Quantum spin mixing in a binary mixture of spin-1 atomic condensates

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Quantum spin mixing in a binary mixture of spin-1 atomic condensates

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We study quantum spin mixing in a mixture of two spin-1 condensates including coherent interspecies mixing processes, using condensates of ^{87}Rb and ^{23}Na atoms in the ground lower hyperfine $F = 1$ manifold as prototype examples. Adopting the single spatial mode approximation for each of the two spinor condensates, we find the mixing dynamics reduce to that of three coupled nonrigid pendulums with clear physical interpretation. With suitably prepared initial states, the spin mixing dynamics allow for the determination of interspecies singlet-pairing as well as spin-exchange interaction parameters.

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I. INTRODUCTION

A topical area in physics today concerns the control and manipulation of the spinor degrees of freedom associated with electrons or atoms. Two highly visible subfields attracting tremendous theoretical and experimental interests are spintronics in condensed matter systems [1] and spinor atomic quantum gases [2]. The latter becomes possible due to optical trapping, which provides equal confinement for all Zeeman states within a fixed manifold of hyperfine spin F. As a result, spin-related phenomena are exhibited and detected in cold atoms, including various quantum phases [3–8] and quantum magnetism studies [9], the observations of spin domain formation [10, 11], as well as the dynamics of spin mixing [5], and spin squeezing [12, 13], *etc.*

According to the formulation of spinor Bose-Einstein condensate [3–8], its mean field order parameter in the hyperfine F state is generally described by a spinor of $2F + 1$ components, strongly influenced by the atom-atom interaction. Within the low energy limit of interests to atomic quantum gases, when described by contact interactions, effective atomic interactions must stay invariant with respect to both spatial and spin rotation, a property for isotropic interaction when only s-wave is involved. Depending on the value of the spin-dependent interaction parameters, the ground state of a spinor condensate can be ferromagnetic or anti-ferromagnetic (polar) for $F = 1$ [3–5], while an additional cyclic phase appears when $F = 2$ [6–8]. Higher spin cases are generally more complicated and so far with limited experimental accesses.

Law *et al.* pioneered the study of atomic spin mixing [5]. They adopted numerical approaches and studied quantum spin mixing dynamics in the absence of an external magnetic (B-) field [5]. Subsequent theoretical and experimental efforts have contributed to observations and controls of coherent quantum spin mixing dynamics, tasks rarely feasible in other quantum many body systems [14–22].

In the semiclassical picture, using mean-field approximation and adopting the single spatial mode approximation (SMA) [5, 23], coherent spin mixing dynamics in a spin-1 condensate is described by a nonrigid pendulum, displaying periodic oscillations and resonance behavior in an external B-field [24, 25]. This picture proves to be widely popular with experimentalists and has been very successful [14–19]. Analogous efforts were applied to spin-2 condensates, for instance, in the higher hyperfine manifold of the ground state ^{87}Rb atoms [19–22]. An interesting application suggested by Saito *et al.* [26] provides a practical method for determining the unknown spin coupling parameters (polar or cyclic) relying on the mixing dynamics with suitably prepared initial states.

In recent years, several groups studied intensively mixtures of atomic spinor condensates [27–33]. Many interesting properties for mixture spinor condensates are by now reasonably well understood, both for when an external B-field is absent or present. Like in the treatment for a single species spinor condensate, the semiclassical mean field approximation is usually adopted for the mixture, while the full quantum approach is limited to atom number dynamics in a few restricted spatial modes of spinor condensates. The ground state properties for the mixture, is found to a large extent, determined by the yet unknown interspecies spin exchange interaction parameter. If it is antiferromagnetic and is sufficiently strong, interesting phases, such as highly fragmented ground states, could arise [29, 30]. Additionally, there exists a so-called broken-axisymmetry phase when an external B-field is present [31]. Within the degenerate internal state approximation [34], which considers atomic interaction potentials as a appropriately weighted contribution from potential curves associated with the coupled electronic spins of the two valence electrons: one for each atom (alkali atoms as considered here) [27, 35, 36], the interspecies singlet-pairing interaction vanishes as all interspecies interaction parameters are determined by a total of only two scattering lengths for the electronic singlet and triplet channels respectively. This approximation provides a zeroth order estimates for the ^{87}Rb and ^{23}Na atom mixture we study. Experiences with spin exchange interactions within each species show otherwise, i.e., the need for more atomic interaction parameters.

We therefore propose to study analogous spin mixing dynamics as considered before in the $F = 2$ spinor condensates to calibrate the interspecies spin-exchange and singlet-pairing interactions with suitably prepared initial states [26].

II. THE MODEL OF A BINARY SPIN-1 CONDENSATE MIXTURE

The binary mixtures of atomic spin-1 condensates have been discussed in several earlier studies [28–31]. In addition to the individual Hamiltonian for each species of the two spinor condensates, additional contact interactions exist between the two species which can be decomposed into spin-independent and spin-dependent terms as well, described by $V_{12}(\vec{r}_1-\vec{r}_2) = \frac{1}{2}(\alpha+\beta\mathbf{F}_1\cdot\mathbf{F}_2+\gamma\mathcal{P}_0)\delta(\vec{r}_1-\vec{r}_2)$ [29, 30] with appropriate interactions parameters α, β and γ [29, 30]. Take spin-1 condensates of ^{87}Rb and ^{23}Na atoms as examples, the total Hamiltonian is then given by

$$
\begin{aligned}
\hat{H} &= \hat{H}_1 + \hat{H}_2 + \hat{H}_{12},\\
\hat{H}_1 &= \int d\mathbf{r}\,\Big\{\hat{\Psi}_m^\dagger\Big(-\frac{\hbar^2}{2M_1}\nabla^2 + V_1^o - p_1 m + q_1 m^2\Big)\hat{\Psi}_m \\
&\quad +\frac{\alpha_1}{2}\hat{\Psi}_i^\dagger\hat{\Psi}_j^\dagger\hat{\Psi}_j\hat{\Psi}_i + \frac{\beta_1}{2}\hat{\Psi}_i^\dagger\hat{\Psi}_k^\dagger\mathbf{F}_{1ij}\cdot\mathbf{F}_{1kl}\hat{\Psi}_l\hat{\Psi}_j\Big\},\\
\hat{H}_2 &= \int d\mathbf{r}\,\Big\{\hat{\Phi}_m^\dagger\Big(-\frac{\hbar^2}{2M_2}\nabla^2 + V_2^o - p_2 m + q_2 m^2\Big)\hat{\Phi}_m \\
&\quad +\frac{\alpha_2}{2}\hat{\Phi}_i^\dagger\hat{\Phi}_j^\dagger\hat{\Phi}_j\hat{\Phi}_i + \frac{\beta_2}{2}\hat{\Phi}_i^\dagger\hat{\Phi}_k^\dagger\mathbf{F}_{2ij}\cdot\mathbf{F}_{2kl}\hat{\Phi}_l\hat{\Phi}_j\Big\},\\
\hat{H}_{12} &= \frac{1}{2}\int d\mathbf{r}\,\Big\{\alpha\hat{\Psi}_i^\dagger\hat{\Phi}_j^\dagger\hat{\Phi}_j\hat{\Psi}_i + \beta\hat{\Psi}_i^\dagger\hat{\Phi}_k^\dagger\mathbf{F}_{1ij}\cdot\mathbf{F}_{2kl}\hat{\Phi}_l\hat{\Psi}_j \\
&\quad +\frac{1}{3}\gamma\,(-)^{i+j}\hat{\Psi}_i^\dagger\hat{\Phi}_{-i}^\dagger\hat{\Psi}_j\hat{\Phi}_{-j}\Big\},
\end{aligned}
\tag{1}
$$

where $\hat{H}_1$ and $\hat{H}_2$ describe a single species system of ^{87}Rb and ^{23}Na atoms respectively with the interspecies interaction described by H_{12}. V_1^o, M_1, p_1, and q_1 (V_2^o, M_2, p_2, and q_2) respectively denote the optical trap, atomic mass, linear, and quadratic Zeeman shifts of a ^{87}Rb (^{23}Na) atom. Both the nuclear spins and the valence electron spins are the same for the two species. In the subspace of hyperfine spin $F = 1$, the linear Zeeman shifts for both ^{87}Rb and ^{23}Na atoms are thus almost equal: $p_1 \simeq p_2$ ($\equiv p$). $\hat{\Psi}_i(\vec{r})$ ($\hat{\Phi}_i(\vec{r})$) annihilate a ^{87}Rb (^{23}Na) atom at the position $\vec{r}$.

The $F = 1$ states for both ^{87}Rb and ^{23}Na atoms are well studied, and their respective atomic collision parameters are known precisely. References [37, 38] provide respectively the numerical values for the a_0 and a_2 parameters of the $F = 1$ state for ^{87}Rb and ^{23}Na atoms, which then gives $\alpha_{1/2}$ and $\beta_{1/2}$. While a number of experimental and theoretical studies have previously addressed collisions between ^{87}Rb and ^{23}Na atoms [35, 36], the most recent one by A. Pashov *et al.* [36] provides a well converged data set for singlet and triplet scattering lengths of $a_s = 70(a_0)$ and $a_t = 109(a_0)$. This can be used to predict the required set of atomic intraspecies collision parameters α, β, and γ, although the actual process is complicated and therefore yet to be completed. What is certain concerns the value of spin exchange interaction γ, it will be actually strong, instead of being weak or vanishing. Thus our study described below provides a worthy alternative approach.

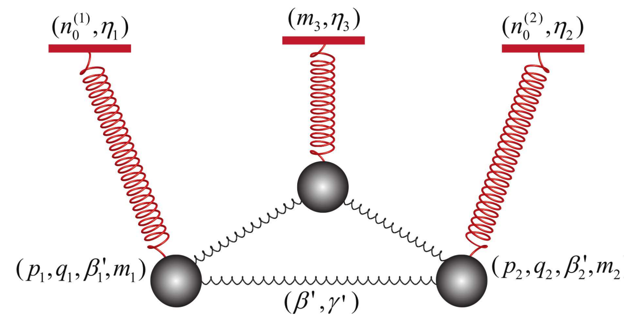

FIG. 1: (Color online). A schematic illustration for the three coupled nonrigid pendulums, with three pairs of canonical variables: $(n_0^{(1)}, \eta_1)$, $(n_0^{(2)}, \eta_2)$, and $(m_3 = m_1 - m_2, \eta_3)$. The first two pairs describe intraspecies spin mixing dynamics, while the remaining third pair denotes interspecies spin mixing dynamics.

We adopt the mean-field approximation and define for each condensate species a mode function $\psi(\vec{r})/\phi(\vec{r})$, justified by the fact spin independent density interaction terms are usually much stronger than spin dependent ones. We therefore take $\Psi_i(\vec{r}) \equiv \langle\hat{\Psi}_i(\vec{r})\rangle = \sqrt{n_j^{(1)}}\,e^{i\theta_j}\psi(\vec{r})$ and $\Phi_i(\vec{r}) \equiv \langle\hat{\Phi}_i(\vec{r})\rangle = \sqrt{n_j^{(2)}}\,e^{i\varphi_j}\phi(\vec{r})$. The spin dynamics are then governed by the spin-dependent energy functional

$$
\begin{aligned}
\mathcal{E} &= \sum_{j=1,2}\mathcal{E}_j + \mathcal{E}_{12},\\
\mathcal{E}_j &= -p_j m_j + q_j(n_j - n_0^{(j)}) + \frac{1}{2}\beta_j' m_j^2 \\
&\quad +\beta_j' n_0^{(j)}\Big[(n_j - n_0^{(j)}) + \sqrt{(n_j - n_0^{(j)})^2 - m_j^2}\cos\eta_j\Big],\\
\mathcal{E}_{12} &= \frac{1}{2}\beta' m_1 m_2 + \frac{1}{6}\gamma'(n_1^{(1)}n_{-1}^{(2)} + n_0^{(1)}n_0^{(2)} + n_{-1}^{(1)}n_1^{(2)}) \\
&\quad +\frac{1}{3}\gamma'\sqrt{n_1^{(1)}n_{-1}^{(1)}n_1^{(2)}n_{-1}^{(2)}}\cos\eta_3,\\
&\quad +(\beta' - \frac{1}{3}\gamma')\sqrt{n_0^{(1)}n_{-1}^{(1)}n_1^{(2)}n_0^{(2)}}\cos(\frac{\eta_1+\eta_2+\eta_3}{2})\\
&\quad +(\beta' - \frac{1}{3}\gamma')\sqrt{n_1^{(1)}n_0^{(1)}n_0^{(2)}n_{-1}^{(2)}}\cos(\frac{\eta_1+\eta_2-\eta_3}{2})\\
&\quad +\beta'\sqrt{n_0^{(1)}n_{-1}^{(1)}n_0^{(2)}n_{-1}^{(2)}}\cos(\frac{\eta_1-\eta_2+\eta_3}{2})\\
&\quad +\beta'\sqrt{n_1^{(1)}n_0^{(1)}n_1^{(2)}n_0^{(2)}}\cos(\frac{\eta_1-\eta_2-\eta_3}{2}),
\end{aligned}
\tag{2}
$$

where $n_{1,2} = \sum_j n_j^{(1,2)}$, $m_{1,2} = n_1^{(1,2)} - n_{-1}^{(1,2)}$, $\eta_1 = \theta_1 + \theta_{-1} - 2\theta_0$, $\eta_2 = \varphi_1 + \varphi_{-1} - 2\varphi_0$, and $\eta_3 = \theta_{-1} - \theta_1 + \varphi_1 - \varphi_{-1}$. The interaction parameters are now redefined to absorb the relevant multipliers: $\beta_1' = \beta_1 \int |\psi(\vec{r})|^4 d\vec{r}$, $\beta_2' = \beta_2 \int |\phi(\vec{r})|^4 d\vec{r}$, and $(\beta', \gamma') = (\beta, \gamma) \int |\psi(\vec{r})|^2 |\phi(\vec{r})|^2 d\vec{r}$. We note that $\int |\psi(\vec{r})|^2 d\vec{r} = \int |\phi(\vec{r})|^2 d\vec{r} = 1$. When the two species are immersible, the overlap between $\psi(\vec{r})$ and $\phi(\vec{r})$ is significantly reduced, leading to diminished β' and γ', essentially reducing the system to two stand-alone spin-1 condensates.

Although complicated in form, the above Hamiltonian gives rise to dynamics that can be interpreted simply in terms of three coupled nonrigid pendulums, with three pairs of canonical conjugate variables: $(n_0^{(1)}, \eta_1)$, $(n_0^{(2)}, \eta_2)$, and $(m_3 = m_1 - m_2, \eta_3)$. Their corresponding equations of motion are given by

$$\begin{aligned} \dot{n}_0^{(1)} &= -\frac{2}{\hbar}\frac{\partial \mathcal{E}}{\partial \eta_1}, & \dot{\eta}_1 &= \frac{2}{\hbar}\frac{\partial \mathcal{E}}{\partial n_0^{(1)}}, \\ \dot{n}_0^{(2)} &= -\frac{2}{\hbar}\frac{\partial \mathcal{E}}{\partial \eta_2}, & \dot{\eta}_2 &= \frac{2}{\hbar}\frac{\partial \mathcal{E}}{\partial n_0^{(2)}}, \\ \dot{m}_3 &= -\frac{4}{\hbar}\frac{\partial \mathcal{E}}{\partial \eta_3}, & \dot{\eta}_3 &= \frac{4}{\hbar}\frac{\partial \mathcal{E}}{\partial m_3}, \end{aligned} \tag{3}$$

as illustrated schematically in Fig. 1.

III. DETERMINING INTERSPECIES SPIN-DEPENDENT INTERACTIONS

When discussing spin mixing in a spin-2 condensate, Saito *et al.* [26] proposed a scheme to determine the value of intra-species spin singlet-pairing interaction by choosing an elementary process $(M_F^{(1)} =)0 + (M_F^{(2)} =)0 \leftrightarrow 2 + (-2)$ which occurs only when the spin singlet-pairing interaction is non-vanishing. With a suitably chosen initial state of zero magnetization, the mixing dynamics is governed by coupled first-order ordinary differential equations, which contain unknown parameters like singlet-pairing interactions and quadratic Zeeman shifts. The analytic solutions can then be compared with the experimental measured dynamics to determine the unknown atomic interaction parameters.

The present study show that analogous approach can be taken to determine the values of interspecies spin-exchange and singlet-pairing interaction parameters: β' and γ', for a binary mixture of spin-1 ^{87}Rb and ^{23}Na atom condensates, by making use of selected elementary collision processes. In order to determine β' and γ', from the Hamiltonian of Eq. (1), we know their relevant collision processes can be categorized into three types with respective interaction strengths:

(i) β: $\Psi_1 + \Phi_0 \leftrightarrow \Psi_0 + \Phi_1$, $\Psi_{-1} + \Phi_0 \leftrightarrow \Psi_0 + \Phi_{-1}$;

(ii) $\beta - \gamma/3$: $\Psi_1 + \Phi_{-1} \leftrightarrow \Psi_0 + \Phi_0$, $\Psi_{-1} + \Phi_1 \leftrightarrow \Psi_0 + \Phi_0$;

(iii) γ: $\Psi_1 + \Phi_{-1} \leftrightarrow \Psi_{-1} + \Phi_1$.

Elementary processes in the first and third types are driven only by interspecies spin-exchange interaction or by the singlet-pairing interaction. Therefore they are potential candidates for determining β' and γ' respectively. As a result, our proposal for their determination consists of two steps.

In the first step, we try to determine γ' from spin mixing dynamics based on the elementary process of the third type by preparing an initial state,

$$\Psi = \begin{pmatrix} \sqrt{n_1^{(1)}} e^{i\theta_1} \\ 0 \\ \sqrt{n_{-1}^{(1)}} e^{i\theta_{-1}} \end{pmatrix} \psi, \quad \Phi_j = \begin{pmatrix} \sqrt{n_1^{(2)}} e^{i\varphi_1} \\ 0 \\ \sqrt{n_{-1}^{(2)}} e^{i\varphi_{-1}} \end{pmatrix} \phi. \tag{4}$$

The reason why we first choose $\Psi_1 + \Phi_{-1} \leftrightarrow \Psi_{-1} + \Phi_1$ is because this elementary process is decoupled from the intraspecies spin-exchange processes: $\Psi_1 + \Psi_{-1} \leftrightarrow \Psi_0 + \Psi_0$ and $\Phi_1 + \Phi_{-1} \leftrightarrow \Phi_0 + \Phi_0$ when the quadratic Zeeman shifts q_j are tuned to large negative values, for instance with off-resonant microwave field [39, 40], or to large positive values with an enhanced uniform B-field. Therefore, the population of the $M_F = 0$ state remains at zero, and the spin mixing dynamics of Eq. (3) reduce to the following pair of equations,

$$\begin{aligned} \dot{m}_3 &= \frac{\gamma'}{12\hbar}\sqrt{[4n_1^2 - (m+m_3)^2][4n_2^2 - (m-m_3)^2]} \\ &\quad \times \sin\eta_3, \\ \dot{\eta}_3 &= \frac{\beta_1' - \beta_2'}{\hbar} m + \frac{\beta_1' + \beta_2' - \beta' + \gamma'/6}{\hbar} m_3 \\ &\quad - \frac{\gamma'}{6\hbar}\frac{2(n_1^2+n_2^2)m_3 - 2(n_1^2-n_2^2)m + m^2 m_3 - m_3^3}{\sqrt{[4n_1^2-(m+m_3)^2][4n_2^2-(m-m_3)^2]}} \\ &\quad \times \cos\eta_3. \end{aligned} \tag{5}$$

Based on the dynamics following Eq. (5), we can deduce the sign of γ' from subsequent spin mixing dynamics. By preparing an initial state with $\sin\eta_3 > 0$, we infer $\gamma' > 0$ if m_3 increases during the initial short time period of the spin mixing dynamics; whereas if it decreases, we know $\gamma' < 0$. To determine γ', we can prepare a suitable initial state, for example, $\eta_3 = \pi/2$ and $m_1 = m_2 = 0$, which leads to

$$(\dot{m}_3)^2 = \frac{\gamma'^2}{144\hbar^2}\Big[(m_3^2 - 4n_1^2)(m_3^2 - 4n_2^2) - \mathcal{C}^2 m_3^4\Big], \tag{6}$$

with $\mathcal{C} = |6(\beta_1' + \beta_2' - \beta')/\gamma' + 1|$. If $\mathcal{C} < 1$, $\dot{m}_3 = 0$ gives four roots $-x_2$, $-x_1$, x_1, and x_2, where $x_{1/2} = \sqrt{2\big(n_1^2 + n_2^2 \mp \sqrt{(n_1^2-n_2^2)^2 + 4\mathcal{C}^2 n_1^2 n_2^2}\big)/(1-\mathcal{C}^2)}$. For $\mathcal{C} \geq 1$, however, only two solutions $-x_1$ and x_1 exist. The solution for the mixing dynamics is expressed in terms of

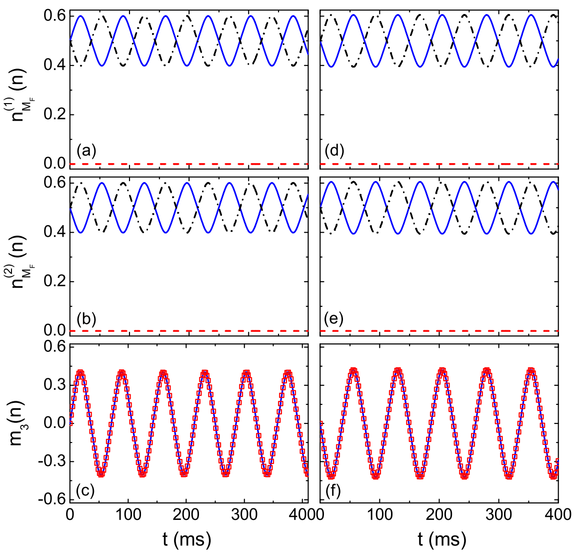

FIG. 2: (Color online). Population dynamics for every spin component. In the left panels of (a)-(c), the interspecies interaction parameters used are $\beta' = 5|\beta_1'|$ and $\gamma' = 2|\beta_1'|$. For the panels of (d)-(f), $\beta' = 5|\beta_1'|$ and $\gamma' = -2|\beta_1'|$ are used. (a) For ^{87}Rb atoms, where the blue solid line, red dashed line, and black dotted-dash line represent the $M_F = 1, 0, -1$ components, respectively. (b) As in (a), but for ^{23}Na atoms. (c) Time dependent m_3 with blue solid line and red square symbols denote numerical and analytical solutions respectively. (d) As (a), but with $\gamma' = -2|\beta_1'|$. (e) As in (b), but with $\gamma' = -2|\beta_1'|$. (f) As in (c), but with $\gamma' = -2|\beta_1'|$.

the Jacobian elliptic functions sn(.) and cn(.) as

$$\begin{aligned} m_3(t) &= x_1 \,\mathrm{sn}\left(\frac{x_2\gamma' t\sqrt{1-\mathcal{C}^2}}{12\hbar}, \frac{x_1}{x_2}\right), \text{for } \mathcal{C} \le 1, \\ m_3(t) &= x_1 \,\mathrm{cn}\left(K\Big(\frac{x_1}{\sqrt{x_1^2+x_3^2}}\Big) - \frac{\gamma' t\sqrt{(x_1^2+x_3^2)(\mathcal{C}^2-1)}}{12\hbar},\right. \\ &\qquad \left.\frac{x_1}{\sqrt{x_1^2+x_3^2}}\right), \text{for } \mathcal{C} \ge 1, \end{aligned} \tag{7}$$

where $K(.)$ is the complete elliptic integral of the first kind, and $x_3 = \sqrt{2\big(n_1^2+n_2^2+\sqrt{(n_1^2-n_2^2)^2+4\mathcal{C}^2n_1^2n_2^2}\big)/(\mathcal{C}^2-1)}$.

Next, comparing to the analytic formulae of Eq. (7) for the subsequent spin mixing dynamics, we can determine the value of γ'. First, we evaluate $\mathcal{C}$ from the experimentally measured oscillation amplitude x_1 of $m_3(t)$ as $\mathcal{C}^2 = x_1^2 - 4n_1^2 - 4n_2^2 + 16n_1^2n_2^2/x_1^2$. Second, based on the known parameters $\mathcal{C}$, x_1, x_2, and x_3, and experimentally measured oscillation period of $m_3(t)$, we could then deduce the value of γ'.

In Fig. 2, we illustrate our numerical results which confirm the stability of the third type elementary interaction processes and the validity of the analytic solutions Eq. (7). The initial state is taken as $\Psi = \psi\sqrt{n_1}(1,0,1)^T/\sqrt{2}$, $\Phi = \phi\sqrt{n_2}(1,0,-i)^T/\sqrt{2}$, with $n_1 = n_2 = n$ and $n = 2\times 10^4$. We further choose $\beta_1'/\hbar = -22.4893\times 10^{-4}$ Hz and $\beta_2'/\hbar = 303.816\times 10^{-4}$ Hz. A noise at the level of 10^{-5} in the population of the $M_F = 0$ spin state for both atomic species is also included. The B-field is set at a large enough value to suppress the intraspecies spin-exchange process with the quadratic Zeeman shifts satisfying $q_1 = 40|\beta_1'|n$ and $q_2 = q_1\Delta E_1/\Delta E_2$, where ΔE_1 and ΔE_2 are the hyperfine splittings of ^{87}Rb and ^{23}Na atoms respectively. In Fig. 2(a-c), $\beta' = 5|\beta_1'|$ and $\gamma' = 2|\beta_1'|$ are used, while $\beta' = 5|\beta_1'|$ and $\gamma' = -2|\beta_1'|$ are used for Fig. 2 (d-f). The time evolution for each condensate species is shown in Fig. 2(a,d) and (b,e) respectively for ^{87}Rb and ^{23}Na atoms. We indeed confirm that intraspecies spin mixing dynamics are suppressed by the large quadratic Zeeman shifts as there is essentially no atomic population remaining in the $M_F = 0$ spin component eventually. The evolutions for m_3 are shown in Fig. 2(c) and (f), they also confirm our predictions based on the insights gained from the analytical solutions that m_3 increases/decreases in the beginning when $\gamma' > 0/\gamma' < 0$. The numerical simulations denoted by solid blue lines agree well with the analytical solutions of Eq. (7) denoted by red square symbols. We further note that $\dot n_1^{(1)} = -\dot n_{-1}^{(1)} = -\dot n_1^{(2)} = \dot n_2^{(1)} = \dot m_3/4$ with the initial state used in this case. As a result, we can determine the sign of γ' from the population of any spin component or species.

In the second step, we determine the value of β', making used of the results from the first step. From the known value of $\mathcal{C}$ after the first step, β' becomes partially determined to within the following two choices

$$\beta'_{\mp} = \beta_1' + \beta_2' \mp (\mathcal{C}\mp 1)\gamma'/6. \tag{8}$$

To fully determine β', we can choose a related elementary interaction process, for example, the elementary process as described in the first type $\Psi_1 + \Phi_0 \leftrightarrow \Psi_0 + \Phi_1$, and prepare the initial state

$$\Psi = \begin{pmatrix} \sqrt{n_1^{(1)}}e^{i\theta_1} \\ \sqrt{n_0^{(1)}}e^{i\theta_0} \\ 0 \end{pmatrix}\psi, \quad \Phi = \begin{pmatrix} \sqrt{n_1^{(2)}}e^{i\varphi_1} \\ \sqrt{n_0^{(2)}}e^{i\varphi_0} \\ 0 \end{pmatrix}\phi. \tag{9}$$

As before we can isolate the elementary collision process from other ones by suppressing intraspecies spin mixing dynamics, such as using a sufficiently strong uniform external magnetic field to guarantee $\Psi_{-1} = \Phi_{-1} = 0$. The interspecies spin mixing process is only induced by the β' term, thus the spin mixing dynamics is governed by the evolution of m_3 through

$$\begin{aligned} \dot m_3 &= -\frac{\beta'}{2\hbar}\sqrt{(m^2-m_3^2)(2n_1-m-m_3)(2n_2-m+m_3)} \\ &\quad \times \sin\frac{\eta_1-\eta_2-\eta_3}{2}, \\ \dot\eta_3 &= \frac{\beta_1'-\beta_2'}{\hbar}m + \frac{\beta_1'+\beta_2'-\beta'}{\hbar}m_3 + \frac{\gamma'}{6\hbar}m_3 \\ &\quad - \frac{\beta'}{2\hbar}\frac{m_3\sqrt{(2n_1-m-m_3)(2n_2-m+m_3)}}{\sqrt{m^2-m_3^2}}. \end{aligned} \tag{10}$$

β' can then be fully determined as follows. First we can infer the sign of β' from the initial stage of the time evolution for m_3, as in the earlier section on determining the sign of γ'. For an initial state with $\Psi = \psi\sqrt{n_1}(1,1,0)^T/\sqrt{2}$ and $\Phi = \phi\sqrt{n_2}(1,-i,0)^T/\sqrt{2}$, where $(\eta_1 - \eta_2 - \eta_3)/2 = \theta_1 - \theta_0 - \varphi_1 + \varphi_0 = -\pi/2$, we confirm $\beta' > 0$ ($\beta' < 0$) if m_3 initially increases (decreases). The actual value of β' is determined by comparing the analytic or numerical solutions using the two choices of β' from Eq. (8) to experimental measurements. Again we assume $n_1 = n_2 = n = 2\times 10^4$, $\beta'_1/\hbar = -22.4893\times 10^{-4}$ Hz, $\beta'_2/\hbar = 303.816\times 10^{-4}$ Hz, $\beta' = 5|\beta'_1|$, $\gamma' = 2|\beta'_1|$, $q_1 = 30|\beta'_1|n$, and $q_2 = q_1\Delta E_2/\Delta E_1$, with the analytic solution for m_3

$$\begin{aligned} m_3(t) &= \frac{x_1(x_2-x_4)+(x_1-x_2)x_4x^2}{(x_2-x_4)+(x_1-x_2)x^2}, \\ x &= \mathrm{sn}\left(d_4 - t\sqrt{d_3}/d_2, d_1)\right), \end{aligned} \tag{11}$$

where $x_{j=1,2,3,4}$ are the four roots of $\dot{m}_3 = 0$, arranged in descending order $x_1 > 0 > x_2 > x_3 > x_4$, and $d_1 = \sqrt{(x_1-x_2)(x_3-x_4)/(x_1-x_3)/(x_2-x_4)}$, $d_2 = 2/\sqrt{(x_1-x_3)(x_2-x_4)}$, $d_3 = [4\beta'^2 - (\beta'_1 + \beta'_2 + \beta' + \gamma'/3)^2]/16\hbar^2$, and $d_4 = \mathrm{F}(\arcsin\sqrt{-(x_2-x_4)x_1/(x_1-x_2)/x_4}, d_1)$, with F(.) the elliptic integral of the first kind.

Figure 3 shows population dynamics for all spin components. Due to the large yet unequal quadratic Zeeman shifts q_1 and q_2, suppression of the intraspecies spin mixing dynamics leads to a suppressed amplitude for the interspecies spin-exchange dynamics. As a result, the quadratic Zeeman shifts cannot be tuned to too large a value, causing nonzero population in the $M_F = -1$ spin component especially for ^{23}Na atoms as is illustrated in Fig. 3(d).

The other related elementary channels can be employed as well to determine the interspecies spin-exchange interaction. Among them, two are capable of determining the combined parameter $\beta' - \gamma'/3$, which can be used further aided by a determination of the sign of $\beta' - \gamma'/3$.

Before conclusion, we hope to stress that the special mixture illustrated in this study involves a spin-1 condensate with ferromagnetic interaction (^{87}Rb) and a polar spin-1 condensate (^{23}Na) with antiferromagnetic interaction. More generally the procedure we suggest for determine the interspecies interaction parameters remains applicable for mixtures with two spin-1 ferromagnetic condensates or two antiferromagnetic condensates.

IV. CONCLUSION

We discuss coherent spin mixing dynamics for a binary mixture of spin-1 condensates. Under mean field approximations, the dynamics are found to reduce to a simple one corresponding to three coupled nonrigid pendulums,

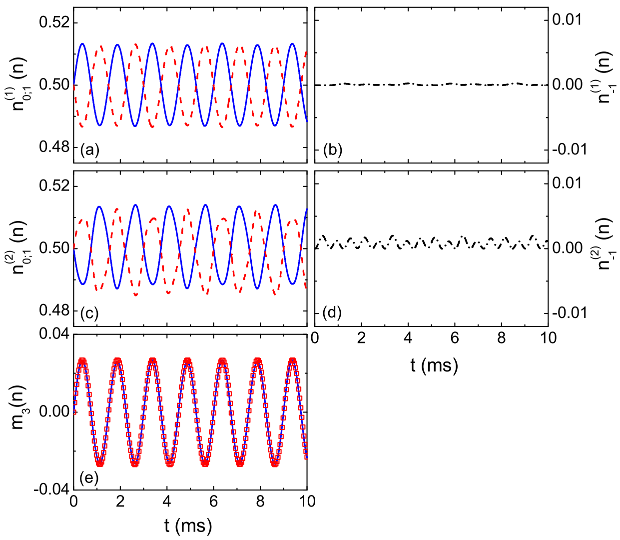

FIG. 3: (Color online). Population dynamics for all spin components, with the interspecies interaction parameters $\beta' = 5|\beta'_1|$ and $\gamma' = 2|\beta'_1|$, and the quadratic Zeeman shifts $q_1 = 30|\beta'_1|n$ and $q_2 = q_1\Delta E_2/\Delta E_1$. (a) For ^{87}Rb atoms, where the blue solid line and red dashed line represent the $M_F = 1, 0$ components, respectively. (b) As in (a), but for the $M_F = -1$ component in black dotted dash line. (c)/(d) as in (a)/(b) respectively, but for ^{23}Na atoms. (e) Time evolution of m_3, where blue solid line and red square symbols denote numerical and analytical solutions respectively.

one for each of the two spin-1 condensates as modeled previously for a single stand-alone spin-1 condensate [25], and a third one for the difference in the magnetization between the two atomic species. By tuning quadratic Zeeman shift to a large enough value, intraspecies spin mixing dynamics can be suppressed, resulting in a pure interspecies spin mixing dynamics. Using suitably prepared initial states with zero population in the $M_F = 0$ for both species, we can determine the value of the interspecies singlet-pairing interaction by comparing analytic formulae for the dynamics to experimental measurements, and at the same time we can partially determine the value of the interspecies spin-exchange interaction parameter β'. Next, starting with an alternative initial state containing no population in the $M_F = -1$ state for both atomic species, and using the two possible values for β' partially determined in the first step, we can numerically or analytically solve the dynamics and compare them with experimental measurements to determine the correct value of β'.

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