3(a)]. The third (fourth) term is the off-resonant coupling between all idling cavities and the $|1\rangle \leftrightarrow |2\rangle$ ($|0\rangle \leftrightarrow |1\rangle$) transition, where

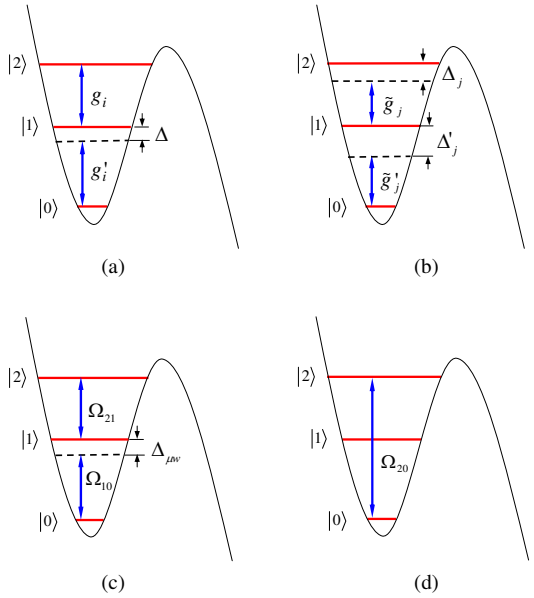

FIG. 3: (Color online) Illustration of qutrit-cavity or qutrit-pulse interaction. (a) Cavity i is resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition of qutrit A when $\omega_{c,i} = \omega_{21}$ with a coupling constant g_i but off-resonant with the $|0\rangle \leftrightarrow |1\rangle$ transition with a coupling constant g_i' and detuning $\Delta = \omega_{10} - \omega_{c,i}$. (b) Cavity j of frequency $\widetilde{\omega}_{c,j}$ is off-resonant with the $|1\rangle \leftrightarrow |2\rangle$ ($|0\rangle \leftrightarrow |1\rangle$) transition of qutrit A with a coupling constant $\widetilde{g}_j$ ($\widetilde{g}_j'$) and detuning $\Delta_j = \omega_{21} - \widetilde{\omega}_{c,j}$ ($\Delta_j' = \omega_{10} - \widetilde{\omega}_{c,j}$). (c) Represents the situation when a microwave classical pulse of frequency $\omega = \omega_{21}$ is applied to qutrit A but off-resonant with the $|0\rangle \leftrightarrow |1\rangle$ transition with detuning $\Delta_{\mu w} = \omega_{10} - \omega$. The corresponding Rabi frequencies are Ω_{21} and Ω_{10}, respectively. (d) A microwave pulse of frequency $\omega = \omega_{20}$ is applied to qutrit A with the corresponding Rabi frequency Ω_{20}. Note that for (c), the coupling of the pulse to the $|0\rangle \leftrightarrow |2\rangle$ transition is negligible due to the fact that the pulse is highly detuned from the $|0\rangle \leftrightarrow |2\rangle$ transition frequency. For the same reason, for (d), the coupling of the pulse to the $|0\rangle \leftrightarrow |1\rangle$ transition and the $|1\rangle \leftrightarrow |2\rangle$ transitions is negligible as well.

$\widetilde{g}_j$ ($\widetilde{g}_j'$) is the coupling constant between cavity j and the $|1\rangle \leftrightarrow |2\rangle$ ($|0\rangle \leftrightarrow |1\rangle$) transition, with detuning $\Delta_j = \omega_{21} - \widetilde{\omega}_{c,j}$ ($\Delta_j' = \omega_{10} - \widetilde{\omega}_{c,j}$) [Fig. 3(b)]. The last term represents the inter-cavity crosstalk between any two cavities k and l, where Δ_{kl} is the frequency detuning for the two cavities k and l.

(ii) The second one involves pulse-qutrit interaction by applying a microwave pulse (with frequency $\omega = \omega_{21}$ and initial phase φ) to qutrit A. Note that when the pulse is on, all cavities are required to be decoupled from qutrit A by a prior detuning of their frequencies

from ω_{21}. The interaction Hamiltonian for this basic transformation is given by

$$\begin{aligned} H_{I,2} \;=\; & \Omega_{21}\left(e^{-i\varphi}S_{12}^{+}+h.c.\right)+\Omega_{10}\left[e^{i(\Delta_{\mu w}t-\varphi)}S_{01}^{+}+h.c.\right] \\ & +\sum_{j=1}^{n}\widetilde{g}_{j}\left(e^{i\Delta_{j}t}a_{j}S_{12}^{+}+h.c.\right)+\sum_{j=1}^{n}\widetilde{g}_{j}'\left(e^{i\Delta_{j}'t}a_{j}S_{01}^{+}+h.c.\right) \\ & +\sum_{k\neq l;k,l=1}^{n}g_{kl}\left(e^{i\Delta_{kl}t}a_{k}a_{l}^{+}+h.c.\right), \end{aligned} \tag{8}$$

where Ω_{10} is the pulse Rabi frequency associated with the $|0\rangle \leftrightarrow |1\rangle$ transition, and $\Delta_{\mu w} = \omega_{10} - \omega$ is the detuning between the pulse frequency ω and the $|0\rangle \leftrightarrow |1\rangle$ transition frequency ω_{10} [Fig. 3(c)].

(iii) The last one requires that during the operation of step n (the final step operation above), a microwave pulse (with frequency $\omega = \omega_{20}$ and initial phase φ) is applied to qutrit A while each cavity is decoupled from qutrit A. The interaction Hamiltonian governing this basic transformation is given by

$$H_{I,3} = \Omega_{20}\left(e^{-i\varphi}S_{02}^{+}+h.c.\right)+\varepsilon, \tag{9}$$

where ε is the sum of the last three terms of Eq. (8), $S_{02}^{+} = |2\rangle\langle 0|$, and the terms describing the pulse induced coherent $|0\rangle \leftrightarrow |1\rangle$ and $|1\rangle \leftrightarrow |2\rangle$ transitions are negligible because $\omega \gg \omega_{10}, \omega_{21}$ [Fig. 3(d)].

For each of the three basic types of transformation described above, the dynamics of the lossy system, composed of all cavities and qutrit A, is determined by

$$\begin{aligned} \frac{d\rho}{dt} \;=\; & -i\left[H_{I},\rho\right]+\sum_{i=1}^{n}\kappa_{i}\mathcal{L}\left[a_{i}\right]+\left\{\gamma_{\varphi,21}\left(S_{21}^{z}\rho S_{21}^{z}-\rho\right)+\gamma_{21}\mathcal{L}\left[S_{21}^{-}\right]\right\} \\ & +\left\{\gamma_{\varphi,20}\left(S_{20}^{z}\rho S_{20}^{z}-\rho\right)+\gamma_{20}\mathcal{L}\left[S_{20}^{-}\right]\right\}+\left\{\gamma_{\varphi,10}\left(S_{10}^{z}\rho S_{10}^{z}-\rho\right)+\gamma_{10}\mathcal{L}\left[S_{10}^{-}\right]\right\}, \end{aligned} \tag{10}$$

where H_I is the $H_{I,1}$, $H_{I,2}$ or $H_{I,3}$ above, $\mathcal{L}\left[a_i\right] = a_i\rho a_i^{+} - a_i^{+}a_i\rho/2 - \rho a_i^{+}a_i/2$, $\mathcal{L}\left[S_{ij}^{-}\right] = S_{ij}^{-}\rho S_{ij}^{+} - S_{ij}^{+}S_{ij}^{-}\rho/2 - \rho S_{ij}^{+}S_{ij}^{-}/2$ $(ij = 21, 20, 10)$, $S_{21}^{z} = |2\rangle\langle 2| - |1\rangle\langle 1|$, $S_{20}^{z} = |2\rangle\langle 2| - |0\rangle\langle 0|$, and $S_{10}^{z} = |1\rangle\langle 1| - |0\rangle\langle 0|$. In addition, κ_i is the decay rate of the mode of cavity i, $\gamma_{\varphi,21}$ $(\gamma_{\varphi,20})$ and γ_{21} (γ_{20}) are the dephasing rate and the energy relaxation rate of the level $|2\rangle$ of qutrit A for the decay path $|2\rangle \to |1\rangle$ $(|0\rangle)$, respectively and $\gamma_{\varphi,10}$ and γ_{10} are those of the level $|1\rangle$ for the decay path $|1\rangle \to |0\rangle$. The fidelity of the operation is given by

$$\mathcal{F} = \langle\psi_{id}|\,\widetilde{\rho}\,|\psi_{id}\rangle, \tag{11}$$

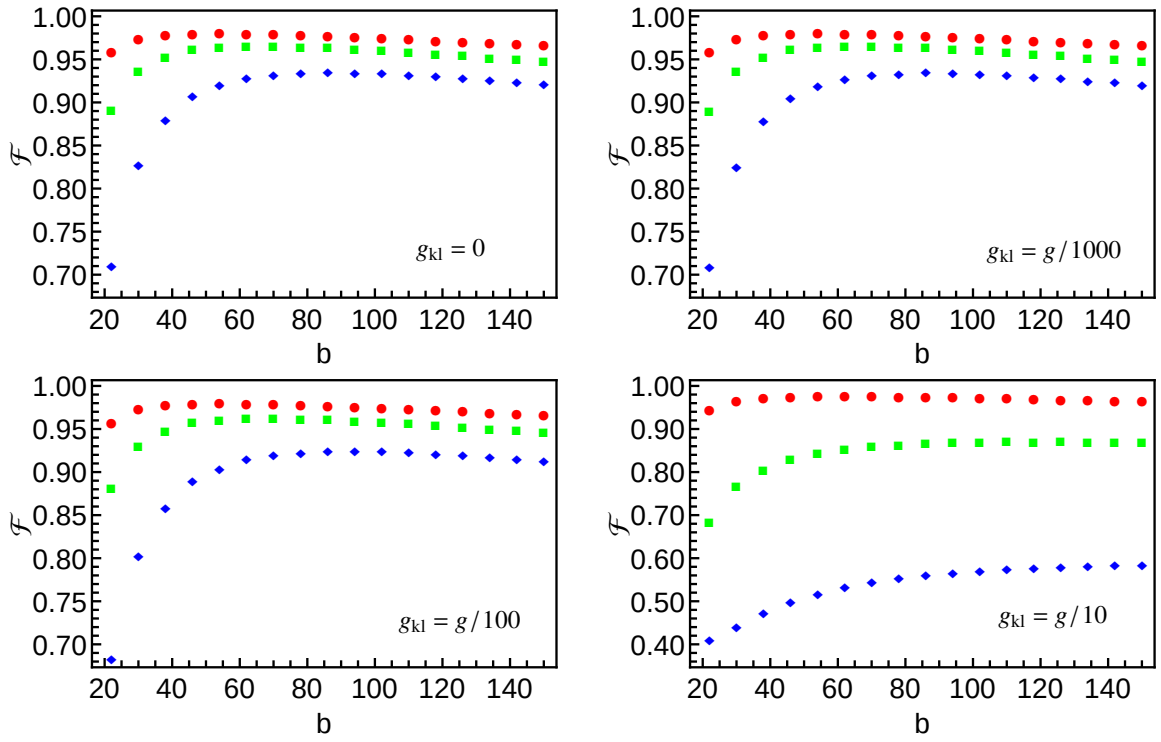

FIG. 4: (Color online) Fidelity versus $b = \Delta/g'$. Refer to the text for the parameters used in the numerical calculation. Here, g_{kl} is the coupling strength between cavities k and l ($k \neq l$; and $k, l = 1, 2, 3, 4$), which are taken to be the same for simplicity. In each figure, the red, green, and blue lines correspond to $n = 2, 3$, and 4, respectively.

where $|\psi_{id}\rangle$ is the state (3) of an ideal system (i.e., without dissipation, dephasing, and crosstalks) and $\widetilde{\rho}$ is the final density operator of the system when the operation is performed in a realistic physical system.

We now numerically calculate the fidelity of the prepared GHZ state of photons in up to four cavities. Without loss of generality, let us consider a phase qutrit with three levels in the metastable potential well, for which $\omega_{10}/2\pi \sim 6.8$ GHz and $\omega_{21}/2\pi \sim 6.3$ GHz [33]. The frequency $\omega_{c,i}/2\pi$ of the *active* cavity i ($i = 1, 2, 3, 4$) is thus ~ 6.3 GHz, resulting in $\Delta/2\pi \sim 500$ MHz. For the *idling* cavity j ($j = 1, 2, 3, 4$), we choose $\widetilde{\omega}_{c,j}/2\pi \sim 5.6$ GHz [47], which leads to $\Delta_j/2\pi \sim 700$ MHz and $\Delta'_j/2\pi \sim 1.2$ GHz. For the phase qutrit here, one has $g_i \sim \sqrt{2}g'_i$, $\widetilde{g}_j \sim \sqrt{2}\widetilde{g}'_j$ and $\widetilde{g}_j \sim g_i\sqrt{\widetilde{\omega}_{c,j}/\omega_{c,j}}(i, j = 1, 2, 3, 4)$. For simplicity, assume that $g_1 = g_2 = g_3 = g_4 \equiv g$ and thus $g'_1 = g'_2 = g'_3 = g'_4 \equiv g'$. Other parameters used in the

numerical calculation are as follows: (i) $\Delta_{\mu w}/2\pi = 500$ MHz, $\Omega_{21} \sim \sqrt{2}\Omega_{10}$, $\Omega_{10}/2\pi = 50$ MHz, and $\Omega_{20}/2\pi = 200$ MHz (which is available in experiments [48]), (ii) $\gamma_{\varphi,21}^{-1} = \gamma_{\varphi,20}^{-1} = \gamma_{\varphi,10}^{-1} = 5$ μs, $\gamma_{21}^{-1} = 25$ μs, $\gamma_{20}^{-1} = 200$ μs [49], $\gamma_{10}^{-1} = 50$ μs, $\kappa_1^{-1} = \kappa_2^{-1} = \kappa_3^{-1} = \kappa_4^{-1} = 20$ μs. For the parameters chosen here, the fidelity versus $b \equiv \Delta/g'$ is shown in Fig. 4, from which one can see that for $b = 50$, 60 and 85, a high fidelity $\sim 98\%$, 97%, and 93% can be respectively achieved for $n = 2$, 3, and 4 when $g_{kl} \leq g/100$ ($k \neq l$; and $k, l = 1, 2, 3, 4$). Interestingly, it is noted from Fig. 4 that the effect of direct coupling between cavities on the fidelity of the prepared GHZ states is negligible when the inter-cavity coupling strength (g_{kl}) is smaller than g by two orders of magnitude. This condition, $g_{kl}/g \leq 0.01$, is not difficult to satisfy with typical capacitive cavity-qutrit coupling illustrated in Fig. 2(b). In this case, because very little field could leak out of each cavity it can be shown that as long as the cavities are physically well separated, the inter-cavity crosstalk coupling strength is $g_{kl} \approx g(C_c/C_\Sigma)$, where $C_c \sim 1$ fF and $C_\Sigma = nC_c + C_q \sim 10^2$ fF are the typical value of the cavity-qutrit coupling capacitance and the sum of all coupling capacitance and qutrit self capacitance, respectively. Therefore, it is straightforward to implement designs with sufficiently weak direct inter-cavity couplings.

Let us focus on the case of four cavities. For $b = 85$, we have $g/2\pi \sim 8.3$ MHz, $g'/2\pi \sim 5.9$ MHz, $\widetilde{g}_j/2\pi \sim 7.8$ MHz, and $\widetilde{g}'_j \sim 5.5$ GHz ($j = 1, 2, 3, 4$). Note that a qutrit-cavity coupling constant $g/2\pi \sim 220$ MHz can be reached for a superconducting qutrit coupled to a one-dimensional standing-wave CPW (coplanar waveguide) resonator [30], and that T_1' and T_2' can be made to be a few tens of μs for the state of art superconducting qutrits at the present time [50]. For the cavity resonant frequency ~ 6.3 GHz chosen here and for the $\kappa_1^{-1}, \kappa_2^{-1}, \kappa_3^{-1}$, κ_4^{-1} used in the numerical calculation, the required quality factor for the four cavities is $Q \sim 7.9 \times 10^5$. Note that superconducting CPW resonators with a loaded quality factor $Q \sim 10^6$ have been experimentally demonstrated [51,52], and planar superconducting resonators with internal quality factors above one million ($Q > 10^6$) have also been reported recently [53]. Our analysis given here demonstrates that preparation of the GHZ state of photons in up to four cavities is feasible within the present circuit QED technique.

Before ending this subsection, we point out that the non-monotonic dependence of fidelity $\mathcal{F}$ on the dimensionless parameter b observed in Fig. 4 are essentially an artifact of the numerical procedure. In our numerical calculation, b is on the increase by keeping the detuning Δ (~ 500 MHz) constant while reducing g' which corresponds to decreasing the

qutrit-cavity coupling capacitance C_c. Since the ratio g/g' is determined by the qutrit's level structure and thus remains constant irrespective the value of coupling capacitance C_c, the protocol would thus take a longer time to complete as g', and thus g, is reduced to a value below which the adverse effects of cavity decay and qutrit decoherence take over.

C. Discussion

In principle, the method presented above can be used to create a GHZ state of n photons in n cavities. However, it should be pointed out that in the solid-state setup scaling up to many cavities coupled to a single superconducting qutrit will introduce new challenges. For instance, the coupling constant between the coupler qutrit A and each cavity decreases as the number of cavities increases. As a result, the operation becomes slower and thus decoherence, caused due to qutrit-environment interaction and/or cavity decay, may become a severe problem. Since g_i is inversely proportional to n, the number of cavities coupled to qutrit A may be limited to about 4 to 6 to maintain sufficiently strong qutrit-cavity couplings.

Tunable resonators usually come with a non-linearity [54,55]. Details on how to tune the frequency of a resonator can be found in Refs. [54,55]. We remark that how to tune frequency of a resonator is not the main focus of this paper, which is beyond the scope of this theoretical work. In addition, the energy relaxation time of qutrit A can be shortened by the Purcell decay of the resonators, which however can be made negligible with a high-Q resonator [56]. A detailed discussion on this issue is out of the scope of this work.

It should be mentioned that three-level superconducting qutrits were earlier used for quantum operations within cavity QED [10,18,19]. We stress that the present work is quite different from the previous one [33]. As discussed in [33], the NOON state of the two resonators was created by first preparing a Bell state of two superconducting qutrits (connecting to the two resonators separately) and then swapping the prepared Bell state of the two qutrits to the two resonators. Thus, if the protocol in [33] is applied to generate a GHZ state of n cavities, one will need to first prepare a GHZ state of n superconducting qubits (each connecting to a resonator) and then swap the prepared GHZ state of the n qubits to the n cavities. However, as shown above, prior preparation of a GHZ state of n superconducting qubits is not required by the present proposal. Moreover, by using the protocol in [33] to implement the current task, n superconducting qubits are required; while only a coupler qutrit A is needed by the present proposal.

III. GENERATION OF A N-PHOTON GHZ STATE IN THE N CAVITIES USING AN ATOM

During the past decade, much attention has been paid to the generation of highly entangled states with atomic systems. Two-atom entangled states and three-particle GHZ entangled states (with two atoms plus one cavity mode) have been experimentally demonstrated in microwave cavity QED [4,5]. In addition, based on cavity QED, numerous theoretical proposals have been presented for entangling atoms coupling to the mode (s) of a single cavity [57] and atoms in two or more cavities [58]. In principle, an entangled state of n photons in n cavities ($n \geq 2$) can be created, by first preparing an n-atom entangled state using the previous proposals [57,58], and then transferring the prepared n-atom entangled states onto n photons in the n cavities via the state transfer from an atom to a photon in a cavity. In the following, we will present an alternative way to implement an n-photon GHZ state, which, as shown below, does not require prior preparation of atomic entangled states. The scheme presented here is actually a generalization of the method described in Sec. II to GHZ-state generation of photons in multiple cavities through an atom.

Consider n identical cavities $(1, 2, ..., n)$ and an atom A with three levels as depicted in Fig. 1. The atom A is initially prepared in the state $(|0\rangle + |2\rangle)/\sqrt{2}$ and each cavity is in a vacuum state, i.e., $|0\rangle_{c,i}$ for cavity i ($i = 1, 2, ..., n$). In addition, assume that the cavity mode of each cavity is resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition but highly detuned (decoupled) from the transition between any other two levels of the atom A. The procedure for generating a GHZ state of n photons in the n cavities is illustrated in Fig. 3. The total operation time τ is given in Eq. (4), in which τ_d is now a typical time for moving atom A into or out of a cavity. The number of cavities to be prepared in an entangled state is limited by the decay of atom A and decay of each cavity.

The present scheme has the following advantages: (i) Only one atom is needed; (i) Neither measurement on the states of the atom A nor measurement on the cavity photons is needed; (ii) No adjustment of the atomic level spacings or the cavity mode frequency is needed during the entire operation.

We should mention that the atom-cavity interaction time can be tuned by changing the atomic velocity in the case when the atom A is sent through each cavity [59]. In addition, it can be tuned by controlling the duration of the atom in each cavity, for the case when the

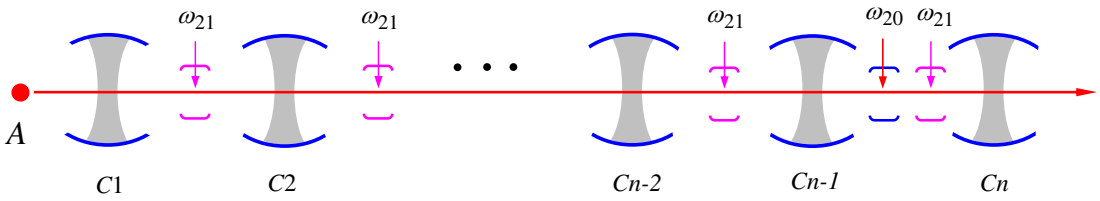

FIG. 5: (Color online) Diagram of n identical cavities and an atom A (a red dot). The atom A is sent through or moved into each cavity for an interaction time $\pi/(2g)$. Before arriving in cavity $n-1$, the atom A is addressed by a classical pulse (with frequency $\omega = \omega_{21}$, initial phase π, and duration $\pi/(2\Omega_{21})$) after it leaves each cavity (see the pink-color frame with an arrow). When the atom A exits the cavity $n-1$, two pulses are applied to it. The first pulse has frequency $\omega = \omega_{20}$, initial phase $-\pi/2$, and duration $\pi/(2\Omega_{20})$ (see the blue-color frame with an arrow) while the second pulse has frequency $\omega = \omega_{21}$, initial phase $\pi/2$, and duration $\pi/(2\Omega_{21})$. Here, g is the coupling constant between the cavity mode and the $|1\rangle \leftrightarrow |2\rangle$ transition of the atom A; ω_{20} and ω_{21} are the $|2\rangle \leftrightarrow |0\rangle$ transition frequency and the $|2\rangle \leftrightarrow |1\rangle$ transition frequency of the atom A, respectively. In addition, Ω_{21} (Ω_{20}) is the Rabi frequency of the pulse associated with the $|1\rangle \leftrightarrow |2\rangle$ transition ($|0\rangle \leftrightarrow |2\rangle$ transition) of the atom A.

atom is loaded into or out of a cavity by trapping the atom in a linear trap [60], inside an optical lattice [61], or on top of an atomic chip [62]. Note that the approach for trapping and moving atoms into or out of a cavity has been employed in the earlier work for quantum computing with atoms in cavity QED [63-66].

To investigate the experimental feasibility of this scheme, let us consider preparation of a GHZ state for 10 photons in ten cavities using a single Rydberg atom. The atom A is chosen as a Rydberg atom with principal quantum numbers 50 and 51 (respectively corresponding to the levels $|1\rangle$ and $|2\rangle$). For the Rydberg atom chosen here, the $|1\rangle \leftrightarrow |2\rangle$ transition frequency is $\omega_{21}/2\pi \sim 51.1$ GHz [67], the coupling constant is $g = 2\pi \times 50$ KHz [68], the energy relaxation time of the level $|2\rangle$ is $T_r \sim 3 \times 10^{-2}$ s [69], and the dephasing time $T_\varphi \sim 10^{-3}$ s of the level $|2\rangle$ can be reached in the present experiment [70]. With the choice of $t_d \sim 1\mu$s and $\Omega_{21} \sim \Omega_{20} \sim 10g$, we have $\tau \sim 7.5 \times 10^{-5}$ s $\ll T_r, T_\varphi$.

In the present case, the mode frequency of each cavity is ~ 51.1 GHz. One can see from the above discussion that each cavity was occupied by a single photon during the GHZ-state

preparation. For a cavity with $Q = 10^{10}$, we have $\min\{T_{cav}^1, T_{cav}^2, ..., T_{cav}^{10}\} \sim 3.1 \times 10^{-2}$ s, resulting in $T_{cav} \sim 3.1 \times 10^{-3}$ s for $n = 10$, which is much longer than τ. Note that cavities with a high $Q \sim 3 \times 10^{10}$ was previously reported [71]. Thus, generating a GHZ state of 10 photons in ten cavities with assistance of an atom is possible within the present cavity QED technique.

By Using *linear optics elements* and *single photon detectors*, many schemes for creating entangled multi-photon states have also been proposed [72]; and experimental realization of an eight-photon GHZ state [1] and a three-photon W state [73] has been reported. However, this type of approaches is much more difficult to implement than cavity QED for hybrid systems consisting of photons and matter qubits of nature made and/or engineered. The present work represents a significant advancement in circuit and atom QED because it provides a simple and fast approach for deterministically creating a multi-photon GHZ state, which needs only a single coupler qubit and does not require measurement or detection on photons.

We noticed that two previous works [74,75] are relevant to ours. Ref. [74] presents a scheme for preparation of a GHZ-type entangled *coherent state* of n cavities by having an atom interacts with each of the cavities dispersively and then measuring the state of the atom. We are aware of that a GHZ entangled Fock state of photons in multiple cavities can in principle be generated using the same procedure described in [74]. However, the method has the following drawbacks: (i) the operation is rather slow because of the dispersive atom-cavity interaction, (ii) a measurement on the state of the atom is required, and (iii) since the prepared GHZ state depends on the measurement outcome on the atomic states, the GHZ-state preparation is not deterministic. In contrast, our proposal mitigates these problems effectively: the operation is much faster because of the resonant atom-cavity interactions; there is no need to measure the state of the atom; and the generation of the GHZ state is deterministic. Ref. [75] proposes a method for preparing a cluster state of photons in n cavities via resonant atom-cavity interactions. However, our proposal is significantly different from that of [75]. First, we focus on preparing a GHZ entangled Fock state of photons in multiple cavities. Second, an n-qubit cluster state cannot be transformed into a GHZ state (for $n > 3$) [76]. Last, the method proposed in [75] requires an atom to interact with *two* classical pulses after it leaves each cavity (except the final one) while our proposal only requires the atom interacting with *one* classical pulse after it exits each cavity (except

the final one).

After a thorough search, we found that three schemes [77-79] were previously proposed for implementing the GHZ state of photons in n cavities by sending an atom through n cavities. However, these schemes require measuring the state of the atom and/or using n levels of the atom (i.e., the number of the atomic levels used needs to be equal to the number of the cavities).

Finally, our work is different from the previous one in [80], in which a matrix-product state (i.e., a generalized version of the GHZ state) was produced through sequential interaction between atomic and photonic qubits. In [80], the authors discussed how to create different entangled states of photons at the output of a cavity, while in our case we consider how to generate entangled states of photons among multiple cavities. In addition, the approach presented in [80] for creating entangled states of photonic qubits, which were encoded in both orthogonal polarization states and energy eigenstates, was based on adiabatic passage techniques. In contrast, as shown above, our present approach is based on resonant interaction.

IV. CONCLUSION

We have presented a method to generate a GHZ state of n photons in n cavities coupled by a superconducting qutrit. By local operations on a qubit (e.g., an atom etc.) placed in each cavity, the created GHZ states of photons can be transferred to qubits for the storage for a long time. This proposal is easy to be implemented in experiments since only resonant qutrit-cavity interaction and resonant qutrit-pulse interaction are needed, and no measurement is required. In addition, we have shown how to apply the present method to create a GHZ state of n photons in n cavities via an atom. We note that neither adjusting the atomic level spacings nor adjusting the cavity mode frequency is needed during the entire operation and only one atom is needed for the entanglement preparation of photons in multiple cavities. In addition, our analysis shows that generating a GHZ state of photons in up to four cavities by a coupler superconducting qutrit or a GHZ state of photons in ten cavities via an atom is possible within the present experimental technique. Finally, it should be mentioned that this proposal is quite general, which can be applied to create a GHZ state of photons in multiple cavities or resonators, when the coupler qutrit is a different physical system, such as a quantum dot or an NV center.

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[1] Y. F. Huang *et al.*, Nature Communications **2**, 546 (2011); X. C. Yao *et al.*, arXiv:1105.6318.

[2] H. Häffner *et al.*, Nature (London) **438**, 643 (2005).

[3] D. M. Greenberger, M. A. Horne, and A. Zeilinger, Going beyond Bells theorem. In Kafatos, M. (ed.) *Bells theorem, quantum theory and conceptions of the universe* (Kluwer Academic, Dordrecht, 1989).

[4] E. Hagley *et al.*, Phys. Rev. Lett. 79, 1 (1997); S. Osnaghi *et al.*, *ibid.* **87**, 037902 (2001).

[5] A. Rauschenbeutel *et al.*, Science **288**, 2024 (2000).

[6] G. Chen, N. H. Bonadeo, D. G. Steel, D. Gammon, D. S. Katzer, D. Park, and L. J. Sham, Science **289**, 1906 (2000).

[7] M. Steffen *et al.*, Science **313**, 1423 (2006).

[8] M. Neeley *et al.*, Nature (London) **467**, 570 (2010).

[9] G. Sun, X. Wen, B. Mao, J. Chen, Y. Yu, P. Wu, and S. Han, Nature Communications **1**, 51 (2010).

[10] C. P. Yang, Shih-I Chu, and S. Han, Phys. Rev. A **67**, 042311 (2003).

[11] J. Q. You and F. Nori, Phys. Rev. B **68**, 064509 (2003).

[12] A. Blais *et al.*, Phys. Rev. A **69**, 062320 (2004).

[13] M. A. Sillanpää, J. I. Park, and R. W. Simmonds, Nature (London) **449**, 438 (2007).

[14] J. Majer *et al.*, Nature (London) **449**, 443 (2007).

[15] C. P. Yang, Shih-I Chu, and S. Han, Phys. Rev. Lett. **92**, 117902 (2004).

[16] A. Wallraff *et al.*, Nature (London) **431**, 162 (2004).

[17] I. Chiorescu *et al.*, Nature (London) **431**, 159 (2004).

[18] C. P. Yang and S. Han, Phys. Rev. A **70**, 062323 (2004).

[19] C. P. Yang, Shih-I Chu, and S. Han, J. Phys.: Condens. Matter **16**, 1907 (2004).

[20] S. L. Zhu, Z. D. Wang, and P. Zanardi, Phys. Rev. Lett **94**, 100502 (2005).

[21] K. H. Song, Z. W. Zhou, and G. C. Guo, Phys. Rev. A **71**, 052310 (2005)

[22] X. L. Zhang, K. L. Gao, and M. Feng, Phys. Rev. A **74**, 024303 (2006);

[23] Z. J. Deng, K. L. Gao, and M. Feng, Phys. Rev. A **74**, 064303 (2006).

[24] K. H. Song, S. H. Xiang, Q. Liu, and D. H. Lu, Phys. Rev. A **75**, 032347 (2007).

[25] C. P. Yang, Phys. Rev. A **83**, 062302 (2011).

[26] P. J. Leek *et al.*, Phys. Rev. B 79, 180511(R) (2009).

[27] L. DiCarlo *et al.*, Nature **460**, 240 (2009).

[28] M. Ansmann *et al.*, Nature **461**, 504 (2009)

[29] J. M. Chow *et al.*, arXiv:0908.1955.

[30] L. DiCarlo *et al.*, Nature (London) **467**, 574 (2010).

[31] M. Mariantoni *et al.*, Phys. Rev. B **78**, 104508 (2008).

[32] F. W. Strauch *et al.*, Phys. Rev. Lett. **105**, 050501 (2010).

[33] H. Wang *et al.*, Phys. Rev. Lett. **106**, 060401 (2011).

[34] M. Hillery, V. Buzek, and A. Berthiaume, Phys. Rev. A **59**, 1829 (1999).

[35] S. Bose, V. Vedral, and P. L. Knight, Phys. Rev. A **57**, 822 (1998).

[36] R. Cleve, D. Gottesman, and H. K. Lo, Phys. Rev. Lett. **83**, 648 (1999); V. Scarani and N. Gisin, *ibid.* **87**, 117901 (2001); G. A. Durkin, C. Simon, and D. Bouwmeester, *ibid.* **88**, 187902 (2002).

[37] C. P. Yang, Shih-I Chu, and S. Han, Phys. Rev. A **70**, 022329 (2004)

[38] D. P. DiVincenzo and P. W. Shor, Phys. Rev. Lett. **77**, 3260 (1996); J. Preskill, Proc. R. Soc. London, Ser. A **454**, 385 (1998).

[39] J. J. Bollinger, W. M. Itano, D. J. Wineland, and D. J. Heinzen, Phys. Rev. A **54**, 4649 (1996); S. F. Huelga, C. Macchiavello, T. Pellizzari, A. K. Ekert, M. B. Plenio, and J. I. Cirac, Phys. Rev. Lett. **79**, 3865 (1997).

[40] J. Clarke and F. K. Wilhelm, Nature (London) **453**, 1031 (2008); P. R. Johnson *et al.*, Phys. Rev. Lett. **94**, 187004 (2005).

[41] J. Koch, , T. M. Yu, J. Gambetta, A. A. Houck, D. I. Schuster, J. Majer, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A **76**, 042319 (2007); A. A. Houck, J. Koch, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Quant. Inf. Proc. **8**, 105 (2009).

[42] I. Chiorescu *et al.*, Science **299**, 1869 (2003); J. Q. You, X. D. Hu, S. Ashhab, and F. Nori,

Phys. Rev. B **75**, 140515(R) (2007); C. P. Yang and S. Han, **72**, 032311 (2005).

[43] S. Han, J. Lapointe, and J. E. Lukens, Single-Electron Tunneling and Mesoscopic Devices (Springer-Verlag, Berlin-Heidelberg, 1991), Vol. 31, pp. 219-222.

[44] J. Clarke and F. K.Wilhelm, Nature (London) **453**, 1031 (2008).

[45] M. Neeley *et al.*, Nature Phys. **4**, 523 (2008).

[46] J. Q. You and F. Nori, Phys. Today **58**, 42 (2005).

[47] M. Sandberg, C.M.Wilson, F. Persson, T. Bauch, G. Johansson, V. Shumeiko, T. Duty, and P. Delsing, Appl. Phys. Lett. **92**, 203501 (2008).

[48] M. Baur *et al.*, Phys. Rev. Lett. **102**, 243602 (2009).

[49] For a phase qubit with the three levels considered, the $|0\rangle \leftrightarrow |2\rangle$ dipole matrix element is much smaller than that of the $|0\rangle \leftrightarrow |1\rangle$ and $|1\rangle \leftrightarrow |2\rangle$ transitions. Thus, $\gamma_{20}^{-1} \gg \gamma_{10}^{-1}, \gamma_{21}^{-1}$.

[50] J. Bylander *et al.*, Nature Phys. **7**, 565 (2011); J. M. Chow *et al.*, arXiv:1202.5344.

[51] W. Chen, D. A. Bennett, V. Patel, and J. E. Lukens, Supercond. Sci. Technol. **21**, 075013 (2008).

[52] P. J. Leek *et al.*, Phys. Rev. Lett. **104**, 100504 (2010).

[53] A. Megrant *et al.*, arXiv:1201.3384.

[54] F. R. Ong *et al.*, Phys. Rev. Lett. **106**, 167002 (2011).

[55] J. R. Johansson, G. Johansson, C. M. Wilson, and F. Nori, Phys. Rev. Lett. **103**, 147003 (2009).

[56] M. D. Reed *et al.*, Appl. Phys. Lett. **96**, 203110 (2010).

[57] J. I. Cirac and P. Zoller, Phys. Rev. A **50**, R2799 (1994); C. C. Gerry, *ibid.* **53**, 2857 (1996); M. B. Plenio, S. F. Huelga, A. Beige, and P. L. Knight, *ibid.* **59**, 2468 (1999); A. S. Søensen and Klaus Mømer, *ibid.* **66**, 022314 (2002); G. P. Guo, C. F. Li, J. Li, and G. C. Guo, *ibid.* **65**, 042102 (2002); Y. F. Xiao, X. B. Zou, and G. C. Guo, *ibid.* **75**, 012310 (2007); L. M. Duan and H. J. Kimble, *ibid.* **90**, 253601 (2003); S. B. Zheng, Phys. Rev. Lett. **87**, 230404 (2001); Phys. Rev. A **66**, 060303 (2002).

[58] J. I. Cirac, P. Zoller, H. J. Kimble, and H. Mabuchi, Phys. Rev. Lett. **78**, 3221 (1997); S. J. van Enk, J. I. Cirac, and P. Zoller, *ibid.* **78**, 4293 (1997); L. M. Duan and H. J. Kimble, *ibid.* **90**, 253601 (2003); X. L. Feng, Z. M. Zhang, X. D. Li, S. Q. Gong, and Z. Z. Xu, *ibid.* **90**, 217902 (2003); P. Xue, C. Han, B. Yu, X. M. Lin, and G. C. Guo, Phys. Rev. A **69**, 052318 (2004); J. H. An, M. Feng, and C. H. Oh, *ibid.* **032303** (2009); X. L. Zhang, K. L.

Gao, and M. Feng, *ibid.* **75**, 034308 (2007); P. Peng and F. L. Li, *ibid.* **75**, 062320 (2007); J. Lee, J. Park, S. M. Lee, H. W. Lee, and A. H. Khosa, *ibid.* **77**, 032327 (2008); J. Cho, D. G. Angelakis, S. Bose , *ibid.* **78**, 022323 (2008); S. B. Zheng, C. P. Yang, and F. Nori, *ibid.* **82**, 042327 (2010).

[59] J. I. Cirac and P. Zoller, Phys. Rev. A **50**, R2799 (1994); C. C. Gerry, *ibid.* **53**, 2857 (1996); S. B. Zheng, Phys. Rev. Lett. **87**, 230404 (2001).

[60] G.R. Guthöhrlein, M. Keller, K. Hayasaka, W. Lange, H. Walther, Nature **414**, 49 (2001).

[61] S. Kuhr, W. Alt, D. Schrader, M. Müller, V. Gomer, D. Meschede, Science **293**, 278 (2001); J. Cho, Phys. Rev. Lett. **99**, 020502 (2007); H. Zoubi, H. Ritsch, Phys. Rev. A **76**, 013817 (2007); D. Jaksch, quant-ph/0407048.

[62] P. Horak, B.G. Klappauf, A. Haase, R. Folman, J. Schmiedmayer, P. Domokos, E.A. Hinds, Phys. Rev. A **67**, 043806 (2003).

[63] A. Beige, D. Braun, B. Tregenna, P.L. Knight, Phys. Rev. Lett. **85**, 1762 (2000).

[64] B. Tregenna, A. Beige, P. L. Knight, Phys. Rev. A **65** 032305 (2002).

[65] A. Beige, H. Cable, P. L. Knight, Proc. SPIE **5111** 370 (2003).

[66] L. M. Duan, B. Wang, and H. J. Kimble, Phys. Rev. A **72**, 032333 (2005).

[67] M. Brune, E. Hagley, J. Dreyer, X. Maître, A. Maali, C. Wunderlich, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **77** 4887 (1996).

[68] S. Osnaghi, P. Bertet, A. Auffeves, P. Maioli, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **87**, 037902 (2001).

[69] J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. **73**, 565 (2001).

[70] M. Brune, private communication.

[71] G. Rempe, F. SchmidtKaler, and H. Walther, Phys. Rev. Lett. **64**, 2783 (1990).

[72] H. Lee, P. Kok, N. J. Cerf, and J. P. Dowling, Phys. Rev. A **65**, 030101(R) (2002); J. Fiurásek **65**, 053818 (2002); X. B. Zou, K. Pahlke, and W. Mathis, *ibid.* **66**, 014102 (2002); *ibid.* **66**, 044302 (2002); T. Yamamoto, K. Tamaki, M. Koashi, and N. Imoto, *ibid.* **66**, 064301 (2002); H. Jeong and N. Ba An, *ibid.* **74**, 022104 (2006); Y. Sagi, *ibid.* **68**, 042320 (2003).

[73] M. Eibl *et al.*, Phys. Rev. Lett. **92**, 077901 (2004).

[74] C. C. Gerry, Phys. Rev. A **54**, R2529 (1996).

[75] X. B. Zou and W. Mathis, Phys. Rev. A **72**, 013809 (2005).

[76] H. J. Briegel and R. Raussendorf, Phys. Rev. Lett. **86**, 910 (2001).

[77] Z. M. Zhan, J. H. Li, W. X. Yang, and W. B. Li, Commun. Theor. Phys. **45**, 151 (2006).

[78] Z. H. Chen, Z. B. Yang, C. G. Liao, and C. L. Luo, Opt. Commun. **284**, 297 (2011).

[79] S. B. Zheng, Chin. J. Laser B **7**, No. 6, 534 (1998).

[80] C. Schön, K. Hammerer, M. M. Wolf, J. I. Cirac, and E. Solano, Phys. Rev. A **75**, 032311 (2007).