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Topological Superfluid Transition Induced by Periodically Driven Optical Lattice

Guocai Liu,[1] Ningning Hao,[2,3] Shi-Liang Zhu,[4,5] and W. M. Liu[2]

[1]*School of Science, Hebei University of Science and Technology, Shijiazhuang 050018, China*
[2]*Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Science, Beijing 100190, China*
[3]*Department of Physics, Purdue University, West Lafayette, Indiana 47907, USA*
[4]*Laboratory of Quantum Information Technology, School of Physics and Telecommunication Engineering, South China Normal University, Guangzhou, China*
[5]*Center for Quantum Information, IIIS, Tsinghua University, Beijing, China*

We propose a scenario to create topological superfluid in a periodically driven two-dimensional square optical lattice. We study the phase diagram of a spin-orbit coupled s-wave pairing superfluid in a periodically driven two-dimensional square optical lattice. We find that a phase transition from a trivial superfluid to a topological superfluid occurs when the potentials of the optical lattices are periodically changed. The topological phase is called Floquet topological superfluid and can host Majorana fermions.

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I. INTRODUCTION

The study of topological phase transition has attracted much attention following the discovery of quantum Hall effect [1, 2] and has now extended into two- and three-dimensional toplogical insulators [3, 4]. This novel phase transition is closely related to the topological characteristics of the system as a whole and can not be interpreted by use of the Landau theory because there is not an order parameter to describe the transition. One remarkable feature linking to the topological phases is the gapless edge mode protected by the topological properties of the ground-state wave function, which is robust to the disorder [5] or other perturbations. Specially, the topological superconductors based on spin-orbit (SO) coupling are one particularly interesting class of systems that could host Majorana fermions which can be used for topological quantum computation. However, subject to the compounds' natural properties [6], we have to rely on serendipity in looking for topological materials in solid-state structures [7–10]. Nevertheless, the ultracold atoms system provides a more convenient research platform, where one can create various optical lattices, adjust the hoppings and the inter-atom interaction [11], even add impurities into optical lattice [12]. So, this flexible tunability gives rise to more oppertunities to engineer the lattice Hamiltonian and realize the novel topological phases in cold atom systems [13–16]. Furthermore, gauge field simulation proposed in theory [17–20] and realized successfully in experiment [21, 22] makes it a hot spot to study topological quantum states in cold atoms system [23–26].

In this paper, we show that periodically driven perturbations may give rise to a phase transition from a trivial superfluid to a topological one, which carries the hallmark with topological protected gapless edges on the boundaries of the system. Time-periodic dependent Hamiltonian can be described by Floquet's theorem, which is used to explain quantized adiabatic pumping phenomena [27–30]. Recently, it demonstrated that the phase transition from a superfluid to a Mott insulator in one-dimensional Bose-Hubbard model can be induced by a periodically driven optical lattice [31]. We extend this phase transition mechanism to explore the topological phase transition in a two-dimensional optical lattice. We study the phase diagram of a spin-orbit coupled s-wave pairing superfluid in a periodically driven two-dimensional (2D) square optical lattice. We find that a topological phase transition from a trivial superfluid to a topological superfluid can be induced in periodically modulated optical lattices. The topological phase is called Floquet topological superfluid [32–36] and can host Floquet Majorana fermions. It was proposed that a topological phase can be realized in a BCS s-wave superfluid of ultracold fermionic atoms in the presence of both a Rashba spin-orbit (SO) interaction and a large perpendicular Zeeman field [37–40]; however, the Rashba spin-orbit coupling and a large perpendicular Zeeman field are hard to be simultaneously realized for cold fermionic atoms [39, 40]. We will prove that if one replaces the Zeeman field by a periodically driven optical lattice [31], a spin-orbit coupled BCS s-wave superfluid will still allow a realization of topological superfluid through modifying the oscillating amplitude (or modulation strength) of optical lattice. Therefore, we provide an alternative method to create an important topological superfluid which can host Majorana fermions.

The paper is organized as follows: In Sec. II, we introduce the s-wave superfluid model in a square optical lattice in the presence of both a Rashba SO coupling and a periodically modulated optical lattice potential. A Zeeman-magnetic-field-like term will be derived under the first order approximation; In Sec. III we present a two-band approximation and explain the topological phase transition at the Γ point in the first Brillouin zone (BZ). At last, we give a brief summary in Sec. IV.

II. MODEL

The tight-binding Hamiltonian, which describes an s-wave superfluid of neutral fermionic atoms in a 2D optical square lattice, is given by

$$H(t) = H_0 + H_d(t), \tag{1}$$

where

$$H_0 = -t_0 \sum_{\langle ij \rangle} c_i^\dagger c_j - i\lambda \sum_{\langle ij \rangle} c_i^\dagger \left(\sigma \times \hat{\mathbf{d}}_{ij}\right)_z c_j + \mu \sum_i c_i^\dagger c_i + U \sum_i c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger c_{i\downarrow} c_{i\uparrow}, \tag{2}$$

and

$$H_d(t) = \mathbf{K}(t) \cdot \sum_i \mathbf{r}_i c_i^\dagger c_i. \tag{3}$$

Here t is the hopping amplitude between the nearest neighbor link $\langle i,j\rangle$, $c_i^\dagger{=}\left(c_{i\uparrow}^\dagger, c_{i\downarrow}^\dagger\right)$ with $c_{i\alpha}^\dagger$ ($c_{i\alpha}$) denoting the creation (annihilation) operator of a fermionic atom with pseudospin α (up or down) on lattice site i. The second term in Eq. (2) represents a Rashba SO coupling interaction which can be obtain by laser-induced-gauge-field method, λ is the coupling coefficient, σ are the Pauli matrices and $\hat{\mathbf{d}}_{ij}$ is a unit vector along the bond that connects site j to i. μ is the chemical potential and $U < 0$ denotes an on-site attractive interaction which is easy to obtain via an s-wave Feshbach resonance in cold atom system. The oscillating Hamiltonian H_d, with $\mathbf{K}(t) = K(\cos(\omega t), \sin(\omega t))$, mimics a monochromatic electric dipole potential with frequency ω and amplitude K. This term can be realized experimentally by periodically shifting the position of a mirror employed to generate the standing laser waves along x- and y-directions, and transforming to the comoving frame of reference [31]. We choose $t_0{=}1$ as the energy unit and the distance a between the nearest sites as the length unit throughout this paper. It was demonstrated in Ref. [37] that the Hamiltonian H_0 in a mean field approximation and combination with a perpendicular Zeeman field can support a topological superfluid. On the other hand, replaced the Hamiltonian H_0 with an one-dimensional Bose-Hubbard Hamiltonian, it was shown in Ref. [31] that a phase transition from a superfluid to a Mott insulator can be induced by H_d in its one-dimensional form.

Generally, for a quantum system with time-independent Hamitonian H, the solutions of time-dependent Schrodinger equation be called stationary states with real eigenvalues E. When a Hamiltonian of quantum system has a periodic dependence on time, i.e., $H(t){=}H(t+T)$ with period $T{=}2\pi/\omega$, the time-dependent Schrodinger equation can not be devided into two different parts according to spatial degree of freedom and temporal degree of freedom. The Hamiltonian, satisfying the discrete time translational symmetry, $t \to t+T$, can been described by Floquet's theorem [31, 41, 42]. Floquet's theorem tell us that the Schrödinger equation with time-periodic dependent Hamiltonian has a complete set of solutions with the form $|\psi_n(t)\rangle{=}|u_n(t)\rangle \exp(-i\varepsilon_n t/\hbar)$. Here, the characteristic exponent ε_n is called quasienergy. Just like that the quasimomentum $\mathbf{k}$ is used to characterizing the Bloch eigenstates in the spatially periodic crystal system, the quasienergy ε_n can characterize the Floquet states in a system with the time variable satisfying the translational symmetry, $t \to t+T$. The periodic function $|u_n(t)\rangle = |u_n(t+T)\rangle$, a same analogy to Bloch states, is called Floquet mode and satisfies the eigenvalue equation

$$[H(t) - i\hbar\partial_t]\, |u_n(t)\rangle = \varepsilon_n |u_n(t)\rangle. \tag{4}$$

We call $\mathcal{H}(t){=}H(t) - i\hbar\partial_t$ as the Floquet Hamiltonian. It is easy to notice that the Floquet modes $|u_n(t)\rangle \exp(im\omega t)$ is also the solution of Eq. (4) with the shifted quasienergy $\varepsilon_n + m\hbar\omega$. Here, $\hbar\omega$ is like the reciprocal lattice vector in $\mathbf{k}$ space and define the width of Brillouin zone with the sense of time. The integer $m = 0, \pm1, \pm2, \cdots$ indexes the different zone structure. Because of the coupling between spatial degree of freedom and temporal degree of freedom in this time-dependent system, it is convenient to introudce the Floquet basis

$$|\{n_i\}, m\rangle = |\{n_i\}\rangle \exp\left[-\frac{i}{\hbar\omega}\int_{-\infty}^{t} dt' \mathbf{K}(t') \cdot \sum_i \mathbf{r}_i n_{\mathbf{i}} + im\omega t\right], \tag{5}$$

with $|\{n_i\}\rangle$ indicating a Fock state with n_i particles on the ith site and m accounts for the zone structure [31], which consist of an extended Hilbert space of T-periodic functions with the scalar product given by

$$\langle\langle\cdot|\cdot\rangle\rangle = \frac{1}{T}\int_0^T dt\, \langle\cdot|\cdot\rangle, \tag{6}$$

i.e., by the usual scalar product $\langle\cdot|\cdot\rangle$ combined with time-averaging. Hence, the quasienergies are obtained by computing the matrix elements of the Floquet operator $H(t) - i\hbar\partial_t$ in the basis (5) with respect to the scalar product (6), and diagonalizing. By a straightforward calculation, we can obtain the matrix elements of some operators in Floquet Hamiltonian $\mathcal{H}(t)$:

$$\left\langle\langle\{n_i'\}, m'|\, c_{i\alpha}^\dagger c_{j\alpha'}\, |\{n_i\}, m\rangle\right\rangle = e^{-i(m'-m)\theta_{ij}} J_{m'-m}(z_{ij}), \tag{7}$$

$$\left\langle\langle\{n_i'\}, m'|\, c_{i\alpha}^\dagger c_{i\alpha}\, |\{n_i\}, m\rangle\right\rangle = n_{i\alpha}\delta_{m,m'}, \tag{8}$$

$$\left\langle\langle\{n_i'\}, m'|\, c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger c_{i\downarrow} c_{i\uparrow}\, |\{n_i\}, m\rangle\right\rangle = n_{i\uparrow} n_{i\downarrow}\delta_{m,m'}, \tag{9}$$

where $J_{m'-m}(z_{ij})$ is the Bessel function of the $(m'-m)$th order and $z_{ij}{=}\frac{K}{\hbar\omega}\sqrt{x_{ij}^2 + y_{ij}^2}$. Here, $x_{ij} = (\mathbf{r}_i)_x - (\mathbf{r}_j)_x$, $y_{ij} = (\mathbf{r}_i)_y - (\mathbf{r}_j)_y$ and $\tan\theta_{ij} = x_{ij}/y_{ij}$.

In the above matrix, the diagonal block of the Floquet Hamiltonian, $\mathcal{H}^{(mm)}$, is the n-photon sector, i.e., the subspace with n photons and the non-diagonal blocks $\mathcal{H}^{(m'm)}$ with $m' \neq m$ correspond to the interaction between different subspaces [34]. When the parameter K is relative small and the adiabatic condition, $t_0 \ll \hbar\omega$, is met, the admixture of neighboring photon sectors is negligible, which enables us to focus on the zero-photon sector. In this case, we can follow the calculations in Ref.[34] to show that, the driven system (1) behaves similar to the undriven system (2), but with the tunneling matrix element t_0 and the SO coupling λ of the latter being replaced by the effective matrix element $t_0 \sim t_0 J_0(z_{ij})$ and $\lambda \sim \lambda J_0(z_{ij})$, respectively.

Now, suppose that we enhance the modulation strength K, then we have to consider the coupling of other photon sectors. For simplify, we only consider coefficient of subspace with $n = 1$ photon on the subspace with $n = 0$ photon. When K is strong enough but still satisfy $z_{ij} << 1$, the system has the effective Hamiltonian [36]

$$\mathcal{H}_{eff} = \mathcal{H}^{(00)} + \frac{1}{\hbar\omega}\left[\mathcal{H}^{-1}, \mathcal{H}^{+1}\right]. \tag{10}$$

Here, $\mathcal{H}^{-1}$ ($\mathcal{H}^{+1}$) denotes the non-diagonal block with $m\prime - m = -1$ ($+1$) around the 0-photon sector. According to Eqs. (7-9), we can obtain

$$\begin{aligned}\mathcal{H}^{(00)} = &-t_0 \sum_{\langle ij \rangle} J_0(z_{ij})\, c_i^\dagger c_j - i\lambda \sum_{\langle ij \rangle} J_0(z_{ij})\, c_i^\dagger \left(\sigma \times \hat{\mathbf{d}}_{ij}\right)_z c_j \\ &+ \mu \sum_i c_i^\dagger c_i + \psi_s \sum_i \left(c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger + \text{H.c.}\right),\end{aligned} \tag{11}$$

$$\begin{aligned}\mathcal{H}^{-1} = &-t_0 \sum_{\langle ij \rangle} e^{i\theta_{ij}} J_{-1}(z_{ij})\, c_i^\dagger c_j \\ &- i\lambda \sum_{\langle ij \rangle} e^{i\theta_{ij}} J_{-1}(z_{ij})\, c_i^\dagger \left(\sigma \times \hat{\mathbf{d}}_{ij}\right)_z c_j,\end{aligned} \tag{12}$$

$$\begin{aligned}\mathcal{H}^{+1} = &-t_0 \sum_{\langle ij \rangle} e^{-i\theta_{ij}} J_{+1}(z_{ij})\, c_i^\dagger c_j \\ &- i\lambda \sum_{\langle ij \rangle} e^{-i\theta_{ij}} J_{+1}(z_{ij})\, c_i^\dagger \left(\sigma \times \hat{\mathbf{d}}_{ij}\right)_z c_j.\end{aligned} \tag{13}$$

In the derivation of Eq. (11), we have made a mean field approximation and ψ_s is the gap function.

By using the Fourier transform of atomic operators $c_{i\sigma}^\dagger$, i.e.,

$$c_{i\sigma}^\dagger = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger e^{-i\mathbf{k}\cdot\mathbf{R}_i}, \tag{14}$$

the Hamiltonian (10) in square lattice system can be rewritten in the momentum space as

$$\mathcal{H}_{eff} = \sum_{\mathbf{k}} \psi_{\mathbf{k}}^+ (\mathcal{H}_{eff}(\mathbf{k}) \psi_{\mathbf{k}}, \tag{15}$$

where we have defined the four-component basis operator $\psi_{\mathbf{k}} = (c_{\mathbf{k}\uparrow}, c_{\mathbf{k}\downarrow}, c_{-\mathbf{k}\uparrow}^\dagger, c_{-\mathbf{k}\downarrow}^\dagger)^{\mathrm{T}}$. The effective Hamiltonian in momentum space is given by

$$\mathcal{H}_{eff}(\mathbf{k}) = \begin{pmatrix} \varepsilon_{\mathbf{k}} - \Gamma(\mathbf{k}, z) & 2\lambda J_0(z)\,\alpha(\mathbf{k}) & 0 & \psi_s \\ 2\lambda J_0(z)\,\alpha^*(\mathbf{k}) & \varepsilon_{\mathbf{k}} + \Gamma(\mathbf{k}, z) & -\psi_s & 0 \\ 0 & -\psi_s & -\varepsilon_{-\mathbf{k}} + \Gamma(\mathbf{k}, z) & 2\lambda J_0(z)\,\alpha^*(\mathbf{k}) \\ \psi_s & 0 & 2\lambda J_0(z)\,\alpha(\mathbf{k}) & -\varepsilon_{-\mathbf{k}} - \Gamma(\mathbf{k}, z) \end{pmatrix}, \tag{16}$$

where $\alpha(\mathbf{k}) = \sin k_y + i \sin k_x$, $z = \frac{Ka}{\hbar\omega}$, $\Gamma(\mathbf{k}, z) = \frac{16\lambda^2 J_{+1}(z) J_{-1}(z)}{\hbar\omega} \cos k_x \cos k_y$, and $\varepsilon_{\mathbf{k}} = -2t_0 J_0(z)(\cos k_x + \cos k_y) - \mu$. Following the method outlined in Ref. [37], one can obtain a "dual" Hamiltonian

$$\mathcal{H}^D(\mathbf{k}) = \begin{pmatrix} \psi_s - \Gamma(\mathbf{k}, z) & 2\lambda J_0(z)\,\alpha(\mathbf{k}) & 0 & -\varepsilon_{\mathbf{k}} \\ 2\lambda J_0(z)\,\alpha^*(\mathbf{k}) & -\psi_s + \Gamma(\mathbf{k}, z) & \varepsilon_{\mathbf{k}} & 0 \\ 0 & \varepsilon_{\mathbf{k}} & \psi_s + \Gamma(\mathbf{k}, z) & -2\lambda J_0(z)\,\alpha^*(\mathbf{k}) \\ -\varepsilon_{\mathbf{k}} & 0 & -2\lambda J_0(z)\,\alpha(\mathbf{k}) & -\psi_s - \Gamma(\mathbf{k}, z) \end{pmatrix}, \tag{17}$$

where the unitary transformation $\mathcal{H}^D(\mathbf{k}) = D H_{eff} D^\dagger$ with

$$D = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}.$$

It is easy to obtain the eigenvalues of Eq. (17) with

$$E_1=-\sqrt{f_1+f_2};E_2=-\sqrt{f_1-f_2},$$
$$E_3=+\sqrt{f_1-f_2};E_4=+\sqrt{f_1+f_2},$$

where we have defined

$$f_1=\Gamma^2(\mathbf{k},z)+\left(\varepsilon_{\mathbf{k}}^2+\psi_s^2\right)+4J_0^2(z)\lambda^2\left|\alpha(\mathbf{k})\right|^2,$$
$$f_2=2\sqrt{\Gamma^2(\mathbf{k},z)\left(\varepsilon_{\mathbf{k}}^2+\psi_s^2\right)+4\lambda^2J_0^2(z)\varepsilon_{\mathbf{k}}^2\left|\alpha(\mathbf{k})\right|^2}.$$

It is obvious that if only $\psi_s\neq 0$, i.e., the system lies in the superfluid phase, the energy levels E_1 and E_4, denoting the lowest and the highest band, will not touch each other. Next, we discuss the levels E_1 and E_2 (or E_3 and E_4). These two levels can touch each other if only the following two relations

$$\Gamma^2(\mathbf{k},z)=0$$

and

$$J_0^2(z)\varepsilon_{\mathbf{k}}^2\left(\sin^2k_x+\sin^2k_y\right)=0.$$

are simultaneously satisfied. From $\Gamma^2(\mathbf{k},z)=0$, we have $k_x=\pm\frac{\pi}{2}$ or $k_y=\pm\frac{\pi}{2}$ in the first BZ. So, when $J_0(z)\neq 0$, we have

$$-\mu=2t_0J_0(z)\left(\cos k_x+\cos k_y\right).$$

When $-\mu>|2t_0J_0(z)|$, the levels E_1 and E_2 (or E_3 and E_4) will not touch each other for ever. Else, when $-|2t_0J_0(z)|\leq-\mu\leq|2t_0J_0(z)|$, E_1 and E_2 (or E_3 and E_4) will touch each other at points $\left(k_x=\pm\frac{\pi}{2},k_y=\arccos\left(\frac{-\mu}{2t_0J_0(z)}\right)\right)$ and $\left(k_x=\arccos\left(\frac{-\mu}{2t_0J_0(z)}\right),k_y=\pm\frac{\pi}{2}\right)$. In the following, we only consider the case, $-\mu>|2t_0J_0(z)|$, which means bands E_1 and E_4 will be off away from the other two levels E_2 and E_3, and then the topological properties of bands E_1 and E_4 will not change if we vary some parameters. Therefore, we will only consider the topological properties of band E_2 and E_3, which have chances to contact each other at the high symmetry points, $\mathbf{K}_{i=1,\ldots,4}=(0,0);(0,\pi);(\pi,0);(\pi,\pi)$ in the first BZ, when satisfying the condition

$$\Gamma^2(z)=\varepsilon_{\mathbf{K}_i}^2+\psi_s^2,\tag{18}$$

because band-gap closing is an essential condition for the topological characteristic changes. We denote $\Gamma_{\mathbf{K}_i}^2(\mathbf{K}_i,z)=\left(16\lambda^2J_{+1}^2(z)/\hbar\omega\right)^2=\Gamma^2(z)$. Considering $z<<1$ and $-\mu>|2t_0J_0(z)|$, the two bands can only touch at point $\mathbf{K}_1=(0,0)$ when varying the parameter z.

III. TOPOLOGICAL PHASE TRANSITION

We now study the topological properties of these two bands by two-band approximation at point $\mathbf{K}_1=(0,0)$ in the first BZ. We will not consider the other points since the gaps at other high symmetry points will not shut down, leaving no influence to the topological changes. To have a basic idea of the topological features of the system, we explore it by using a two-band approximation at point $\mathbf{K}_1$ in the first BZ. We expand the Hamiltonian (17) at point $\mathbf{K}_1$ and obtain

$$\mathcal{H}_{\mathbf{K}_1}^D(\mathbf{q})=\begin{pmatrix}\psi_s+\Gamma(z) & 2\lambda J_0(z)q_+ & 0 & 4t_0J_0(z)+\mu\\ 2\lambda J_0(z)q_- & -(\psi_s+\Gamma(z)) & -(4t_0J_0(z)+\mu) & 0\\ 0 & -(4t_0J_0(z)+\mu) & \psi_s-\Gamma(z) & -2\lambda J_0(z)q_-\\ 4t_0J_0(z)+\mu & 0 & -2\lambda J_0(z)q_+ & -(\psi_s-\Gamma(z))\end{pmatrix}\tag{19}$$

with $q_\pm=q_y\pm iq_x$. Because $\psi_s<0$ and $\Gamma(z)>0$, when taking $|4t_0J_0(z)+\mu|\ll 1$, we can see that the major contribution to bands E_2 and E_3 comes from the up-diagonal sector in above matrix and thus we may treat the others as a perturbation. Under this condition, we obtain an effective two-band Hamiltonian given by

$$\mathcal{H}_{eff}^{\mathbf{K}_1}(\mathbf{q})=2\lambda J_0(z)q_y\sigma_x-2\lambda J_0(z)q_x\sigma_y+M(z)\sigma_z,\tag{20}$$

where the corresponding mass term $M(z)=\psi_s+\Gamma(z)+\left(4t_0J_0(z)+\mu\right)^2/\left(\psi_s-\Gamma(z)\right)$ and $\sigma_{\nu=x,y,z}$ the Pauli matrices. Let $M(z)=0$, we obtain the gapless condition Eq. (18) again at $\mathbf{K}_1$ point. Eq. (20) can be written as $\mathcal{H}_{eff}^{\mathbf{K}_1}(\mathbf{q})=\sigma\cdot\mathbf{d}$, where the vector $\mathbf{d}=\{2\lambda J_0(z)q_y,-2\lambda J_0(z)q_x,M(z)\}$. The topological features of the system can be characterized by the winding number (first Chern number) of the Berry phase gauge field $C=\frac{1}{4\pi}\int dk_x\int dk_y\hat{\mathbf{d}}\cdot(\frac{\partial\hat{\mathbf{d}}}{\partial k_x}\times\frac{\partial\hat{\mathbf{d}}}{\partial k_y})$ in the first Brillouin zone, where $\hat{\mathbf{d}}=\mathbf{d}/|\mathbf{d}|$. When $M(z)\neq 0$, it is straightforward to obtain the winding number for the effective system described by Eq. (20), i.e.,

$$C=\frac{1}{2}\mathrm{sign}\left(M(z)\right).\tag{21}$$

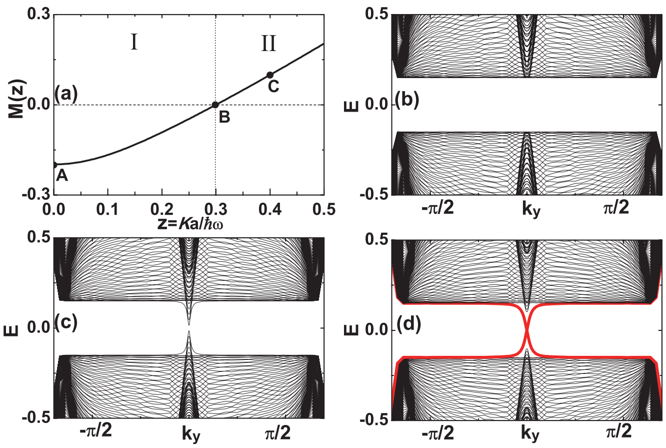

FIG. 1: The phase diagram and the band structures of the system. (a) The mass $M(z)$ as a function of z. The region I with $M(z)<0$ is a trivial superfluid, while the region II with $M(z)>0$ is a topological superfluid. The band structures of the effective Hamiltonian (10) in a striped geometry with 300 sites in x direction are shown in (b), (c) and (d) corresponding to the points A ($z=0$), B ($z=0.299$), and C ($z=0.4$) in (a), respectively. Other parameters $t_0=1, \lambda=1.6, \psi_s=-0.15, \hbar\omega=6.0$, and $\mu=-3.91$.

This non-integral winding number appears since the deviations from this two-band approximation model at large momenta are not included in the above calculation of the winding number. So it can not be directly related to the topological features of the system; however, the change in the winding numbers is independent of the large-momentum contribution [43]. Let us discuss the change of the topological properties of superfluid system when we adjust the oscillating amplitude K of optical lattice. It is obvious that the initial non-driven system is in a trivial state which corresponds to $z=0$ and $M(z)<0$. Now, we apply the driven field to the system and make $M(z)<0$ to $M(z)>0$, the change in Chern number is:

$$\Delta C=\frac{1}{2}\left[\mathrm{sign}\left(M(z)_{>0}\right)-\mathrm{sign}\left(M(z)_{<0}\right)\right]=+1. \quad (22)$$

So, we get a topological superfluid state with $C=+1$ for $M(z)>0$.

It is notable that gapless chiral edge states are usually the hallmark of a topological system. Therefore, to further prove the above argument, we will show the phase diagram and the band structures of the effective Hamiltonian (10) in a striped geometry in Fig. 1. To be consistent with the aforementioned adiabatic condition $t_0 \ll \hbar\omega$, we choose $\hbar\omega=6.0$, which is sufficient larger than $t_0=1$. This condition actually places some restrictions on the parameter values for which the transition is expected to be observable. When $\hbar\omega$ is large enough, the Zeeman-magnetic-field-like term $\Gamma(z)=16\lambda^2 J_{+1}^2(z)/\hbar\omega$ would be large enough to induce the topological phase transition only under the condition that the order parameter ψ_s is the same order with $\Gamma(z)$. In order to see the chiral edge states clearly, we choose the parameter $\psi_s=-0.15$. In Fig. 1(a), we plot the mass $M(z)$ as a function of z, which can be adjusted by changing the modulation strength K. The point **B** denotes $M(z)=0$ with $z=0.299$, where the bands E_2 and E_3 contact each other at Γ point. The band structures in a striped geometry with 300 sites in x direction are shown in (b), (c) and (d) corresponding to the points A ($z=0$), B ($z=0.299$), and C ($z=0.4$) in Fig. 1(a), respectively. In the numerical calculation, we take the typical parameters $t_0=1, \lambda=1.6, \hbar\omega=6.0$, and $\mu=-3.91$. It is clear that there is no edge state in the region I where the mass $M(z)<0$ with $0\leq z<0.299$, so it is a trivial superfluid. In contrast, there are a pair of edge states in the region II where the mass $M(z)>0$ with $z>0.299$, so it is a topological superfluid. There is a phase transition from a trivial superfluid to a topological superfluid occurs at the point **B** where the gap is closed.

Generally, there exists Majorana Fermionic excitation bounded with the vortex structure in the nontrivial topological superfluid phase. Hence, we can obtain the 2-D Foquet Majorana fermions [32] if there have the vortex structures in our system. The vortex structure can be produced from two different routes, one of which can be realized through the phase twist of the SO-produced lasers: $\lambda \rightarrow \lambda e^{im\theta}$ with m the vorticity [37]. Another route is that the vortex structure can come from initial rotation of the atomic cloud [44]. Then the vortex structure is coupled with the superfluid order parameter: $\psi_s \rightarrow \psi_s e^{im\theta}$, which is similar with the case in the topological superconductor [45]. Both cases give the similar Majorana fermion obviously confirmed from the Eq. (16) and Eq. (17) connected by the unitary transformation D. The zero mode solutions of the Majorana fermion can be obtained from the Bogoliubov-de Gennes (BdG) equation, which has the similar form compared with that in Ref. [37]. Moreover, such Majorana fermion excitations can be detected by the standard Raman spectroscopy [40, 46].

IV. CONCLUSION

In summary, we have discussed the topological superfluid phase transition in a periodically driven square optical lattice. By using Floquet's theorem, we find that a Floquet topological superfluid will be created when the two-dimensional square optical lattice potentials are periodically driven. This topological phase is interesting in hosting a Majorana fermion excitation which can be detected by Raman spectroscopy in cold atom system. Therefore we propose a novel scenario to create Majorana fermions which may play a key role in topological quantum computation.

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